Numerical Methods and Computation
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Lecture No - 24
Solution of a System of Linear Algebraic Equations (Continued)
Eigen Value Problems

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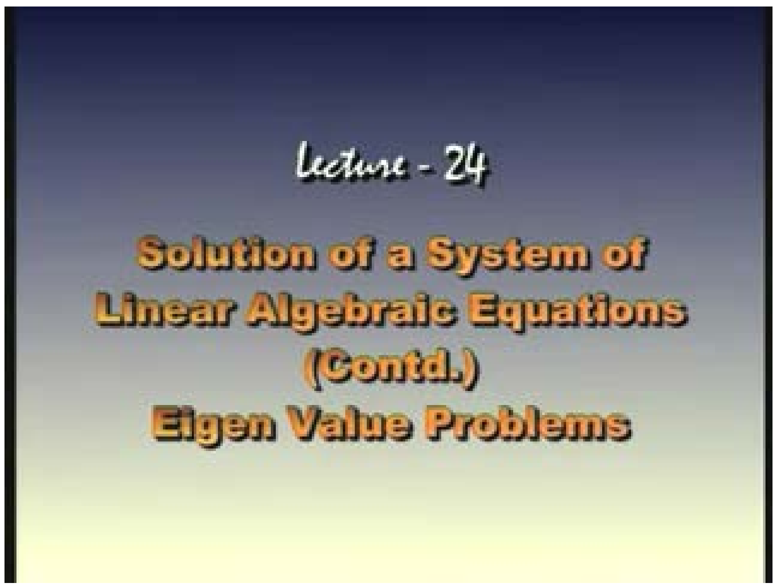

In our previous lecture we have derived the power method for finding the largest Eigen value in magnitude of a given matrix. We produce automatically the required Eigen vector corresponding to that Eigen value also as we proceed on with the power method. Now let us just briefly describe what we have, what is a power method.

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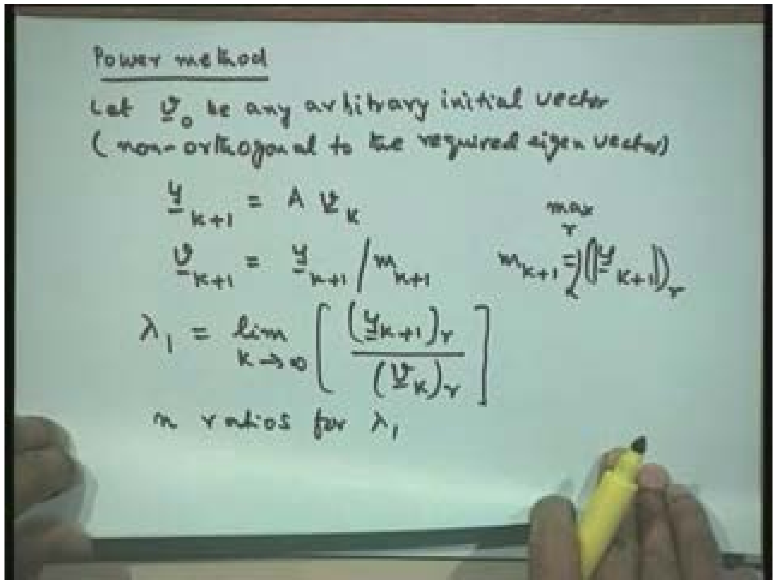

The power method we had written it in this form, now we start with any arbitrary initial vector, so let us take v_0 be any arbitrary initial vector, of course we shall assume that is non-orthogonal to the required Eigen vector, so it is non-orthogonal to the required Eigen vector. Then we have defined the power method as follows, we form the vector y_{k+1} as A into v_k, so therefore setting K is equal to 0, i can get y_1 from here. Now before we proceed with the next step, would like to normalize such that the largest component of this y is 1, therefore i would now write v_{k+1} as the vector y_{k+1} divide this by the number m_{k+1}, where this m_{k+1} is the largest element of y_{k+1}, so i will take this vector, take the largest component with respect to this, i should write this as maximum with respected to r, so this is the largest component in magnitude, i can, i will, you can take it magnitude or you can take even sign also, it is immaterial for us, so we take the largest element, mostly in magnitude we can make it and then divide it out by this, then the largest element of y_{k+1} is going to be 1. Therefore there is no chance of the round of errors increasing every time, because we are every time, we are now normalizing it, then the Eigen value after certain number of iterations we shall test this value of lambda that is, limit K tending to infinity y_{k+1} component divided by v_k component this.

So we will test the ratios of the corresponding components of the current value of y_{k+1} and the previous v_k, this is nothing but the ratios of components of A_{k+1} and A_k in the normalized form, therefore these will give you n ratios, n ratios for lambda 1. Now the iteration is stopped when the difference between the any two ratios is less than given tolerance, so stopping criteria is, when magnitude of difference between any two ratios is less than a given tolerance epsilon, so we are given a tolerance that you find the solution accurate to 4 decimal places or 5 decimal places and we will find this Eigen value lambda 1 such that the difference between any 2 of these ratios is less than epsilon, that is the required power method. Now let us apply this on a example, let us take an example.

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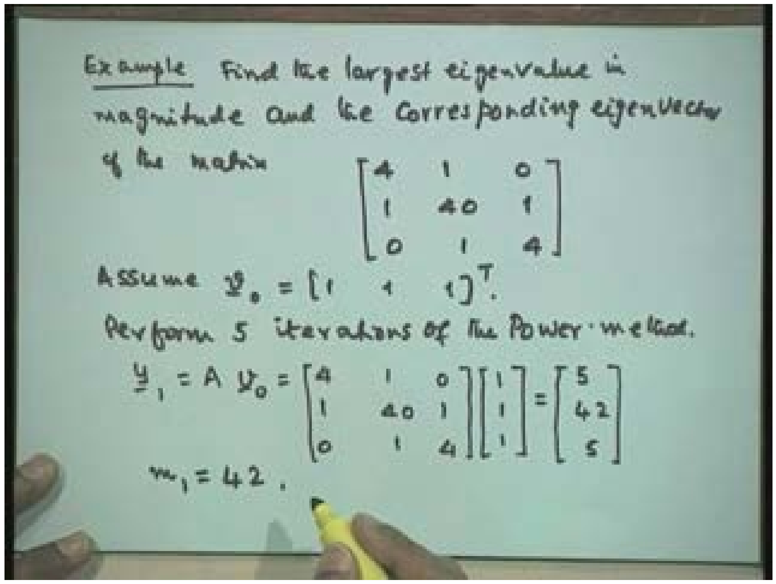

Let us write the example as, find the largest Eigen value in magnitude and the corresponding Eigen vector, corresponding Eigen vector of the matrix 4, 1, 0, 1, 40, 1, 0, 1, 4. Let us also give, assume v_0 is equal to 1 1 1 and let us also give the number of iterations, perform 5 iterations of the power method. Let us write down the first step, the first step would be to write down y_1 is equal to a v_1, normalize it and then repeat this procedure until the 5 iteration that is required. So we start with y_1 is equal to A into v_0, therefore that will give us 4, 1, 0, 1, 40, 1, 0, 1, 4, though we have given the initial vector as 1 1 1 so we so this, so i get this as 5, 42, and 5. Now i find the largest element in magnitude of this, that is 42, so i will call this as m_1, m_1 is equal to 42.

