Numerical Methods and Computation

Prof. S.R.K. Iyengar

Department of Mathematics

Indian Institute of Technology Delhi

Lecture No # 11

Solution of a System of Linear Algebraic Equations

In today's lecture we shall start the discussion on the solution of a system of linear algebraic equations and the Eigen value problem.

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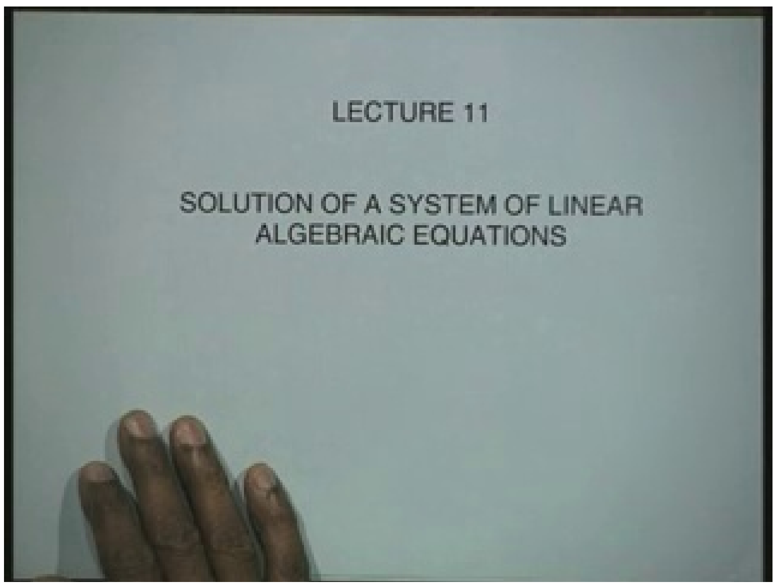

The topic of the solution of the linear algebraic equations or Eigen value problem is one of the very important topics of numerical analysis. The reason being that if we are solving a linear or nonlinear ordinary differential equation or a linear or nonlinear partial differential equation or a system of differential equations, by any one of the numerical methods like finite difference methods or other methods. We produce correspondingly a system of linear algebraic equations or nonlinear algebraic equations. Now we know that nonlinear algebraic equations can be solved by Newton's method or a number of variants methods are available for us. This reduces the given nonlinear system to a linear system and then solves this in an iterative procedure. Therefore the basic problem is how to solve a linear system of algebraic equations when such a system converges, how do we solve, what are the techniques for this, if the system is sufficiently small how we solve, if the system is very large then how to solve. For example if you are designing a particular object and it governs a system of partial differential equations and the number of points at which we need to find the solutions goes in millions of points, therefore the system of equation that we get will be few millions of equations; how do you solve such huge system of equations. So we would like to have discussion on such problems.

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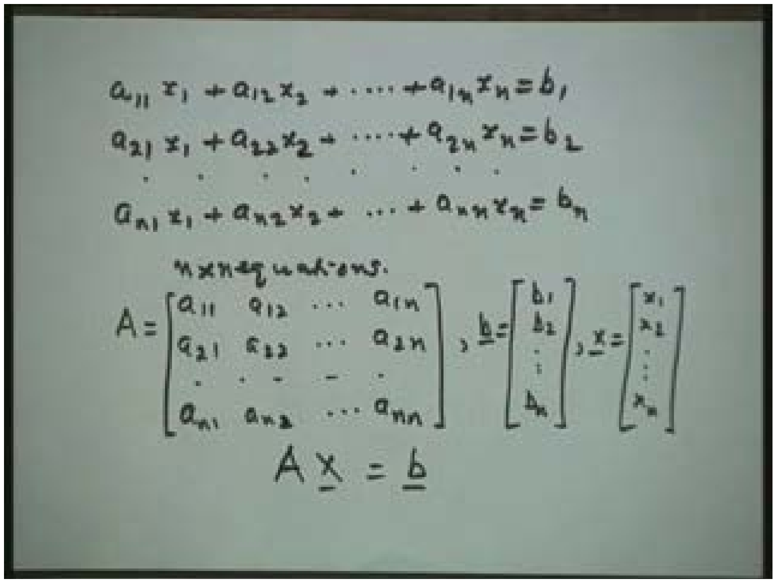

What we are really looking for is a linear system of equations like a_{11} x_1 a_{12} x_2 … a_{1n} x_n is equal to b_1; a_{21} x_1 plus a_{22} x_2 plus … a_{2n} x_n is equal to b_2 and a_{n1} x_1 a_{n2} x_2 plus… a_{nn} x_n is equal to b_n. We are discussing here a system of n into n equations. It is possible for us to find the solution for a system of equations but we shall not be discussing here at the moment. Now here the variables x_1 x_2 x_n corresponds to interpretation which depends on the problem that we are solving but in general we shall not be bothered about what x_1 x_2 x_n are with respect to corresponding problem. So depending on the physical problem they could be stresses, they could be velocities, they could be accelerations or they could be any other variable that is there in the physical problem. Now in order to have a discussion on this let us write in the matrix representation. So what I would write is the matrix A as a coefficient matrix. So I will write this $a_{11,}$ $a_{12,}$ a_{1n}. This is a_{21}, $a_{22,}$ $a_{2n,}$ a_{n1}, $a_{n2,}$ a_{nn}. So I will take this as matrix A and we will write down vector with an underscore as the vector b_1, b_2, b_n and the variable x as the solution vector x_1 x_2 x_n. If I write these coefficients matrix as A, the right hand side vector as b and the solution vectors as x then I can write down the given system of equations as AX is equal to b. So this will be my basic system of equation that we would like. I am interested in the matrix formulations and use the matrix formulation to derive the methods for the numerical solution of these equations. Now before we actually derive the numerical methods, let us just briefly review what we have done in the properties of matrices because we shall be using those properties of matrices in deriving the methods. A particular method suiting a particular type of system of equations like a symmetric system or Hermitian system like, depending on the type of equations that we have, we shall choose a particular method. So let us just briefly write down what are the properties of a matrix.

