Numerical Methods and Computation
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Lecture - 17
Solution of a System of Linear Algebraic Equations (Contd…)

In the previous lecture we have given a brief introduction on the iteration methods for solving the system of equations A x is equal to b. Let us just define what we have explained last time.

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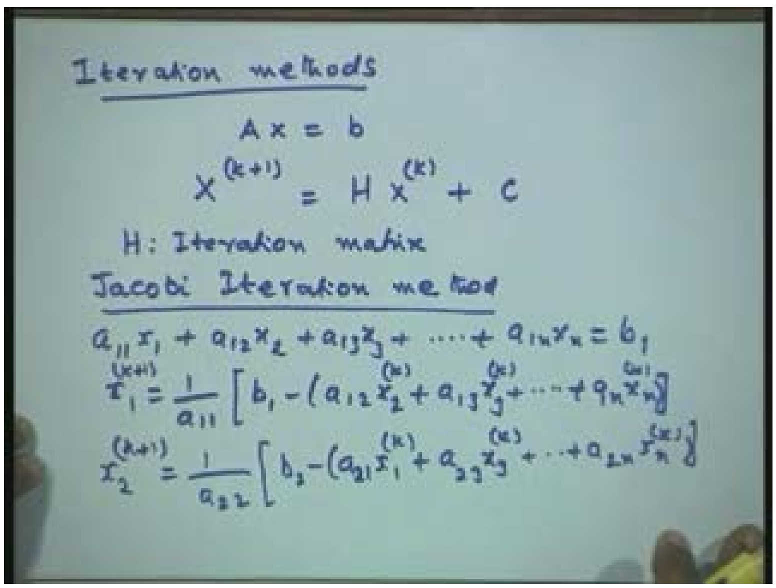

We have defined the iteration methods, we have again the system of linear system of algebraic equations A x is equal to b then we want to write an iteration method which iterates a given estimate that is an initial approximation and then iterates further and further that means we would like to write down an iterative procedure of the form X k plus 1 is H X k plus some C where, H contains the component of A and we call this H as the iteration matrix, iteration matrix the reason being the convergence or divergence of the iteration method would depend on the properties of this matrix and hence it is call the iteration matrix. Anyway once you have given the this C as a 1 that contribute the contribution of b is in c now the first method that to we would like to describe in this is call the Jacobi iteration method, we call this as the Jacobi iteration method. This method is a very straight forward and simple method in all iteration methods we solve the equations one by one.

So we iterate it first equation then iterate solve for the second equation for third equation and so on. So this is how an iteration go procedure goes 1, so let us take the first equation in this set the first equation reads a 11, x 1 plus a 12, x 2, a 13, x 3 plus so on a 1n, x n is equal to b 1 this is our first equation. The method proposes that we take all the variables x 2, x 3, x n to the right hand side keep the first variable from the first equation on the left hand side then take the iterate on the right side as of the previous iterate and the 1 on the left as the current iterates that means I would write this as x 1 is equal to 1 upon a 11, I have taken everything to the right hand side and divided by the pivot.

So that will be b 1 minus a 12, x 2 plus a 13, x 3 plus so on a 1n, x n. Now when once you take this to the right hand side and write in this form I would now define a new iteration as now you can put this super fixe a k plus 1 here put k over here, put k over here and k over here that means all these values on the right hand side we have using at the previous iterates and we are now obtaining the current iterate x 1 at k plus 1. Similarly, the second equation we shall retain the second and known on the left hand side and take all the other variable to the right hand side that means I would like to write down x 2 k plus 1 is equal to 1 over a 22 that is a pivot a 22 and that is b 2 minus a 11, x 1k plus a 2, a 23, x 3k plus so1, a 2n, x n at k.

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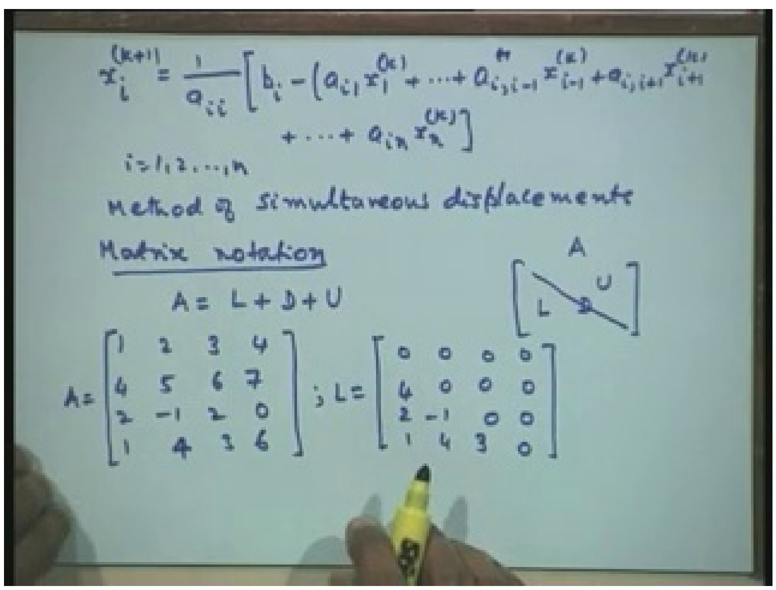

So we are taking a 21, x 1, a 23, x 3, a 2n, x n to the right hand side and all of them shall be evaluated all of them are evaluated at the previous iterate k and we obtain the present iterate x 2k, x 2k plus 1 from here. So in general, in general therefore we have] I can write down any ith variable if I take the current iterate is a plus 11 upon a ii, the right hand side is b i minus all the values up to x i that means a i1, x 1 of k plus so on a ii minus 1 that is the previous one to this that is x i minus 1 of k, k then the this particular value is missed therefore I will have the next one as the ii ith plus 1 x i plus 1 of k plus all the remaining values a in, x n of k.

So this is the value corresponding to x i is missed here. So a ii term is missed here and we have the all the other variables on the left of x i and on the right of x i brought to the right hand side and written this particular form if you write down the entire system in this particular fashion I is equal to 1, 2, 3 n if you write all of them are simultaneously being displaced on the right hand side by the previous estimates the previous value therefore sometimes the Jacobi iteration method is also called method of simultaneous displacements. So we are defining them replacing the right hand side simultaneously therefore it is called method of simultaneous displacements because some books do use this word simultaneous displacement in place of the Jacobi iteration method.

Now a important thing to note here is that the systems this equations are being sold first equation for first variable second equation for second variable and so on nth equation for nth variable therefore the convergence property of this would depend on the order which you are solving. If you interchange in equation the properties are going to change therefore is very important that we retain the equations as given in that particular one unless we have a specific reason that we have pivot is become 0 or you want improve convergence by theoretically showing that if I should, if I interchange the equation this particular fashion I get better convergences, so I can interchange equations. Otherwise, the given equation as given shall be used the procedure which you have adopted can be used for actual computation purposes but you found to analyze it the methods would like to have matrix notations for this Jacobi iteration method.

