Numerical Methods and Computation
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Lecture No - 25
Interpolation and Approximation

Now in today's lecture, we shall start discussion on interpolation and approximation.

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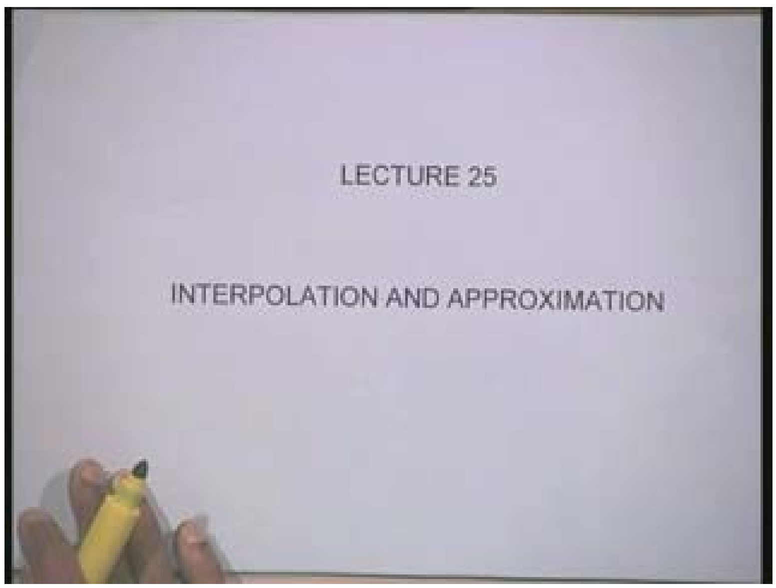

Let us just define what we mean by interpolation and what we mean by approximation. Now let us consider the case when we are given a data, so the data could be a table of values, now the various types of data can exist.

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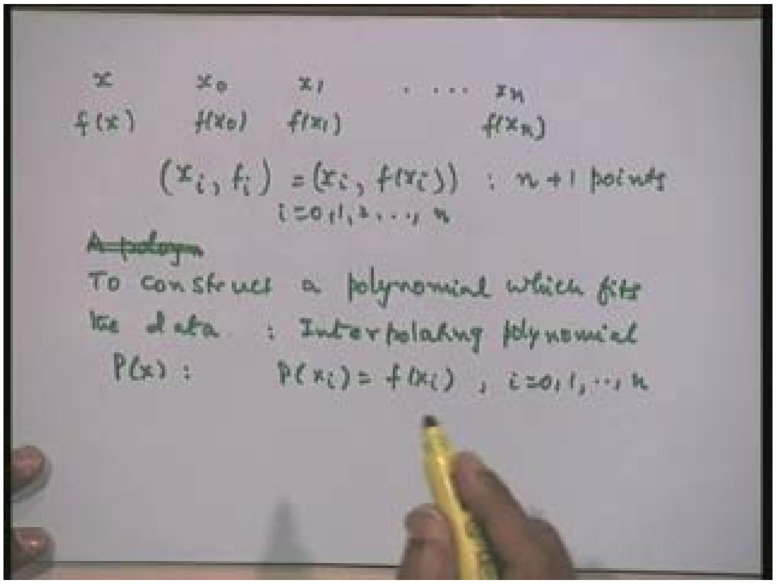

So for example, i may have simply, if i take x as a variable and the f(x) as the dependent variable, i may have set of values given to me, at a point x_0 i have been given a value of x_0, x_1 have been given a value of at x_1 and so on i have given, given these n plus 1 values. So this is the, values are given by x_i f_i, which i am denoting x_i f at x_i this and these n plus 1 points at which the data is given to us, so i is going from 0, 1, 2 to n. Now the problem is to construct a polynomial which fits this data, so that shall be called an interpolating polynomial, to construct a, a polynomial which fits the data and this is called the interpolating polynomial, that means if the polynomial is say, p(x) then what we are saying is that p at x_i will be equal to f at x_i, so this is what we mean by saying by fitting this, a polynomial to the given data at n plus 1 points, so the, if i substitute x is equal to x_i, i would get back my value of x_i, so it exactly fits the given data.

Now the use of the interpolating polynomial are many, one is, i can estimate or predict (Refer Slide Time: 03:25) the value of the dependent variable at any intermediate point from here. Let us say an experiment has been conducted in an intervals of say 15 seconds and you have done it, experiment over a few hours, now you want to find out approximately what is a value of the dependent variable at a certain instant of time, which is not one of this points, then we will able to predict the value of the dependent variable using the interpolating polynomial. Secondly, consider this as a point in a two dimensional plane, this is a point in two dimensional, a point in two dimensional plane, it will represent a curve in the two dimensional plane. You may like to have the slope of the curve at any point, that means you want to, like to know dy by dx or d (Refer Slide Time: 04:17) by dx at a point. Now if i have the interpolating polynomial, which is a function of x, i can just differentiate this by with respect to x and then use that for finding the value of the slope at any, at any point, at any intermediate point or at these points. Alternatively you may like to integrate the function f(x) over this entire range, i can use this interpolating polynomial and again find the value of the integral from there, so the purpose of the interpolating polynomial are many and its application is in many areas.