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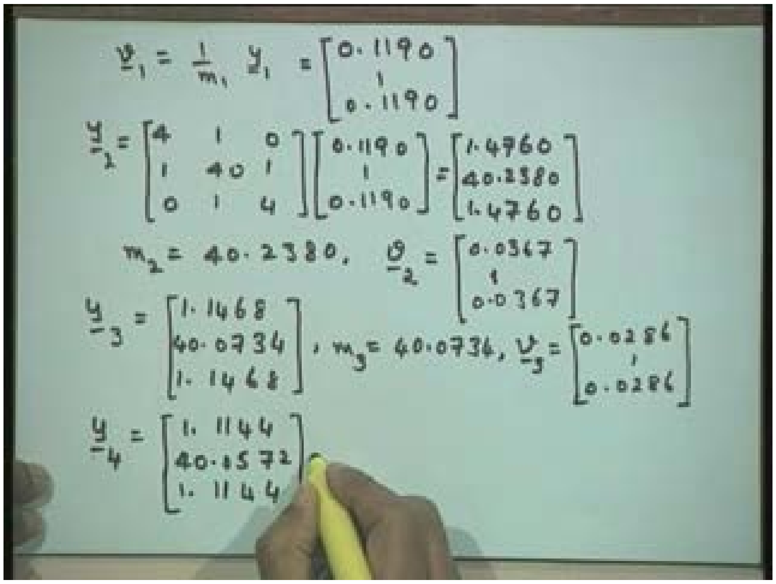

Therefore v_1, now v_1 is 1 upon m_1 of y_1, so we have to divide this by the largest element in magnitude, so that will give me 5 by 42, let us divide it out, i will give the value of this, that is 0 point 1 1 9 0, 1, 0 point 1 1 9 0. Now we go to the next iterations, first iteration is over, second iteration will give us y_2 is equal to a v_1 1, 40, 1, 0, 1, 4 into this value 1 1 9 0, 1, 1 1 9 0. So this gives me 4 times 1 1 9 0 plus 1 so, and that is one point 4 7 6 and the middle value is 40 times plus 2 times this, that is 40 2 3 8, 1 point 4 7 6 0. Now we find the largest element in magnitude here, that is your m_2, that is equal to 40 point 2 3 8 and hence our v_2 is, we divide this by 40 point 2 point 2 3 0, so we will have here 0 point 0 3 6 7, 0 point 0 3 6 7, now at this stage second iteration is complete. So we go to third iteration, so let me give the values of the iterates, this is 1 point 1 4 6 8, 40 point 0 7 3 4, 1 point 1 4 6 8, this is what i would get by multiplying with A this particular vector. Now again i find the largest element in magnitude is this one, that is your m_3 is 40 point 0 7 3 4, division by this element would give me the next vector v_3 and that is, that gives me 0 point 0 2 8 6, this completes our third iteration. So let us go to fourth iteration, the fourth iteration gives me 1 point 1 4 4, 40 point 0 5 7 2, 1 point 1 4 4. Now i find the largest element in magnitude, therefore this is 40 point 0 5 7 2 and v_4 is, we divide by this element and i will have this as 0 point 0 2 7 8, 0 point 0 2 7 8, at this stage the fourth iteration is complete.

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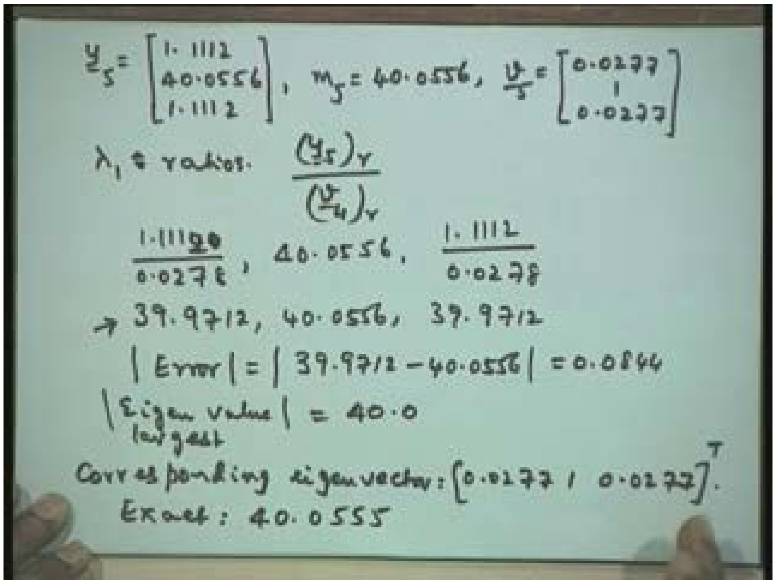

Now let us go to fifth iteration, this gives me 1 point 1 1 2, 40 point 0 5 5 6, 1 point 1 1 2, therefore our largest element in magnitude is 40 point 0 5 5 6 and v_5 is 0 point 0 2 7 7 (Refer slide time: 11:47) 0 point 0 2 7 7. Now we have completed the 5 iterations that is required here, therefore we can now find out the ratios, now lambda 1 would be the, the ratios for this will be the ratio of y_5 component by v_4 components, i want the ratio of y_5 and the previous v, that is the ratio of these two components that is required, that is the components therefore are 1 point 1 1 4 4 divided by, now this is y, triple 1 2, this is triple 1 2 divided by 0 point 0 2 7 8 and 40 point 0 5 5 6 divided by 1 and the third one is 1 point 1 1 2 by 0 point 0 2 7 8. This is the ratio of the component of the present y and the previous v, that will give this ratios and this is 39 point 9 7 1 2, 40 point 0 5 5 6, 39 point 9 7 1 2. Now i can find the, what is the error between this values, so i can find out error, magnitude of the error is 39 point 9 7 1 2 minus 40 point 0 6, that is approximately 0 8 4 4 (Refer slide time: 13:36), therefore we will take the, to this error will take the Eigen value, largest Eigen value as this. Therefore the largest Eigen value in magnitude is equal to, if you just round it off to one digit, it is going to be 39 point this or we just take it as 40, we can just round it off to one decimal place, so that will be 40 point, 40 point 0 can be taken here, this is of course the largest Eigen value. Now the corresponding Eigen vector is a current v_5 that we have found, therefore the corresponding Eigen vector, corresponding Eigen vector is this, that is your 0 point 0 2 7 7, 0 point 0 2 7 7 is the (Refer Slide Time: 14:48). Now the exact Eigen value of this is, now before we make the exact, if you want the sign of this Eigen value we must substitute in A minus lambda I and see that it is equal to 0, to see whether it is plus 40 or it is a minus 40, so if it is not correct, if we have taken it, if the Eigen value was minus 40 and we have taken it as plus 40 then A minus lambda I is equal to 0 will not be satisfied, it will be a big number but if you have the, correct sign is taken then will have the correct Eigen value from there, therefore the sign of the Eigen value it tested by putting in A minus lambda I. The exact Eigen value of this is, 40 point 0 5 5 5, this is the exact Eigen value for this particular problem.