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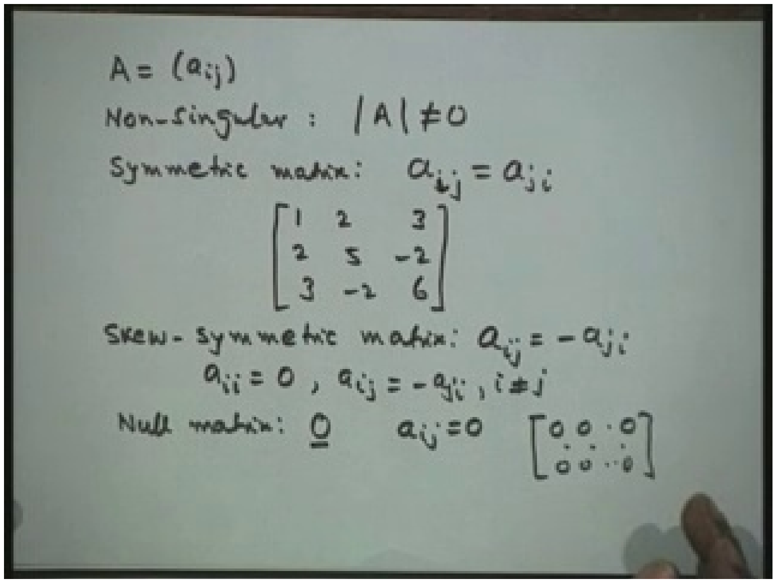

We shall take the matrix A as the elements a_{ij}; so that's what we have written. The elements are a_{11}, a_{12} … a_{1n}. Now first of all we define a a non singular matrix; if the determinant of matrix A is not equal to zero then we shall call it as a non-singular matrix. So I can find the value of the determinant and determinant is not equal to zero. Then I define a symmetric matrix; if the elements of A satisfy the property that a_{ij} is equal to a_{ji}, and then the matrix is a symmetric matrix. For example I can easily construct a symmetric matrix. I can fill these elements here. For example this is our symmetric matrix. So you have a_{12} is a_{21}; a_{13} is a_{31}; a_{23} is equal to a_{32}. Diagonal elements are given to us. So this is a symmetric matrix; whereas if it satisfies the property that there is a minus sign here then we shall call it as a skew symmetric matrix. In other words these elements a_{ij} is equal to minus a_{ji}. Now we can obviously see by by setting a_i is equal to j we can get a_{ii} is equal to minus a_{ii}, that means the diagonal elements are always zero and obviously a_{ij} is equal to minus a_{ji} for all other elements i_0 is equal to j. So the diagonal elements for a skew symmetric matrix are always zero. Then we define a null matrix as the matrix with all elements, zero elements. So all a_{ij} is equal to zero. So we just have the null matrix as 0, 0, 0 … 0, 0, 0, and 0. So it is just a null matrix with all elements of the matrix A as zero. Then we shall say it is a null matrix and we shall use the notation zero with underscore to say it is a vector we are talking of or a matrix we are talking of.

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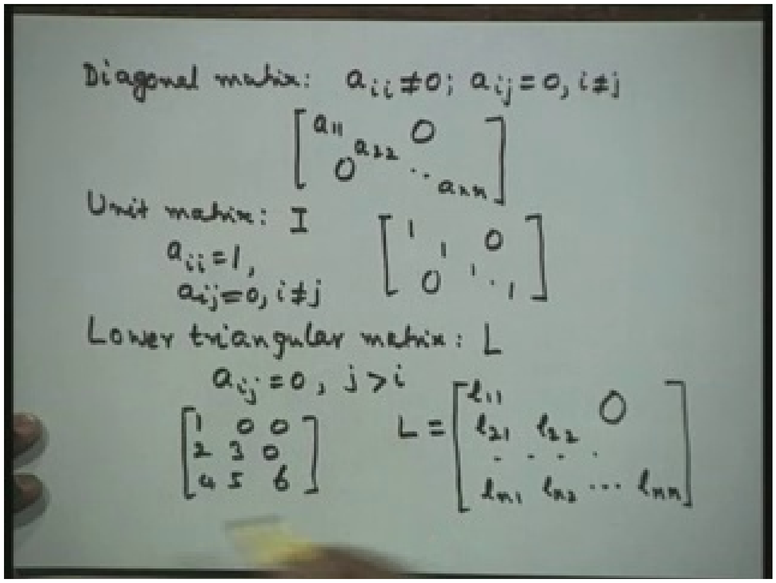

Then we define a a diagonal matrix ; a diagonal matrix is a matrix in which only diagonal elements are existing and half diagonal elements are all zeros. So it will mean that a_{ii} is not equal to zero and a_{ij} is equal to zero; for all i_0 equal to j. That means it will look like a matrix a_{11}, a_{22}…a_{nn} and these are all zeros. So the diagonal matrix will be simply the elements in the diagonal. They are all non-zero elements and the half diagonal elements are zero elements. Then we need a definition of unit matrix also; unit matrix is a diagonal matrix with all diagonal elements as one. So we shall denote this by I, so that the definition of this is simply 1, 1, 1, 1 and these are all zeros. That means we are taking all elements a_{ii} is equal to one and all half diagonal elements is equal to zero for i_0 equal to j. We have this as a unit matrix. Then we define a lower triangular matrix; in a lower triangular matrix all the elements on the diagonal and below the diagonal, they are non-zero and the elements above the diagonal they are all zero. That means we are defining a_{ij} is equal to zero for j greater than I, that means we are talking of a_{12} a_{13} all are zero. That means I can denote this lower triangular matrix by L, so I will write down the matrix L as $l_{11,}$ l_{21}, $l_{22,}$ l_{n1}, $l_{n2,}$ l_{nn}. All the elements above the diagonal are zero. So this is a lower triangular matrix. The elements in the lower triangular, they are non-zero and all the elements above the diagonal they are all zero. For example, we can take this as follows; so this is a three by three lower triangular matrix in which the elements above the diagonals are all zero. Similarly we define an upper triangular matrix.

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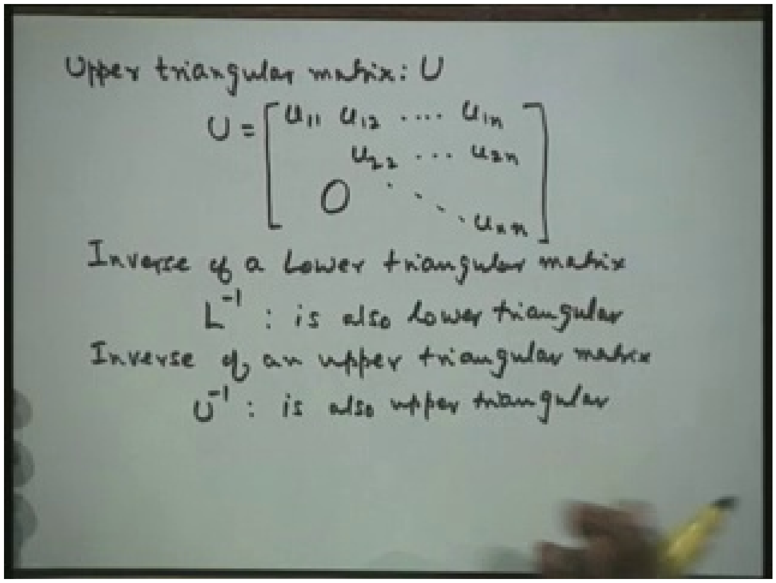