So what we let us call it as matrix notations, the coefficient matrix A given this coefficient matrix here I will write it into some of 3 parts the diagonal part the diagonal matrix I will call it as D, the strictly lower triangular part I will call as L, this I will call it as U that means I would like to write down A is equal to L plus D plus U. For example, let us take I say a matrix 1, 2, 3, 4, 4, 5, 6, 7, 2, minus 2, 2, 0, 2, 0, 1, 2, 3, 6, let us change it 1, 4, 3, 6. Now if this is the matrix given to us A then I am defining L as a strictly lower triangular part that means everything will be 0 here this and this strictly lower triangular part is 4, 2 minus 1, 1, 4, 3.

So this is the diagonal and above is removed and whatever is strictly lower triangular I will call this as L that is strictly triangular lower part. Now D is simply the diagonal part of the matrix, so it will be simply 1, 5, 2, 6. So this will be the diagonal matrix diagonal part 1, 5, 2, 6 and U is the remaining part, the upper triangular part so here we will have all 0s in the lower triangular part including the diagonal and the this part is 2, 3, 4, 6, 7 and of course here it also 0, so I have 0 over here. This is the break up I would like to take for the given matrix, so as strictly lower triangular part the diagonal part and strictly upper triangular part.

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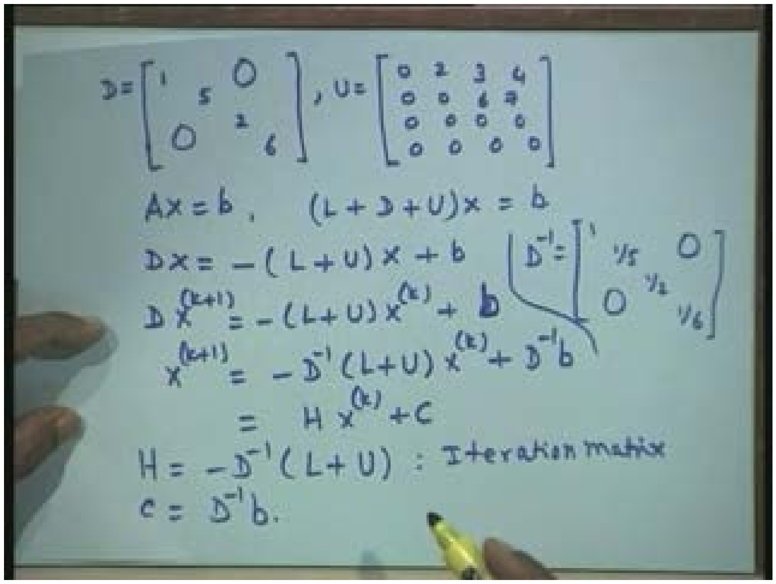

Now if I write it this way then the system of equations can be written as A x is equal to b so I can write this as L plus D plus U of x is equal to b. Now if you just look back at this this particular way in which we are written it you can see that these are all division pivots a 11 here, a 22 here, a ii here that mean these pivots are coming from the diagonal elements therefore I would keep D on this left hand side and take the remaining things to right hand side minus L plus U of x plus b. So I take this contribution of the lower triangular part and the upper triangular part to the right hand side and write it as D x on the left hand side.

Now the iteration reads D of x k plus 1 is minus L plus U of x k plus b, plus b. Now I take D to the right hand side invert D. So I will write this as b plus 1 x k plus 1 is minus D inverse of L plus U x k plus D inverse of b this is our H x k plus C which were written here therefore H is minus D inverse L plus U, this is our iteration matrix, this is our iteration matrix and C is D inverse of b. Now we can see that here we are using D inverse, D is a diagonal matrix therefore its inverse simply will be one upon a 11, one upon a 22 and so on therefore in this case D inverse will be simply 1, 1 by 5, 1 by 2, 1 by 6. So in this case, in this example will have simply D inverse as 1 by 5, 1 by 2, 1 by 6.

So that is how the when we written, when we have written this method for simultaneous displacements here. You can see this again this is D inverse to the component corresponds to the one upon a 11, one upon a 22, one upon a ii. Therefore, the that division is coming from the multiplication by D inverse therefore the this iteration matrix will play the complete part in the whether this method is going to converge or it is going to diverge and of course in the limit we would expect that this will go to x is equal to A inverse b. However, when the if the numbers in your system is very large then we may expect lot of round off errors in the procedure.

Now we can write down a very interesting alternative way of writing this method for computation purposes wherein we would like to tackle numbers which become smaller and smaller in suppose the solution is minus 10 here there is 15 other is 50 like that. So that those solutions are going to come from the Jacobi iterations but we will modify this and write a alternative method, alternative way of doing it which allows us to work with smaller and smaller numbers that means we tackle the errors and not the solutions. Now what we do is we just make a simple manipulation from this procedure. So let us start with this.

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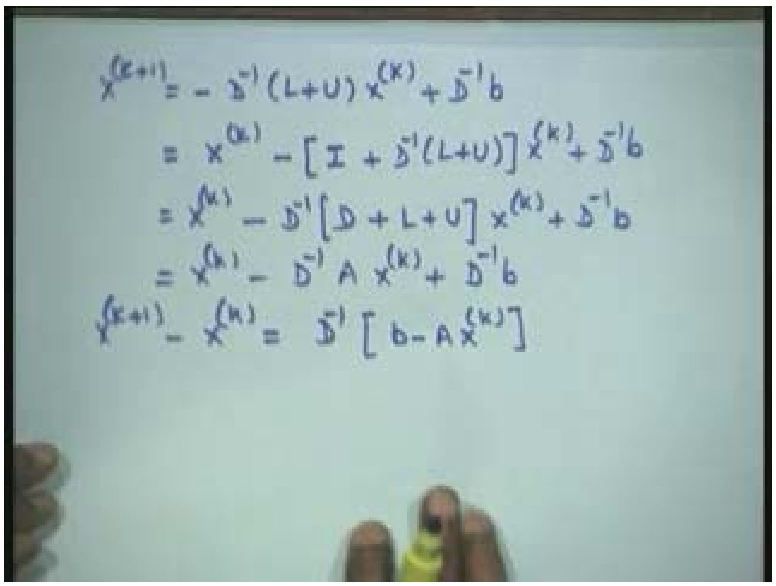

Let us start with the x k plus 1 is equal to minus D inverse L plus U x k plus D inverse of b. So I am starting with the method that is given over here what I do here is I add and subtract x k I add x k here and subtract x k whatever I have subtract x k I will combine with this x k. So I will insert it inside as I, I into x k plus D inverse L plus U of x k plus D inverse of b. So I have added x k and subtracted x k, so this minus x k included in this one. Now I make a small manipulation here I will take D inverse outside, so I will be D into D inverse, so DD inverse so write down D inverse of D L plus U D inverse I brought it on the left multiplication factor. So I will have this as x k plus1 x k and D inverse of b but by definition D plus L plus U is A, so I bring back A here so this I will write is as x k minus D inverse of A x k plus D inverse of b.

Now I bring x k to the left hand side so I will write down x k plus 1 minus x k is equal to D inverse b minus A x k. I have combine these2 I taken D inverse out so b minus A x k. Now we can see this is a very very interesting equation the left hand side is the difference between the 2 estimates, if it is converging this is going to be 0 therefore this is nothing but our error approximation, this is error of approximation. If you look at this right hand side this particular part this is how well the system has been equations have been satisfied this is A x minus b or b minus A x, our system of equation is A x is equal to b.