[Student: Sir why do we take up 40 point 0 5 5 6] Actually the, the, this also converges to, because this is the factor that we have taking out the, that factor is divisible, this also tends to the required Eigen value but as defined, we have taken at a particular level A_k, A_{k+1}, the ratio of the corresponding components is your lambda 1, therefore we would like to take the ratios of this one, yes it is true, this also would converge to the Eigen value, required Eigen value, when we take that common factor out, then that Eigen value, that also will correspond to the Eigen value, that is also comes to the Eigen value but in general we take this particular thing but that is also correct, that is also correct.

Attempts have been made to make this power method faster, now the, the most important application of power method is, as we mentioned earlier, we want to find out the rate of convergence of an iterative method, that is Gauss Jacobi or Gauss Seidel or successive over relaxation, we need the largest Eigen value in magnitude of iteration matrix. Now we would not like to find all the Eigen values of the given matrix by the Rutishauser method or if it is symmetric by Jacobi or the Givens method, therefore the alternative for that is a power method which takes very little amount of time, it is simply multiplication of a, of a matrix and a vector repeated number of times, may be a 50 times 100 times that is to be done and then we have the correspond ratios of the corresponding components and we can get the Eigen value of that, that is a largest Eigen values in magnitude which will give the rate of convergence as minus log of spectral radius of the iteration matrix, therefore this is the method to be used whenever you want to find the spectral radius of the iteration matrixes.

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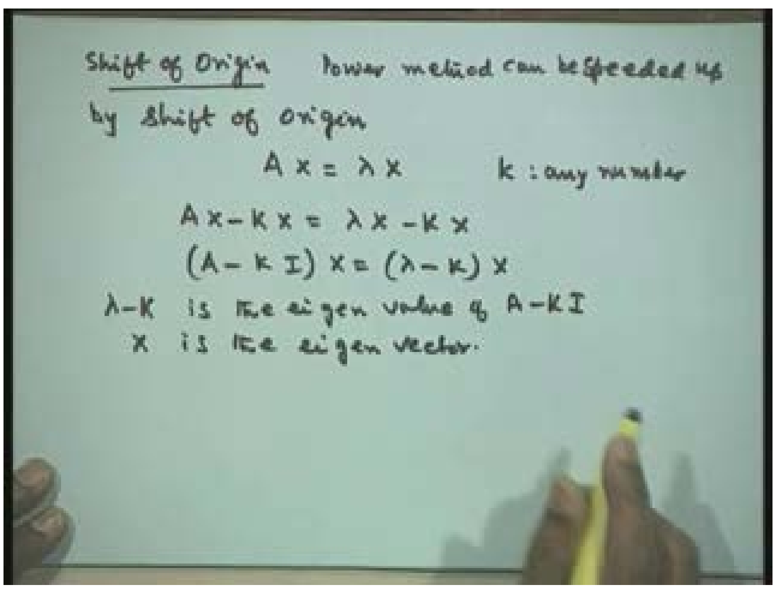

One can speed up the convergence of the power method by using a very simple technique and that is called as shift of origin. We can say that power method can be speeded up by shift of origin, now what you mean by the shift of origin is this, so let us take our Eigen value problem, A X is equal to lambda X and let us take K as any number, K is any number, K is any number, then let us subtract K X from both sides, A X minus K X is lambda X minus K X, that means A minus K I of X is equal to lambda minus K of X, therefore this states that lambda minus K is Eigen value of A minus K I and X is the corresponding Eigen vector, therefore this implies that lambda minus K is the Eigen value, is the Eigen value of A minus K I and X is the Eigen vector. Therefore what has happened is that, if i subtract K, this is A minus K I, that means we are subtracting K from the diagonal elements, this is K into I, so we are subtracting the same number from the diagonal elements, say you are subtracting 5, so all the elements, then what happens is all Eigen values are subtracted by the same number. So if the Eigen values of a matrix, we have lambda 1, lambda 2 lambda 3, we have given a shift by say number 5, then the new Eigen values are going to be lambda minus 5, lambda 2 minus 5, lambda 3 minus 5 but Eigen vector is not going to change, Eigen vector is going to be the same thing, now we can apply the power method on A minus K I.

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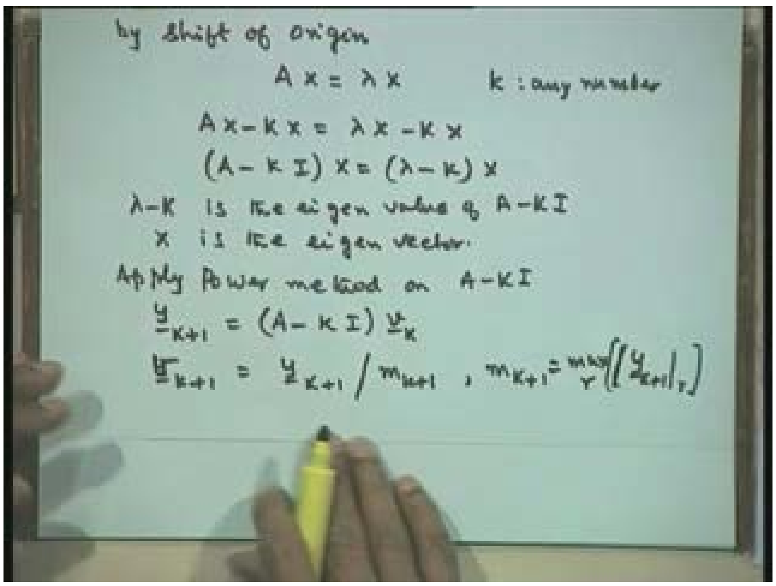

Now apply power method, power method on A minus KI, that means we are going to write y_{k+1} is A minus K I v_K, v_{k+1} is equal to y_{k+1} divided by m_{k+1}, m_{k+1} is maximum of, of r, of this element of r, that means the power method gives the dominant Eigen value of A minus K I, therefore power method gives the largest Eigen value in magnitude of A minus K I.

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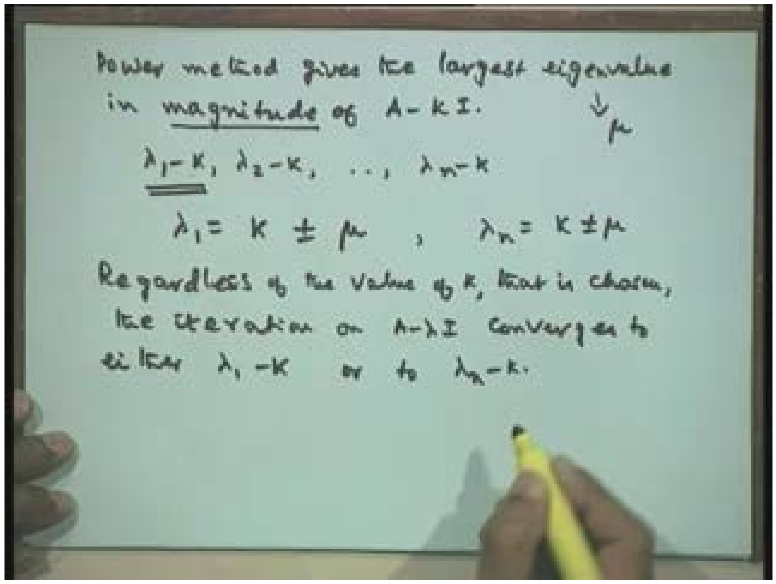