We denote an upper triangular matrix by U; all the elements on or above the diagonal are non-zero and all the elements below the diagonal are zero, that means we have U is equal to a matrix like u_{11}, u_{12}, u_{1n}, $u_{22,}$ u_{2n}, u_{nn} and this is all zeros. All the elements on or above the diagonal are non-zero and all the elements below the diagonal are zero. The important point we would like to note here is we know the inverse. So if I have an inverse of a lower triangular matrix, that means L inverse, I want the L inverse is also the lower triangular. Therefore if I have a lower triangular matrix and its inverse, its inverse will also be a lower triangular matrix; L inverse is also a lower triangular. This is an important property that we shall be using later on. Similarly the inverse of an upper triangular matrix is also an upper triangular matrix. So the inverse of an upper triangular matrix which we shall denote as U inverse is also upper triangular. Both these properties we shall be using in constructing some of the numerical methods for the solution of the algebraic equations.

Then we would like to define what is known as tridiagonal matrix. So tridiagonal matrix is one in which only the leading diagonal, one above and one below, that means the three diagonals are non-zero, all other elements of the matrix are zero's. This means what we have here is a matrix in which this is a diagonal, this is a diagonal, this is a diagonal, all the others are zeros; these are non-zero elements. Sometimes it is called the leading diagonal, the super diagonal and sub diagonal. So it is sometimes it is also called as the super diagonal and sub diagonal. So the elements of these three errors are non-zero and all other elements are zeros. Mathematically of course we can say that this a_{ij} is equal to zero, for all ij is greater than one. So if the magnitude of the difference is one, which means only these three diagonals are going to be non-zero, otherwise it is going to be zero for all other values. So the tridiagonal system is a very important thing that comes by solving the ordinary differential equations by any difference method that we get. We get a system of linear algebraic equations in which the system is the only tridiagonal and the solution of tridiagonal system is very important and also the simplest of all the systems that we can get it.

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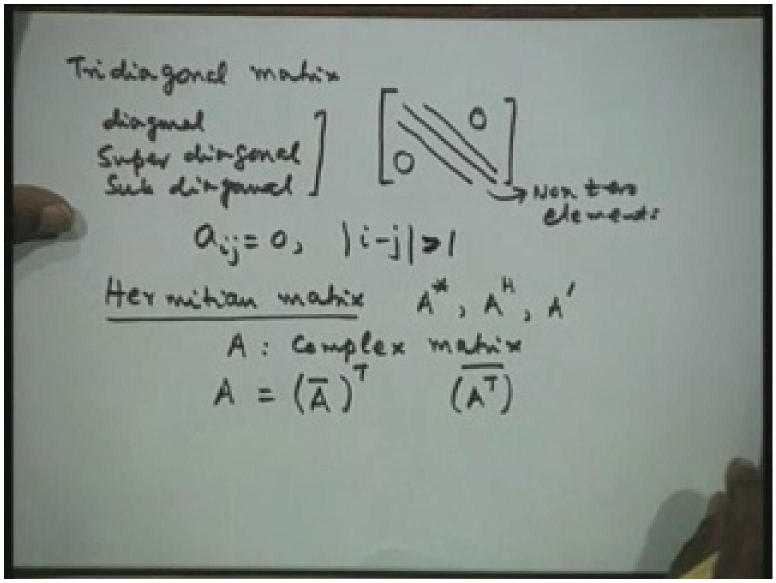

Then we also define the Hermitian matrix. Now we are talking of complex matrices; the elements of the matrix are complex. So A is a complex matrix; there are various notations for Hermitian matrix for A depending on the books. It is denoted by A* and some books write it as A^H, some books even write it as A with a prime; so all these three are the notations for a Hermitian matrix. What this defines is that given a matrix A we construct it's complex conjugate, we take it's transpose and if this is equal to the given matrix A then A is a Hermitian matrix. We can of course take the transposition first and then we can take the conjugate also. It would not matter whether we take the conjugate first or transpose next. Then if this property is satisfied, then the given matrix is said to be Hermitian matrix.

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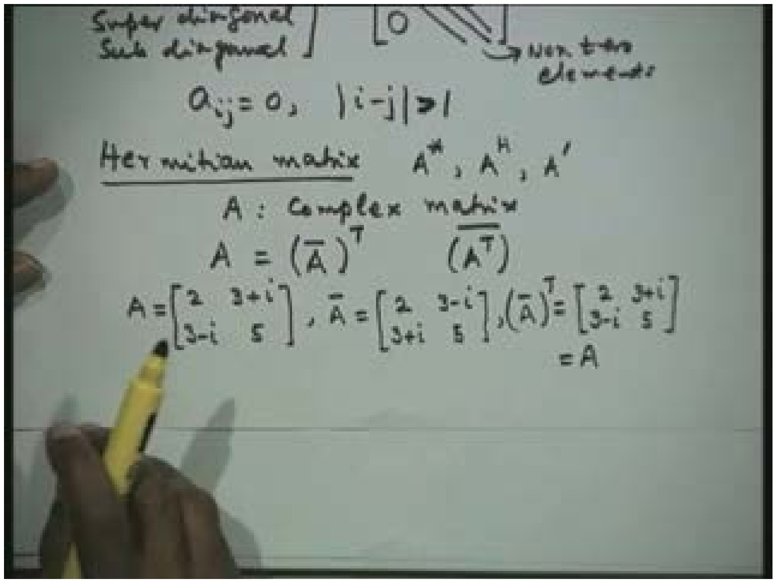

Now let us take an example. A is equal to 2 plus $3_{i;}$ 3 – 2_{i} is equal to this. Then I can construct its conjugate that is the conjugate of all the elements. This is 2 3 –i; this is 3 + i and this is 5.Then let us take the transpose of this matrix. The transpose of this is 2 3 -i; 3 +i and 5 which is same as A. Therefore this is a Hermitian matrix. Now you can observe two properties here. The diagonal elements are all real, so if the diagonal elements are not real then its conjugate will not be equal to that. So once the conjugate is equal to this, then these are all real elements because this will then be read as some A plus ib is equal to A minus ib, so that we can simplify and then show that the diagonal element should all be real elements. These are all complex conjugate of each other. So this is 3 +i; this is 3 –i. So they are conjugate of each other, so that when we take the conjugate and then transpose it and then get back to matrix as A.

