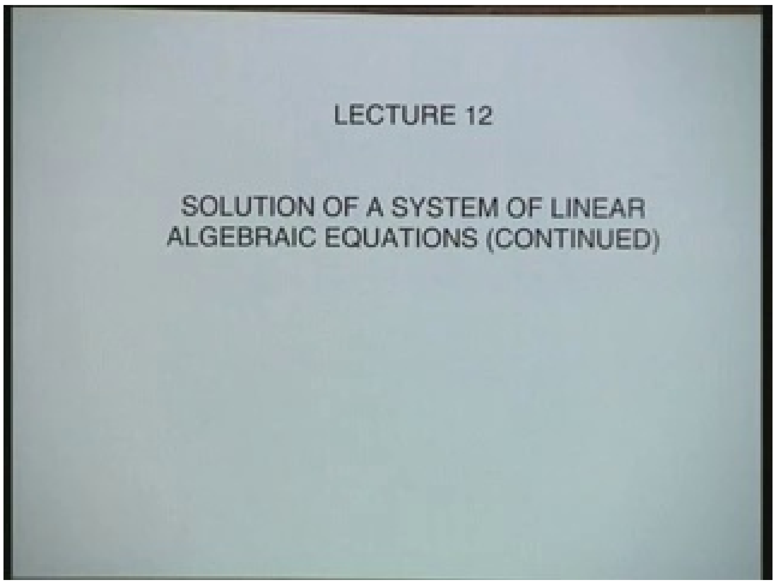
Numerical Methods and Computation
Prof. S. R. K. Iyengar
Department of Mathematics
Indian Institute of Technology, Delhi
Lecture - 12
Solution of a System of Linear Algebraic Equations (Contd…)

Now in our previous lecture, we have introduced the concept of a linear system of algebraic equations and an Eigen value problem.

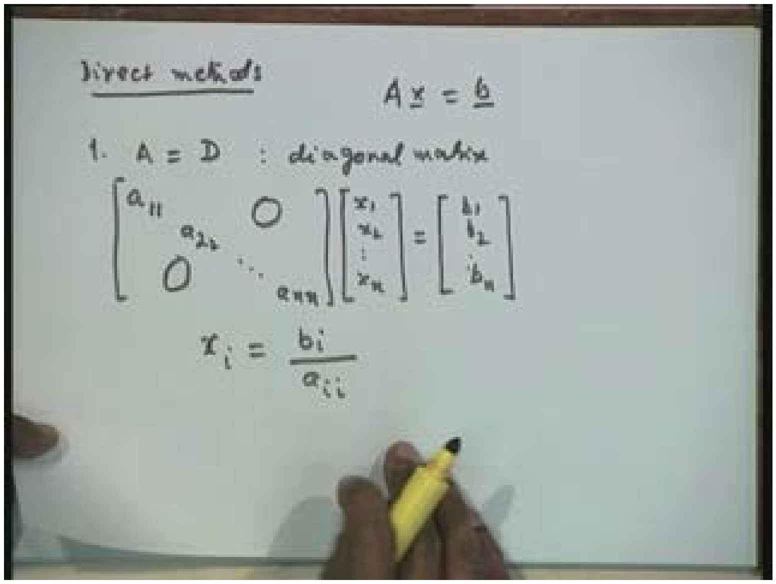
(Refer Slide Time: 01:08)



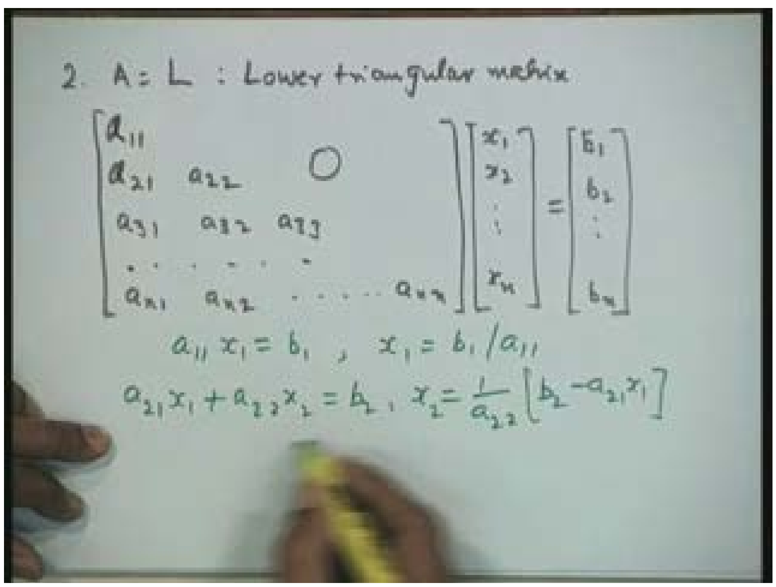
We have also given some properties of the matrices which we shall be using in our applications. We have also defined a direct method and an iterative method, the let us first of all start with a direct methods before we actually discuss or derive some methods, we would like to state that if the matrix A of this A x is equal to b our system of linear algebraic equations has some special properties then this solution can be obtained almost trivially, directly you can apply without any computations. Why we want to discuss those cases is, all the numerical method that we are going to construct would reduce the given system to that particular form, from this solution can be obtained easily.

Now let us first fall take the case when A is a diagonal matrix. So A is equal to D, a diagonal matrix that means what we are talking of here is the matrix a 11, a 22 so on a nn, this a and we have the vector x 1, x 2, x n and a right hand side vector we have b 1, b 2, b n. Now we can see that this is a very trivial case because from the first equation we have a 11 is x 1, b 1 therefore x 1 is determined immediately similarly, x 2 is determined from the next equation. So as the solution of the problem is simply x i is equal to b i by a ii. Now this is a direct application of these that when a is of the form of a diagonal matrix this solution is very trivially obtained.

(Refer Slide Time: 01:29)

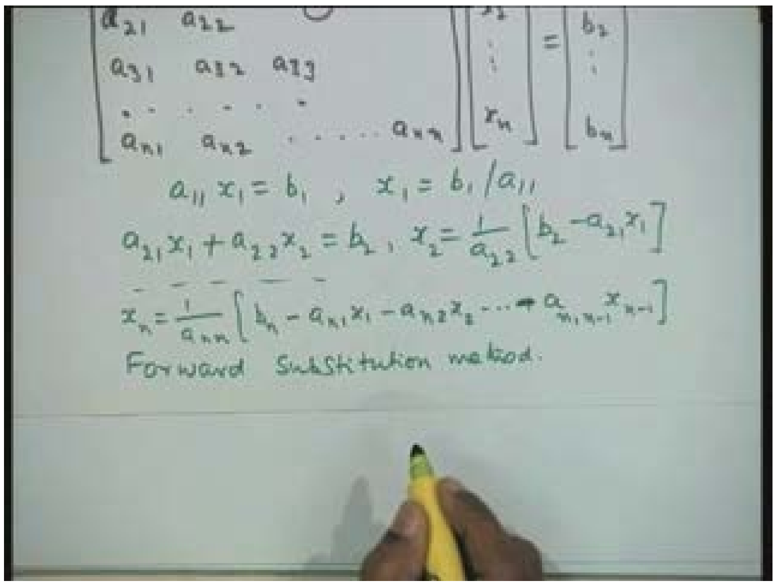


(Refer Slide Time: 03:26)