When once we find the largest Eigen value of A minus K I, we can go back and find out what is the correct Eigen value, the Eigen values are now lambda 1 minus K, lambda 2 minus K and so on Lambda n minus K. Therefore the original Eigen values will be lambda 1, lambda 2, we to take to the right hand side and add to this largest Eigen value. Therefore i can now find out the largest Eigen value in magnitude as original matrix, as lambda 1 is equal to the K, i have taken to the right hand side and whatever this value that we have got here as mu, mind you this is a, it is given in magnitude therefore the original Eigen value will be either with a plus sign or with a negative sign, we do not know its sign so it will be plus minus sign. So i can take K to the right hand side and K plus minus mu and this Eigen value, am calling it as mu. The correspondent to the Eigen value of this or we can also have this is equal to K plus minus mu. Now why i had written only these two is, that it has been shown that regardless of the value that we take for K here, this power method would converge either to the largest Eigen value or to the smallest Eigen value only, regardless of the value of K, regardless of the value of K that is chosen, the iteration on A minus lambda I converges to either lambda 1 minus K or to lambda n minus K.

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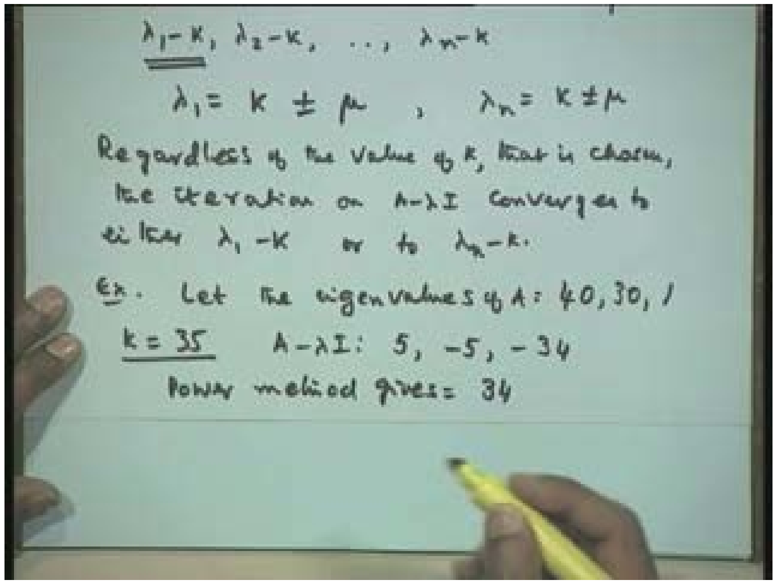

Now let me illustrate this little bit further, what we mean by this, let us take a very simple example. Let us suppose a matrix has an Eigen values, let us take them as some numbers, so let us say 40, 30, 1, so these are the Eigen values of given matrix. Let me choose K is equal to some number, 35 i will choose, so let me choose K as 35, therefore the Eigen values of A minus lambda I would be 40 minus 35 that is 5, 30 minus 35 minus 5, 1 minus 35 that is minus 34. Now the iteration on A minus lambda I would converge to plus 34, the largest Eigen value in magnitude of this, therefore power method [Student: A minus K I] yes, power method gives the Eigen value as 34, the Eigen value is magnitude of this, it will give you, so it will give you the Eigen value of the A minus lambda I as 34, from which we can determine. Now if you look at this 34, this corresponds to 1, this is the Eigen value, when you go back it is going to give you the Eigen value as this, this is the, so we are taking this K plus minus mu, so it is going to converge, therefore it converges to the smallest Eigen value, if i taken K is equal to 35.

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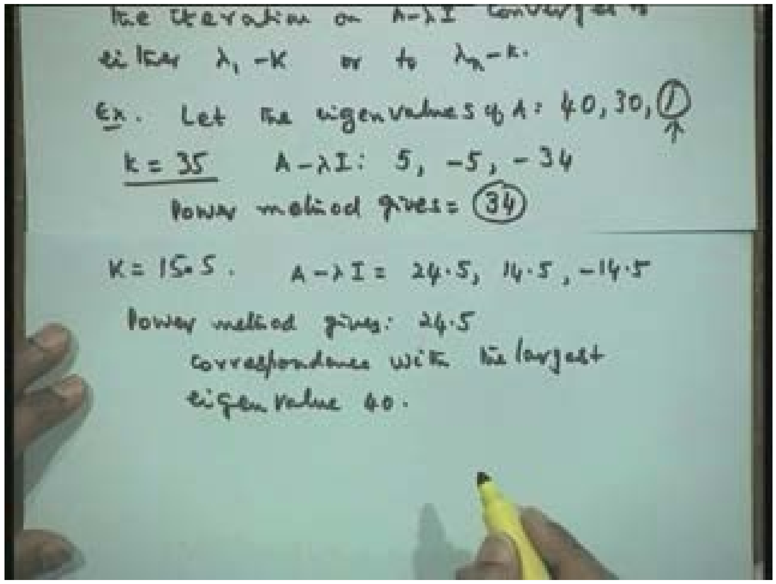

Let us take some other value, let us take K is equal to some like 15, say 15 point 5, let us take 15 point 5, then A minus lambda I will be 40 minus 15 point 5 that is 24 point 5, 30 minus 15 point 5 that is 14 point 5 and 1 minus 15 point 5 minus 14 point 5. Now the power method will give the largest Eigen value in magnitude of A minus lambda I, that is 24 point 5, therefore now the power method gives, gives 24 point 5, which will now correspond to the largest Eigen value, because this correspondence is with respect to largest Eigen value. Therefore the correspondence is with the largest Eigen value, correspondence is with the largest Eigen value, which is 40. Therefore irrespective of, therefore the value that we choose here, the iteration is going to converge either to the largest Eigen value in magnitude or the smallest Eigen value in magnitude but the value of, now why, why, what was the need actually to introduce to this particular shift of the origin. When we started it, we said we wanted to speed up the iteration, we can show that by choosing K suitably we can achieve faster convergence, we also said that everything depends on the ratios, as K tends to infinity lambda I by lambda 1 in magnitude is less than 1, it will go to 0 faster if lambda I, that means all other Eigen values are smaller in magnitude by leading Eigen value.