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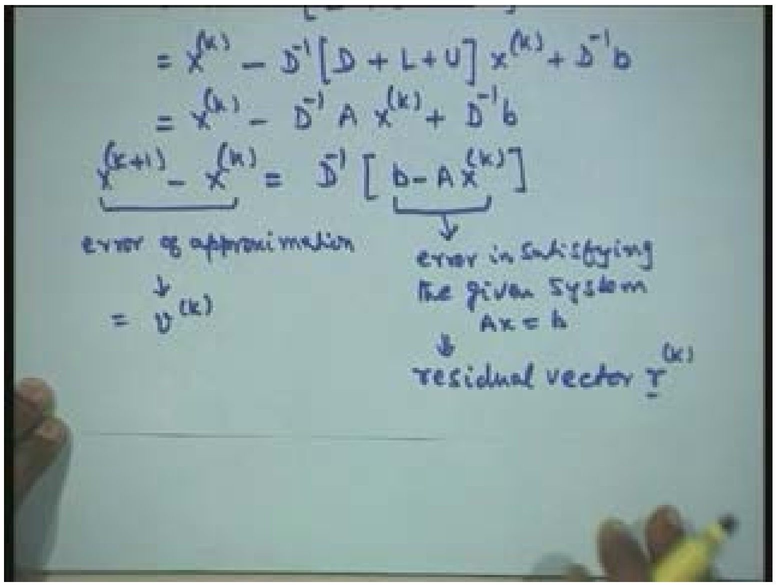

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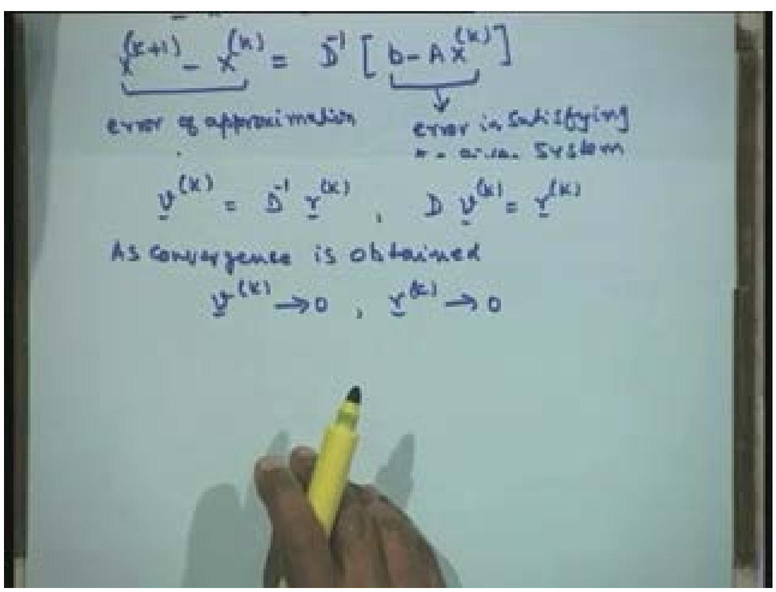

Now this shows how well the equations given in system is satisfied. So this gives you the error in satisfying the given equations error in satisfying the given system A x is equal to b. Now this we shall call this is called as a residual residual of this system. So we call this as residual vector we call this as a residual vector and give a notation to it as vector r k, this is a vector r of k and this is error approximation this we shall denote it by some v k will denote it by v k. Therefore I can write this system as v k is also vector of course these are all vector that is equal to D inverse of r k or if I want to write D on the left hand side I could as well write D of v k is equal to r k.

Now I would prefer this particular system to be solved computationally because as the convergence is achieved x k plus 1, tends to x k, this x k plus1, 1, tends x k and A x k also tends to b that means v k, tends to 0 and r k also both tends 0 therefore as convergence is obtained as convergence is obtained v k tends to 0, r k tends to 0 therefore we are computing with smaller and smaller numbers therefore the amount of round off error also would go on reducing therefore in this way we will be controlling round off errors completely we are not saying for a trivial problem of 10 by 10 or 15 by 15. We are talking of 1000’s of equations where in you will have to use it so many times and this so that the round off error can effect few places of your of your solution.

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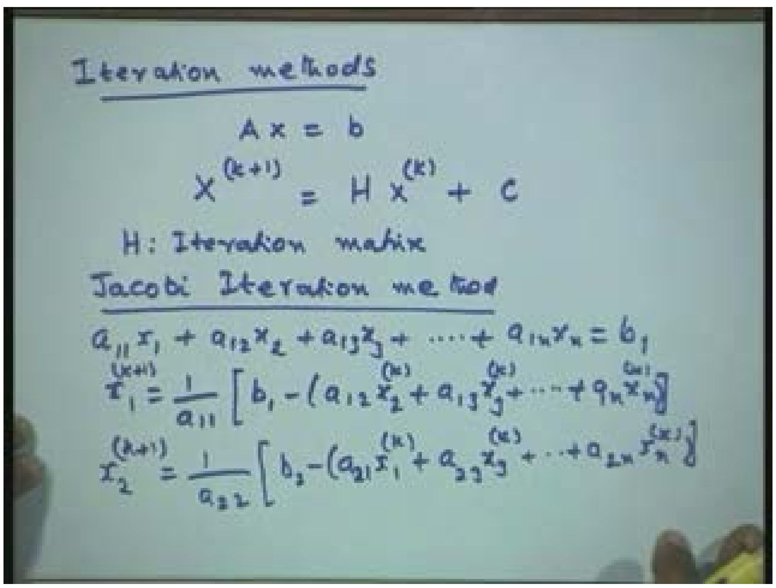

In this case the solution would not be effected because the much lesser the round of error then what it would be in the other case of the problem. Now this is called the Jacobi iteration method. This was one of the first method that was discover for the solution there is a an obvious disadvantage in this particular method as we have done and the disadvantage is we know we are solving equation by equation if I solve the first equation I am now found out x 1 k plus 1 but when I went to the next equation x 2 k plus 1 even though the solution x 1 k plus 1 is available to me, I am not using it here I am using only x 1 k of this one and when I come to the last equation for example, when I come to the last equation xn i have already computed x 1, x 2, x n minus 1.

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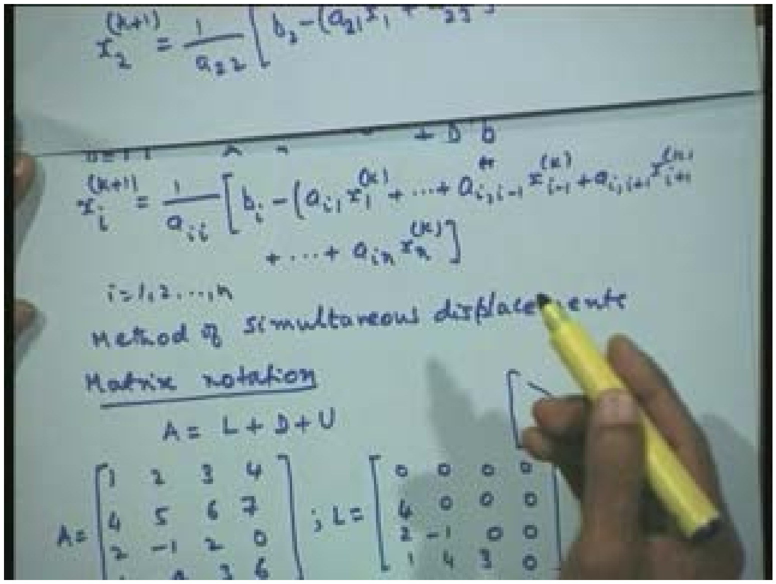