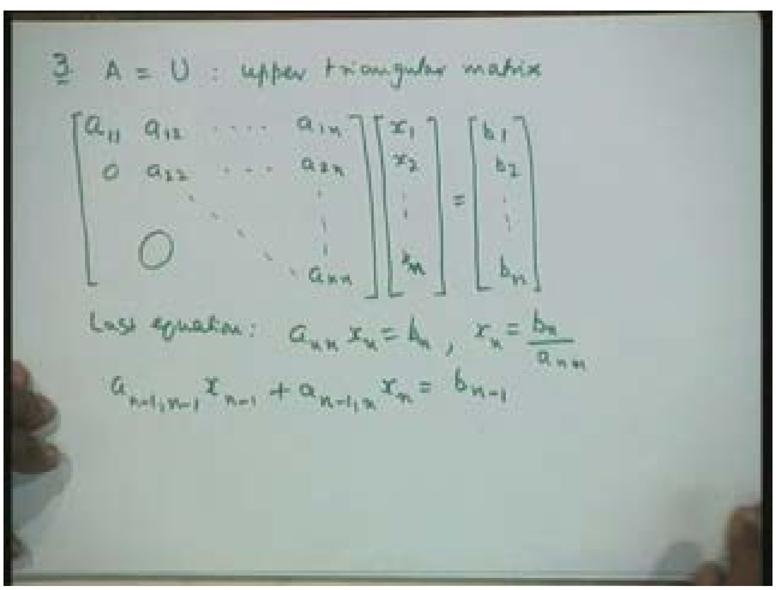


Now let us take the another case when, the case when A is of the form L, a lower triangular matrix L, a lower triangular matrix and therefore the system is of the form l 11 or let us take a 11, a 21, a 22, a 31, a 32, a 33, a n1, a n2 so on a nn. We have the vector x 1, x 2, x n, b 1 right hand side vector b 1, b 2, b n. Now you can see that we can solve the first equation as a 11, x 1 is equal to b 1 that is x 1 is equal to b 1 upon a 11 then I can take the second equation that is a 21, x 1 plus a 22, x 2 is equal to b 2 therefore x 1 is known. So it goes to the right hand side and x 2 will immediately be determined x 2 is 1 upon a 22 of b 2 minus a 21, x 1. So I can take this a 21, x 1 to the right hand side since it is known and I can proceed the computation like this. Now I can obtain the third unknown we can obtain the last unknown. So the last equation by the time we have arrived at the last equation x 1, x 2, x n minus 1 all of them have been computed. So all of them can be taken to the right hand side therefore from the last equation I would get x n is 1 upon a nn and the right hand side b n and all these terms go to the right hand side a n1, x 1 minus a n2, x 2. So on the last but one term is a minus sign this is a minus sign an n minus one x n minus 1. Therefore, in this case also we are able to solve the system without any computation except that we start with the first variable and then come forward therefore we shall call this as a forward substitution method, it is a substitution, so will call this as a forward, forward substitution method.

(Refer Slide Time: 05:22)



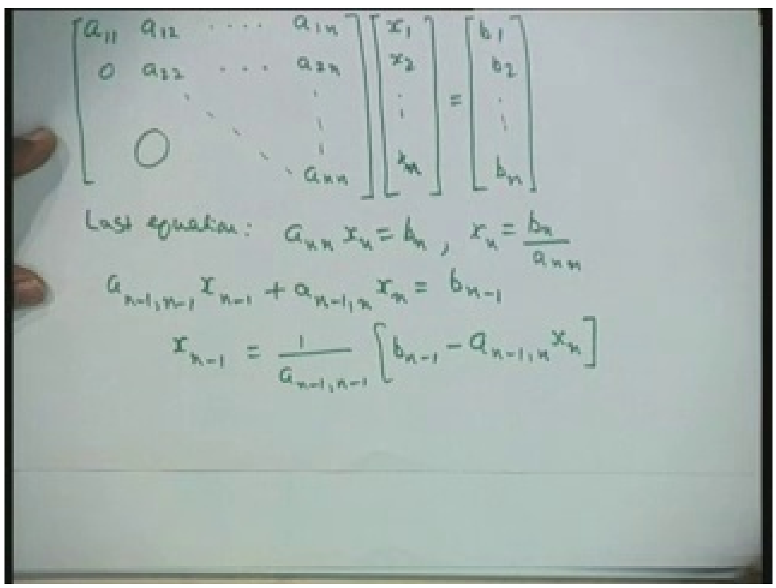
(Refer Slide Time: 06:54)



The only computation that are involved is these n divisions that we are making and the multiplications and subtractions that we are making. We are not doing any further computation, special computation except these divisions and multiplications straightforward multiplications, so in this case also the solution is trivial. Now let us take the third case we will take now the matrix A is of the form U an upper triangular matrix. Therefore, the matrix system would now look like a 11, a 12 so on a 1n, a 22, a 2n so on. We have a nn we have the vector x 1, x 2, x n and the right hand side vector b 1, b 2, b n.

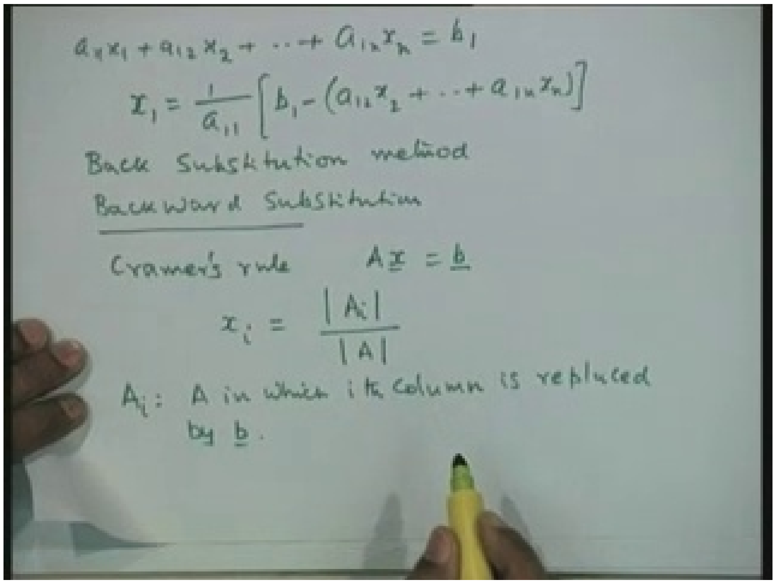
Now we can see that this system can also be obtained the same way but we start from the last equation. So if I take the last equation, let us take the last equation which gives me a nn, x n is equal to b n that means I can solve the last equation for the variable x n, b n upon a nn. Then, I go backwards then take the previous equation, the previous equation will read a n minus 1, n minus 1, x n minus 1 that is the diagonal element and the last element is an minus 1n, x n is equal to b n minus 1.

