Numerical Methods and Computation

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Lecture - 21
Solution of a System of Linear Algebraic Equations (Continued)
Eigen Value Problems

In the last lecture, we have derived the Gershgorin theorem for finding bounds of Eigen values. We call these bounds as Gershgorin circles. If the matrix is symmetric, then these Gershgorin circles reduce to real intervals. So, we can exactly tell the bounds or the end points in which the Eigen values lie. We would now derive some numerical methods which give all the Eigen values or all the Eigen vectors or if you need a particular Eigen value, that particular Eigen value and the corresponding Eigen vector.

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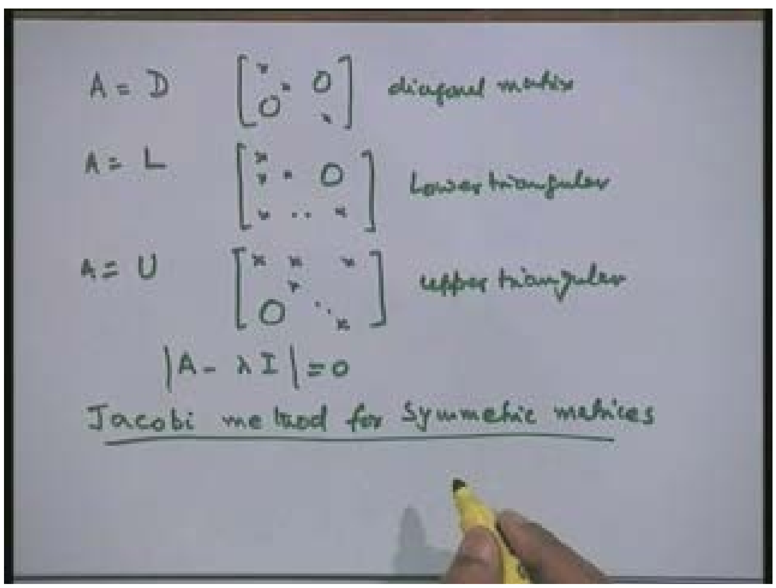

Suppose that the matrix A is one of the three special forms, if A is say a diagonal matrix, of this particular form or if A is a lower triangular matrix or if A is an upper triangular matrix, in these three cases the Eigen values are trivial because all the Eigen values are placed on the diagonal of this particular matrix. We can write down the characteristic equation of this matrix as, A minus lambda I determinant is equal to 0. It would immediately tell us that the Eigen values are placed on the diagonal, in all these three cases. Therefore, most of the numerical methods would attempt to reduce the given matrix using the similarity transformations to one of these forms or some other forms are also possible. One particular method that we have is called the Jacobi method, Jacobi method for symmetric matrices. The discovery of this method actually started with the problems that arose from the mathematical models for the physical problems.

In most of the cases the differential equations, ordinary or partial differential equations, the Eigen value problems there, had given rise to symmetric systems and have some special properties. Therefore the original methods that were discovered were always for the symmetric matrices with some special properties, which later on when the problems are much more generalized, the symmetry was lost and we have general matrices. We have methods also for general matrices. But even today most of the practical problems which we have and the modern mathematical models do give rise to symmetric systems. We know that, we have earlier discussed some other properties of symmetric matrices; one property is that, all the Eigen values are real.

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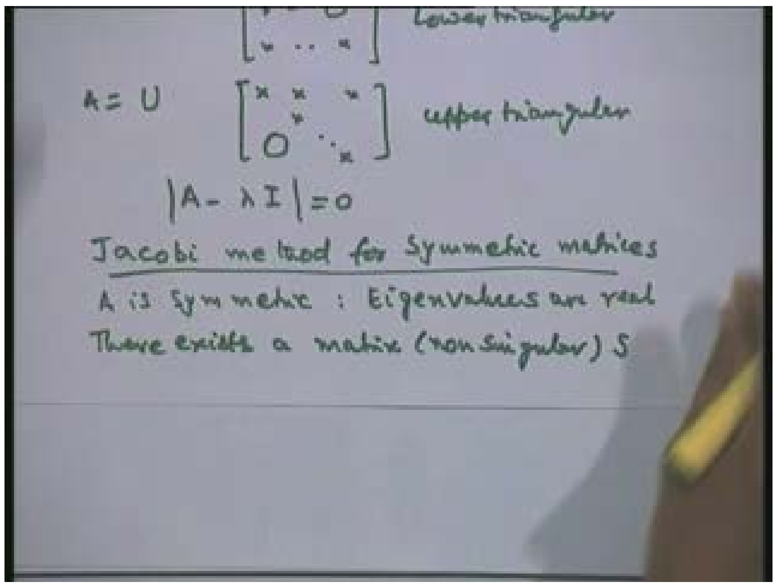

If A is symmetric, if A is symmetric then Eigen values are real. Secondly we have mentioned that there exists a similarity matrix, which reduces A to a diagonal form. Therefore there exists a matrix, there exists a matrix which is non-singular, will call it as S.

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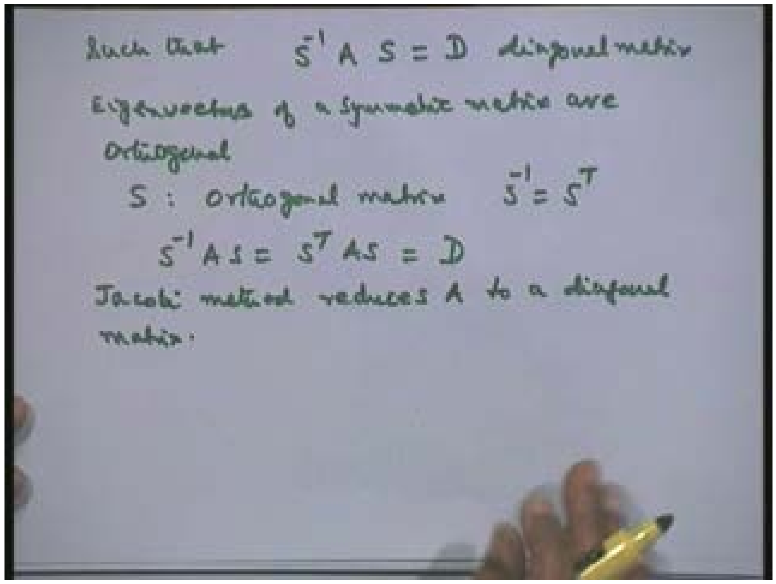

There exists a matrix S such that, S inverse A S is equal to D, D is our diagonal matrix. Therefore, all the Eigen values are now located on the diagonal of this matrix D. Now there is another property of the Eigen vectors of symmetric matrices; the Eigen vectors of symmetric matrices are all orthogonal. The Eigen vectors of a symmetric matrix that means, corresponding to the Eigen values that we have, they are orthogonal, they are orthogonal. Therefore the matrix that we are constructing, we are talking about this non-singular matrix which is now reduced to the diagonal form, is a matrix which is an orthogonal matrix. Therefore, this matrix that we are talking here is an orthogonal matrix that means, S inverse is equal to S transpose, so we will have S inverse is equal to S transpose. Therefore this similarity transformation that brings A to D, will be S inverse A S, same as S transpose A S and that will be equal to D. This is the fundamental property of the Eigen values of a symmetric matrix.