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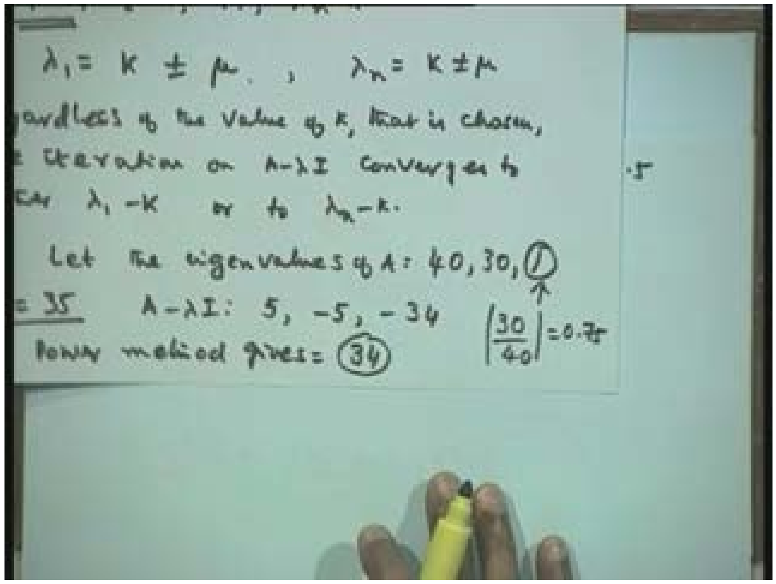

Now if i take this example, if i apply the iteration method on this, the Eigen value ratio of the next Eigen value to this is this, 30 by 40 that is lambda 2 by lambda 1 and this is 3 by 4 that is point 7 5, therefore this is the number by which it going to reduce the leading one as we go the iteration, that is point 7 5 whole square, point 7 whole cubed and so on that is how the reduction is going to come. Now let us see, when we have done this, when we are done here, now the leading Eigen value is 34, next Eigen value is 5 therefore in this case the ratio is 5 upon 34, right, in this case it is 5 upon 34 and if you now look at the third case, this is 14 point 5 by 24 point 5 this is approximately equal to 0 point 5 9.

Now we can see that the ratio of lambda 2 by lambda 1 in this case is 0 point 5 9, it is much smaller than 0 point 7 5, therefore for this value of K the convergence is faster. Hence the convergence is faster, if the ratio of the next Eigen value in magnitude and the largest Eigen value is magnitude is smaller. Therefore we conclude that by choosing the suitable shift to the Eigen value, we can obtain the largest or the smallest Eigen value in magnitude faster that is the reason why one can use the shift or the origin to speed up the power method and another case is that you have in a particular problem, that you want to locate a particular Eigen value, what is the Eigen value near a particular number, what i can do is, i will take that number, subtract from A, find the Eigen value then that Eigen value is going to be the smallest. Suppose the Eigen value is 3 point 5, you have taken the number as K is 3, now when once K is equal to 3, the new, final Eigen value should be 3 point 5 minus 3 that will be point 1 and that is going to be the smallest Eigen value of your new system, therefore the, this will now shift, will now converge to the smallest Eigen value of the system. Now, now we said that it is going to converge to the smallest Eigen value, now how do i find, power method gives me only the largest Eigen value not the smallest Eigen value, how do i find the smallest Eigen value. Now we have a modification in the power method called the inverse power method, which will give the smallest Eigen value instead of the largest Eigen value.

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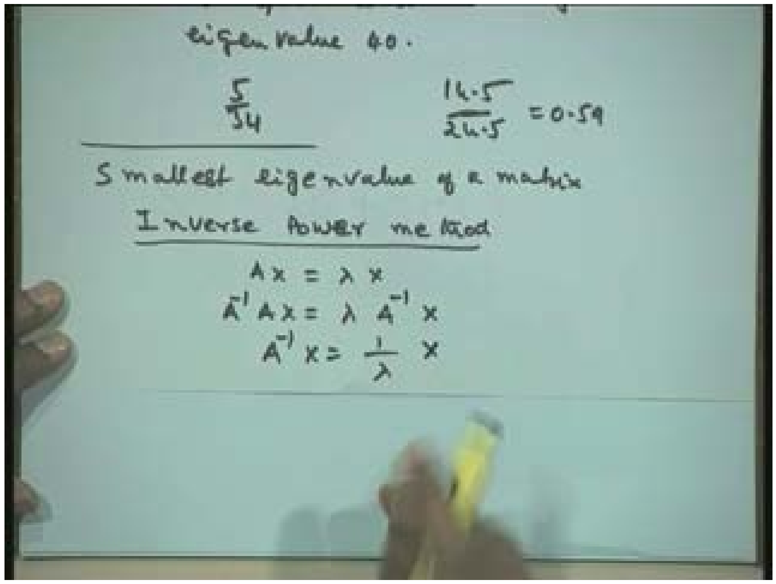

So let us find out how we get the smallest Eigen value of a matrix and this is called the inverse power method, inverse power method. The idea behind inverse power method is very simple, let us first review the definition of the Eigen value problem, now i pre multiplied by A inverse, so i will have A inverse A x is equal to lambda A inverse X, therefore i bring lambda to this side and i will write right hand side first, so i will write A inverse X one upon lambda of X.

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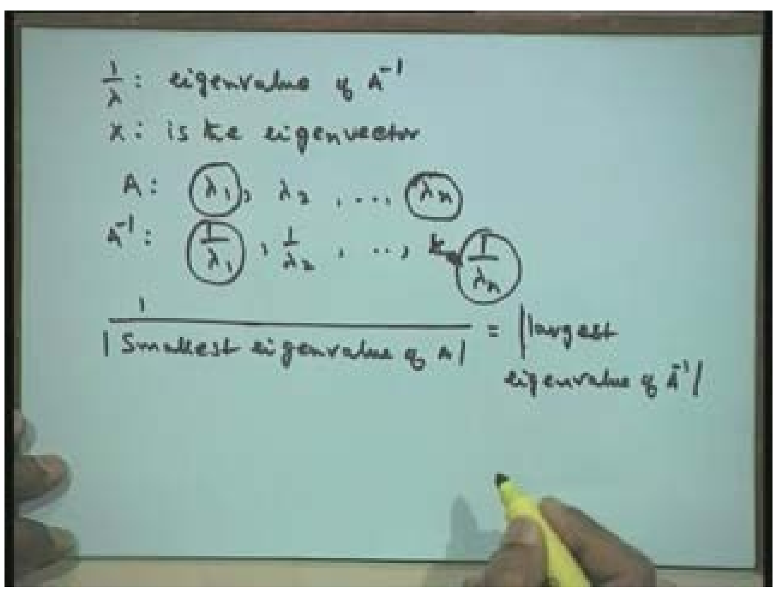

I have discussed this earlier but at this stage it is very relevant for us again, now therefore 1 upon lambda is Eigen value of A inverse and X is Eigen vector is same, therefore 1 upon lambda gives the Eigen value of A inverse. X is again the Eigen vector, X is the Eigen vector, the Eigen vector has not changed, the Eigen vector is the same. Now therefore if, if i take a matrix A it has got the Eigen values of lambda 1 lambda 2 lambda n in order of course, in the order of this, then A inverse has got the Eigen values, 1 upon lambda 1, 1 upon lambda 2 and so on 1 upon lambda n. Now the leading Eigen value of A, that is the largest Eigen value in magnitude of A, will now become the smallest Eigen value of A inverse. This is the largest Eigen value and therefore its inverse is going to be the smallest Eigen value of A inverse, that means if i, now the smallest Eigen value, sorry i should have written 1 upon, now the smallest Eigen value of A will become the largest Eigen value of A inverse. Therefore if i want the smallest Eigen value of A, i need to find the largest Eigen value in magnitude of A inverse, that means i just need to apply power method on A inverse, because that will give me the largest Eigen value of A inverse and hence the smallest Eigen value of inverse. Therefore the, the 1 upon the smallest Eigen value will be the largest Eigen value of A inverse. Therefore the leading Eigen value will be this, this will be the largest Eigen value and this will be 1 upon lambda n of this.