(Refer Slide Time: 08:51)



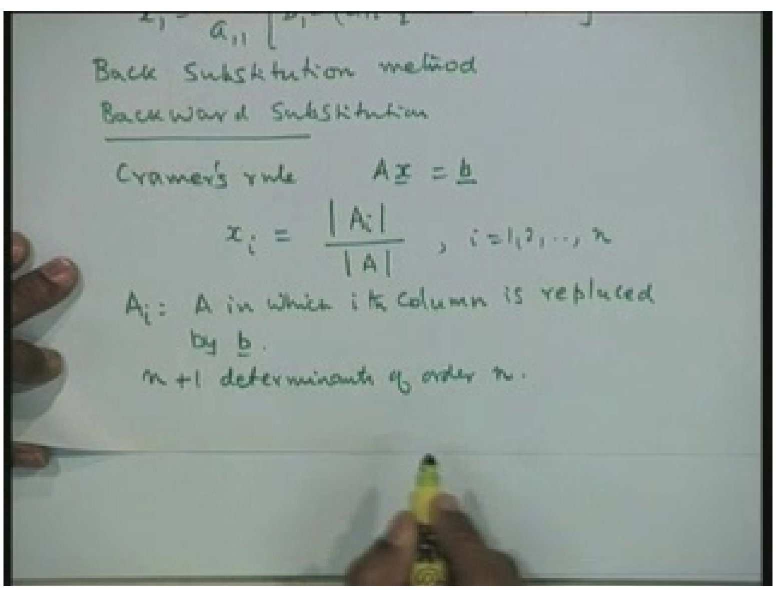
Now x n is determined so this goes to the right hand side I can find the x n minus 1 from here as x n minus 1 is 1 upon a n minus 1, n minus1 and the right hand side b n minus 1 minus a n minus 1n x n. So we can proceed on like this we can work backwards and get the previous value x n minus 2 and so on x 2 then x 1. So when i land up into the first equation it will then read like, the first equation will read like a 11, x 1, a 12, x 2, a 1n, x n is equal to b 1 that is your our first equation. Now all these variables x 2, x 3, x n they all been determined by now so they all go to the right hand side and therefore I have the solution for x 1 is 1 upon a 11, b 1 minus a 12, x 2 plus so on a 1n x n. Now in this case also therefore the solution is very straight forward almost trivial start starting from the last equation going backwards we shall give a name for it as the back substitution method or backward substitution method, So we shall call this as back substitution or we shall also sometimes is backward substitution method.

(Refer Slide Time: 09:25)



Now before we take up the actual methods, we would also interesting to know that we from our earlier years we know that there are 2 ways of solving a problem one is the Cramer's rule that we can apply and other is the inverse method and why that methods, those methods should not be is not applicable for larger systems let us have a look at it. If I solve the system of equations by Cramer's rule, we are solving the system A x is equal to b if we are solving we know that this solution of this problem can be written as x i is equal to determinant of A i divided by determinant of A i, A where, A i is A in which the i th column is replaced by b.

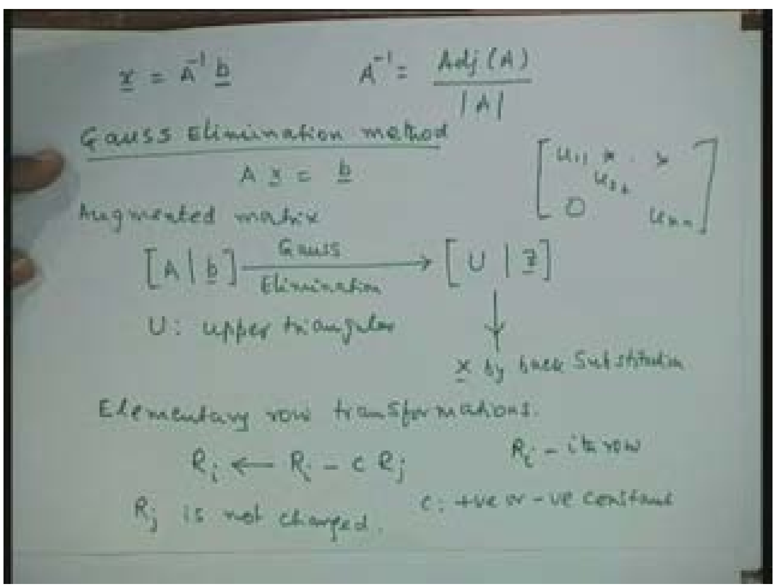
(Refer Slide Time: 11:45)



We are denoting the matrix A i as A in which i th column is replaced by b and I can now find out the values of all the variables x 1, x 2, x n by just evaluating this determinants therefore to use Cramer's rule I need to evaluate n determinants which are in numerator and one determinant which in the denominator. Therefore, I need to evaluate n plus 1 determinants, n plus 1 determinants of order n. So each of them is of order n, now we know that if I want to expand the determinant by the method that we know that means finding the finding the cofactors minus multiplying and simplifying it, it would take almost factorial n operations for the total computation.

We can see that if the matrix system is of the order 50 by 50 or 100 by 100 almost impossible for us to solve the by the Cramer's rule that many operations. Therefore as by the method that we know the Cramer's rule cannot be applied therefore, if I want to apply Cramer's rule I must know alternative method of evaluating the determinant in a much more simpler fashion, if I want to use a Cramer's rule. However, if I if I can finds such a method I can finds still simpler methods which give me the solution itself directly rather than using the Cramer's rule and the same thing holds if you are using the other concept of finding x is equal to A inverse of b. The discussion that we made for Cramer's rule holds also here because A inverse has got the determinant the definition of A inverse is adjoint of A divided by determinant of A.

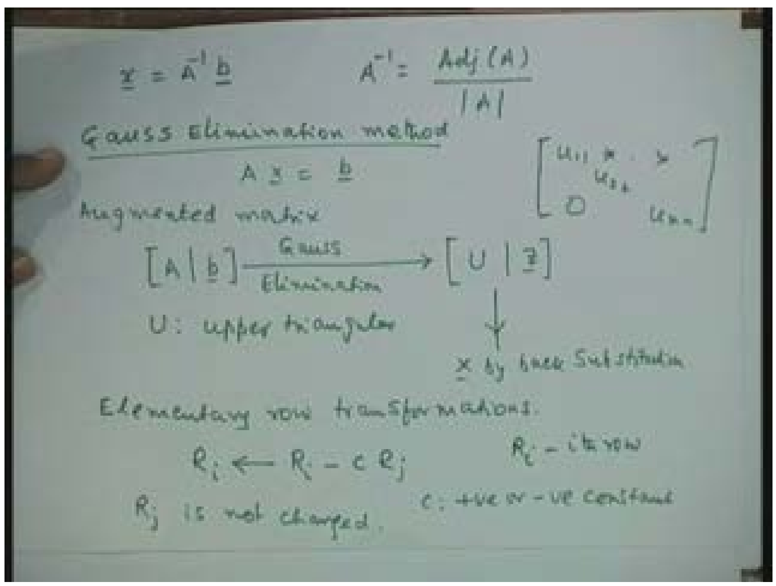
(Refer Slide Time: 13:11)



Therefore here also we have got a evaluation of one determinant and all the cofactors of the matrices that we have n square cofactors to be determined therefore the computation as we know also is of enormous magnitude and is almost impossible for us to use for a 50 by 50 or a 100 by 100 system. Therefore, we need alternate methods to even compute inverse or the determinant or this solving this system of equations itself, no none of them would work for last systems if we take 50 by system and by the method we know finding the all the co-factors then writing the adjoint and a divide determinant if you go by 60 by system even few days of computer time would not be sufficient for us to solve such a system. Therefore, we need much simpler methods which can give us inverse as well as determinant and any one of the methods can be chosen.