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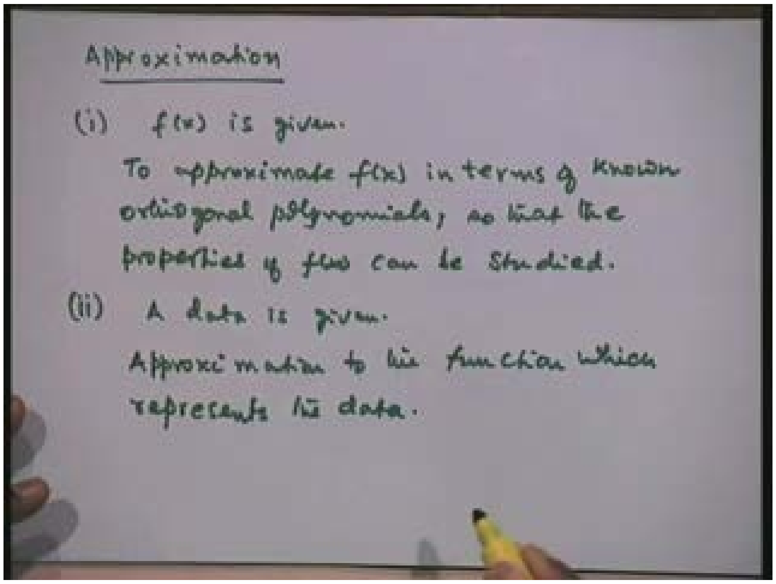

The second problem is approximation; the problem of approximation is slightly different from interpolation. The problem of approximation is of two types, one type is, that we have been given a function f(x) already, so a continuous function is given, so f(x) is given, f (x) is given and it has got certain properties but the f(x) could be very complicated to, to actually find out what are the properties of this function or like what is the maximum ,what is its minimum, i mean how it is behaving, all this properties is unknown when its f(x) is complicated.

What we would like to do is, we would like to approximate f(x) in terms of known polynomials, the known, known polynomials are mostly orthogonal polynomial. The orthogonal polynomials which we know are the Legendre polynomials, Chebyshev polynomials and there are other polynomials over the interval, given in an interval, we know the orthogonal corresponding polynomial. I would like to, when once f(x) is defined in a particular interval, i would suitably chose the orthogonal polynomials and then approximate f(x) in terms of those orthogonal polynomials, a finite series of them, so therefore the problem of approximation is what, that means to approximate f(x) in terms of known orthogonal polynomials, in terms of known orthogonal polynomials. Now when once we approximate it, now since it is a known orthogonal polynomial, i know all the properties of orthogonal polynomials and hence i can able to describe the properties of f(x) using the polynomial, so that, so that the properties of f(x) can be studied.

Now will come back to this problem and take the suitable interval and then construct the approximation problem. The second type of problem is that we are given a data; instead of a continuous function we are given a data, so a data is given. Now this data represents a function f(x) but we do not know the function f(x) but i would like to write an approximation to the polynomial f(x) which may be representing this data, for example in the interpolation, we construct a polynomial exactly which fits the data. Now i want to construct an approximation here, which is an approximation to the f(x), which will be representing the data that means the approximation here is not fitting the data, it is only approximating the function which represent the data.

So this is an approximation, approximation to the function which represents the data. We are going to consider both of them through, simple examples also we shall take it up and see how the approximation is useful for us and how interpolation is also useful for us. So let us first start with interpolation, so let us define interpolation.

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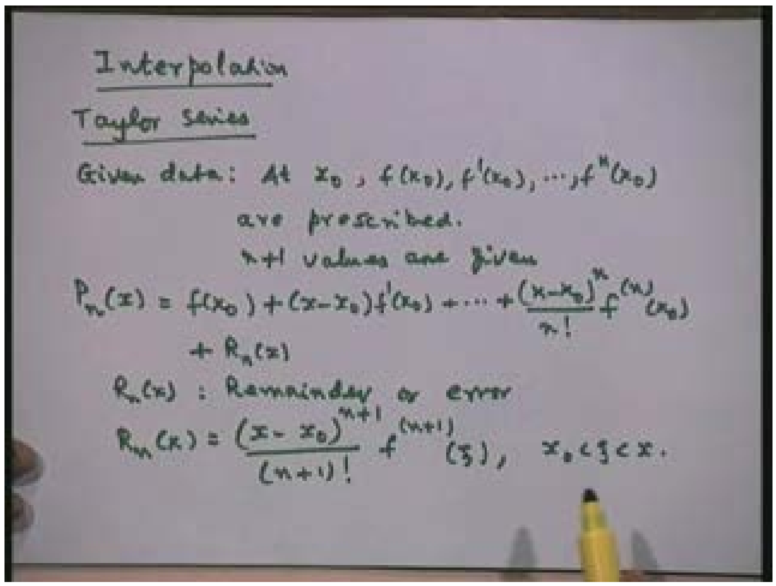

Now here the Taylor series which you already know can be looked after as an interpolation polynomial. So for example let us describe how it is an interpolating polynomial. Now we will take the given data as this, so the given data is at x_0, at x_0, f at x_0, f dash at x_0 so on f^n at x_0 are prescribed. So we are taking the ordinate of x_0 and its first end derivate, they are all prescribed for us, so we have data f n plus 1values prescribed, this is n derivative plus 1, so there are n plus one values, n plus values are given. Then i can write down the Taylor series, which is a polynomial of degree n at x, as f at x_0 plus x minus x_0 f dash of x_0 plus 1 x minus x_0 to the power of n by factorial n nth derivative at x_0 plus off course $R_n(x)$, we can write down the error also, plus $R_n(x)$. $R_n(x)$ is the remainder, $R_n(x)$ is the remainder or will call it as error and the value of $R_n(x)$ error is given by x minus x_0 to the power of n plus 1 by n plus 1 factorial f $^{(n+1)}$ plus some intermediate point zhi, where x_0 zhi less than x, zhi is a point between x_0 and x which we are considering.

Now this Taylor series can be taken as an interpolating polynomial but interpolating this type of data, so the values at x_0 or given n derivatives and then we can write down this as an interpolating polynomial. In all the interpolating problems the error place a very important role, the error will govern what is the step size that we can use in a particular data or what is the step size you can use for a particular problem for solving it or what is the error bound, what is the largest error which you are committing in a particular interpolation problem, you may be like given a data of say thousand points, you may like to use only set of few points to construct a polynomial, now what will be the error in that particular approximation or interpolation will be known through this error term, that is we have got here or the reminder term, therefore we can always bound this.