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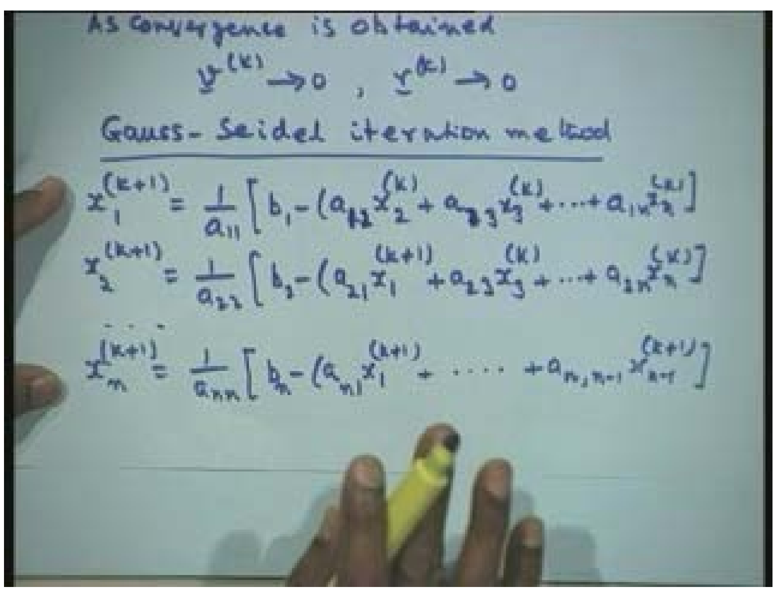

But none of these information's are being used in computing the present iterate we are still using all the previous iterate and hence the convergence would be slow and if I now include that at every stage information if I am solving ith equation i minus 1 variables have already been solved. So therefore that information can be used now to get a better estimate and if I do that it is called the Gauss Seidel iteration methods and it turns out to one of the most powerful methods even today. So that is called the Gauss Seidel iteration method, Gauss Seidel iteration method.

So let us straight away write down since we have just explained therefore will have x 1 k plus 1 will be the same equation as we have in the Jacobi. So the first equation remains the same b 1 minus a 21, x 1, x 2 of k plus a 2 sorry a 12 of this a 13, x 3 of k plus so on a 1 n, x n of k. Now we write x 2 k plus 1, one upon a 22, b 2 now I have x 1 k plus1 available to me therefore I will now use that current estimate of x 1 k plus 1, a 23 I do not have x 3 as yet so I will use a previous estimate only and write down a 2n x nk like that then finally x nk plus 1 will be 1 upon a nn, b n. Now all the previous values have been computed, so I will have all of them from the previous x n minus 1 of k plus 1.

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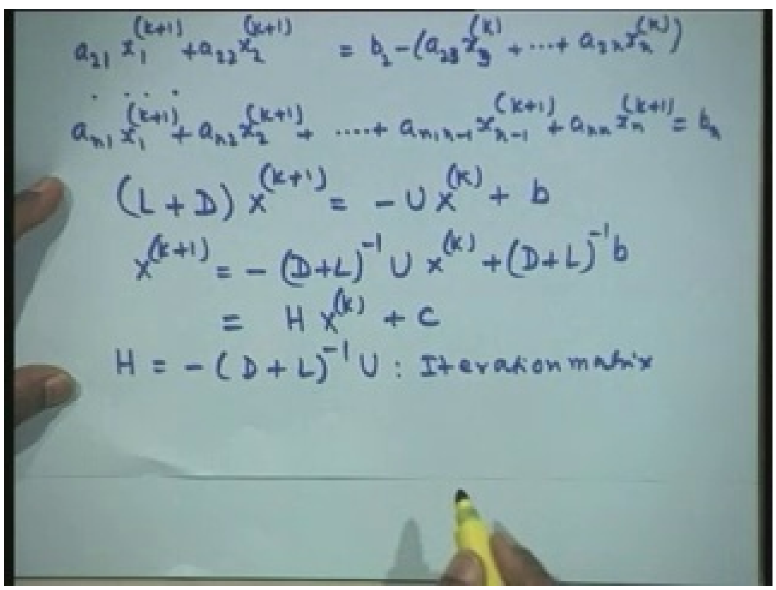

Now when we have reached this last equation all the previous variables solutions has been obtained and hence I used the whatever current available values that shall be used on right hand side to produce this particular x n, this is called the Gauss Seidel iteration method. We shall show latter on that theoretically we can prove if a is symmetric matrix for example or in general in the general case also that Gauss Seidel is at least 2 times faster than the Jacobi iteration, I mean rate of convergence we are talking it is actual computed time you can also count of it at least 2 time faster than the Gauss Jacobi to be iteration method. Now for analyzing it for a convergence analysis I would like to write down in the matrix format, what I would do is I will have to retain all the current iterates on the left hand side.

So I will bring everything to left hand side and write this equation. So let us rewrite this equation first equation this is a 11, x 1, x 1k plus 1, a 11, x k plus 1 and the let us leave some space here. So that its visibility i can live it, so this will be equal to b1 minus a 12, x 2 k plus a1n, x nk. So I am cross multiplying it and taking if there is a current iterate I will take the left hand side here there is nothing now you can see if I go the next equation now if I take the second equation this is a 22, x k plus 1 and this is also the current iterate.

So I will bring it to left hand side, so I write this as a 21, x 1k plus 1 plus a 22, x 2k plus 1 and the right hand side is b 2, a 23, x 3k, x 3k plus a 2n, x nk. Now if I go the last equation I am multiplying this all of them belong to the current iterate, so all of them come to the left hand side. So I will have here a n1, x1k plus 1, a n2, x 2k plus 1 plus so on a nn minus 1 x n minus 1 k plus 1 plus a nn, x n k plus 1 is bn so all of them are current iterate only, so all of them come to the left hand side. Now let us look at the coefficient matrix, the coefficient matrix is a 11 plus a 21, a 22 the last row gives you a n1, a n2, a nn that is nothing but L plus D.

So both of them are on the left hand side so I have L here D on this side, so I have got here x k plus 1 here and I have got here only the upper triangular part has gone to the right hand side. So minus U x k plus b so this is a 11, a 21, a 22, a n1, a n2, a nn therefore this L plus D on the left hand side on the right hand side you have the contribution of the upper triangular part that minus U into x k plus b. Therefore, I can write this as x k plus 1 is minus L plus D or D plus L inverse of U x k plus D plus L inverse of b which is our H x k plus C where now the iteration matrix, H is the iteration matrix is now given by minus D plus L inverse of U so this is our iteration matrix. Again the the convergence property of the Gauss Seidel iteration method will depend on these iteration matrix minus D plus L inverse of U.

Now again for computational purposes I would, I would still not pay for this I would use the error format of the methods. So the just as we have done for the Jacobi let us also do for the for the Gauss Seidel also let us call this as the error format of the of the sum of equations. I will do the same manipulation that we done for the previous method, so we shall write this as x k plus 1, I add and subtract x k here, let us bring a minus sign here, minus sign here, I plus D plus L inverse of U x k plus D plus L inverse of b. I have added and subtracted x k from this particular side. So x k here minus of x k and D plus L inverse of U so I retained it as it is. So I just added and subtracted x k again I will manipulate it in such a way I take this D plus L inverse out on the left then this will be D plus L, D plus L, D plus L inverse gives us I. So this is our I and I have taken this from left hand side, so U will be left out x k plus D plus L inverse of b.