In fact if you have problem in which you are solving a problem with more than one right hand side then it would be easier for me to solve for A inverse once keep it store it and then the right hand sides can be different sides can be used that is A inverse b 1, A inverse b 2, A inverse b 3 so on. Once if we determine it we can have the solution that case and that is a practical problem also. Suppose you take a very simple problem of Poisson equation the solving in a circular rectangle with different boundary conditions by a finite difference method what it would produce you linear system of equations since the boundary conditions are changed right hand sides only change therefore the coefficient matrix A is always the same therefore in such problems I would prefer to find an inverse by a suitable procedure then I can find A inverse of b 1 as a solution.

(Refer Slide Time: 15:47)



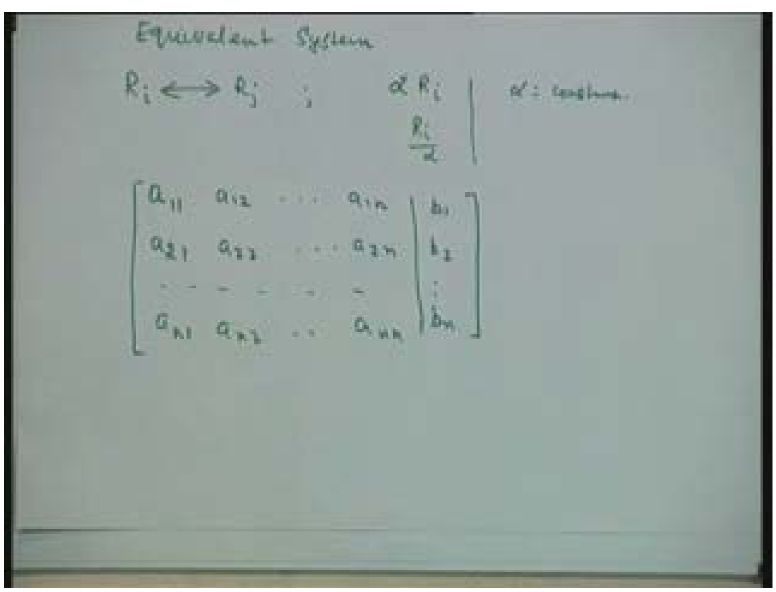
So for different boundary conditions, so I can have A inverse b 2 and so on but for a have a single problem then I will not go for this method or for the Cramer's rule I would going for a better method which would give me the computation in much less amount of time. The first method that we would do in this direction is called the Gauss elimination method, Gauss elimination method. So what we have here is our system of equations A x is equal to b then I will write down the augmented matrix, so I will write the augmented matrix for the system, the augmented matrix is A b, what the Gauss elimination procedure tries to do is to reduce this system to an equivalent system, what this will do is the Gauss elimination will attempt to do is to reduce A into an upper triangular form and in this procedure b gets changed to a new vector z where, U is upper triangular.

So we shall reduce so an augmented matrix into this particular form where U is upper triangular and z is the changed vector of b. Now when once this is a equivalent system that means the solution of this is same as the solution of this then I can find the solution of this by the back substitution method which we have just now described because U is upper triangular therefore I can get from here the solution x by back substitution. I can obtain the solution from here by using the back substitution method that we have just now discussed. Therefore, the procedure would be how to get the this particular augmented matrix from the original system A b.

We shall obtain this particular augmented matrix by using simply the elementary row transformations. So the elimination procedure uses elementary transformations, elementary row transformations. Now let us describe what is an elementary row transformation the elementary rows transformation is some of the transformation which you already know that from any given row of a matrix I can add or subtract a constant multiple of another row that means if I have a row R I, we will call it row R i is the i th row if I describe R i is the i th row then R i getting replaced by R i minus c R j where c is a a constant positive or negative constant, positive or negative number. From row R i, we are subtracting a constant multiple adding or subtracting constant multiple c of R j another row and this shall be called an elementary row transformation. Obviously, R j is not changed because R j is now it is only R i that is changing R j is not changed.

Now I would use this elementary transformation to reduce the matrix A b augmented matrix to the new augmented matrix U and z. The 2 systems are called equivalent, we say that this is equivalent to this that means all the properties that A b this augmented matrix has this augmented matrix also has the same properties including the value of the determinant. If I reduce this A to U the upper triangular matrix the diagonal what a let us now write now what is determinant of U the determinant of U would be product of diagonal elements because U is upper triangular, since U is a upper triangular it will be of the form U 11, U 22 elements here, elements here U nn these are all 0s. So if I find the determinant of this determinant of this simply U 11, U 22, U nn therefore the property of A will be retained here that the determinant of A will be simply equal to u 1, u 23, u n therefore that is what we mean by saying that all the properties of A, B are carried over to this equivalent system, these are equivalent systems.

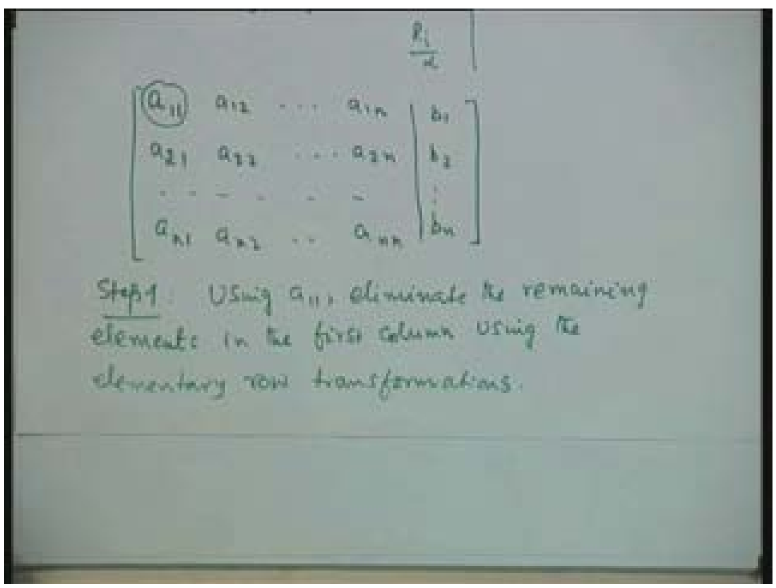
(Refer Slide Time: 20:18)



Now we can also use other elementary row transformations that means the other elementary transformations which we shall use are R i is inter changed with R j that means we are inter changing 2 rows that means we are inter changing 2 equations when we talk of row we are including b also. So we are talking of the row of the augmented matrix, so it would include both A as well as the the component of the right hand side vector. The another elementary row transformation is alpha you can multiply any row by alpha that means alpha is a constant, alpha is a constant multiply means it is also division. So there is R i by alpha this, so you can multiply a row or divide a row by a constant but we must be careful about this 2 transformations in the sense that some properties of A gets changed for example, if you are talking of the determinant if I inter change 2 rows I know the value of the determinant gets changed by a minus sign.