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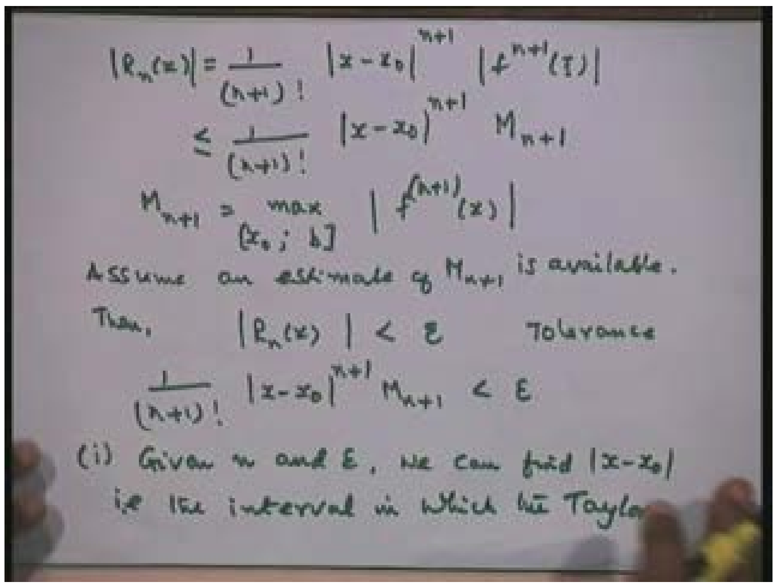

So i can find the bound of the error as, so i can write the magnitude of $R_n(x)$, that will give me the bound of this, that will be 1 upon n plus 1 factorial x minus x_0 magnitude n plus 1 magnitude of $f^{(n+1)}$ of zhi. Zhi is a point which is unknown for us between x_0 and x, therefore we shall replace this by its maximum value, so then i can write down this as less than or equal to 1 upon n plus 1 factorial x minus x_0 magnitude to the power of n plus 1 and this i will write this as M_{n+1} and M_{n+1} is maximum of x lying between, given an interval x_0 to some b that is given to us, as $f^{(n+1)}$ of x. Now we proposed to use the Taylor series starting from the point x_0 to a point b, so the maximum of f n plus 1th derivative over the interval x_0 to x b shall be taken as the quantity that we want to use it here, so M_{n+1} be the quantity that will be using here.

How we can use the knowledge of this error bound to say something about this Taylor series, now let us say that are estimate of M_{n+1} is available to us, assume an estimate of n plus 1 is available, now a way of doing, getting an estimate is, suppose you are ask to construct a 5 term Taylor series, so let us suppose you are ask to construct a 5 term Taylor series, 5 term Taylor series means starting with this 1 2 3 4 5, that means up to fourth derivative so, x or x minus x_0 to the power of 4, factorial 4 f 4. Now the remainder term will be of fifth derivative, so what we do is to find an estimate of the required derivative, of fifth derivative, i will construct the Taylor series with one more non vanishing term that means one more term that we want. We want 5 terms series, so i will construct the sixth term which is a non-zero, because there may be many zeros in between, so will i have the next non-zero. When once i construct the next non-zero term, i will now differentiate this required number of times and take that as an estimate, because you want a certain number of terms, so we construct the next higher order polynomial, differentiate it the required number of times and that can be taken as an estimate, because this, we want to lay a rough estimate of this, so we will assume that an estimate of M_{n+1}, this n plus 1 derivative is available. Let us say you want that, the error should be less than some tolerance epsilon, so this is our tolerance given to us, error tolerance is given some 10 to the power of minus 4, 10 to the power of minus 5 is given to us.

Now interestingly this will then give us, let us write it here, this will be 1 upon n plus 1 factorial x minus x_0 to the power of n plus 1 M_{n+1} is less than epsilon, now here if the tolerance is given to us and number of terms that we have in the Taylor series, that is n, n describe the number of terms in the Taylor series, n plus 1 is the, if you have got a Taylor series of order n number of terms are n plus 1. So when, if n is given and epsilon is given, then i can find out up to which point i can use the Taylor series with that accuracy, because we want to use the Taylor series from x_0 to certain point b. Now when once where given epsilon and n, i can now find out what is this distance, what is this value x minus x_0, that will give me how far i will be able use the Taylor series with this accuracy. Therefore you can say given n and epsilon, we can find x minus x_0 that is, the interval in which the Taylor series holds, in which the Taylor series holds.

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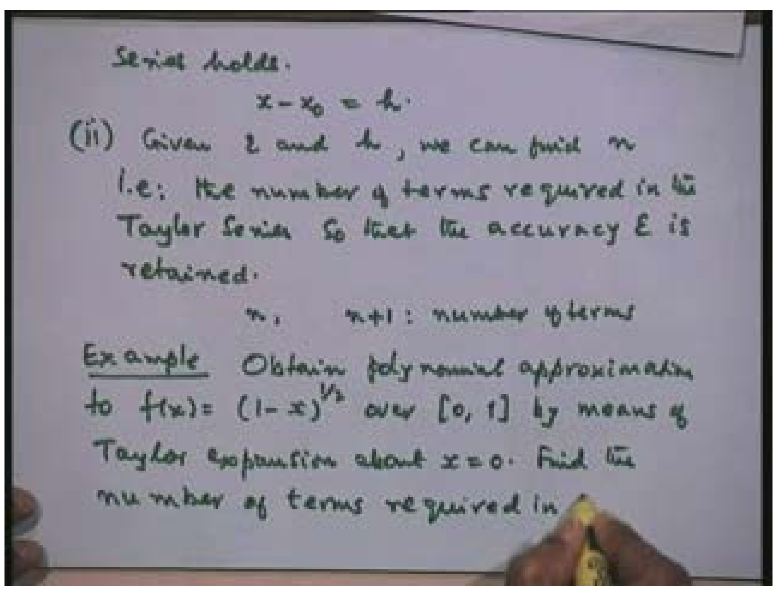