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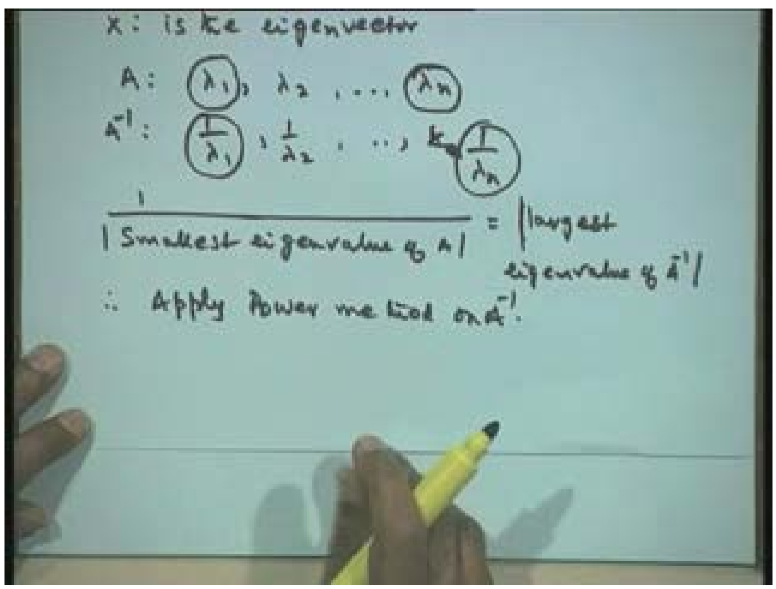

Therefore we conclude, apply power method on A inverse and this is the reason, why it is called inverse power method, we are applying the power method on the inverse, so it is called inverse power method, that is why the name has come as inverse power method. Now let us write down in detail what is the inverse power method, so that we can give the simplest way of getting the iteration.

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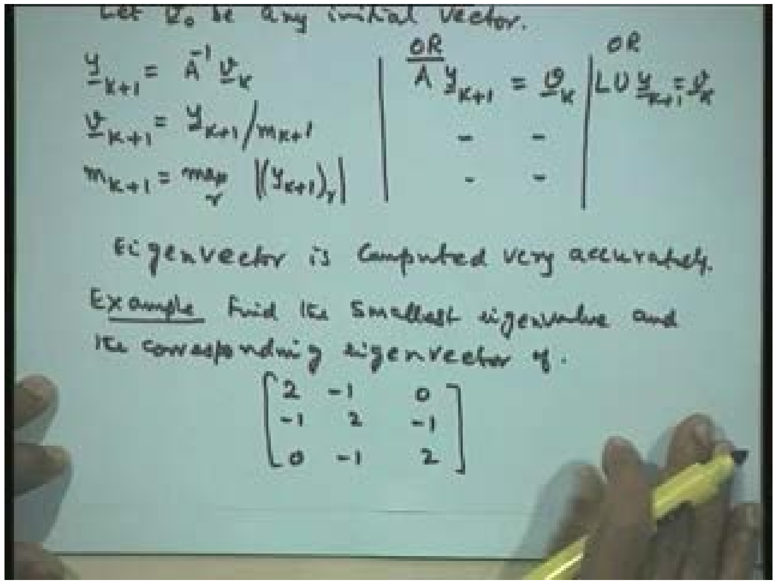

So let us start us again start with any initial vector, let v_0 be any initial vector, of course again non-orthogonal to the actual Eigen vector, then the iteration is, y_{k+1} is A inverse of v_k. I am applying the power method on A inverse, then i have this v_{k+1} is equal to y_{k+1} divided by m_{k+1} and m_{k+1} is maximum of r, the largest element in magnitude of this. Now for large matrices we may have some problems of round of error finding A inverse, because we need A inverse explicitly here, therefore if you are finding the inverse of a matrix which have decimal numbers and it is approximate numbers, then the round of error is very big in A inverse, which can disturb the Eigen vectors very much. Therefore in such cases where we have doubt about the round of errors, we do not like to solve this one but will make a minor modification of this, i will bring, pre multiply this by A, therefore i would write this as A y_{k+1} so, i would now have an alternative procedure this or this is equal to v_k. So i have written this as A of this one, remaining steps remain the same.

Now what has become this, this has become a linear system of algebraic equations A, A y is equal to right hand side that is, you can use any one of your Gauss elimination or the LU decomposition or any method. Now since it you are iterating a number of times, the best way is to write this A is L into U, so i can make it L into U is equal to this, is v_k. why? when once you factorize A as L into U, it is done only forward substitution, back substitution for, for each iteration. So we are going to find the decomposition only once, A is equal to L into U and lay for each iteration one forward substitution and one back substitution is to be done, so we can choose any one of these three methods for using the inverse power method and safest would be to use this, if when you are doubt about the round of errors in the particular problem and it was also shown that the inverse power method is most powerful to get the Eigen vector very accurately, that means the Eigen vector in this particular case comes out very accurately, Eigen vectors are computed, is computed very accurately.

Now let us illustrate this also by an example, let us take an example. Find the smallest Eigen value, find the smallest Eigen value and the corresponding Eigen vector of, let me take this as 2, minus 1, 0, minus 1, 2, minus 1, 0, minus 1, 2. Let us also specify number of iterations, use 4 iterations of inverse power method. Assume, let us give the initial vector, again let us take it as 1 1 1.

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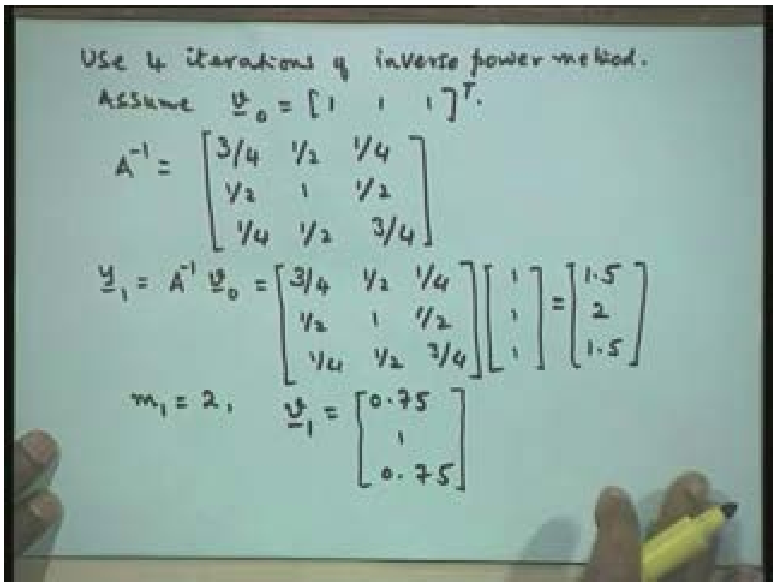

Now let me apply the first method that we have, that is A inverse, let us find A inverse for this matrix and then find it. So i will give the value of A inverse here, value of A inverse is 3 by 4, 1 by 2, 1 by 4, 1 by 2, 1 by 2, 1 by 2, 3 by 4. Now i am applying the power method on this, therefore i will determine y_1 is A inverse v_0, so let us write down the first step, later on i will give the remaining values, 1 upon 4, 1 upon 2, 1 upon 4, 1 upon 2, 3 by 4. Now this i will write it as, i simplify this, 1 point 5, 2, 1 point 5. Now the largest value here is 2, so i will have m_1 is equal to 2, therefore v_1 is 1 point 5 by 2 that is point 7 5, 1, point 7 5.