Now the Jacobi method attempts to reduce the matrix A to a diagonal form, so that is the Jacobi method. Jacobi method reduces A to a diagonal form, reduces to diagonal matrix. Now how it reduces to a diagonal matrix is, we know that by applying a similarity transformation, using an orthogonal matrix finally reduces A to D. Therefore it attempts to find a series of similarity transformations in a plane, which is with the orthogonal rotations.

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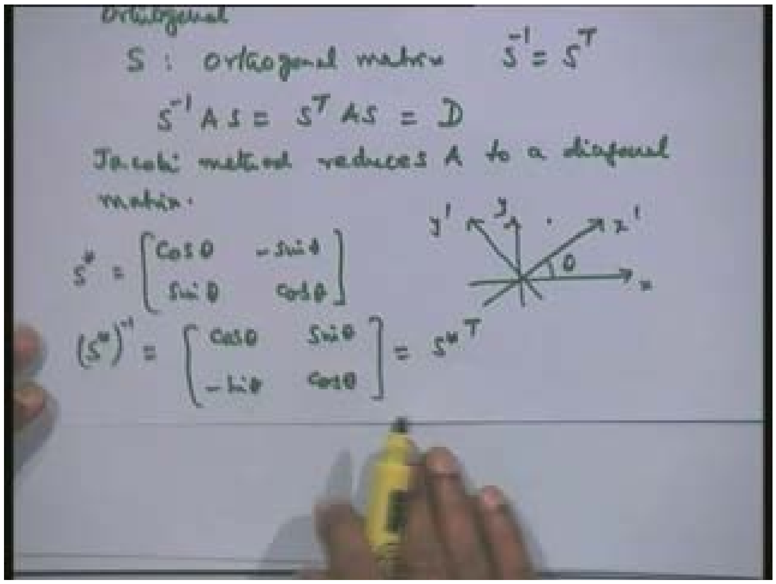

Take a two dimensional coordinate system x y, then rotate it by an angle of theta. If i take the new coordinate system as x dash y dash, original system is x y, new system is x prime y prime, the angle of rotation is theta, then we know that the matrix of transformation that reduces is given by, cos theta minus sin theta sin theta cos theta. Therefore, if you now take any point in the x y, in this plane, then i can connect the coordinates of this point with respect to these two systems through this matrix, which is our rotational matrix. Now we can see that, this matrix is a orthogonal matrix, because we can try to find the inverse of this. If i take this as sum S star, let me put as S star then, S star inverse is equal to cos square theta plus sin square theta, that determinant is 1. Therefore i have cosine theta over here, minus sin theta here, sin theta here, and cos theta over here, this is S star of transpose. Therefore this is an orthogonal matrix that means, this is an orthogonal rotation. Therefore the Jacobi method uses the orthogonal rotations in a plane to reduce the matrix to diagonal form.

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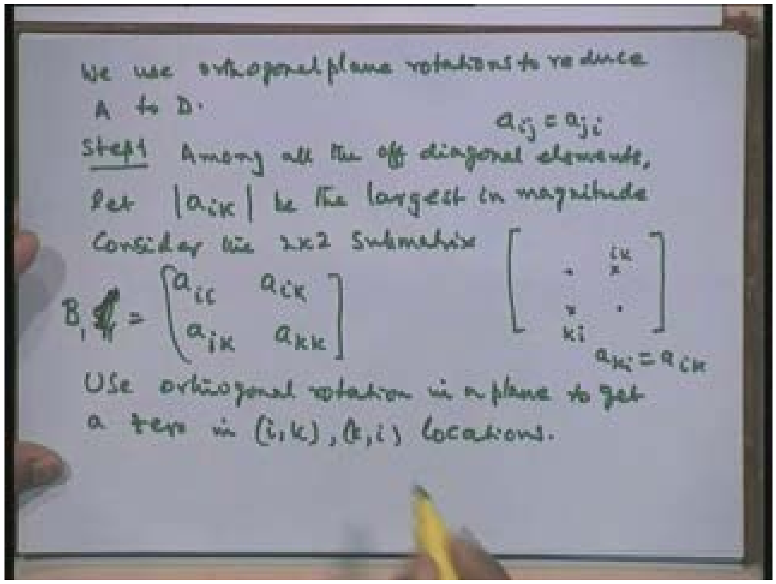

Therefore we can say that, we use orthogonal plane rotations to reduce A to D. Now let us see how to do it. It is to be done in two steps, let us see what is the first step, let us take the first step, now remember we are talking over symmetric matrix, therefore we know a_{ij} is equal to a_{ji}, we shall use this property as we go along. We search, first of all for the largest off diagonal element, that means locate largest off diagonal element, so let us say among, among all the off diagonal elements, let us speak the largest magnitude. So let a_{ik} be the largest in magnitude, so if i take this matrix a_{ij}, now it is symmetric, therefore, if this is the largest element in magnitude, this largest element in magnitude is also located here, because of symmetry. Therefore we have a_{ki} is equal to a_{ik} and these will be the diagonal elements, i have put with a small dot, so this is the location of these four. Then what we do is, we consider this 2 by 2 sub matrix, now consider the 2 into 2 sub matrix, this location is i i, this is a_{ik}, this is a_{ki}, we can again put it as a_{ik} and this is equal to a_{kk}. Now i consider this sub matrix and reduce this element a_{ik} to 0 by using the orthogonal rotation in the plane. Now use orthogonal rotation, let us put some s_1 star, so that we can use it, use orthogonal rotation in a plane to get a 0 in (i, k) and it would mean (k, i) locations. I think let us change, we shall use S_1 for the rotation matrix, let us call this as B_1, i will call it as B_1.