If you can represent this x minus x_0 is equal to h, the distance h, then it is actually giving what is the maximum distance you can go from x_0, so that the Taylor series is valid with this tolerance of this or alternatively, we can say that if you are given the interval and tolerance then, this will tell me what will be the value of n, so that means how many terms of the Taylor series we should take, if you want an accuracy of say ten to the power of minus 6, so i can say that this many terms 10 terms, 11 terms, 12 terms, should be taken in order that we have this particular tolerance. So we can say, given epsilon and h so, this is the h which we are talking of, given epsilon and h, we can find, we can find n, the n is number of terms, number of terms required in the Taylor series, so that accuracy epsilon is retained.

Now we are finding n, when once i find n, i will say that number of terms will be equal to n plus 1, the number of terms will be, because f at x_0 is the 0^{th} term, so it will be, total number of terms will be n plus 1 terms, that will be required in the Taylor series. So a number of observations or number of conclusions can be made when once we have the remainder of this particular Taylor series. Now let us illustrate these two points with an example. Let us take an example on this, so let us say obtain polynomial approximation, let us give the function, to the given function let us take a simple function, 1 minus x to the power of half over 0 1, by means of Taylor series, by means of Taylor expansion, let us take the point x is equal to 0, about x is equal to 0. Find the number of terms required in the expansion; find the number of terms required in the expansion.

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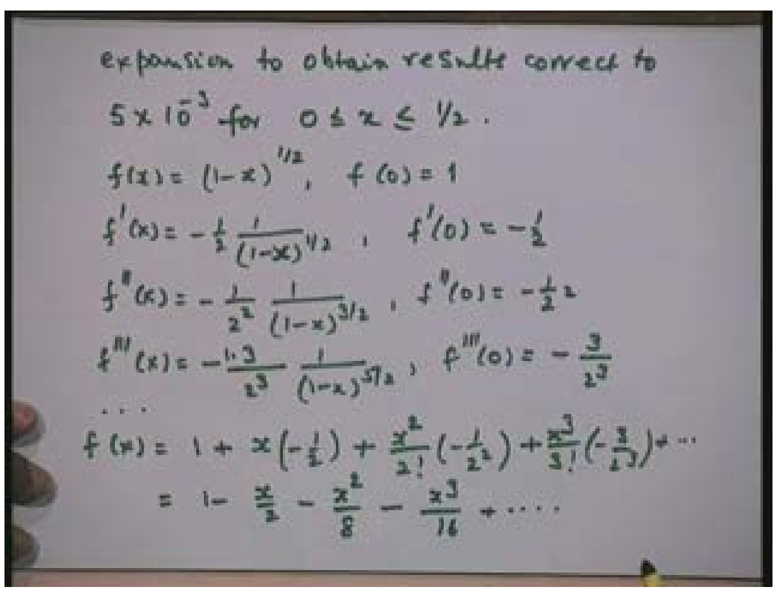

To obtain results correct to 5 into 10 to the power of minus 3, for 0 less than x less than half, less than or equal half. I am taking a particular problem; i am giving the interval and then say, giving the accuracy also and then asking, what is the number of terms in the Taylor series that will be required in order to achieve this accuracy. So the first part let us just do it, let us differentiate it and find out what is the, what are the values. We are given f(x) is equal to 1 minus x to the power of half; therefore f of 0 is equal to 1. Let us differentiate it, f prime of x is half 1 minus x to the power of half, i have taken the minus sign already outside, minus sign i have taken it. So if i said f prime at f is equal to 0, i will get minus half.

Let us write down the second derivative, this i will write it as, 1 up on 2 up, therefore this is plus and again minus sign, so i will have 1 upon 2 square 1 minus x to the power of 3 by 2, so that second derivative is equal to minus 1 upon 2 square. Now let us write down one more and then write down the series, now this is 3 by 2 with a negative sign, so this is plus 3 by 2 cubed with a negative sign, that is 3 by, i will write 1 into 3 by 2 cubed 1 upon 1 minus x to the power of 5 by 2, so that f triple dash 0 is equal to minus 3 by 2 cubed. Therefore our series is f(x) is 1 minus, x minus x_0, x_0 is 0 so, 1 minus x into, i will put it, i will put it, let us put it here 1 plus x into minus half x minus x_0 whole square by factorial 2, second derivative 1 upon 2 square x cubed by factorial 3 minus 3 by 2 cubed plus so on. So that i can write this as 1 minus x upon 2 x squared by 8 minus x cubed by 3, 3 cancels i will have 16 plus so on.

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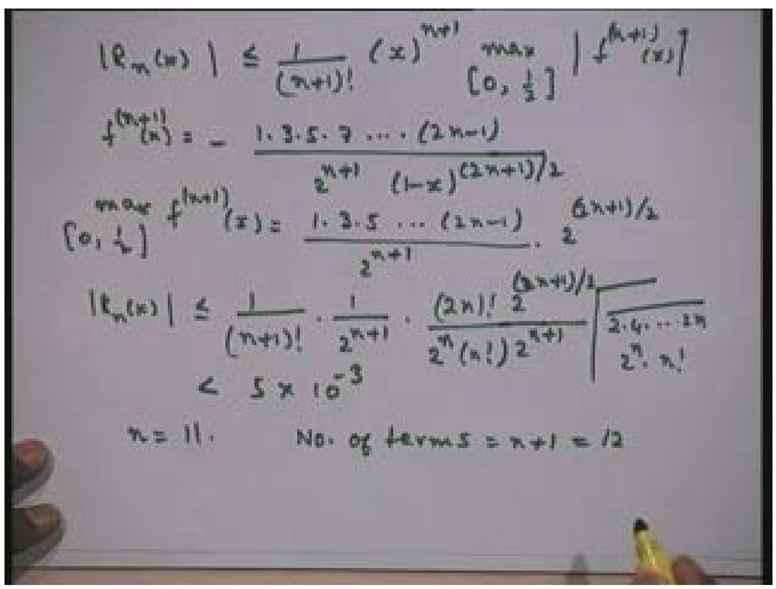