One more observation that we make here is as follows. That is if A is a real matrix, instead of complex, then its conjugate is same as that itself; that means we can drop this conjugate part, that is A is equal to A transpose. Therefore a symmetric matrix, therefore the Hermitian matrix is generalization of symmetric matrix to complex matrices.

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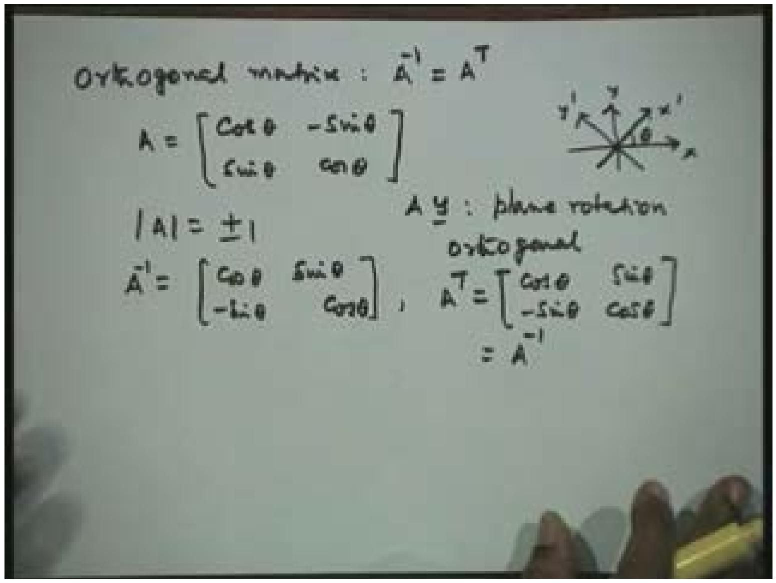

Now let us define an orthogonal matrix which is a very important matrix for our purpose. Finding the Eigen values using numerical methods, you would entirely define a construction of orthogonal matrices. So we define the orthogonal matrix as A inverse is equal to A transpose that means the inverse of the matrix A is same as the transposition of a matrix A. The simplest example of an orthogonal matrix is if you take the two dimensional coordinate system; we take your xy coordinate, rotate this entire system by angle of theta and form a new coordinates system. This is our coordinate system, then the transformation gives you the coordinate change of the coordinate system maximum to x prime, y prime is governed by this particular coefficient matrix cos theta minus sin theta sin theta cos theta. If we apply it on a particular vector it would give me a rotation by an angle of theta in the plane. If you apply it on any vector y, it will give us a plane rotation that means a rotation in a plane. This is the simplest example of an orthogonal matrix because they are orthogonal system of coordinates and we are rotating by an angle of theta. So this will give us an orthogonal rotation. So what this will provide is a plane rotation but this is an orthogonal rotation.

Now we can very easily show some of the properties that determinant of A is always equal to plus minus one. So cos square theta plus sin square theta is one, so the determinant is always one. This is a very important matrix, we can show it. For example, its inverse is co factor of cos theta is cos theta; co factor of this is cos theta; co factor of this is minus sin theta; it is a transposition. So I will have here this is equal to sin theta and this is equal to minus sin theta and the A transpose. If I transpose this I will get here cos theta minus sin theta sin of theta cos of theta, so this is equal to A inverse. So therefore A inverse is equal to A transpose and this is an orthogonal matrix. As I said this is a very important matrix for us.

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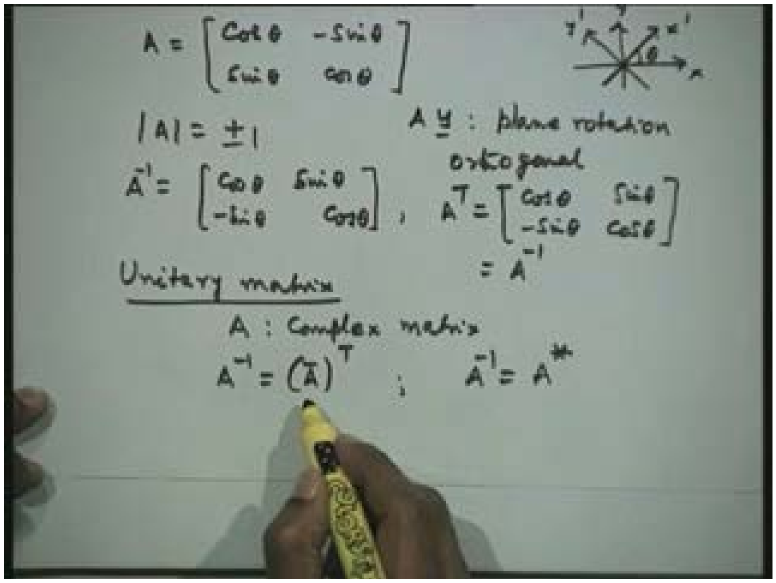

If A is a complex matrix then I would define what is known as unitary matrix. So I will call it as a unitary matrix. Now again we are taking A as a complex matrix. Now the definition is; A inverse is A bar transpose, that means we can also call this is A is equal to A star, where A star is conjugate transpose. Therefore A inverse is equal to A conjugate transpose. Now we shall say that this is a unitary matrix. Now you can see that when A is a real matrix, then A conjugate is same as A. Therefore A inverse is equal to A transpose that means we go back and show that this is an orthogonal matrix. Therefore the definition of an orthogonal matrix generalized to the complex elements would produce a unitary matrix. Hence as I said orthogonal matrix is very important and very often the system that arise in the practical problems, they are all not real coefficients. Many problems are there in which we get complex elements. Therefore instead of using the orthogonal plane rotations we shall use the unitary matrices to produce the required rotations but with the complex elements.

Then we give one more definition of a positive definite matrix. The general definition is if I take an arbitrary vector, (x is an arbitrary vector) then I form its conjugate transpose A. We are talking of a matrix here x transpose Ax. I will put an underscore for this. This is equal to greater than zero; for all x, not equal to zero and this will be equal to zero; only for x is equal to a null vector. That means if I take the arbitrary vector x, then perform this operation this is your conjugate transpose Ax. Then this is going to be a number because if A is a_n into n matrix, then you have vector x is equal to n into one vector; then this will be transpose of this. Therefore this will be one into n, A is n into n and x is equal to n into one, so these are the orders of the matrices. So that I would simply get this as one into one therefore what we get here is a number and this number should be strictly positive for arbitrary x. Then this matrix shall be called as positive definite. Even though this is theoretically possible for us to do it for a two by two or three by three or four by four matrix, for a general matrix it will be difficult. So you need some alternative way of looking at it. An alternative way from this definition is all the leading minors of A should be strictly positive; leading minors means the determinant of one into one leading matrix, the determinant of leading two into two matrix, so that we are taking all the leading minors; not all the minors but only the leading minors (that means all along the diagonal element). So all the leading minors are greater than zero.