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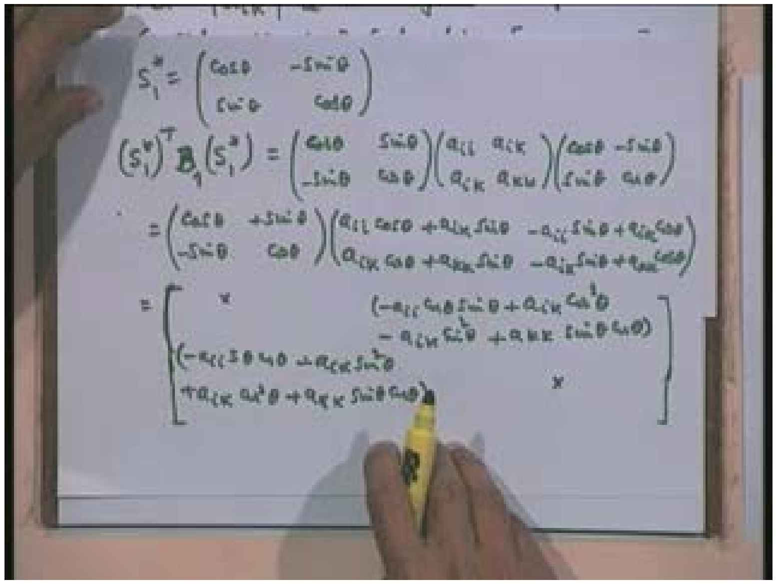

Now let us take the orthogonal matrix in the two dimensional plane, which gives the required rotation as S_1 star is equal to cos theta, minus sin theta, sin theta, cos theta. Now the problem is to find the value of theta such that, the similarity transformation produce a 0 in the (i, k) and (k, i) location, so i want to find the value of theta. Now what is the similarity transformation, similarity transformation is S_1 inverse, here it is transpose A S star and here we are applying it on B S_1 star transpose B_1 S_1 star. Now this will be cos of theta, i am taking the transpose of this, minus sin of theta, this is sin of theta, this is cos of theta that is the transpose of this matrix multiplied by B_1, $a_{ii,}$ $a_{ik,}$ $a_{ik,}$ a_{kk} into S_1 star cos theta, minus sin theta, sin theta, cos theta. Now let us multiply it out, this is cos of theta, sin theta, minus sin theta, cos theta. Now this is a_{ii} cos theta plus a_{ik} sin theta, this is a_{ii} cos theta plus a_{ik} sin theta and this element is minus a_{ii} sin theta plus a_{ik} cos theta and correspondingly this is a_{ik} cos of theta plus a_{kk} sin theta and this is minus a_{ik} sin theta plus a_{kk} cos of theta.

Now since my effort is to determine theta such that, the off diagonal element that is location, this location, this location is 0, i do not bother about what this element is, i shall be interested in what is this element and what is this element. Now this element would be, i multiply the first row by the second column, so i would have here minus a_{ii} cosine theta sin theta plus a_{ik} cos square theta, i multiply the cos theta by this particular element, a_{ii} sin theta cos theta a_{ik} cos square theta then i multiply this by sin theta, so this will be minus a_{ik} sin theta, sin square theta that is, sin theta into sin theta plus a_{kk} sin theta cos theta. Now let us put a small bracket here, so that will have this. Now let us write down this element, this element, because of symmetry it has to be the same, because we started with the symmetric matrix therefore, this will also be the symmetric part. Now we can see that, if i multiply it out, i will get minus a_{ii} sin theta cos theta minus a_{ik} sin square theta, i am multiplying this minus sin theta with this and cos theta plus a_{ik} cos square theta plus a_{kk} sin theta cos theta. As i mentioned, because of symmetry these two elements would be identically be the same.

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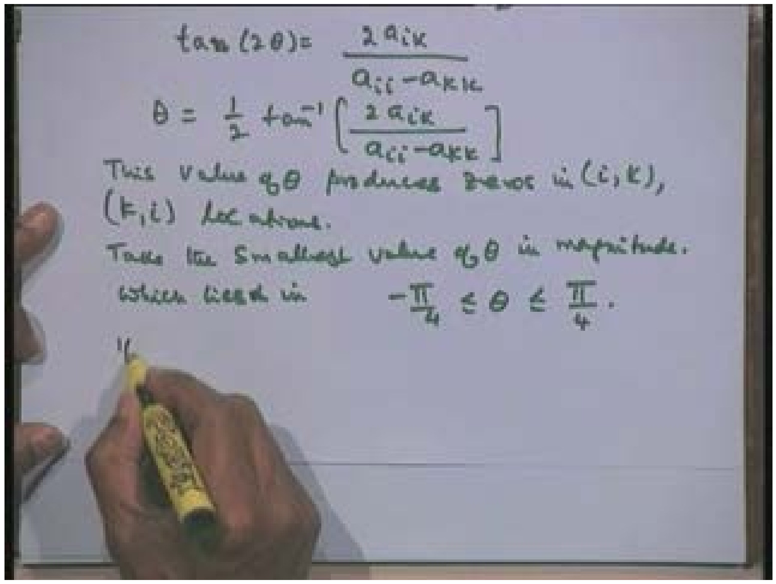

Now the problem is, i want to determine the theta such that, we get location 0 in the location of this (i, k) and (k, i), therefore i will set this as 0 and determine theta. Therefore, that means, set this as 0, so let us simplify this, a_{ik} this is cos square theta minus sin square theta that is cosine of 2 theta plus, these two combine a_{kk} minus a_{ii} sin theta cos theta, therefore i will write it as half of sin 2 theta is equal to 0.

Let us now solve for theta, so i can take this to the right hand side and write this as tan of 2 theta, tan of 2 theta is equal to 2 times a_{ik} and i will change the sign of this, a_{ii} minus a_{kk} or theta is equal to half tan inverse 2 times a_{ik} by a_{ii} minus a_{kk}. This value of theta will produce zeros in (i, k) (k, i) location, so this value of theta produces zeros in (i, k) and (k, i) locations.

Now tan two theta has got a number of solutions for this, more than one solution is available for this, but we would like to choose that value of theta which gives us the smallest rotation of all possible, that means we would like to take the value of theta lying between minus pi by 4 to plus pi by 4. So take the smallest value of theta in magnitude, smallest value of theta in magnitude, which lies in minus pi by 4 less than or equal to theta less than or equal to pi by 4. So it has got, actually four values it will have, of which will have the smallest rotation possible, because we are going to do rotation of rotation, so that we bring the entire matrix to the diagonal matrix, so we would like to choose the smallest rotation in the pair, that means it essentially would imply that in a plane we are rotating the given system, again rotating the system or may be in the positive negative direction, we are rotating the system a number of times so that A is reduced to D.

Now you can see some typical particular cases, if a_{ii} is equal to a_{kk} the denominator is going to be 0, therefore this will be infinity, therefore tan infinity will be pi by 2, so i will have the extreme values in those cases.