Now i need to find the n^{th} derivative and i want to find the bound for this, so i want to find $R_n(x)$ is less than equal to 1 upon n plus 1 factorial, x minus x_0, that is the x to the power of n plus 1 and m_{n+1} that is maximum of, the interval given to us is 0 half, 0 half and i need n plus 1^{th} derivative of x. Now let us write down what is our n^{th} derivative, now we have the expression, i have written this in a particular fashion, you can see, this is next derivative is going to be given me minus 5 by 2, so i absorb this minus sign, always minus sign comes 1 3 then 5, so i will have the products in numerator as 1 3 5 7 9 and so on because, this is 5 by 2, 7 by 2, 9 by 2 and so on and the denominator becomes 2 cube, 2 to the power of 4, 2 to the power of 5 and so on.

Therefore my n^{th} derivative is with a negative sign, 1 into 3 into 5 into 7 into 2 n minus 1, 2 n minus 1, this is 2 to the power of n plus 1 1 minus x to the power of 2 n plus 1 by 2. This is, this is 3, this is 5 by 2, so this will be 2 n minus 1 and this will be 2 n plus 1 by 2. This is our f n plus 1, this will be f n plus 1. Let us find the maximum of this, i want the maximum of this in this interval, so maximum in the interval, 0 to half f n plus 1 of x is 1 3 5 so on 2 n minus 1 by 2 to the power of n plus 1. I need the minimum here, so that it will give the maximum of this, so the minimum of this in 0 half will occur it half, i will have here 1 by 2 of this power, that is 2 to the power of 2 n plus 1 by 2. Now let us put it here, therefore i will have $R_n(x)$ is less than or equal to 1 upon n plus 1 factorial, the maximum of x to the power of n plus 1 in this interval is at half, so 2 to the power of n plus 1 that comes from here and let us do one more simplification, i will insert here 2 into 4 into 6 into 2 n, all the even terms i will supply, so that this numerator will become 2 n factorial, so i can make this as 2 n factorial in the numerator, now i multiplied by, in the denominator 2 into 4, i multiply 2 into 4 into so on 2 n, so i will have this in denominator here but this is, i can take 2 common here each factor, a 2 from here, 2 from here, 2 from here, that is n terms are there, 2 to the power of n, 1 into 3 into 3 into n that is n factorial, so the denominator will be 2 to the power of n, n factorial and this 2 to the power of 2 n, 2 n plus 1 by half and this should be less than, we are given the 5 into 10 to the power of minus 3, 5 into 10 to the power of minus 3. Now it is a matter of simple cancelation and using the calculator because these are all factorials involved, using the calculator by setting in integer value n and we can find out when this particular value is less than 5 into 10 to the power of minus 3. In this case the example comes out to be, n is equal to 11, n is equal to 11 is a value that is required and therefore number of terms is 12, is n plus 1 that is equal to 12 times. This is how the Taylor series can be used, however the Taylor series is not the one that is commonly encountered.

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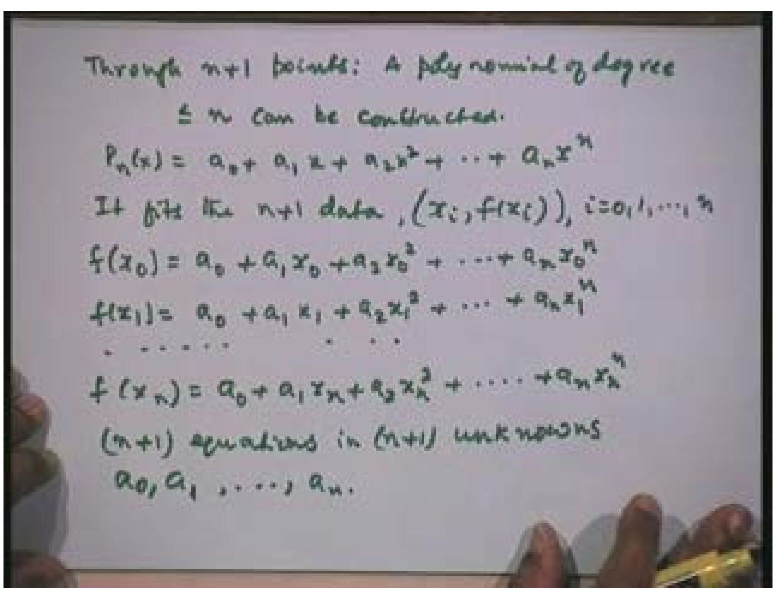

The most commonly encountered problem, we shall call it again as interpolation is the data given to us, a table of values. As i said 2 types of problems are encountered, we are given x f(x), x_0 f at $x_{0,}$ x_1 f at f (x_1) so on x_n f at x_n, therefore we are given n plus 1 data values. We could also call, in the two dimension plane this is a point, so we can also say n plus 1 data points are given. Now first of all, we know that in the two dimensional plane, if 2 points are given i can always pass a straight line through these 2 points, so this is a straight line, that means given two points i can always construct a linear polynomial to go through those two points. Therefore we have through two points, through two points, they are distinct points, these are n plus 1 data points but let us make it clear, they are all distinct points that we are considering. Through these 2 points a straight line, let us call it as p_1(x), polynomial of degree 1 x can be constructed, through 2 points a straight line a p_1(x) can be constructed.