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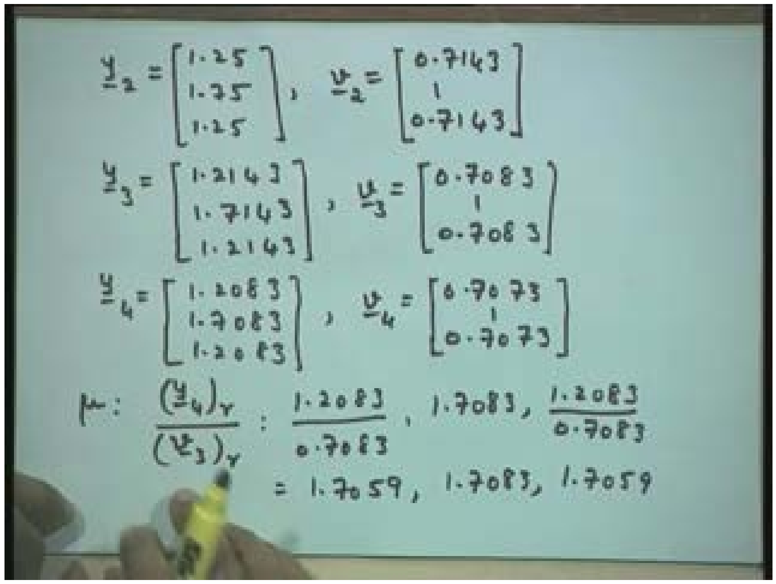

Now i get the remaining iterates, i will give you the remaining iterates, y_2 comes out to be 1 point 2 5, 1 point 7 5, 1 point 2 5 and v_1, v_2 is 1, point 7 1 4, 3 point 7 1 4 3 and y_3 is 1 point 2 1 4 3, 1 point 7 1 4 3, 1 point 2 1 4 3 and v_3 is 1, 0 point 7 8 3, 0 7 8 3 and the fourth iteration gives me this, 1 point 2 0 8 3, 1 point 7 0 8 3, 1 point 2 0 8 3 and v_4 is point 7 0 7 3 point 7 0 7 3. Now i need to find the ratios for this Eigen value mu, which will be the ratios of the components of y_4 and the previous v, that is your v_3 components, so ratio of these components and these are 1 point 2 0 8, 3 by point 7 0 8 3, this divided by this is, 1 point 7 0 8 3 and this is 1 point 2 0 8 3 by point 7 0 8 3 and these values are 1 point 7 0 5 9, 1 point 7 0 8 3, 1 point 7 0 5 9.

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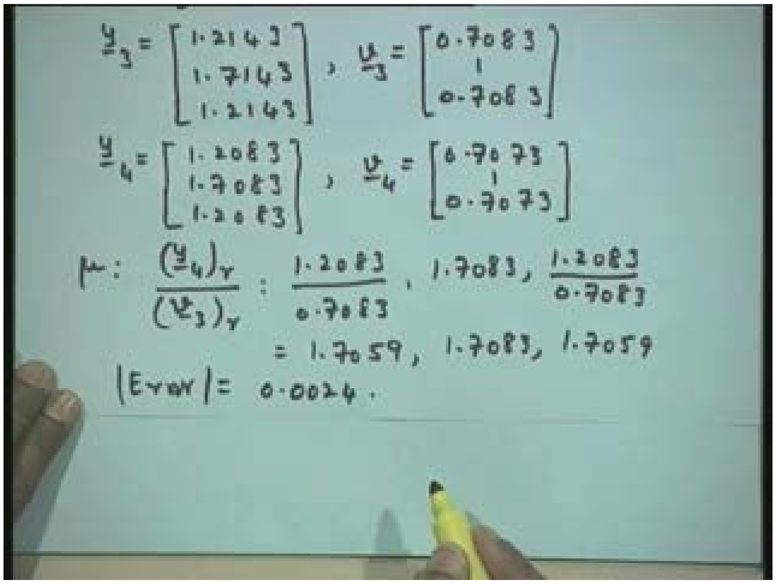

Now we can see the error is point two zeros 2 4, that is your magnitude of error in this case is, subtract these two that is, 0 0 2 4. So the magnitude of error is 2 2 4, so we have got the even 4 iterations, we have got the value quite accurately.

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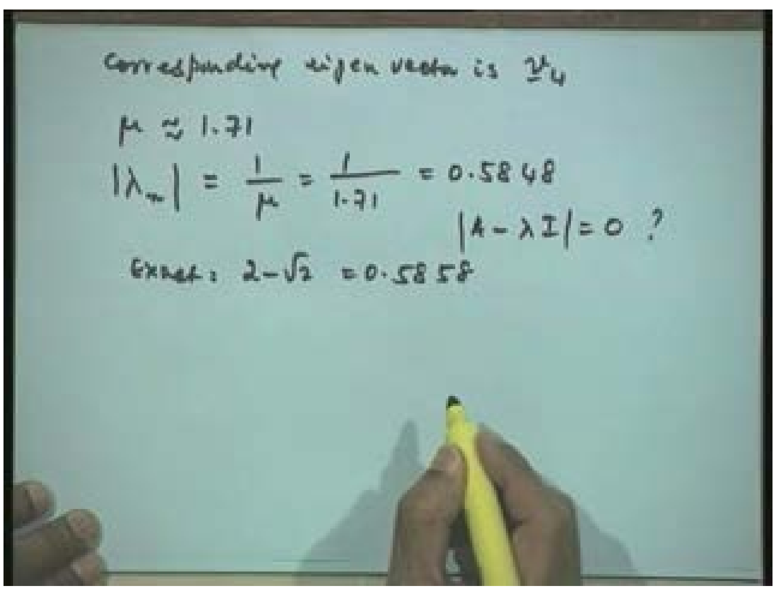

The corresponding Eigen vector is v_4, the corresponding Eigen vector is v_4. Now we have got, let us now take the value, let us approximate it, mu as, say 1 point 7 1, i can approximate it as 1 point 7 1. Then magnitude of lambda, now i want the smallest Eigen value of the original matrix, that is our lambda n, that is equal to 1 upon mu, this is the largest Eigen value of A inverse, therefore this is 1 point, 1 point 7 1 approximately point 5 8 4 8, of course i could have kept all the four decimal places but i just, for the sake of illustration i have just rounded it off to two places. Now if i want to check whether this sign is point 5 8 or not, i must now substitute it in A minus lambda I determinant and see whether this is 0 or not, if this is 0 then this is plus sign if it is not it must be the other sign, that will be negative sign.

Now the exact Eigen value in this problem is 2 minus root 2, that is point 5 8 5 8, this is the Eigen, exact Eigen value for this particular problem. As i said the, the sign must be determined from this, we cannot straight away say that this is the Eigen value; we can only say this is the required Eigen value in magnitude. Now let us, let me close this up with one simple example, which i would give you the, you can complete it, complete it, i will just give this. Let us consider the following example.