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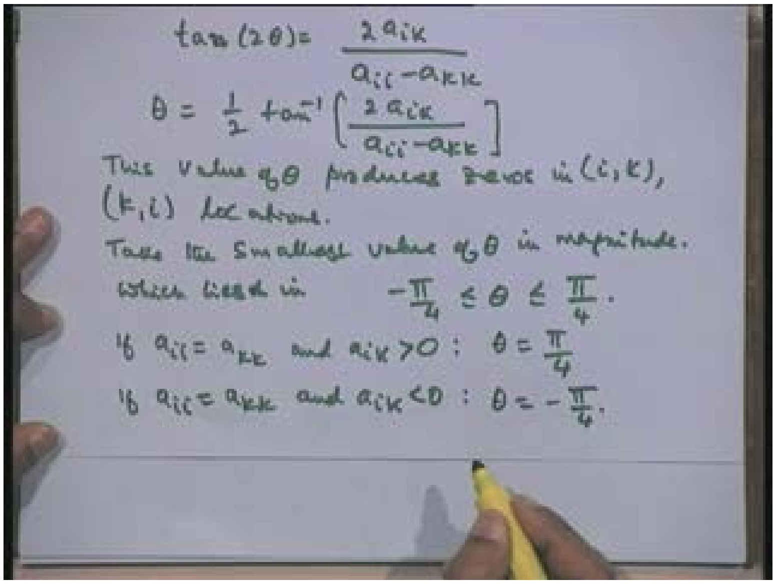

So i have here, if a_{ii} is equal to a_{kk} and a_{ik} is positive, a_{ik} is positive then, this will give you plus infinity. If a_{ik} is positive, denominator is 0, it will give plus infinity. Therefore tan inverse plus infinity will be plus pi by 2, so i will have theta is equal to pi by 4. Whereas, if a_{ii} is equal to a_{kk} but a_{ik} is negative, a_{ik} is negative then, this is a negative by 0, this is minus infinity, therefore tan inverse would be minus pi by 4, so theta will be minus pi by 2 by half that is, minus pi by 4. Therefore, if in a particular case, we are fortunate to have the diagonal elements at that time to be equal, then they are going to cancel, then rotation by an angle pi by 4.

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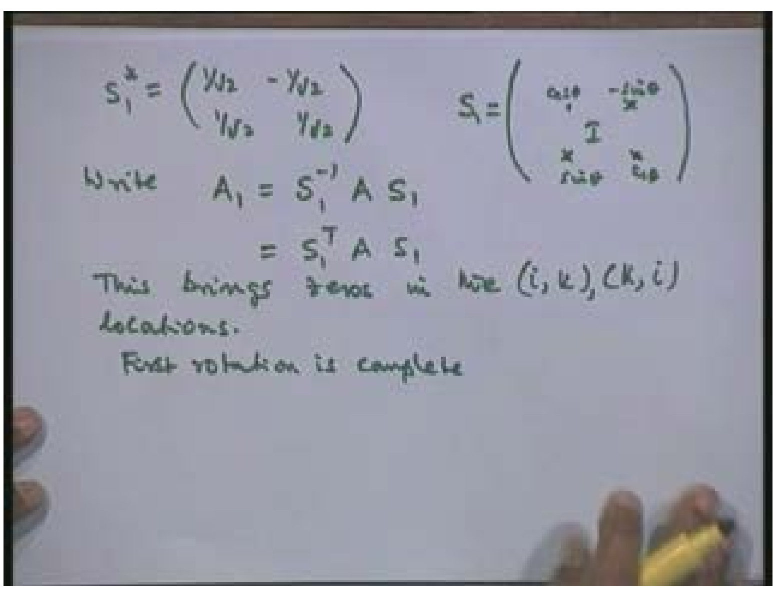

And the matrix S_1 star would look very nice because, in that case S_1 star will be simply cos of pi by 4 1 by root 2, sin of pi by 4 1 by root 2, 1 by root 2, 1 by root 2. Rotation matrix looks very simple for our computational purposes. Now, once the value of theta is determined, we would like to use the rotation on the whole matrix. Let us write A_1 is equal to S_1 inverse A S_1, what is this S_1 matrix will be, the S_1 matrix will be, all will be I, except these four locations. So the location of (i, k) is here, diagonal element is here, (k, i) is here and your (k, k) element is here. So we will locate cos theta here, we will locate minus sin theta here, sin theta here, cos theta here and I in the remaining part. So that means, we are now absorbing this four elements to the correct locations because, when we stared it, these are the four locations we have chosen, that is the sub matrix a_{ii}, (i, k) (k, i) (k, k) locations, therefore in these 4 locations, these 4 values will go, otherwise the entire matrix will be simply I. This will now be applied on the matrix A and this would of course bring your 0 in the (k, i) (i, k) locations. Now if i perform this, that is your S_1 transpose A S_1, now this brings zeros, this brings zeros in the (i, k) (k, i) locations, this brings zeros in the (i, k) (k, i) locations. Now one rotation is complete, so the first rotation is complete, first rotation is complete.

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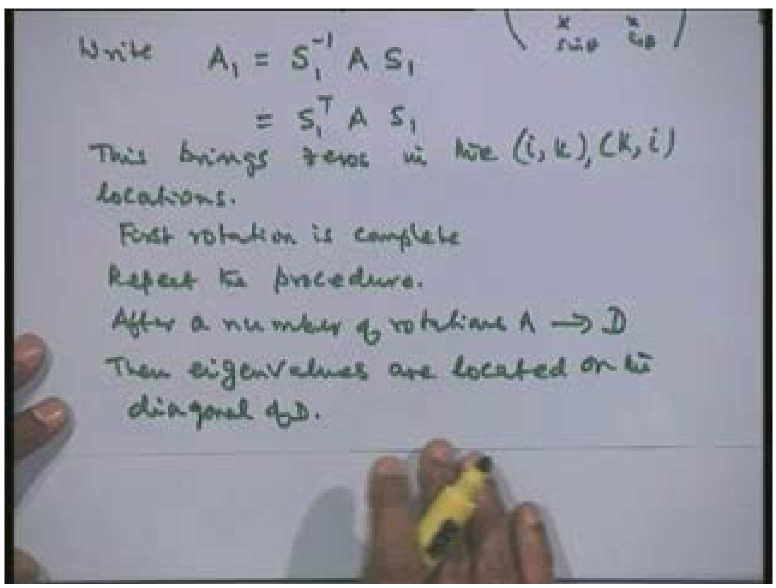

Now repeat the procedure, repeat the procedure, what we mean by repeat the procedure is that, we shall again search for the last search off diagonal element in A_1 and now repeat the whole procedure that we have done. Find the new value of theta which produces 0 in those off diagonal elements and then complete the rotation like this. After a number of rotations, A goes to D, then Eigen values are located on the diagonal of D, on the diagonal of D.