Suppose now we have 3 points, now if i have 3 distinct points, i can try to pass a, a parabola, these are the 3 points, i can try to pass a parabola through this, that means a polynomial of degree 2 can be constructed. Even though there may be a degenerate case in which all the 3 points may lie on this, on a straight line, therefore through 3 points a polynomial of degree less than or equal to 2 can always be constructed. The degenerate case is a straight line otherwise will have a, has a second degree polynomial, that is a parabola, that means a polynomial of degree less than or equal to 2 can be constructed. We can construct a polynomial of degree 2 like this, which is of course a parabola, which is a parabola in 2 dimensions. As i said the degenerate case could be, that it all the points are lying on the straight line, therefore i can have a polynomial of degree 1, which is passing through all the 3 points. Therefore in general, i know have n plus 1 points, therefore through n plus 1 points, through n plus 1 points a polynomial of degree less than or equal to n can be constructed. Now the existence of this can be proved mathematically also, let us suppose this is our, p_n(x) is a polynomial of degree n, which is given by a_0 a_1 x a_2 x square so on a_n x n, let us suppose this is our polynomial. This polynomial is fitting the data, n plus 1 data, so it fits the n plus 1 data, x_i f(x_i), i is equal to 0, 1, n, that means this polynomial is exactly satisfied by this, therefore let us substitute the values x_0 x_1 x_2 x_n here. So will have here f of p_n at x_i, x_0 that is f(x_0) is a_0 a_1 at x_0, a_2 x_0 square a_n x_0 to the power of n. Similarly x_1 also satisfies, therefore will have a_0, a_1 x_1, a_2 x_1 square, a_n x_1 n and so on, will have f of x_n a_0, a_1 x_n, a_2 x_n square plus a_n x_n to the power of n. Now you can see that, this is a system of n plus 1 equations, in n plus 1 variables a_0 a_1 a_2 a_n, we have to determine a_0 a_1 a_2. If the unique solution for a_0 a_1 a_2 a_n exist, then this polynomial exists uniquely. Therefore i need to determine this a_0 a_1 a_2 a_n and for that we may have n plus 1 equations, these are n plus 1 equations in n plus 1 unknowns, in n plus 1 unknowns a_0 a_1 a_n. Now the condition for the existence of this will be, the determinant of this coefficient should not be equal to 0, then the solution exists.

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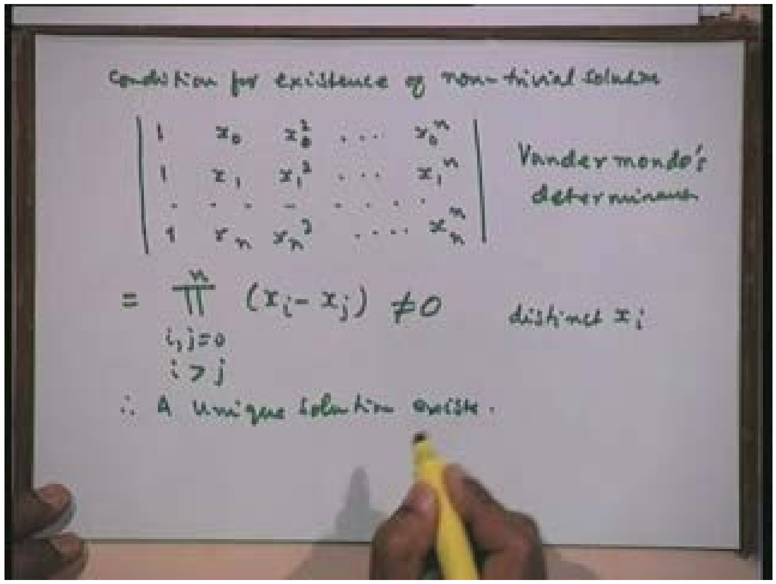

So the condition for this is, therefore will have the existence condition or condition for existence of non-trivial solution is the determinant of 1, i am just writing the coefficients of this system, that is 1, x_0, x_0 square, $(x_o)^n$, 1, x_1, x_1 square, $(x_1)^n$, 1, x_n, x_n square, x_n to the power of n, so that will be 1, x_0, $x_{1,}$ x_0, x_0 square, x_0 to the power of n, 1, x_1, x_1 square, x_1 to the power of n, 1, x_n, x_n square, x_n to the power of n. This determinant has a name, it is called Vandermonde's determinant, it is called the Vandermonde's determinant.

Now we can expand this determinant by the rules of the evaluating the determinant, i can subtract the first row from the second one and then you take common x minus x_0, x_2 minus x_0, x_n minus from the rows and i can simplify this and this is a varies trivial case. Value of determinant product of all x_i x_j, i comma j going from 0 to n but i greater than j, i am taking x_1 minus x_0, x_2 minus everything common, similarly i will have all the factors that comes in to the product is this one. We have started the problem with saying that these points are distinct points, so the data was distinct, so distinct x_i, since there are distinct x_i, x_i minus x_j is never 0, therefore this is not equal to 0. Since this is not equal to 0 we have proved that the, the solution exists, therefore this a unique solution exists. Therefore we have now determined that this polynomial can be determined uniquely but i would now like to show that the polynomial is unique, we are now shown the solution for the a_0 a_1 a_2 a_n exists and it is unique but i want show that this polynomial also, there cannot be more than 1 polynomial which can feed the given data, so the polynomial is also a unique polynomial, the interpolating polynomial is unique.