So when I am doing this inter changes the such inter changes will make the value of the determinant of a different sign possibly. Similarly, if I multiply a row by alpha the determinants gets multiplied by alpha or if I divide a row by alpha the value of the determinant also gets divided by alpha. So the property of the determinant gets changed when you use these 2 elementary transformations but however if you keep track of the number of inter changes you are making or the factors that you are multiplying or dividing then I would be able to give the determinant also, so that the that property also would not change. Now let us describe now, what is the Gauss elimination procedure.

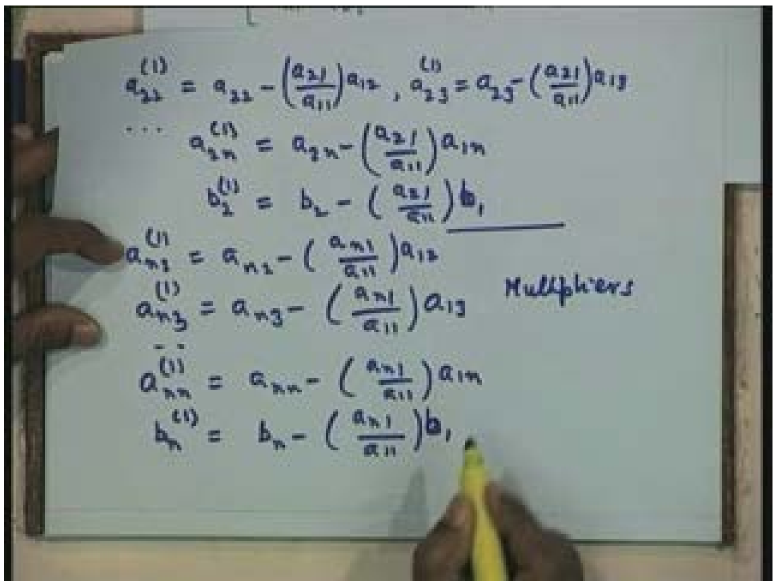
(Refer Slide Time: 23:13)



So let us take our system of equations a 11, a 12, a 1n, a 21, a 22, a 2n, b 1, b 2, a n1, a n2, a nn. Now in the first step what we do, we use this element a 1 and make all these elements as 0 by multiplying by suitable factor I would like to use the operation R i, I would like to use this operation R i minus c R j to bring this 0s over here like for example, if I multiply this by a 21 by a 11 and subtract this becomes 0 then all these elements gets changed. Similarly, I can multiply this by a 31 by a 11 subtract I get a 0, lastly if I multiply this by an1 divided by a 11 subtract I get a 0.

So in the first step I will use this first element or the first equation and then eliminate all the elements in the first column to 0 by using the elementary transformations. So that is our step 1, we will say using a 11, using a 11 eliminate the remaining elements, remaining elements in the first column using the elementary row transformations that is a special case we will discuss it, yes, that is an important case we are going to discuss at the end of this using the elementary row transformations that means if we are eliminating it what we are really doing here is that the new a 21 becomes 0, let us called the new element a 22, let us give as a super fix to it and let us call it as a 11 the new element that is obtained from a 22.

(Refer Slide Time: 25:13)

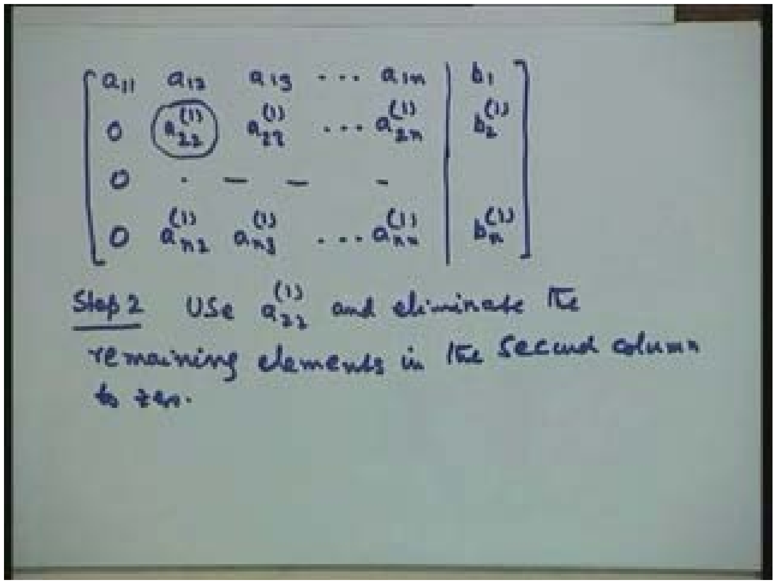


So what we are doing here is a 22 from this we are subtracting we are multiplying this element by this factor, we are multiplying this a 12 by this factor and subtracting from this. So that is a value of this new element so similarly, I will have the this next element a 23, a 23 gets changed as a 21 upon a 11 into a 13. So we are using the same factor a21 by a11 multiplying the next element here, multiplying the next element in this as a 13, a 13 element here and subtract from this to get our new element a 23 and so on. We will have a 2, n 1 that is this element will be a 2n minus a 21 upon a 11 into a 1n and b 2 the right hand side also gets changed as b 2 minus a 21 upon a 11 into b 1.

Now all these elements are changed, now we proceed on this particular elementary row operation procedure until the last. So if go to the last row the last row will now become this factor 0 and this will become a n2 of 1 is a n2 minus a n1 upon a 11 into a 12. So the this is made 0, so I have I am multiplying with a n1 divided by a 1 that is the factor I am using to make this 0. So I will have this an1 upon a 11 into a 12 subtract it have a new element. Similarly, the next element a n2, a n3 of 1 is a n3 minus a n1 upon a 11 the same factor a 13 and lastly I will have a nn is a nn minus a n1 upon a 11 into a 1 and the right hand side b n1 is equal to b n minus a n1 upon a 11 into b 1. Therefore, we have now produced 0s in the first column except for the first element and all the elements are getting changed, now these numbers we can see when we are applying for this first second row that we are completing we are multiplying by the same quantity a 21 by a

11, a 2 by a 11. When I go to the last row I am going to a n1 by a n1 this therefore each row has got 1 multiplier that is being used these put quantities have put in the brackets they are all called multipliers, they are called the multipliers for this particular elimination.