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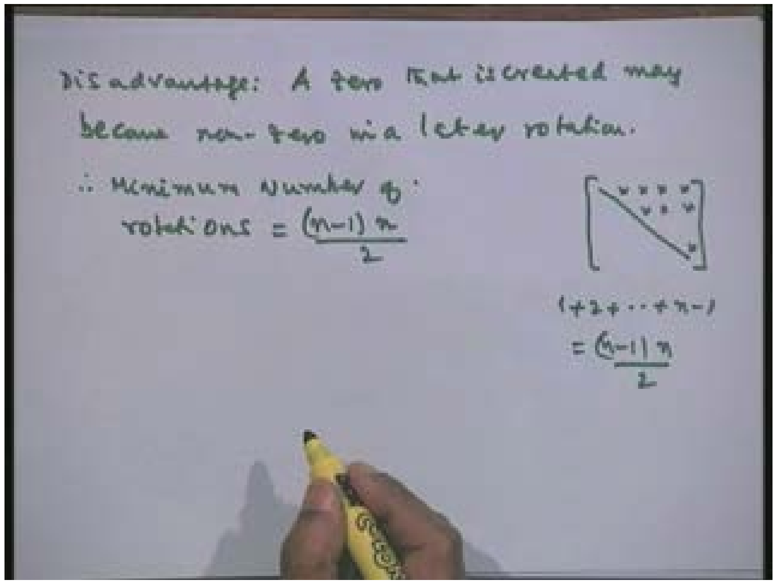

Now, before we make a comment on the number of rotations that is required and what are the Eigen vectors on the system, let us make a comment here, that is about the disadvantage of the Jacobi method, if that disadvantage was not there, then this Jacobi method would have been the best method and we do not need any other numerical methods. The disadvantage in this method is, the 0 that is created once can be disturbed or may be disturbed in the later rotations, that means if i have produced 0, say location (9, 10) and (10, 9) location and if i go to the further rotation, this 0 can be disturbed and it can become a nonzero quantity, that means at a later stage we again have to make this number that has been produced here as a 0 again, that means the number of rotations will not be exact, will not be able to predict, what will be the number of rotations that is required. Therefore, we can only state how many minimum number of rotations that will be required for reducing A to D, therefore this is the main disadvantage. A 0 that is created may become nonzero in a later rotation, therefore a 0 that is created may become nonzero in a later rotation. Therefore, we can only say the minimum number of rotations, minimum number of rotation, now let us see what is the number of elements that will have to bring them to 0. We are now on the diagonal, now because of symmetry, the a_{ij} elements is equal to a_{ji} elements, therefore i need to bring these n minus 1 elements to 0, then i have to bring n minus 2 elements to 0 and finally 1 element to 0. So the number of elements that is to be made to be 0 or let us count from bottom, 1 plus 2 plus 3 so on n minus 1. So this is the number of elements that has to be made as 0, therefore the number of rotations there by will be n minus 1 into n by 2, so this will be the minimum number of rotations, that will be required will be n minus 1 into n by 2. But of course, if you are doing the exact arithmetic by hand, that means do not convert theta into the decimal form and if you are able to write exactly the values of theta then, it is possible for us to retain these zeros that is created. Now let us find out what is the Eigen vector, when we know that, yes i need these many number of rotations and i can now find out all the Eigen value and then let us see what is Eigen vectors, now let us just repeat what we have done in the first rotation.

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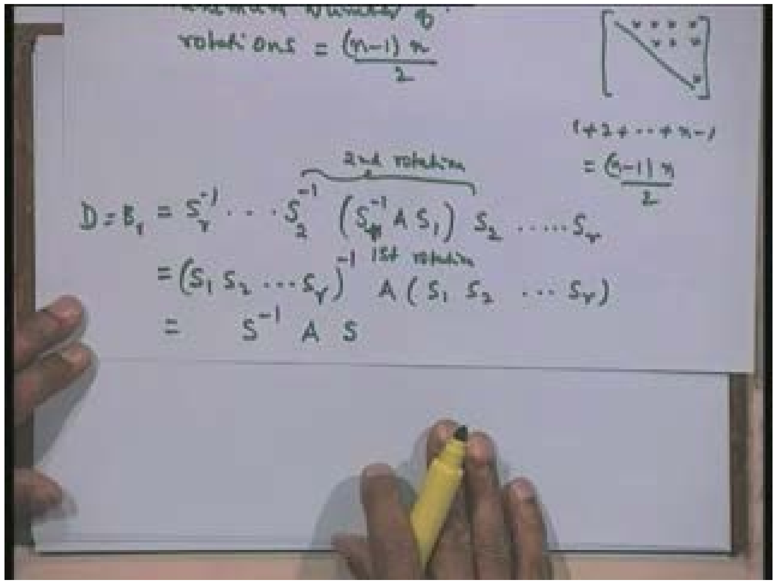

I will start from the middle of this line, we have S_1 inverse A S_1, this is the first rotation, then i have applied another similarity transformation S_2 inverse S_2, this is my second rotation, this is my second rotation so on. Now applying, we know that the minimum number of rotations is this, therefore let us say we need some r rotations, so i can make this as r, i can make this as S_r inverse and this is finally our matrix B_r and that is your D. Therefore this is the first rotation, the next rotation and finally the r^{th} rotation that we needed to reduce the matrix A to the diagonal form D, this is how sequence of rotations looks like. Now let us simplify this, this i can write, let us first of all write down the right hand side, this is S_1 S_2 S_r, so i will take this as S_1 S_2 S_r. If you look at this left hand side, this is S_r inverse S_r minus one inverse S_2 inverse S_1 inverse that is the inverse of this this is your S_1 S_2 S_r inverse. The inverse of this is nothing but this inverse and this is, i will know call this as S inverse AS.