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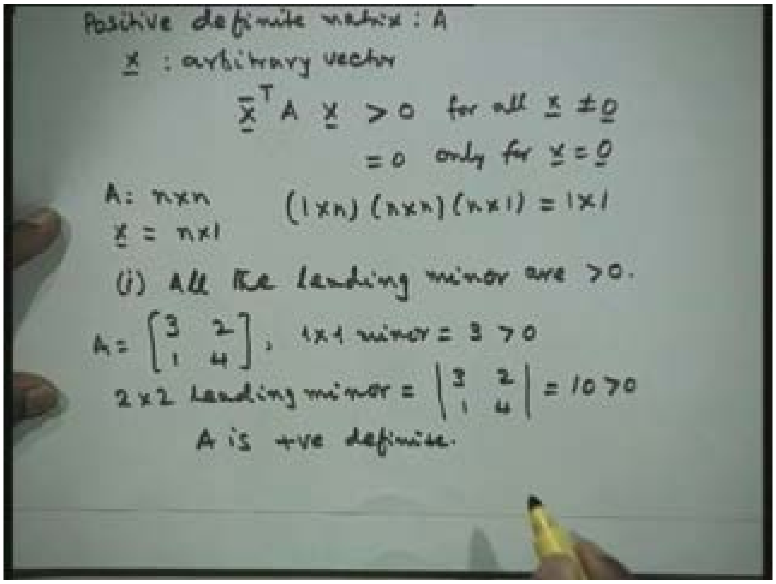

Let us take a very simple example. Let me take this matrix as A. So I would therefore first of all consider one into one minor, which is a leading minor. One into one leading minor is three, greater than zero that is the one into one minor. So then I need two into two leading minor; two into two will be simply determinant of 3, 2, 1, 4 that is equal to twelve minus two; that is ten is greater than zero. Therefore by this definition this is a positive definite matrix; therefore A is positive definite. So this is a simple criterion that one can use to find out the positive definiteness of the matrix; this is possible for a small matrices.

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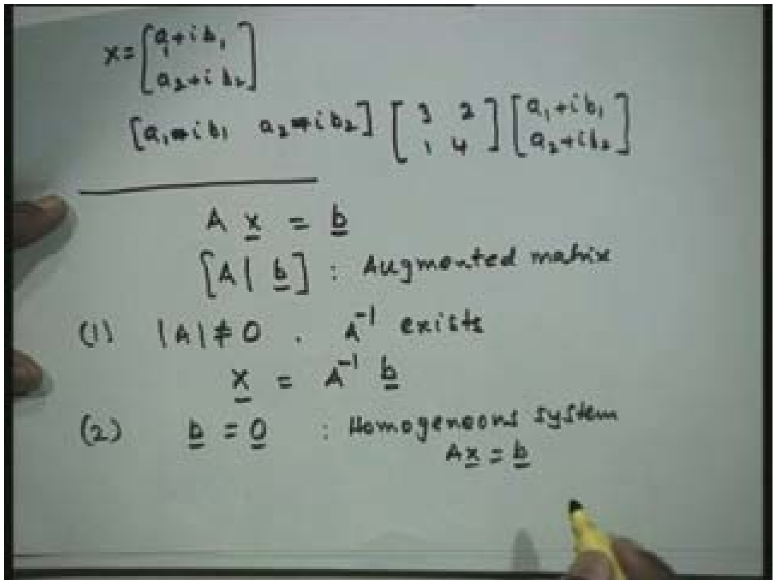

We could as well take here say x is equal to this vector; x can be taken as its two by two matrix. So I can take this sum a plus ib, a_2 plus ib_2 and then construct my x transpose A. So that will be a_1 plus $ib_{1,}$ a_2 plus ib_2 with a negative sign. We are talking of the conjugate transpose. So the conjugate of this will be minus sign; transpose will be a row vector and A is your 3, 2, 1, 4 and multiply this by x; so multiply this by x, so I would therefore multiply this by a_1 plus ib_1 a_2 plus $ib_{2.}$ I multiply it out, simplify the whole thing and show that this is indeed a quantity which is strictly positive, which is possible for us to write it in the perfect square and then simplify. Let us now come back to some more concepts which we require before we construct numerical methods.

Now we start with the matrix system Ax is equal to b. We mentioned earlier that the representation or the definition of x depends on various different problems. So we are not really concerned what is the definition of x; it represents some physical solution. Therefore we shall use only the matrix A and the right hand side vector and then discuss how to solve this particular problem. Whatever solution we get out of this is a physical representation of the problem. So we shall call this matrix as an augmented matrix; that means A is augmented by the right hand side vector b. Some more methods which we learnt earlier for the solution of linear equations, let us briefly discuss and see what are the difficulties in those methods which we cannot apply for large systems. Let’s assume that the determinant of A is not equal to zero; then A inverse exists. If determinant of A is not equal to zero, then A inverse exists. Then I can write down the solution of this problem as x is equal to A inverse of b. Now secondly if I have a right hand side as zero that means it is a homogenous system. Then what we produce is a homogeneous system Ax is equal to b. Then if determinant of A is not equal to zero, then by this definition we will have x is equal to A inverse into null matrix; so that is equal to zero. Therefore this is always a trivial solution. There is no other solution of the problem except a trivial solution. If A is a non-singular matrix and you have a homogeneous system, the only way that this problem can have non-trivial solution is that the determinant of the coefficient is equal to zero; and then it will have non-trivial solutions. So for a homogeneous system we need that the determinant of A should be equal to zero in order that the non-trivial solution for this particular problem exists.