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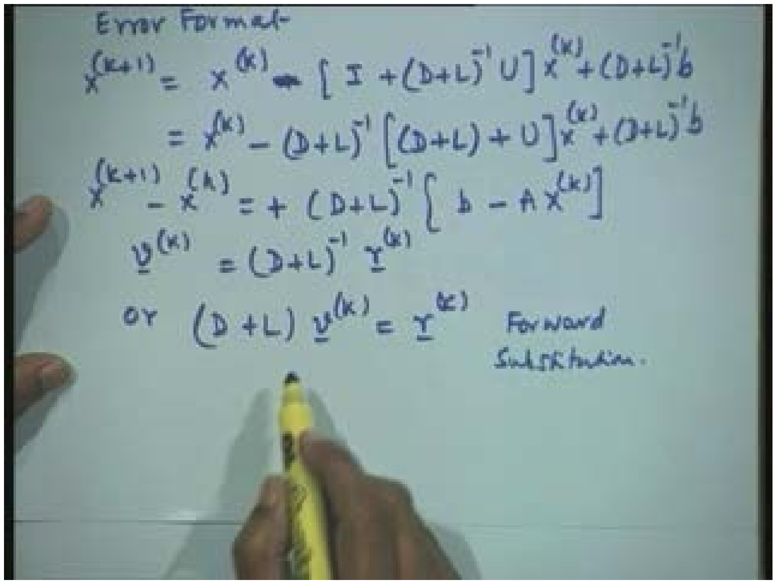

Now again this is D plus L plus U that is a again D plus L inverse common so I will have here b minus a x k common here therefore I can bring x k plus1 minus x k, I bring it to left hand side this is minus D plus L inverse of b minus A x k, I have taken D plus L inverse common of the left here so I have b here and this is our A, A x k okay okay we will put a positive sign will put a positive sign here. Now in the notation of the previous method this will be simply v k is equal to D plus L inverse of r k. So this again the same residual vector and this is the error of approximation or we can use it as D plus L v k is equal to r k, this we are very well versed about this this one nothing but the forward substitution this is D plus L of v k therefore this is a lower triangular matrix therefore this is nothing but our forward substitution. You can look also in the in the sense of this is a forward substitution method.

Now you can see the difference between the 2 methods because it is so trivial I mean discussion of this particular aspect to the error format you can see that both of them would require evaluation of r k, b minus A, x k is residual vector we need evaluation of residual vector in both the methods and where as in the Jacobi we are only having D. So we had only dividing by the1 division for each equation that is a 11, a 22, a 33 and so on 1 division per equation whereas here we have got forward substitution and we know the count for the forward substitution here also.

So if this gives you forward substitution and that is the direct replacement. But theoretically we can show that this method converges at least 2 times faster than the Jacobi iteration method. Now before we analyze it let us just take an example on this. So I will take this has an example example, 1 more just comment before we proceed on in the case of the Gauss Seidel we are replacing the right hand side vector whatever currently is available of the iterates therefore we call it as the successively we are replacing the right hand sides as the solution vector current vector available.

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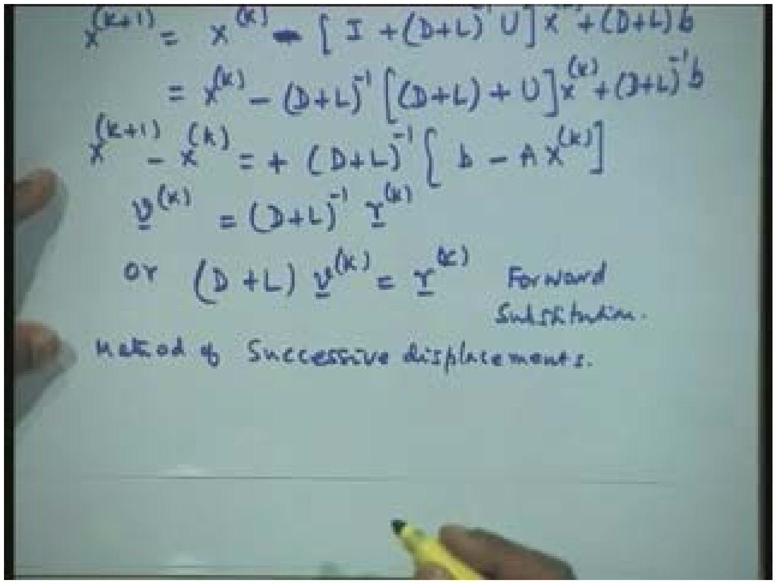

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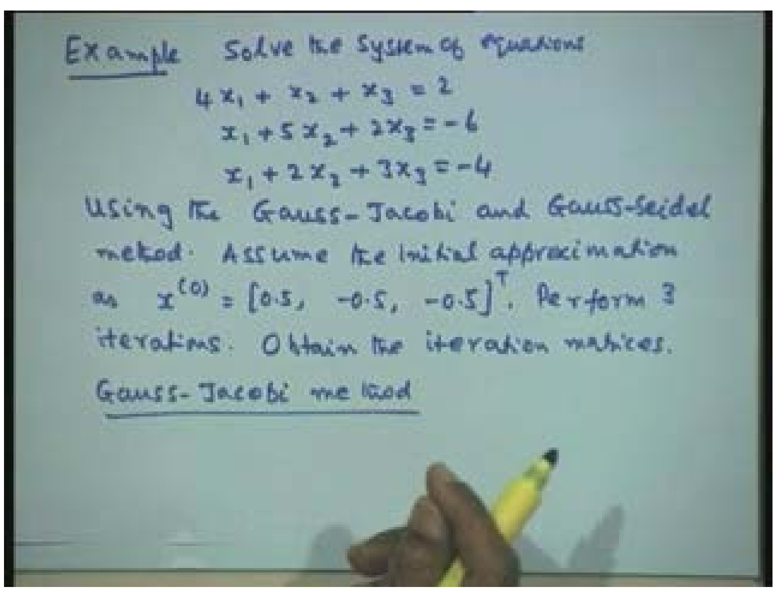

So it is called the successive displacement in the previous case we have called it as the directly all of them are displaced on the right hand side. So this is also called method of there we called simultaneously all of them here it is successively we are doing it method of successive displacements as and when it is available it is used therefore it is successively it is being used displacements are used. Now let me write one example solve the system of equations 4 x 1 plus x 2 plus x 3 is 2 plus 2 x 3 is equal to minus 6 x 1 plus 2 x 2 plus 3 x 3 is minus 4. So using the let us use both of them using the Jacobi or simply Gauss Jacobi, Gauss Jacobi and Gauss Seidel methods.

Let us also give initial approximation, so let us assume the initial approximation initial approximation as x 0 first iterate as 0.5, minus 0.5, minus 0.5. So we have given the initial solution vector also as 0.5, minus 0.5 n minus .5, let us further moment ask perform 3 iterations, perform 3 iterations and also lets obtain, obtain the iteration matrices, obtain iteration matrices.