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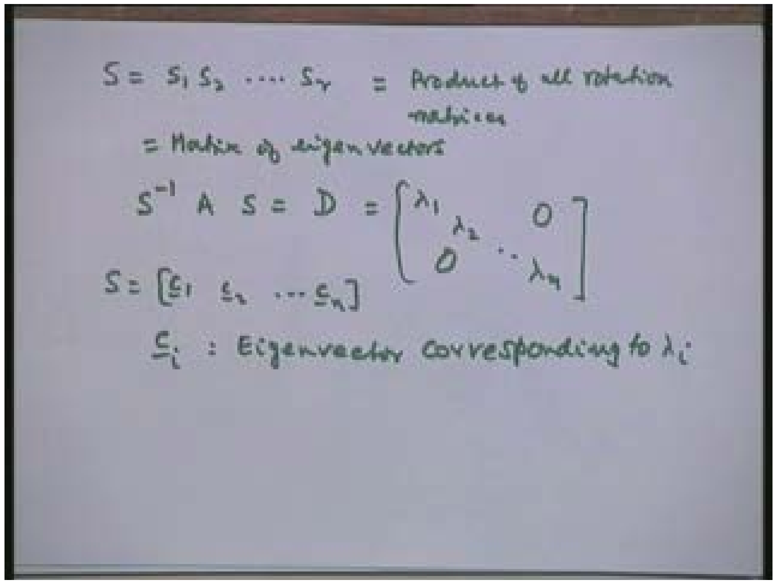

Therefore where our S is equal to S_1 S_2 so on S_r and this is the product of all rotation matrixes that we have used, this is the product of all rotation matrixes. Now i have got the matrix, final matrix, product of all matrixes, this is the matrix of Eigen vectors and this is the matrix of Eigen vectors. We mentioned earlier, if you remember the earlier discussion, we mentioned that if a matrix A is reduced to a diagonal matrix then there is 1 to 1 correspondence between the Eigen values located on the diagonal of D and the columns of S, therefore when once i get the matrix of Eigens vectors then the first column will be the Eigen vector corresponding to the first Eigen value, let us put this as lambda 1, lambda 2 so on lambda n and let us write S is equal to column 1 vector, column 2 vector, column n vector. So let us write S as the column vector C_1, C_2, c_n as the n column vectors of the S, then c_i will be the Eigen vector corresponding to lambda i, then C_i is the Eigen vector corresponding to lambda i. Therefore for finding the Eigen vector we do not have to do anything extra, expect that multiply this matrices S_1 S_2 S_r matrices and then take the columns as the corresponding Eigen vectors and they would almost be in the required form of normalized form also and we will have the Eigen vectors available for us automatically.

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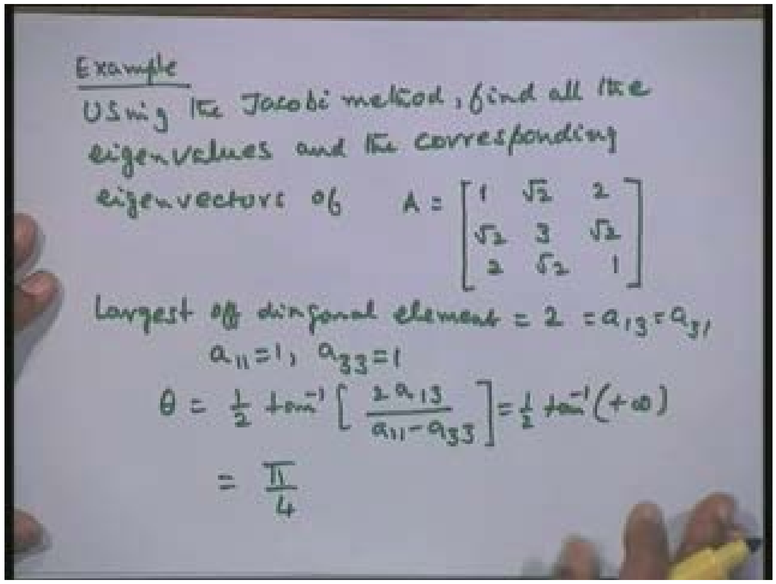

Now let us take a simple example on this, now i will take a very simple example which will do by exact the arithmetic and will also show that, even though we said you need n minus 1 into n by 2 number of rotations, if you are, if the matrix is such that it is, you are doing exact arithmetic you may get its required form earlier than the number of rotations also, so let us take this very simple trivial example. Using the Jacobi method, using the Jacobi method find all the Eigen values, find all the Eigen values and the corresponding Eigen vectors of, let us take A is 1, root 2, 2, root 2, 3, root 2, 2, root 2, 1.

Now first of all i need to locate the largest off diagonal element, the largest element in magnitude is 2 here, so will have here the largest off diagonal element, now here all are positive numbers, so we don't have say magnitude but otherwise, will have to take in the magnitude. Largest off diagonal element is equal to 2 and this is your a_{13} is equal to a_{31}, a_{13} is equal to a_{31} and the corresponding diagonal elements are a_{11} is 1, a_{33} is equal to 1 so with respect to this element, i am now taking the sub matrix that is your diagonal elements are 1 and 1 and this is 2. Now i find theta, theta is equal to half tan inverse 2 times a_{13} by a_{11} minus a_{33} and this is half tan inverse of plus infinity, a_{11} is equal to a_{33} here and a_{13} is positive, therefore plus infinity, therefore the value of theta is phi by 4.

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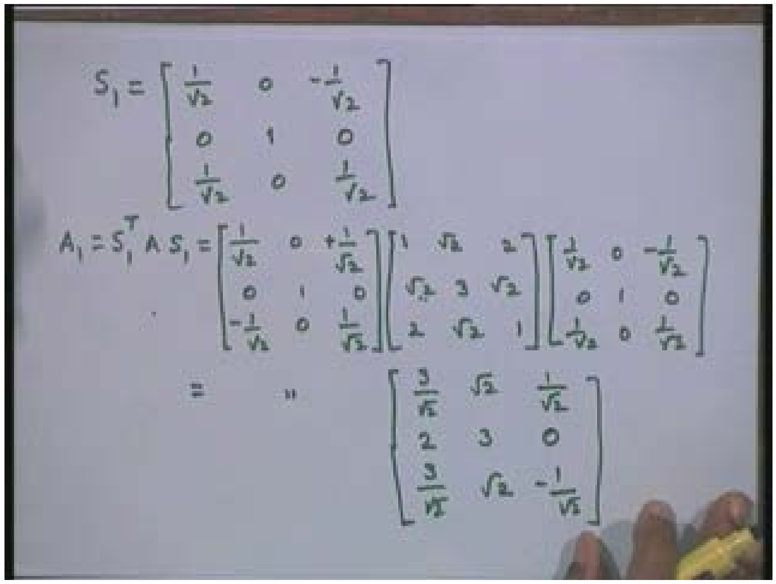