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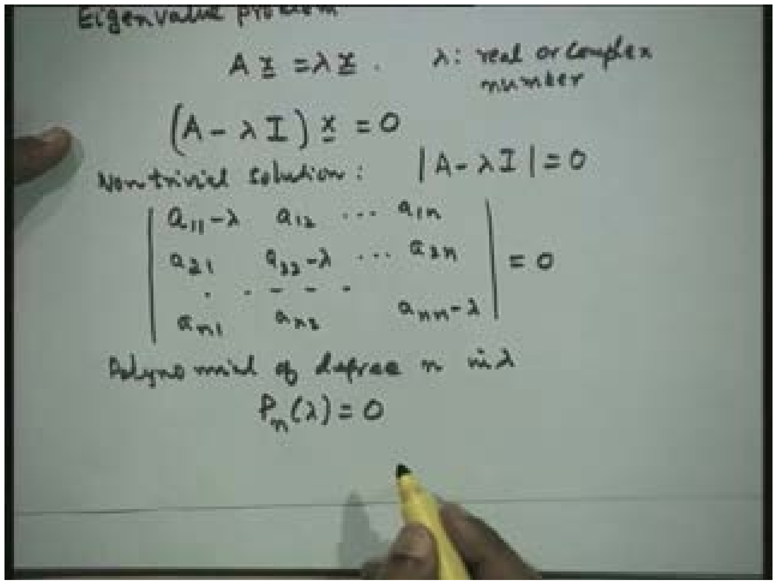

Now let us also define an Eigen value problem which we have learnt earlier.
So let us define what an Eigen value problem is. What we have here is a system A into x is equal to lambda into x, where lambda is a real or complex number. The problem is to determine lambda and the corresponding x. If I bring this to the left hand side, I can write this as A minus lambda into I lambda is a number, therefore I will write down lambda into I into x is equal to zero; so I bring everything to the left hand side. Now this is a homogeneous system of equations. Now we just said that if it is homogenous system, non-trivial solutions exist only if the determinant coefficient matrix is zero. Therefore for non-trivial solutions, for obtaining the non-trivial solutions for this problem we will require A minus lambda I is equal to zero. We require a determinant of A minus lambda I should be zero in order that non-trivial solutions exist. What is A minus lambda I? Let us just represent what it looks like. This is equal to A_1 minus lambda into I that is only lambda in the diagonal elements. So this is $A_{12\ldots}$ $A_{1n,}$ this is A_{21}, A_{22} minus lambda $A_{2n,}$ A_{n1}, A_{n2}, A_{nn} minus lambda determinant is equal to zero.

Now I can expand this determinant. There is lambda in each of the diagonal elements; therefore it will give me a polynomial of degree n in lambda. Therefore this gives us polynomial of degree n in lambda, so that we can write this as polynomial degree n in lambda is equal to zero. That is what we would get here like this. Now we have discussed earlier the methods for finding the roots of a polynomial, therefore we shall use those methods for finding the roots of this particular polynomial which is a polynomial degree n. Therefore it has got n roots, lambda one lambda two so on lambda n which may be real, which may be real and distinct, which may be complex and a combination of real and complex also. Therefore we shall be interested in finding the Eigen values of a given particular problem numerically. If once lambdas are known, I can go back to this particular equation and then find out what will be the values of x.

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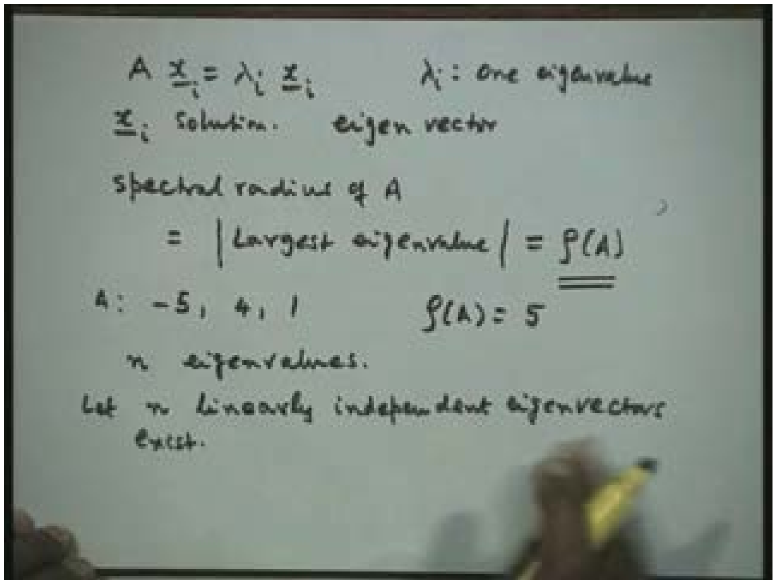

A into x is equal to lambda i into x. I take a particular lambda i as a_1 Eigen value. If I take this as one Eigen value and then construct this particular problem and I solve this particular problem; we know that the non-trivial solution exists now for lambda$_i$. Therefore non-trivial solution exists for this. Therefore x will be the solution and this is called the Eigen vector. So this is called the Eigen vectors. So to the lambda i and the corresponding solution we can now put suffix I; for this also we can put suffix i. Therefore the Eigen value problem is to determine the Eigen values and the corresponding Eigen vectors xi. So that will complete our Eigen value problem. One important property of the Eigen values of a matrix is, we define what is known as spectral radius of matrix. So I would like to define what the spectral radius of a matrix is. The spectral radius of a matrix is the largest Eigen value in magnitude. So I find all the Eigen values and find the largest Eigen value in magnitude. So magnitude of largest Eigen value is equal to the spectral radius and the notation that we normally use is this; rho of A denotes the spectral radius of a matrix. For example, Let us suppose a matrix has got these three Eigen values; three by three matrix has got three Eigen values. So we are talking of spectral radius A as the largest Eigen value in magnitude. Therefore spectral radius of A will be equal to five. The largest Eigen value in magnitude is equal to five. It is possible that a complex pair may turn out to be having the largest magnitude than that would be the largest magnitude of that particular matrix. So this is an important concept that we shall be using while proving the convergence of the iterative method that we are going to construct for the solution of these equations.

Now if I have a Eigen value problem, it has got n Eigen values; so we have got n Eigen values. It is possible that the system that is given to us may have n independent Eigen vectors; that means for each Eigen value we have one Eigen vector or we may be falling short also; that means if you have a repeated Eigen value, say Eigen value is 1, 1, 1; it is possible that the system does not have three linearly independent Eigen vectors and it has got a repeated Eigen value. So let us assume that it has got n linearly independent Eigen vectors. So let n linearly independent Eigen vectors exist.

Now these linearly independent Eigen vectors, we can take it as the basis of a vector space and then we shall say that these forms a vector space which we call it as the Eigen space. Suppose you have got a two by two matrix. I have got two Eigen vectors there. So those two Eigen vectors can form the basis for a two dimensional coordinate system just like your x axis y axis coordinate system. We can have any other vector in two dimensional coordinate system as the basis of the two dimensional system. Therefore since these are n linearly independent Eigen vectors they can form an Eigen vector space called the Eigen space.