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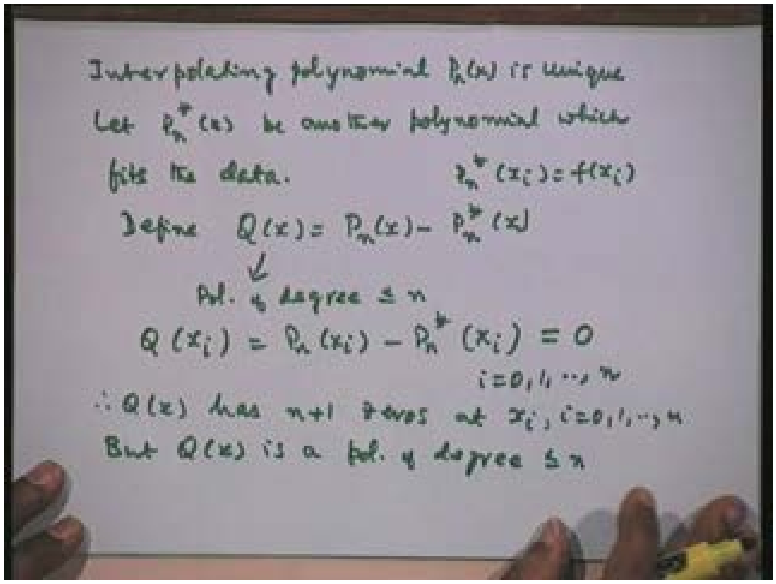

Interpolating polynomial $p_n(x)$ is unique; the proof of this is just 2 lines. Let us assume that there is another polynomial, let P_n some star x be another polynomial which fits this given data, be another polynomial which fits the data, that means we are talking that p_n star of x_i is equal to $f(x_i)$ for all i. Then let us define the difference between these two polynomials as a function Q(x), define Q(x) is equal to $P_n(x)$ minus P_n star of x. Now $P_n(x)$ is the polynomial of degree less than or equal to n, P_n star x is also polynomial f degree less than or equal to n, therefore this is a polynomial of degree less than or equal to n, this is the polynomial of degree less than equal to n.

Now let us see what happens for Q(x) at these values x_i, therefore Q at x_i is equal to P_n at x_i minus P_n star at x_i, both the polynomials are fitting the data, therefore this is f x_i, this is also f x_i, therefore this is equal to 0, for i is equal to 0 1 to n. Now you observe that Q(x) is now vanishing at n plus 1 points, therefore Q(x) has got n plus 1 roots or n plus 1 zeros, therefore Q(x) has n plus 1 zeros but Q(x) is a polynomial of degree less than or equal to n only. Therefore the only possibilities that Q(x) is trivially 0 or identically 0, therefore the, therefore Q(x) has n plus 1 zeros at x_i but Q(x) is a polynomial of degree less than or equal to n and it is having more number of zeros.

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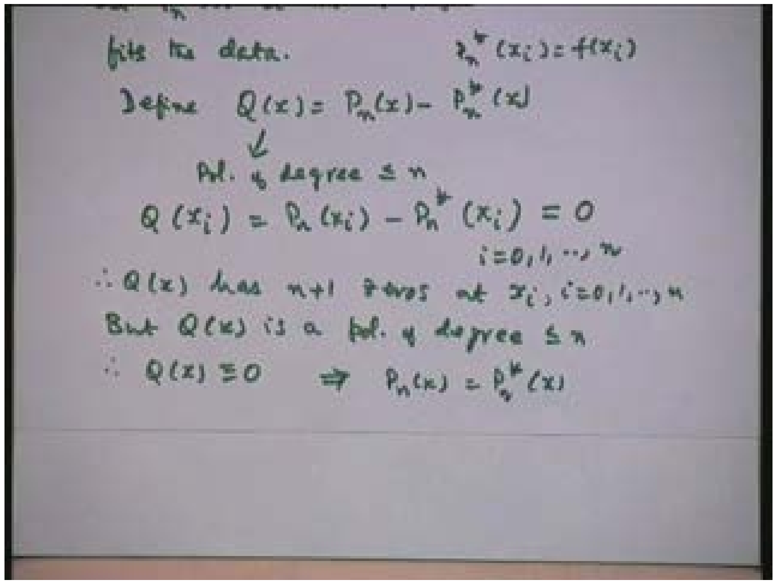

Therefore the only possibility is that Q(x) is identically equal to 0 and when once Q(x) is identically 0 we will get $P_n(x)$ is equal to P_n star x that means $P_n(x)$ is equal to P_n star of x. Therefore the interpolating polynomial unique, in other words what we want to state here is, that we will it see later on also, we will shall be constructing the polynomial using different methods or different ways. The form of the interpolating polynomial can be different but when once you simplify the whole thing and bring it to the form the both will be identical, therefore we cannot have 2 interpolating polynomials for fitting the data but all are them will be the same, but the form in which you write down the polynomial would be different, i mean one may write it in a ratio form, which one simplified this and this or you write down in a different form or in product form it will be in a different form but all of them would identically same when you bring out everything into the expanded form and write it, therefore in that sense interpolating polynomial is unique.

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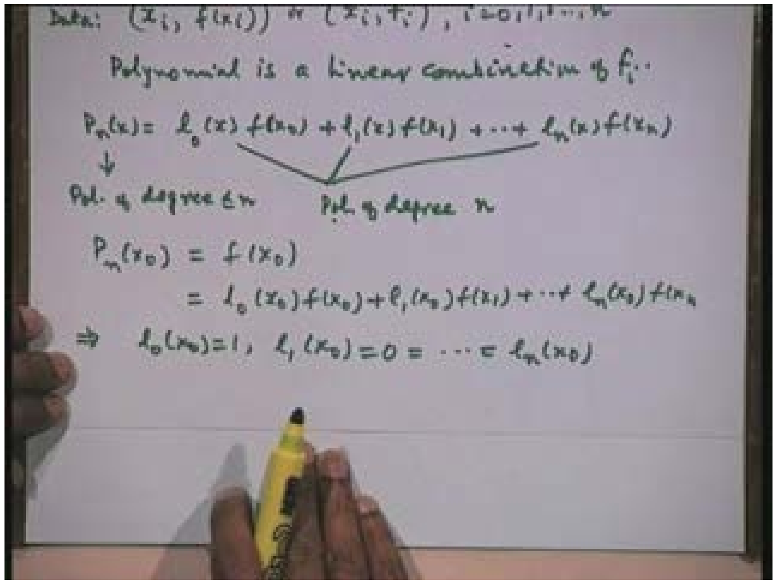