(Refer Slide Time: 28:57)



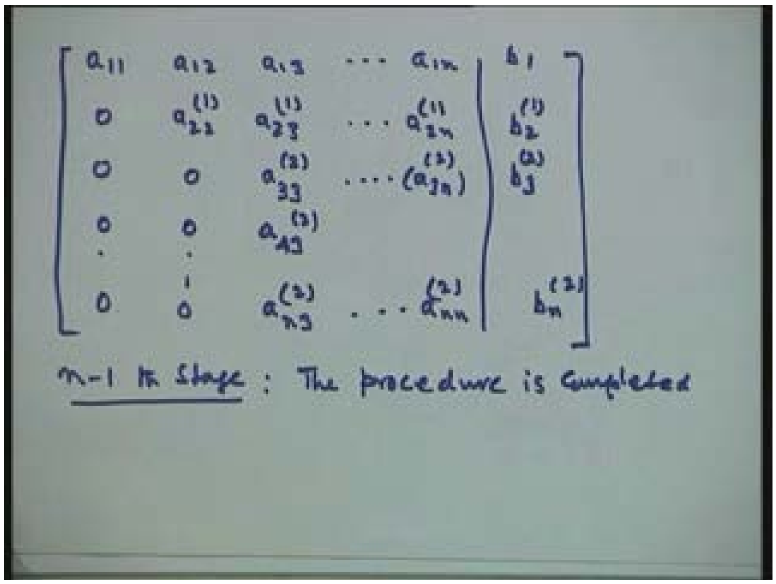
Now if I complete this particular step 1 what I am now having here is a system which would look now like like this. So let us now just write down what is the system that we are going to get from here the first row is unchanged because we have not touched the first row. So a 11, a 12, a 13, a 1n, b 1 that was not touched. So this has become a 0 this has become a 22 of 1 this has become a 23 of 1 this is a 21 of 1, b 2 of 1. So I have a 0 here all these elements so I have a 0 here a n2 of 1, a n3 of 1 a nn of 1, b n of 1.

Now the step 1 is over that I have produced 0s in the first column except for the first element 11 element. Now I proceed to step 2, now I will move to the right that is I move to the second column I will use the diagonal element 22 element leaving the first row because the operation of the first row is over. So I will now take the second row now I will choose this as my main element use this and eliminate the remaining elements in that column 2, 0s bring those elements to 2,0s. Therefore, step 2 will be use a 22 1 and eliminate the remaining elements, remaining elements in the second column, in the second column to 0.

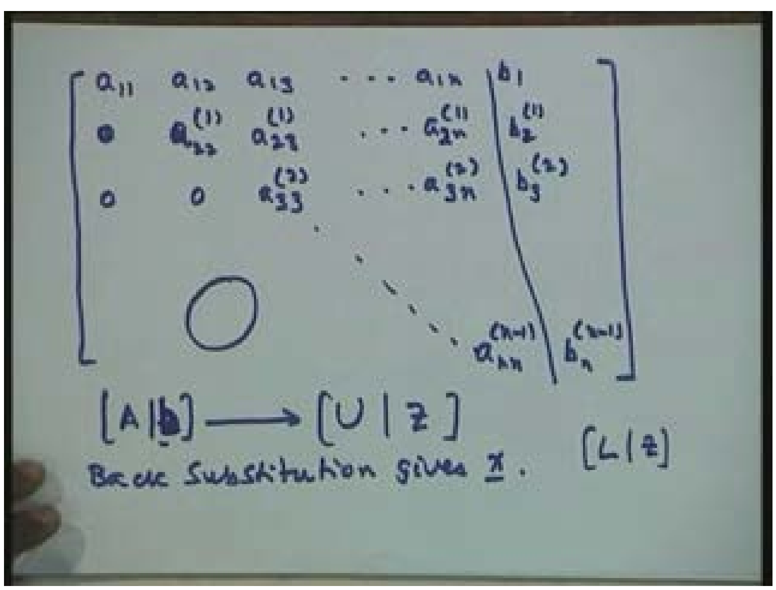
Now let us see how it looks like because I will not repeat it because say what we have done it is repeated. Now let us see how the system is going to look like after this particular step so it is now the first row is unchanged so that stays as it is, the second row is this is this second row is unchanged so it will retain as a 22 of 1, a 23 of 1, a 2n of 1, b 2. Now add a 0 here, now this has been made a0 this element changes as a 33, now we will put super fix as 2 I will have a 3n super fix of 2, this is b 3, now b 3 of 1, b 2 of 1, b 3 of 2.

Now I have got 0s over here I have got 0s over here and now I have got the elements over here, this is you are a 43 2 and so on this is your a n3 2, a nn 2, b n of 2. So after the second step, after the second step I would then have this as my new matrix. Now this is the matrix that has now we shift it to the right to the next column and we repeat this for n minus 1 stages. So when we do n minus 1 stage n minus 1th stage, when we have reached the n minus1th stage we have reached the last 2 by 2 system therefore I will use this pivot that is here to eliminate this bring it to 0 and we will have to do only one less of each iteration.

(Refer Slide Time: 31:13)



(Refer Slide Time: 33:29)

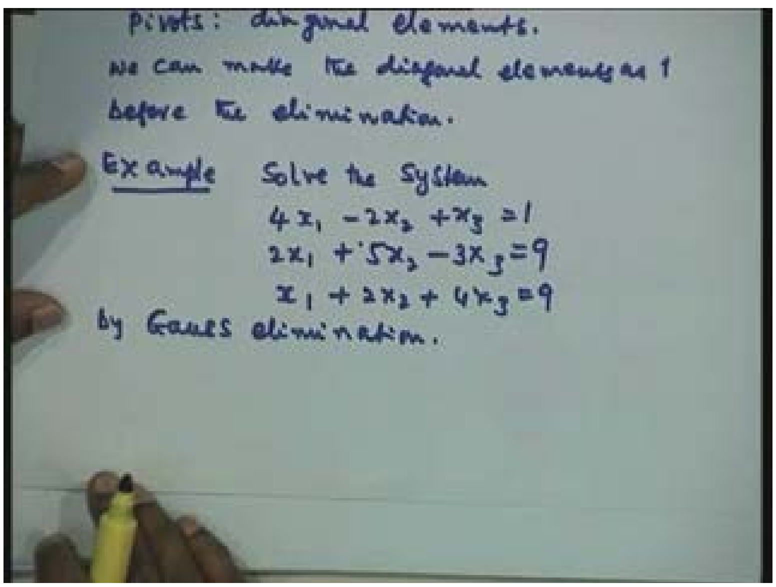


so at n minus 1 th stage the procedure is complete, the procedure is complete. Now when the procedure is complete, let us see how the matrix looks like. The matrix would therefore be I will repeat the same thing let us write down a 11, a 12, a 13, a 1n, b 1 that is the first row 0, a 22 of 1 a 23 of 1, a n1 of 1 that is equal to b 2 of 1, add a 0, add a 0, a 33 of 2 a 3n of 2, b 3 of 2 and so on. I will have here is a nn of n minus 1 and b n of n minus 1. Now this is an upper triangular matrix and this is the what we were saying that a can be reduced to the augmented matrix here A b, A b is now reduced to an upper triangular matrix and the vector as z.