Now i will write down the rotation matrix, first matrix S_1, we said S_1 will be I, matrix I, except the elements corresponding to this four elements, the four elements are this this this and this, therefore cos theta will come here, minus sin theta here, sin theta here and cos theta here, therefore our S_1 will be cos of theta 1 upon root 2, 0, minus 1 upon root 2, 0, 1, 0, 1 upon root 2, 0, 1 upon root 2, therefore this is the rotation matrix that we need. Now we need A_1, A_1 is S_1 inverse that is S_1 transpose AS_1, so let us write this 1 upon root 2, 0, this is, i am taking the transpose so this comes here, this is 1 upon root 2, 0, 1, 0, 1 upon root 2, 1 upon root 2. The matrix A is 1, root 2, 2, root 2, 3, root 2, 2, root 2, 1 then we have got S_1 that is your, 1 upon root 2, 0, minus 1 upon root 2, 0, 1, 0, 1 upon root 2, 1 upon root 2. Now let us retain the matrix, this matrix as it is, let us multiply these two matrices. So i will have 1 into 1 upon root 2 that stays as it is, this is 0, this is 2 times 2 by root 2 so that will be 3 divided by root 2, then 1 into 0, root 2, 2 into 0, so i will have simply root 2 here, then minus 1 upon root 2, this is 0, this is plus 2 upon root 2, so i will have here 1 upon root 2 here and second row i take, this is 1 upon root 2 into root 2 that is 1, 0, this is 1, that is equal to 2. Now the middle row 0, 1, 0, so this multiplies 3, then this is minus 1, 0, plus 1, so will have 0, then 2 by root 2, 0, 1 by root 2 that is 3 by root 2, then this is the middle element, that is root 2, this is minus 2 by root 2 and plus 1 upon root 2, so i will have here minus 1 upon root 2.

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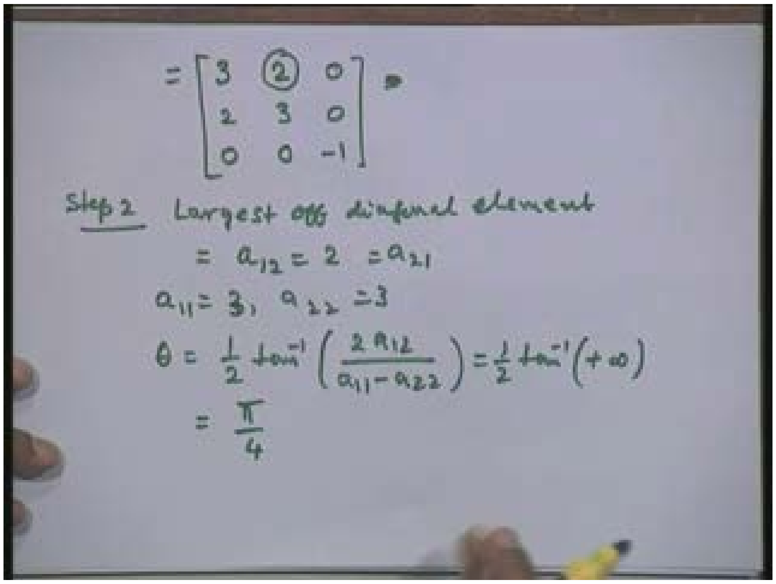

Now let us multiply the whole thing, now i multiplying this row with this column, so will have here 3 by 2 into 0 plus 3 by 2, so i will have 3 by 2 plus 3 by 2 that is 3, then 1 upon root 2 into root 2 that is 1, this is 0 this is 1, that gives me 2, 1 upon root 2 1 upon root 2 that is 1 by 2, 0, minus 1 by 2 so that gives me 0, then middle row 0, 1, 0 therefore it will be middle row comes at 2 0, 2 3 0, this middle row and this one, so we will have simply this and the third row this is minus 3 by 2, 0, plus 3 by 2 so have a 0 here, this is minus 1, this is 0, this is plus 1, so have a 0 here, then this is minus 1 by 2, 0, minus 1 by 2, minus 1 by 2 minus 1 by 2, that gives me minus 1. Now, in one rotation we started with a, trying to get the 0 in these two locations but we now got an extra 0 also here, these two also have become 0 now. Now i need to make only one more element as 0, i know the largest off diagonal element is 2, so that is my step two, step two is now i will take the largest off diagonal element, largest off diagonal element is equal to a_{12} location that is 2 and a_{21}. Now the corresponding diagonal elements are a_{11} and a_{22}, so the corresponding diagonal elements are a_{11} is 3 and a_{22} is equal to 3. I again determine my value of theta from here, theta will be equal to half tan inverse two times a_{12} by a_{11} minus a_{22}, again this is plus infinity, a_{12} is positive therefore this is again plus infinity, therefore have the value as pi by four again. Now the values of cos theta sin theta would lie in this four locations, we made, we are making a_{12} location as 0, therefore cos theta will be here, minus sin theta, sin theta and cos theta will be located here.

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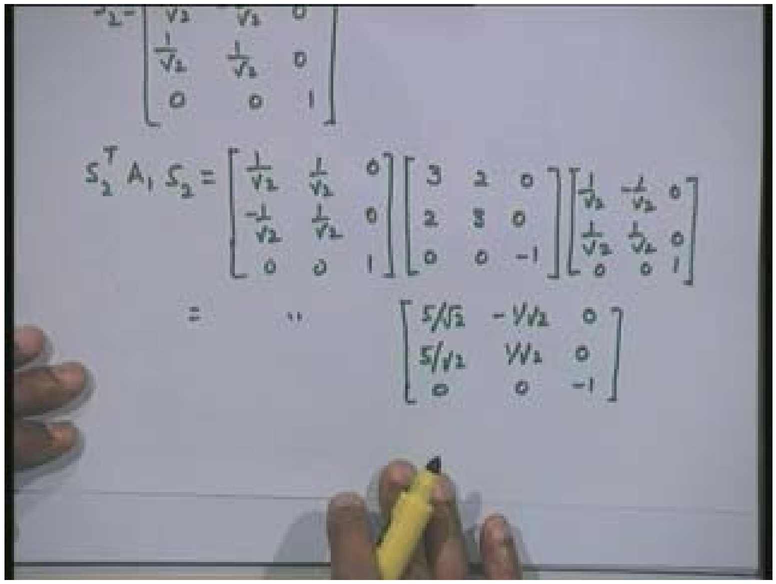

Therefore i can write down matrix S_2 will be cos theta, minus sin theta, sin theta, cos theta, remaining is all I, so remaining is all I. Now we perform our rotation with respect to this new matrix, therefore S_2 transpose A_1 S_2, therefore lets write it as 1 upon root 2, minus 1 upon root 2, 0, 1 upon root 2, 1 upon root 2, 0, 0, 0, 1, that is our inverse and A_1 is 3, 2, 0, 2, 3, 0, 0, 0, minus 1 and S_2 is 1 upon root 2, minus 1 upon root 2, 0, 1 upon root 2, 1 upon root 2, 0, 0, 0, 1. Now let us multiply it out this matrix again, therefore this gives us 3 by root 2 plus 2 by root 2 that is 5 by root 2, 5 by root 2, this is minus 3 by root 2 plus 2 by root 2 that is minus 1 upon root 2, this is 0 this is 0 this is 0, so have a 0 here, again 2 into 1 upon root 2 plus 3 by root 2 that is again 5 by root 2, this is minus 2 by root 2 and this is plus 3 by root 2, so i will have plus 1 upon root 2 and this is again 0. 0 0 minus 1, therefore it will be 0 0 minus 1.