Let us now construct the interpolating polynomial; the most fundamental interpolating polynomial is called the Lagrange interpolating polynomial. Now let us again take the data, as the given data is x_i, $f(x_i)$ or we let us use a short rotation x_i f_i, i is equal to 0 1, this is the data given to us. Now if i want to construct a polynomial that fits this data, this polynomial must be a combination of f_i, it must use all the values of 0 f_1 f_2 f_n therefore you should be linear combination of all f_i. Therefore the polynomial is a linear combination of f_i that means it should be of the form $P_n(x)$ is equal to some; i will write it has l_0 x, some function of x into $f(x_0)$, that is because it is a polynomial of degree n in x, therefore this f x f_1 f_2 f_3 they are all numbers, they are all constants, therefore $l_0(x)$ into $f(x_0)$ plus $l_1(x)$ into f of x_1 plus so on $l_n(x)$ is equal to $f(x_n)$.

Now as i said this is a polynomial, what we want here is a polynomial f degree less than or equal to n, $f(x_0)$ $f(x_1)$ $f(x_n)$ they are all numbers, since these are all numbers the only possibilities is that $l_0(x)$, $l_1(x)$, $l_n(x)$ all these must be also be polynomial of degree n, polynomials of degree n. Now the less than or equal to n we shall not put it here, it is n, if it is less than n when you simplify this $f(x_0)$ $f(x_1)$ $f(x_n)$ they are all numbers, when you simplify automatically the leading terms would cancel and it will agree with the left hand side. Therefore these l_0 l_1 $l_n(x)$ must be polynomial sub degree n in order that the given data can be represented by an n^{th} degree polynomial, but if this is the polynomial this is fitting this data, so let us just satisfy, that is p_n of x_0 must be equal to $f(x_0)$, must be equal to $f(x_0)$ but this is equal to $l_0(x_0)$ $f(x_0)$, $l_1(x_0)$ $f(x_1)$ plus so on $l_n(x_0)$ $f(x_n)$. Now this should be equal to $f(x_0)$ only, therefore i cannot have $f(x_1)$ $f(x_2)$ because this is a identity, therefore this should be containing only this, the only possibilities is that $l_0(x_0)$ must be 1 and all this should vanish, only then this will be equal to this one. Therefore this implies that $l_0(x_0)$ must be 1 and all other values $l_1(x_0)$ must be equal to 0 so on, $l_n(x_0)$ must be equal to 0, only then this is going to be same as this particular polynomial.

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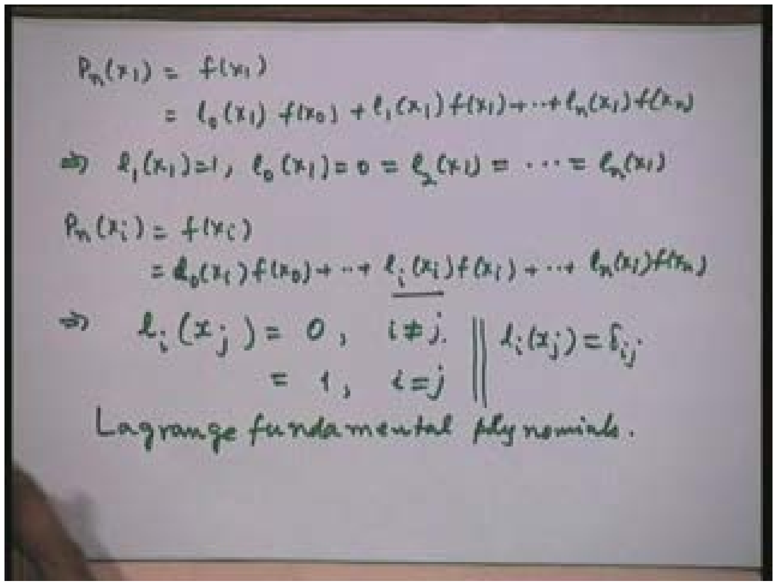

Let us do one more and generalize it, this is $f(x_1)$, $l_0(x_1)$ $f(x_0)$, $l_1(x_1)$ $f(x_1)$, $l_n(x_1)$ $f(x_n)$. Now if this is to be again to be equal, $f(x_1)$ is here, $f(x_1)$ is here, the only possibilities is $l_1(x_1)$ must be 1 and the remaining all l should be 0. So this would imply that $l_1(x_1)$ must be equal to 1, $l_0(x_1)$ is equal to 0 and this is $l_2(x_1)$ is equal to 0 so on $l_n(x_1)$ must be 0. Therefore it is possible for us to immediately generalize it by just writing x_i is equal to $f(x_i)$ that is $l_0(x_i)$ $f(x_0)$ plus $l_i(x_i)$ $f(x_i)$, $l_n(x_i)$ $f(x_i)$. Now i can generalize from here that this $l_i(x_i)$ will be 1 and all the remaining will be 0, therefore i would get the result that $l_i(x_j)$ is equal to 0, for i not equal to j, this is equal to 1, for i is equal to j and those who are used to the rotation of chronicle delta we will can write this as simply delta ij, so this is the chronicle delta, this is definition of this is that is equal to 0, when i not equal to j and when it is will be equal to 1 and when y is equal to j. Therefore all this polynomials, they should satisfy this and these polynomials are called Lagrange fundamental polynomials, Lagrange fundamental polynomials. We shall actually construct the form of this Lagrange fundamental polynomial using this idea from here and we will be able to construct the Lagrange interpolating polynomial from there. We should stop here.