Now when once we do this, now the solution will be trivial now that I will solve that u z now back substitution. So I can get the value x n from here I can get x n minus 1 from here and then therefore back substitution gives the solution x, back substitution gives x, our vector x. We can also eliminate the upper the the, if you do the upper part then the we shall be using elementary column operations then we are going to bring it as L, L into z, L of z. So they those are the you are now applying instead of rows you are applying the columns.

So the that means you are going to bring it to a matrix of the form L z if you do the column operation yes it is possible but however in computer software elementary column operations is not used but it can be used it is defined well defined. So in that can also possible but as a tradition we will not be using elementary column operations. We shall use only the elementary row transformations. Now one more the observation that can be made here is at any particular step in order to make for exam particularly with hand computation it is possible for us to before we do each step we can make this diagonal element as 1.

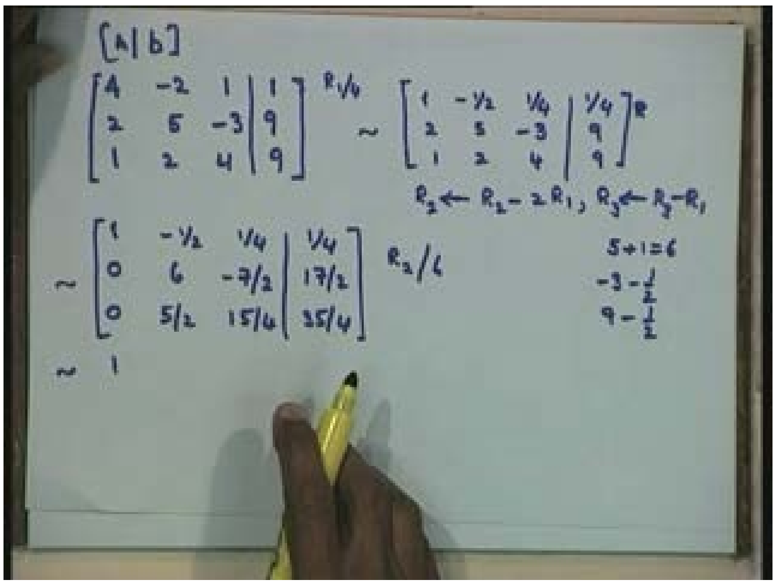
(Refer Slide Time: 36:39)



So I will divide first throughout by a 11 then it will be 1 then I will be multiplying by a 21 subtract, a 31 subtract, an 1 subtract. So it is possible for us to make the diagonal element as 1 before each step. The diagonal element shall be called as a pivot, so we shall call the diagonal elements each diagonal element the as the pivots. So we shall call this as the pivots as the diagonal elements. So what we stated was that we can make the diagonal elements, we can make the diagonal element as 1, all elements diagonal elements as 1 before the elimination. Now as I said that this will be very useful for hand computation when we do this one.

Now the the question which I have asked I will answer it late after taking an example of solving it what happens when you have a pivot as 0 or what we should do other cases. Let us first of all take an example on this and let us see how you can do it. So let us take an example, now let us call it as solve the system. Let us take a 3 by 3 system 4 x 1 minus 2x 2 plus x 3 is 12, x 1 plus 5x 2 minus 3 x 3 is 9, x 1 plus 2x 2, 4x 3 is 9 and by Gauss elimination. Now what I have to do is I have to write down my augmented matrix A b. So I will write this 4 minus 2, 1, 1, 2, 5 minus 3, 9, 1, 2, 4, 9, so I will put a line here and this.

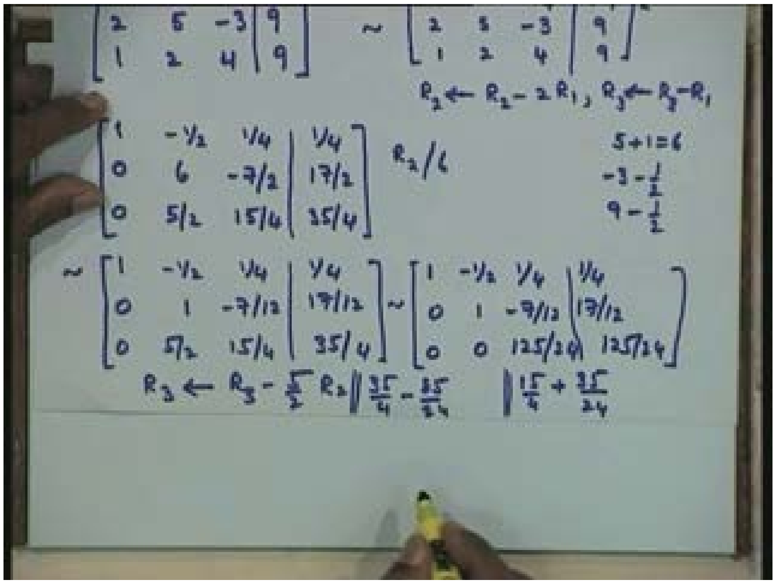
(Refer Slide Time: 38:38)



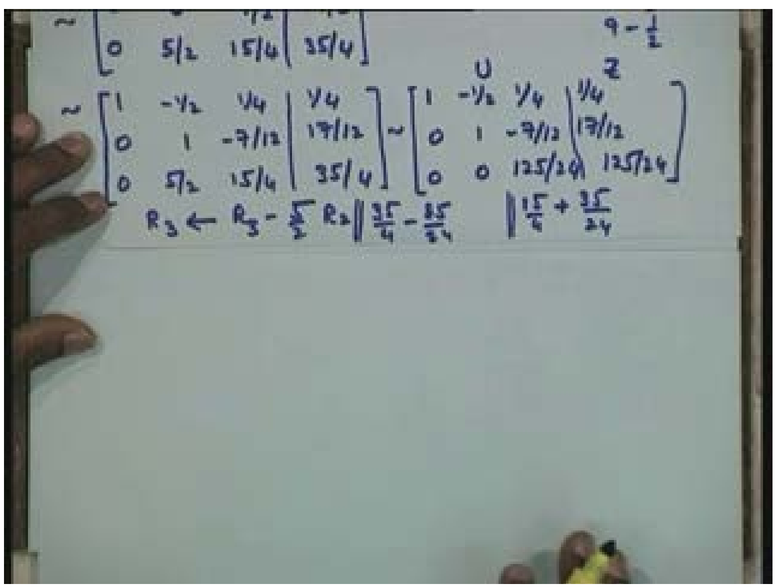
Now let us divide the first row by 4, so I will perform the operation R 1 by 4 then I will have an this notation I shall use for the showing that is an equivalent system. So what I would get an equivalent system as 1 minus half 1 by 4, 1 by 4. So these elements stay as it is now I would perform the elementary row transformations, I should make these 2 as 0. Now I must make this transformation I will write here R 2, R 2 minus 2, R 1. So from R 2 I will subtract 2 times the row R 1 and from R 3, I just need a subtraction R 3 minus R 1 if I subtract these two I will get a 0 over here so R 3 will be replaced by R 3 minus R 1 therefore lets write it 1 minus half 1 by 4, 1 by 4 I might subtract 2 times this, so let us do it here this is 25 and here multiplying by 2 and this 15 plus 1 that is equal to 6, so I will have 6 over here.