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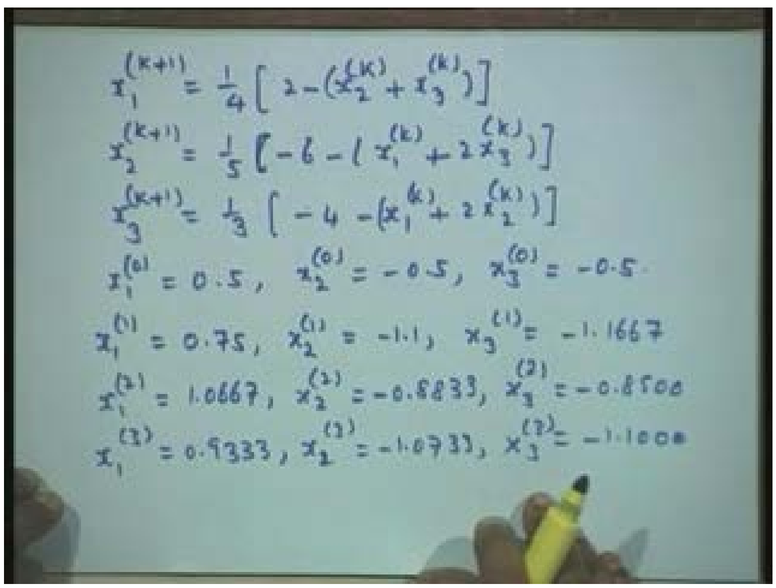

Now let us write down the firstly the Gauss Jacobi method. Now in the Gauss Jacobi, we take the 2 variables to the right hand side and write this x 1 k plus 1 is 1 up on 4, 2 minus x 2 k plus x 3 k, x 2 and x 3 is taken to the right hand side, the current iterate, the previous iterate and on the left hand side we have the current iterate. Second equation gives x 2 k plus 1 is 1 up on 5 minus 6 minus x 1 k plus 2 x 3 k. So x 1 and 2 x 3 are taken to the right hand side these are the previous iterated values and the current iterative is here from the third equation will have x 3 k plus 1 is one upon 3 minus 4 minus x 1 of k plus twice x 2 of k this is both x 1 and x 2 are taken to the right hand side and will have this particular thing.

Now we have given initial approximation as this, so we are given x 1 of 0 is .5 we are given 2 of 0 is minus .5, x 3 of 0 is minus .5. We are given these 3 values initial values, so I just have to substitute these and evaluated it I will give the values that will be obtain over here. So the if I substitute then I would get here x 1 of 1 this simply I am substituting value of x 2 is minus .5, this is minus .5 and then simplify this one I will give the values of this this 0.75 x 2 of 1 is minus 1.1, x 3 of 1 is minus 1, 1.1667 this is the completion of the first iteration by substituting the values of the previous iterated values in this set of equations.

Now this will be our new starting value so these values will substitute on the right hand side get our next iterate equations. So the next iterated values are these x 1 of 2, 1.067, x 2 of 2 is minus .8833, X 3 of 2 is minus .8500. This completes the second iteration, we have got the values of x 1 of 2, x 2 of 2, x 3 of 2 using this set of values and the third iteration gives x 1 of 3 is .933, x 2 of 3 is minus 0733, x 3 of 3 is minus 1.0. The exact solution IU may give we can compare it exact solution is 1, minus 1, minus 1, so this is the solution vector is 1, minus 1, minus 1. Now I need the iteration matrix for this. So we have written iteration matrix for the wait I will take a different color as minus D inverse of L plus U therefore minus the diagonal part of a 4, 5, 3 is the diagonal.

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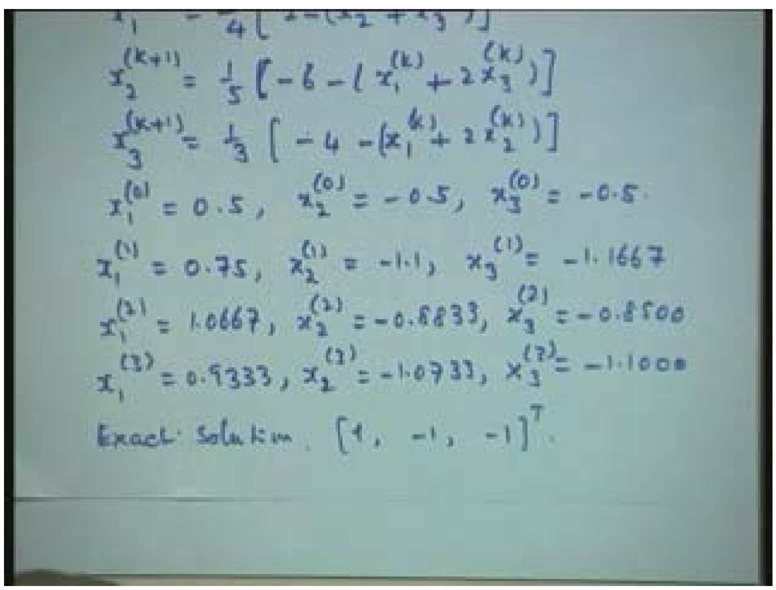

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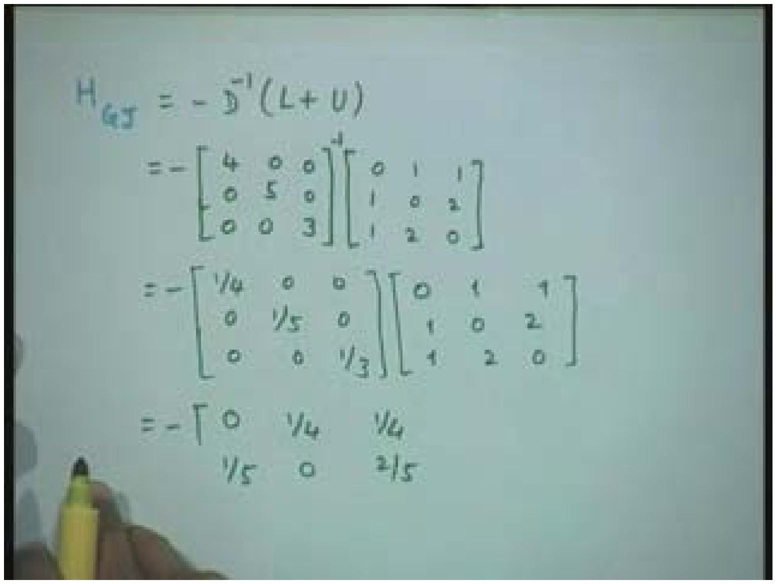