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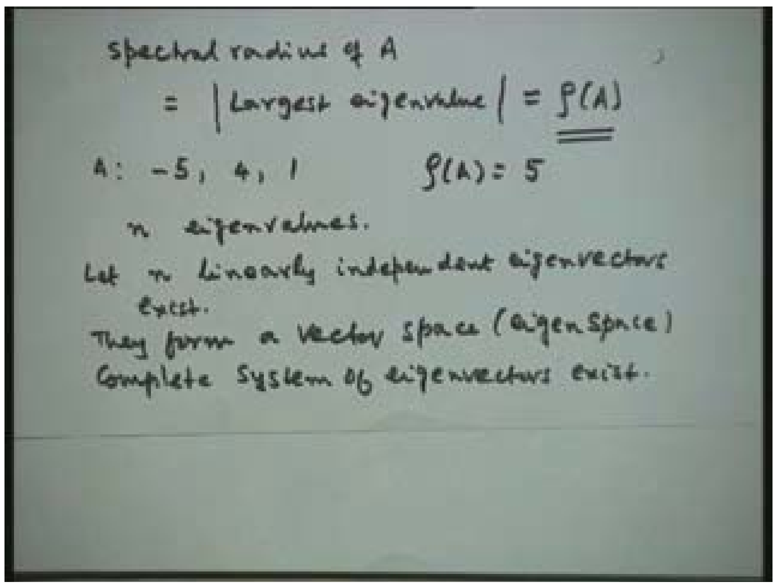

Therefore they form a vector space. They form a vector space which we call as the Eigen space. In this case A has got a complete system of Eigen vector. If it is short we shall say it does not have a complete system of Eigen vectors. If the system has n linear independent Eigen vectors we shall say that the complete system of Eigen vectors exists. If I construct the Eigen vectors, in the solution of the Eigen value problem also we need an initial approximation for Eigen vector because we want to construct the Eigen vectors. I would assume that the Eigen value problem we are talking of has a complete system of Eigen vector, so that any vector in this space can be written as a linear combination of this. For example, in two dimensional coordinate system any vector can be written in terms of the base vector i and j. So we are able to construct any vector in terms of the base vectors. Similarly in this space if I take any vector y, I can write this y as a linear combination of these Eigen vectors $x_{1,}$ x_2, x_1, $x_{3,}$ x_n.

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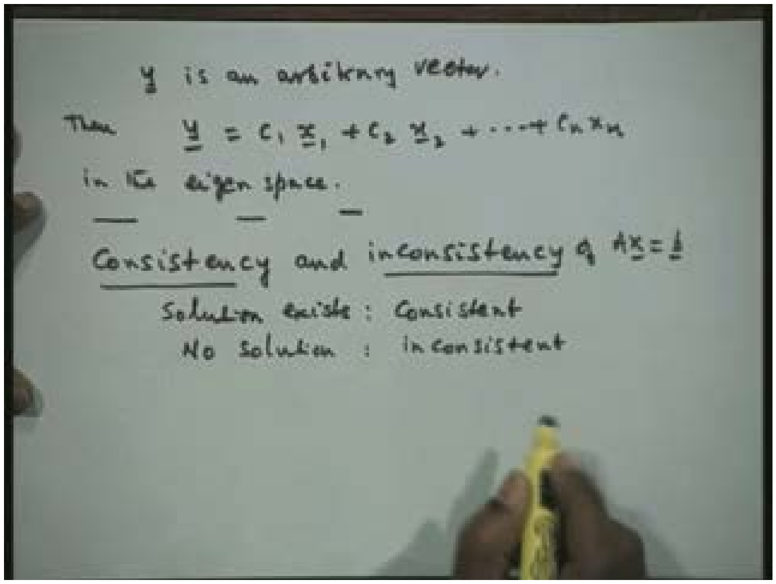

This implies that if y is an arbitrary vector, then I can always write y as c_1 x_1 plus c_2 x_2 so on c_n x_n in the Eigen space. This is a very important concept for us which we shall be using while constructing the numerical methods for finding the Eigen values. Now before we actually define some numerical methods let us also look into the concept of the consistency and inconsistency of a system of equations which we have earlier studied. Let us define what we mean by consistency and inconsistency of a system of linear algebraic equations. So what we have here is Ax is equal to b. Now what we really mean is consistency means solution exists and inconsistency means no solution. So that is a general definition. We can define that if the solution exists, then we shall say it is consistent and if there is no solution, then it is inconsistent.

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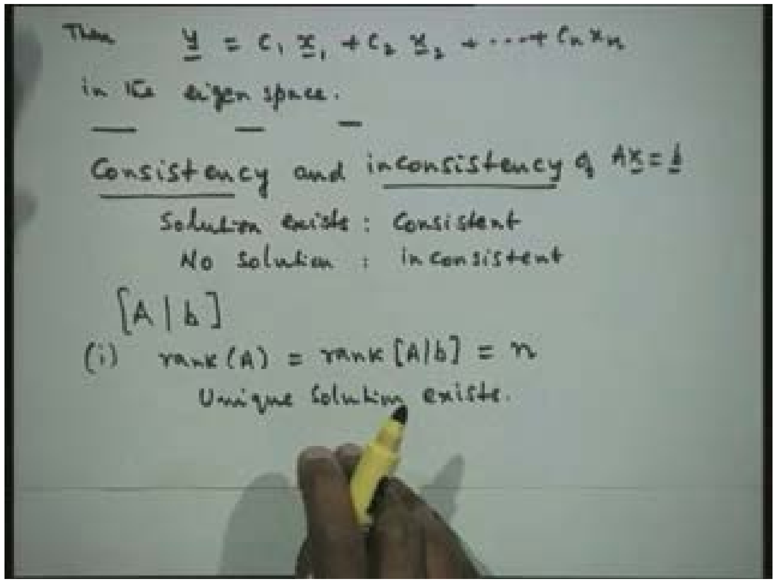

Now the method which we know earlier is that I construct the augmented matrix Ab. Then if the rank of A is same as rank of this augmented matrix, then the system has a unique solution (only one solution). So the first property that you know is that the rank of A is equal to rank of Ab. Rank A is equal to rank of Ab is equal to n. Let us also take it as equal to n. Then a unique solution exists. So unique solution exists means, let's also say therefore it is consistent. So we wanted the definition of consistency.