Then I have the element minus3 here and subtracting with a 2 that is equal to minus half. So I will have here minus7 by 2 then I have 9 and multiplying by 2 and subtracting I have minus half. So I will have 17 by 2 here. Now the next one is subtract these 2 rows so I have 0 here 2 plus half that is 5 by 4 that is 4 minus 1 by 4, 15 by 4, 9 minus 1 by 4 that is 35 by 4, 35 by 4. Now if it again make this diagonal element as 1, so I would now perform the I could, I should put dash here I will perform the operation R 2 divided by 6, row 2 divide row 2 by 6 then I will get a system 1 minus half 1 by 4, 1 by 4, 0, minus 7 by 12, 17 by 12, 0, 5 by 2, 15 by 4, 35 by 4.

(Refer Slide Time: 41:50)

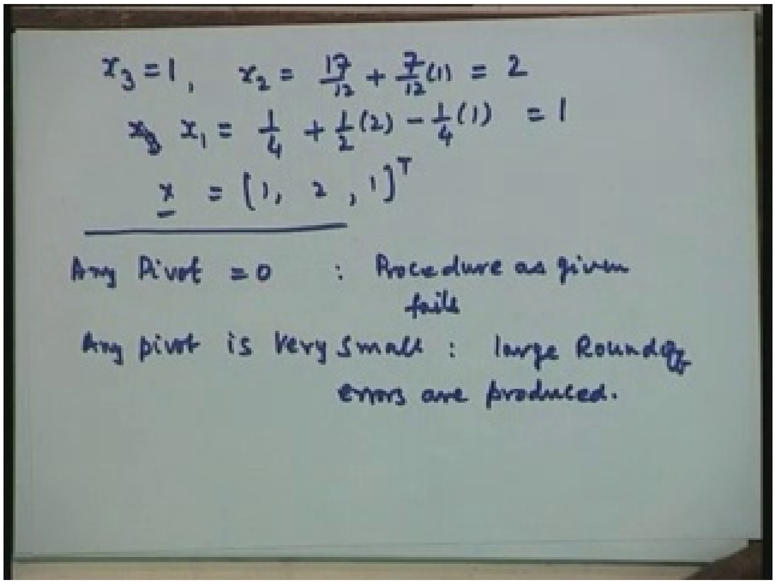


(Refer Slide Time: 43:48)



Now I need to perform the operation row 3 is row 3 minus 5 by 2 of row 2. So I have to subtract from here 5, 5 by 2 times this. So this show that this will become 0 then I will have the equivalent system 1 minus half 1 by 4, 1 by 4, 0, 1, 7 by 12, 17 by 12, 0, 0. So this this number is we can write down here this is by 15 by 4 and we are multiplying by minus this, so we will have plus 35 by 24. So this is 125 by 24, so I will have here is 125 by 24. Then let us look at the last element this is 35 by 4, 35 by 4 and minus 5 by 2 that is 85 by 24. So this is also comes are to be 125 by 24.

(Refer Slide Time: 43:53)



Now the system is complete, we are now got our U and Z therefore the last variable is x 3 is equal to 1, these 2 elements are same so x 3 is equal to 1. We substitute in the backward direction, so I will have here x 2 is equal to right hand side 17 by 12, 7 by 12 into 1 that is taking this elements to the right hand side that gives me 2, x sorry x 1 is coming from here so right hand side is 1 by 4, now I am substituting the values so take them to the right hand side half into 2 minus 1 by 4 into 1. This is equal to 1, therefore the solution is obtained as 1, 2, 1, 2, 1 as our solution vector is your solution vector this one. Of course we could as well have done it without dividing the by the pivot because at every stage we have divided here by the pivot, this pivot 4 we have divided here by pivot 6 then we have divided this by pivot to get the solution problem. We could have done it without dividing by the pivot also the now but but when we are actually doing with hand computation after 1 step I mean 1 or 2 problems you solve you would notice that I need not have written this this particular step 1 step in between.

So I could as well have gone with R 1 by 4 assuming that R 1 by 4 is done then this step when once you write this 1 I need not to write these 2 because this is a repetition. So I could then apply this R transformation itself and get this step straight straight away. So that we need not repeat some of the rows that we are going to write here. For example here also, we are repeated this step we need not have repeated this step. So when once this is performed I could immediately go with the next computation as R 3 is R 3 minus 5 by 2 R 1. So we can avoid the some of the steps and get the computation much more faster. Now you can see the how fast we are able to get solution by using the this Gauss elimination procedure.

Now the, now I would ask the answer the question which you have asked earlier, the procedure would fail if any pivots are 0. At the moment as we are looking at it we can say if pivot is 0 at any stage any pivot, so it is any pivot is 0 the procedure as given fails, as given fails not only that if any pivot is very small, is very small, if the pivot is very small then also we will be having serious probe difficulties in the solution because when you are dividing by very small number that means you are multiplying by very large number say let us suppose a pivot is 10 to the power of minus 6.

So in next step we are dividing by the pivot that means you are multiplying by 10 to power of 6 therefore the numbers have become very large and therefore, you are introducing tremendous amount of round off error, therefore this will produce large round off errors, large round off errors are produced. Now when this is the thing and in fact what, what is it that the computer software has got is that we shall now go in and also use the other elementary row row transformations that is inter change of rows.

So how we perform it is we shall call that as the pivoting procedure, so such a thing is called a pivoting procedure. I would just briefly explain and close it for today if at any stage pivot is 0 or very small element or right from the beginning we can start, so that the if you are tackling in a computation procedure smaller and smaller numbers you are at a great advantage because the amount of round off error will be much less only when we are tackling very large numbers the round off errors becomes more and more, what we do is we start the first column of the in the step I will search for the largest element in magnitude. Then I will bring that largest element as the pivot that means I will inter change the first equation with that equation which has got the largest element in magnitude that means it is simple inter change of the equations the then computation is done I go to the second step at the second step I use the pivot and then compare it with the elements below that in that second column.

I am not bothered about the first element, the elements below that if necessary I will inter change the elements. Suppose we have a 0, as we have say 0 automatically thrown being be thrown down, so it will go to the next some other row. So this procedure is called a pivoting procedure, in this procedure we are only inter changing the equations therefore we are not changing the property of the solutions system except that the first as becomes 100 th equation, 100 th equation as come as the first equation, in the second stage the second equation might have been thrown as the 105 th equation 105 th as come to this portion.

So we are now inter changing the equations only and we are not touching the position of the variables. So the solution of the problem when you have land up finally is going to the same thing and in that case we are now completely avoiding this particular failure that may occur and that is called the partial pivoting procedure. So that we will describe it in next time.