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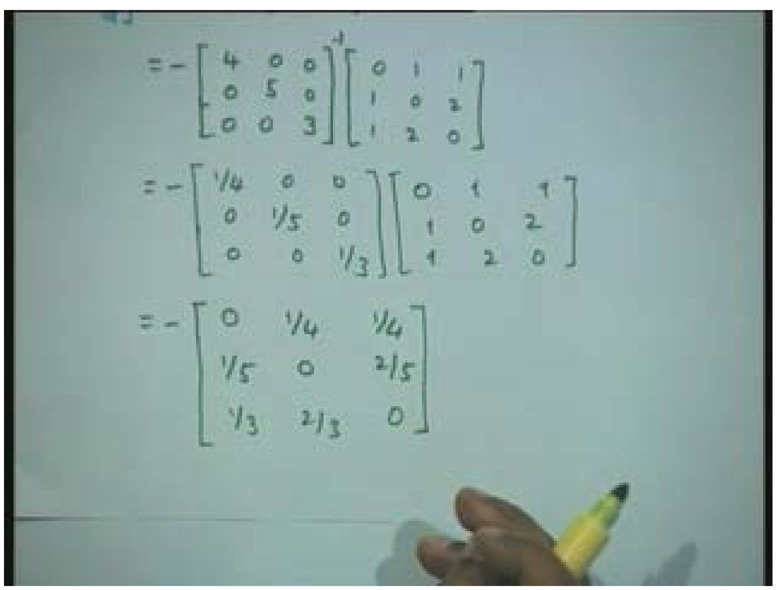

So this will be 4, 5, 3 alright, let us put 0s here, 0s here and the remaining part is L plus U L plus U will give U yes, inverse will put inverse here and then L plus U is the remaining part. So we have diagonal elements as 0's here and the remaining part is 1, 1, 1, 2, 1, 2 that is your 1, 1 here 1 and 2 here 1 and 2 here. Now the inverse of D would simply gives us 1 by 4, 1 by 5, 1 by 3 here. So this gives me 1 by 4, 0, 0, 0, 1 by 5, 0, 0, 0,1 by 3 and you multiply this by 0, 1, 1, 1, 0, 2, 1, 0 this. So I can just multiply this gives me 0, 1 by 4, 1 by 4 that is this row and this 3 columns then the second one1 by 5, 0, 2 by 5 and the third row would give us 1 by 3, 2 by 3, 0. So this is the iteration matrix for our Gauss Jacobi method. Now this we are going to use it later on to study the convergence properties of the Gauss Jacobi and also to obtain what is the rate of convergence and also how fast it is converging.

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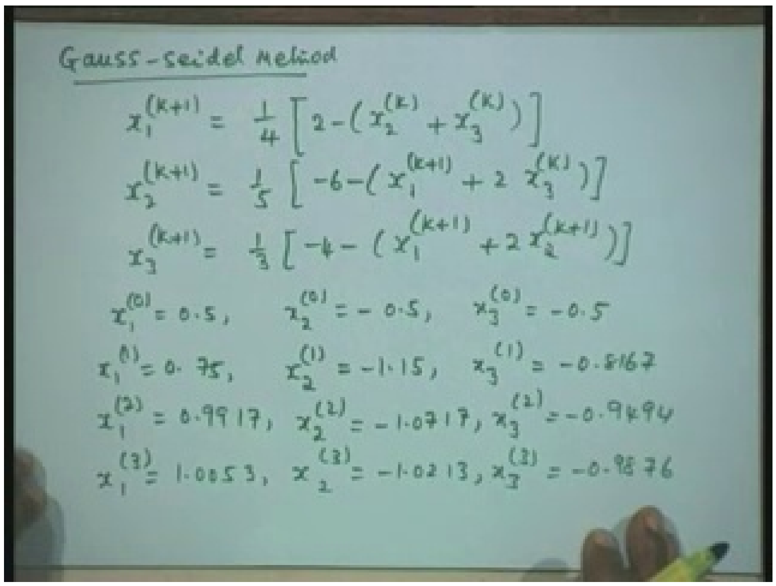

Now let us take the Gauss Seidel, so let us do Gauss Seidel now whatever we are written here will now be written rewritten except that here will have a x 1 k plus1 and here both of them will be at k plus 1th iterate. So let us rewrite this system so this will be x 1 of k plus 1, first equation remains as it is 1 by 4, 2 minus x 2 k plus x 3 k then x 2 k plus 1 is equal to 1 by 5 minus 6, minus x 1 of k plus 1 the current iterate 2 times the x 3 of k, the previous iterate then x 3 k plus 1 is equal to1 by 3 minus 4 minus x 1 k plus 1 plus 2 times x 2 k plus 1.

So here we have got all of them at the current iterate. Now again we start with the iteration starting the initial value is given to us again as x 1 of 0 is .5, x 2 of 0 is given as minus .5 and x 3 of 0 is equal to minus .5 then I substitute it here the first value would be the same as the previous because the first iteration is the same. Now you can see that the current value of x 1 k, x 1 of 1 has to .75 therefore the second solution is going to change substantially because in place of 0.5 which was a previous iterate, now we are using current iterate as .75 over here.

Now this value comes out to be xv2 ofv1 is minus 1.15 and similarly, x 3 of 1 now when I solving for x 3 of 1 I have got x 1 of 1 available to me x 2 of 1 is also available to me. So both these values that are now will be used on the right hand side and this is this value comes out to be minus 8167. So I would give the remaining iterates also what I would get here x 1 of 2 is equal to .9917, x 2 of 2 is minus 0717, x3 of 2 is 9414 wherein I have use in the first step the previous iterated value and the next step I have use this contribution here and in the third value we have using both these values current iterates here and then obtaining this value and the third iteration gives out the values of this 1. 0053, x 2, 0.3 minus 0213, x 3 is minus .9876.

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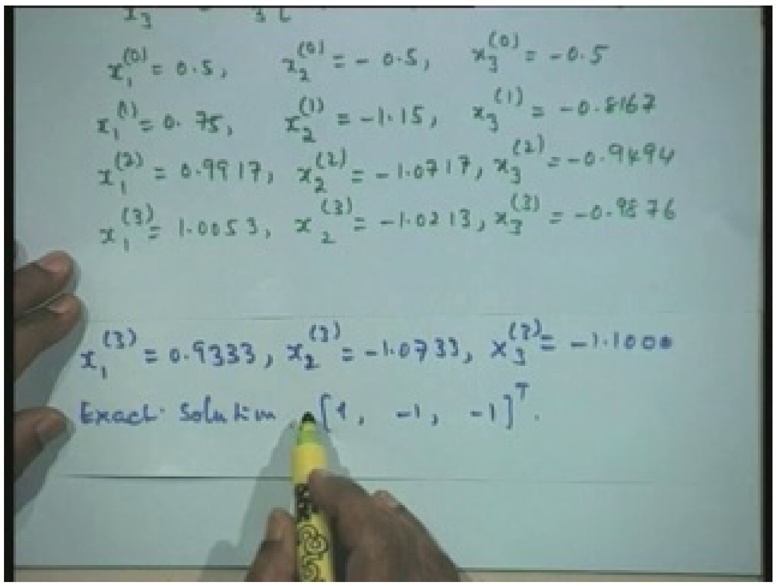

Now this is the solution value that we have obtained after the third iteration. Now it would be interesting for us to compare this value with what we had obtained in the Gauss Jacobi value. If you look at this the Gauss Jacobi values this was .9333 exact solution was 1 and here we got 1.0053 this value is very close this is 0213, 0733, 9816 and this minus 1.1. Now you can see that even for the simple problem of a 3 by 3 system it is so clearly visible that the Gauss Seidel is much faster compare to the Gauss Jacobi scheme.

Now yes, yes error will be yes now after few iterations initially it is it is trying to adjust it values though as the number of iterations suppose you take a 1000 equations when once you come to the x 100, x 1000 value in the few iterations. You will see that this is converging very very fast it actually takes time for iterations to adjust its values and afterwards the convergence will be very fast.