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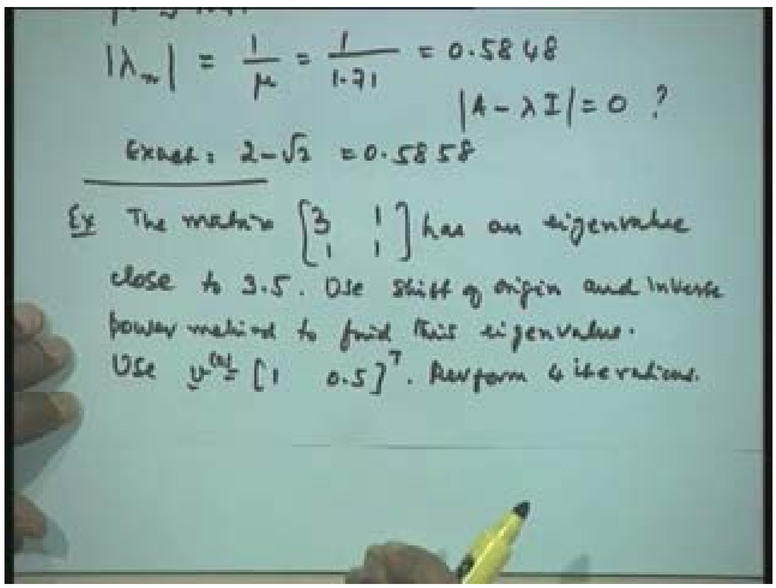

The matrix 3 1 1 1 has an Eigen value close to 3 point 5, has an Eigen value close to this one. Now use shift of origin, shift of origin and power method and inverse power method we need and inverse power method to find this Eigen value, to find this Eigen value. Use v_0 is 1, point 5, yes, perform four iterations. I want to illustrate the shift of the origin, the importance of the shift of the origin in this case; i will give the first few steps which you can complete it.

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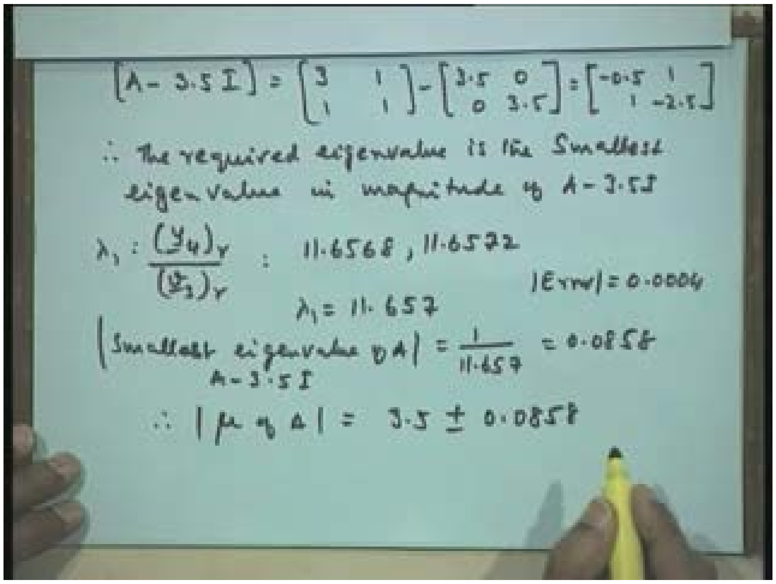

What you really need to find is A minus lambda I, that is A minus 3 point 5 I what is this matrix. This will give you, 3 1 1 1, minus 3 point 5, 0, 3 point 5, which is, which will give us minus point 5, 1, 1, minus 2 point 5. Now the required Eigen value is the smallest Eigen value of this because the Eigen value is close to 3 point 5, therefore the required Eigen value is the smallest Eigen value of this, therefore the required Eigen value is the smallest Eigen value in magnitude, in magnitude of A minus 3 point 5.

Therefore i can find the inverse of this, then go through the previous example that we have done, find the Eigen value and finally you can test whether the Eigen value is the same sign. So i will give you the last step what it gives you, the, the magnitude of lambda 1 comes out to be, y 4 r by v 3 r, these ratios come out to be, it is only 2 by 2 matrix, 6 point 6 5 6 8, 11 point 6 5 7 2, so that the error in this case is three zeros 4, that is three zeros 4, error is three zeros 4. Therefore you can take this Eigen value, lambda 1 as 11 point 6 5 7. This is the dominant Eigen value of this A by inverse therefore, its inverse will be the least Eigen value of A inverse, therefore the smallest Eigen value, let us put magnitude, smallest Eigen value of A is 1 upon lambda minus this, that is equal to point 0 8 5 8. Smallest, yes, yes, yes, therefore our required Eigen value, if you take this as mu of the, of A, smallest Eigen value of A will be 3 point 5, that is the shift that is K, we do not know the sign, it will be plus or minus 0 point 0 8 5 8. We do not know the sign, so will make it as 0 point 5 8, either it is 3 point 5 8 5 8 or if i subtract this 3 point 4 1 4 2.

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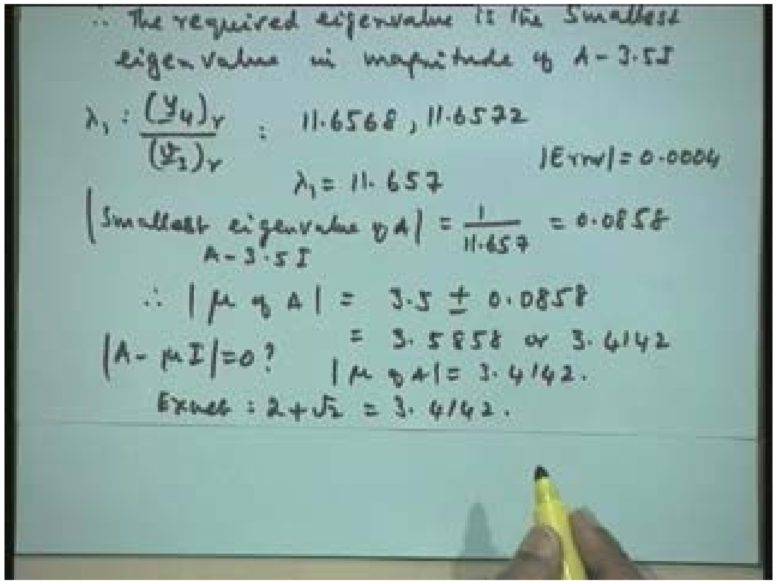

Now, i can now substitute this in A minus lambda I, that means A minus mu i and then check whether this is 0 or not, just evaluate the determinant, evaluate the determinant and it turns out, that the value here is smaller one, 3 point 4 1 4 2. In this case the, the smallest Eigen value is 3 point 4 1 4 2 and the exact Eigen value is 2 plus 2 root 2, that is 3 point 4 1 4 2 is the exact Eigen value. With this we complete the topic of numerical solution of algebraic equations and Eigen value problems and the next lecture we shall start the interpolation and approximation. Okay will stop by this one.