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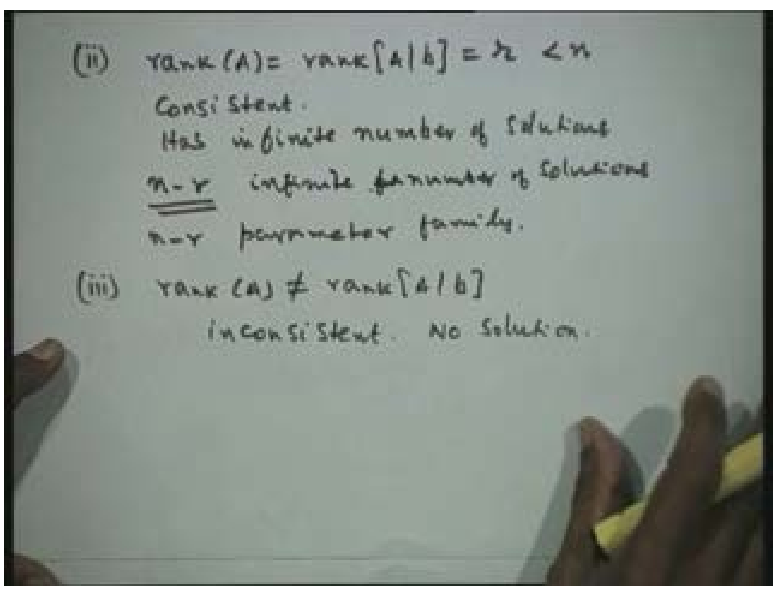

Let us take that rank of A is equal to rank of Ab is equal to some number r but less than n. Then the system is consistent but it has infinite number of solutions. So this system is consistent but has infinite number of solutions. Now the infinite number of solutions can also be characterized further. What really this shows is that your system of equations has many dependent equations. The number of dependent equations depends on what is this value of r. If this value of r is n

minus one, then we have one dependent equation. If this value is n minus two, you have two dependent equations. If this is r, then there are n minus r dependent equations. Now if you have one dependent equation then one variable is extra. So we can take it to the right hand side and then say it gives you one parameter family of solutions. Similarly if you have n minus r as dependent equations it will give you n minus r parameter family of solutions. So it is not only infinite but it is also an infinite parameter family of solution. So this has got n minus r infinite number of solutions. Therefore this is the number of infinite n minus r parameter solutions. We can also call it as n minus r parameter family. And lastly if the rank of A is not equal to rank of Ab then this system is inconsistent and this implies that the problem has no solution. This means rank of A is two; and rank of Ab is three, then in that case the system has no solution. We shall reduce the augmented matrix into an upper echelon form using the elementary row transformations. We shall find the rank of A and the rank of the augmented matrix. Some inconsistency like zero is equal to some five or zero is equal to two, something like that would come out; so that the inconsistency is apparent there and therefore the problem has no solution.
Now when we are constructing numerical methods, our numerical methods should automatically and implicitly tell us that the given system is right or wrong; because we are tackling of thousands of equations. We do not know in advance what errors have been committed, whether we have produced consistent or inconsistent systems. Therefore the numerical method should automatically take care and tell us that the system that we have produced; the inconsistent system is wrong or you can say that it has infinite family of solutions, so you have many dependent equations in that. So these concepts should automatically come out of the numerical method that we are constructing there.

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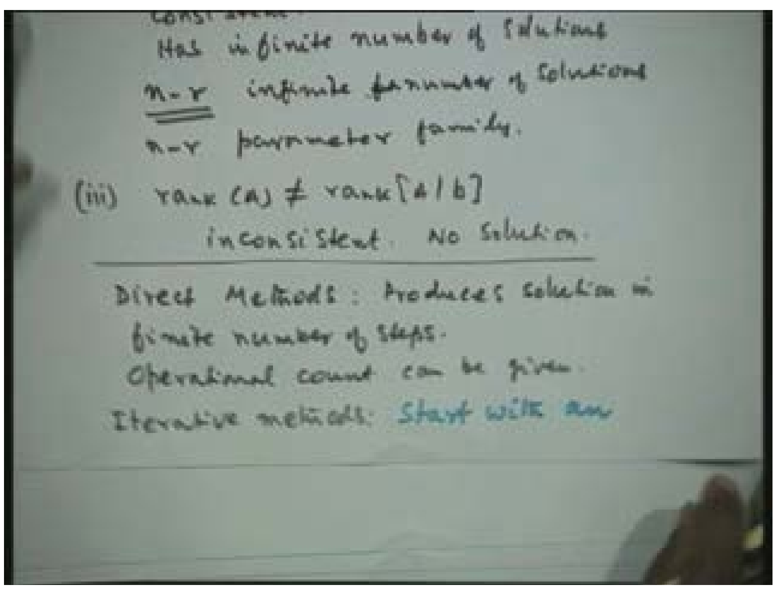

We shall define these methods that we have into to two classes as the direct methods and iterative methods. What we mean by direct method is the same definition that we have given in the case of the solution of a nonlinear equation that these methods produce the solution in finite number of steps, therefore this produces a solution in finite number of steps. Therefore we shall be able to give in this case the operational count or the total number of operations that is there in this particular method. So the operational count can be given. The other methods are the iterative methods. In the iterative methods we start with an initial approximation to the solution of the problem, and then refine it further and further using the numerical method, so that we finally have the solution which converges. So we start with an initial approximation.

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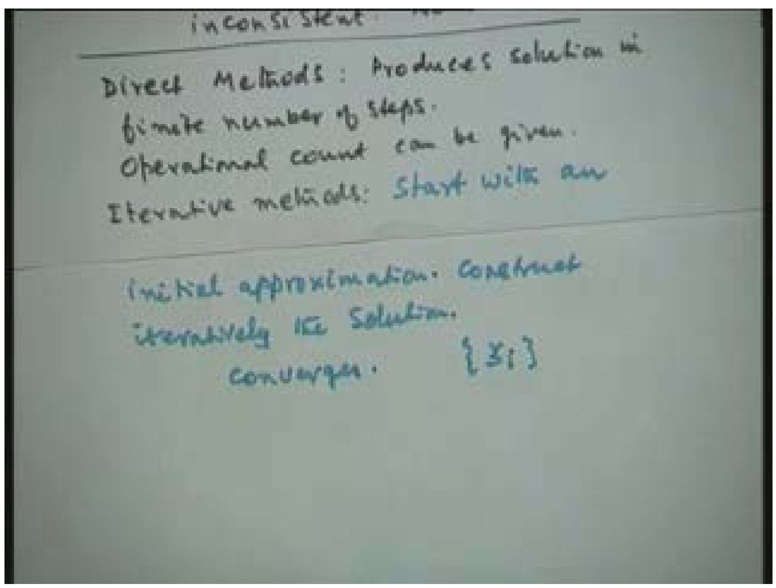

We start with an initial approximation and construct the later approximation and then construct iteratively the solution. Of course we must guarantee that the solution converges. So we must theoretically show that this iteration sequence of the solutions converges; that means we have got the sequence of solutions vectors xi as the solution, they all converge and that requires the analysis of the solution. We can do the analysis of this iterative procedure and then say that our method that is being used is a convergent method, it will always converge or we can say that under this condition the method is going to converge and then once you take the initial approximation, in that format the solution converges.