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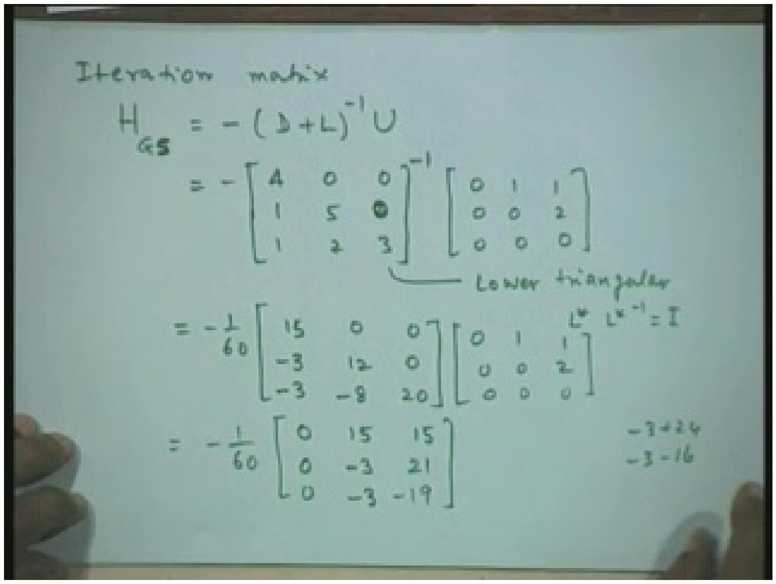

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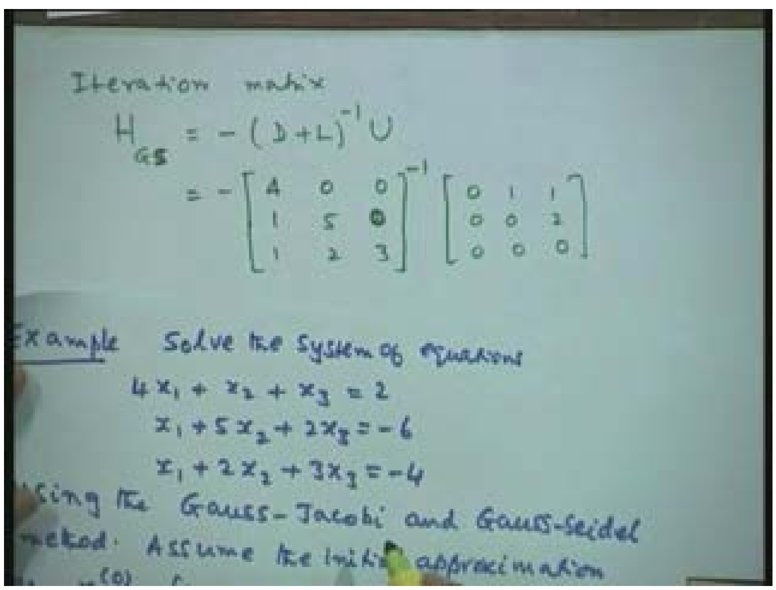

Now let us write down the iteration matrix for the Gauss Seidel also. So let us write down the iteration matrix. Now the iteration matrix let us give a suffix to it will put GS, Gauss Seidel minus D plus L inverse of U. So this our minus D plus L inverse of U and that is equal to let me just go back here and then write from here our D plus L will be 4, 1, 5, 1, 2, 3. So that will be the

D plus L part that means I have here this 4, 0, 0, 1, 5, 2, 1, 2, 3 inverse that is our D plus L is 4, 1, 5, 1, 2, 3.

So this is our D plus L inverse of this and U is the upper triangular part, so this strictly upper triangular so let put them these are 0s, 1, 1 and 2, 1, 1 and 2 this is your 1, 1 and 2, so this is your U part. Now D plus L is always lower triangular therefore I can obtain its inverse by using the properties that LL inverse is equal to I. So I can use that this is a lower triangular, this is a lower triangular matrix, so I would in general use this we are using that notation some let us call it as L star. So I can use this property that LL star is equal to I and forward substitution I am talking for a big matrix a for a small matrix we can, we can do it directly but for if when it is a system is very large I would use this notation and get this one.

So the I can immediately write down find the value of the determinant here that is equal to 60, so will have here minus 1 upon 60 I can write its cofactors this is for this element this is 15, for this element this is 12, for this element this is 20 and the inverse of a lower triangular matrix is a lower triangular, so I can put that 0s over there straight away. Now I can take the cofactors of this remove this row column I will have with 3 here, so I will have minus 3 here I take cofactor of this corner element that is 2 minus 5 that is minus 3.

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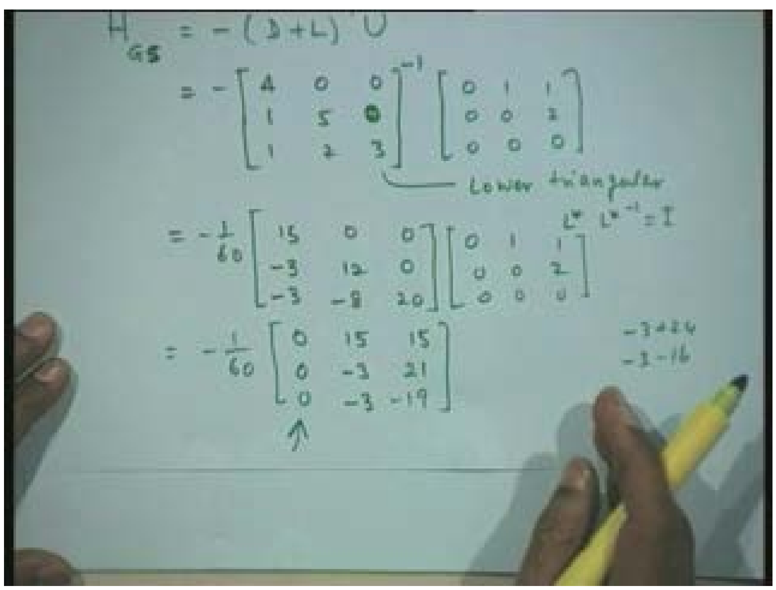

So it stays on there the cofactor this element is 8 therefore I will have a minus 8 and this is being multiplied by 0, 1, 1, 0, 0, 2, 0, 0, 0, so let us multiply it out 1 upon 60. Now you can see that the first column because these are all 0s therefore this also 0, so this is always going to be 0 these are all 0s. So the any row multiply by this this all going to be 0 this is 15 and third row 15, the first row third column gives me15 multiply with this I would get here minus 3 and this is minus3 plus 24 so I will have here 20 one third row second column gives again minus 3 and this and to this gives me minus 3 minus 16, so I will get here minus 19.

Now interestingly the Gauss Seidel method will have the iteration matrix as 0s in the first column, it is trivial because the we are taking only U is the one that is being multiplied here and U has got 0s in the first column always therefore the product will always contain 0s in the first column for the iteration matrix of the Gauss Seidel. This is of course an advantage for us will see as we analyze the method for the finding the convergence rate this will be of something little use for us because it will reduce the amount of working. So this is a fundamental property of the Gauss Seidel that the first column of this will be in the same thing, I think we will stop for this today.