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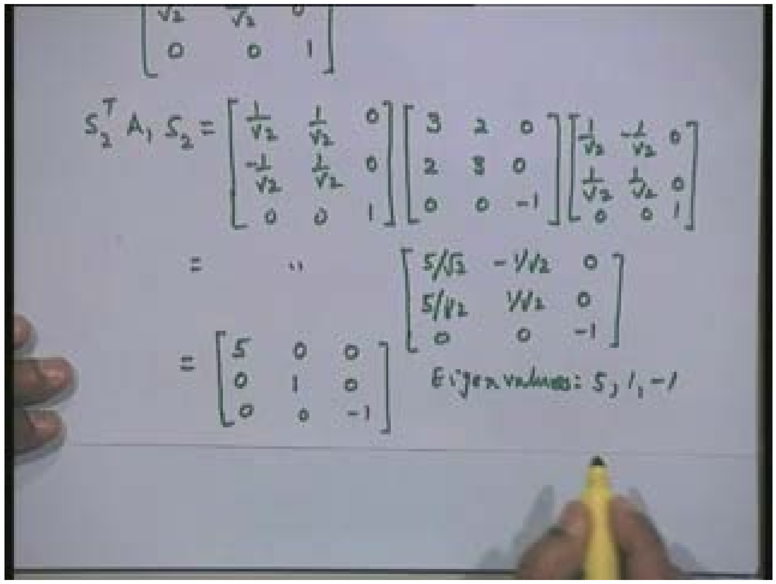

Now let us write it here itself, now i am multiplying this row and this column, this is 5 by 2, this is 5 by 2 so i will have here 5, this is minus 1 by 2, this is plus 1 by 2, so i have 0 here and this is again 0. Now second row, these are opposite signs, they are equal, opposite sign so i will again produce a 0 here, this is plus 1 by 2 plus 1 by 2, so that will give me 1, this is 0, 0 0 minus 1, therefore we have now finally produced the diagonal matrix, therefore here we can immediately write down that our Eigen values or 5 1 and minus 1, these are the Eigen values of our system.

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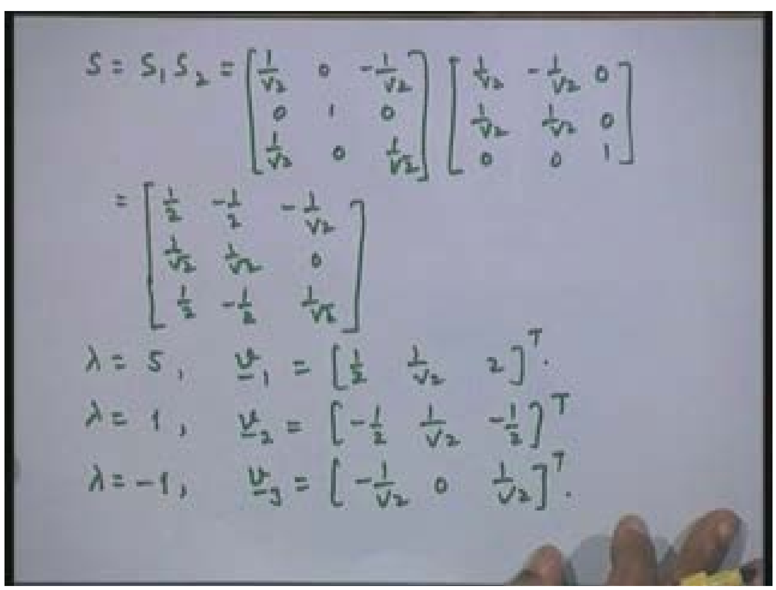

Now i need Eigen vectors, we have S is equal to S_1 S_2, i need to find this product. Now if you just go back and write down what is our S_1 from here, these are the four locations in which these are 1 upon root 2 was located, so let us write this as 1 upon root 2, 0, minus 1 upon root 2, 0, 1, 0, 1 upon root 2, 0, 1 upon root 2 so this was our S_1. Now we can write down the value of S_2 that obtained earlier as, this is 1 upon root 2, minus 1 upon root 2, 0, 1 upon root 2, 1 upon root 2, 0, 0, 1. Now multiply it out, i will have this as 1 by 2 and this is 0, this is 0, so i will have here 1 by 2, this is minus 1 by 2 because this is 0, multiply this is also 0, this 1 and this is minus 1 upon root 2, this row and last column, so middle row it will repeat again so 1 upon root 2, 1 upon root 2, 0, the last row gives us 1 upon 2, these are all zeros, so this is 1 upon 2, this is minus 1 upon 2, 0 0 so minus 1 upon 2 and lastly 1 upon root 2.

we said that the correspondence of the Eigen values and Eigen vectors is the first Eigen value; first column, second Eigen value; second column or the corresponding Eigen vectors, therefore lambda is equal to 5 and lets call V_1 as our Eigen vector, that is your half 1 upon root 2, 2 as the Eigen vector. Lambda second Eigen value was 1, therefore the second column C_2 is equal to minus 1 upon 2, 1 upon root 2, minus 1 upon root 2 as the Eigen vector and the third Eigen value minus 1 is the third column here V_3 is, minus 1 upon root 2, 0, 1 upon root 2. Therefore these are the corresponding Eigen vectors and the Eigen values are located on the diagonal of this and we have this particular, this one. Now in practice what really happens is, when we are finding the value of theta on the computer, the value of theta here we have made the exact arithmetic, cos theta is pi by 4, you retained as pi by 4, cos theta is 1 upon root 2 we retained it, but when once you write it in the format, in the decimal numbers and go to the computer some of these numbers get disturbed, therefore the locations that were zeros will be destroyed and will have non 0 numbers and those nonzero number have again to be made zeros therefore the number of rotations there by increases. Even if i disturb the elements of the matrix slightly and ask you to do it, you will find that you may need about half a dozen iterations or so to reduce into this particular form to a given accuracy, because a computer is going to stop when the off diagonal elements are of accuracy of say 10 to the power of minus 6 and a minus 7 or so, so the number of rotation that will be required depend on the accuracy of the particular problem and thereby the amount of time also computed time also depends on that. Therefore in advance we will not be able to say that the Jacobi method will take this many rotations or this much computed time to solve this particular problem, but it can definitely give us the solution. If you have, if you can give the sufficient amount of time for it, yes Jacobi method will always give us the solution, all the Eigen values and the Eigen vectors. okay will stop it here.