Materials Science
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Lecture 34
Conductors and Resistors

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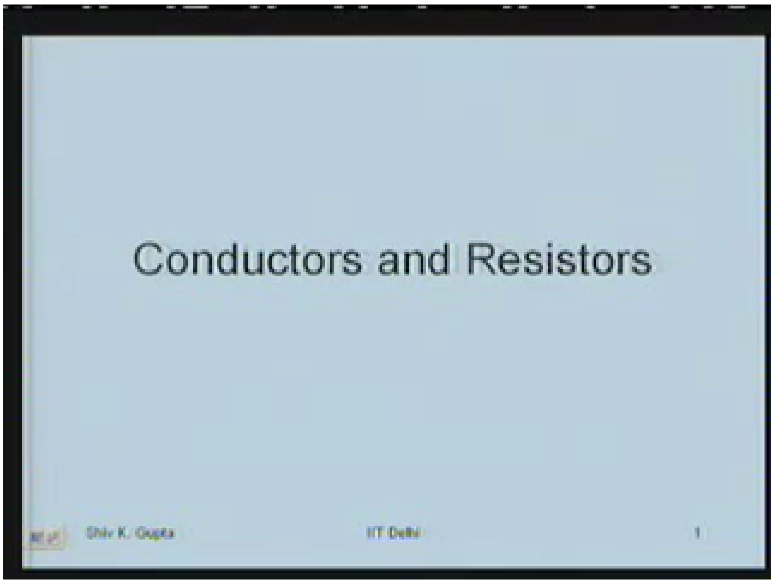

Right so far we had looked at the mechanical behaviour of materials and now we shall look at the electrical behaviour of materials and among this we shall first start with conductors and resistors and before that we will just tell you these materials on the basis of the electrical behaviour how do we classify them.

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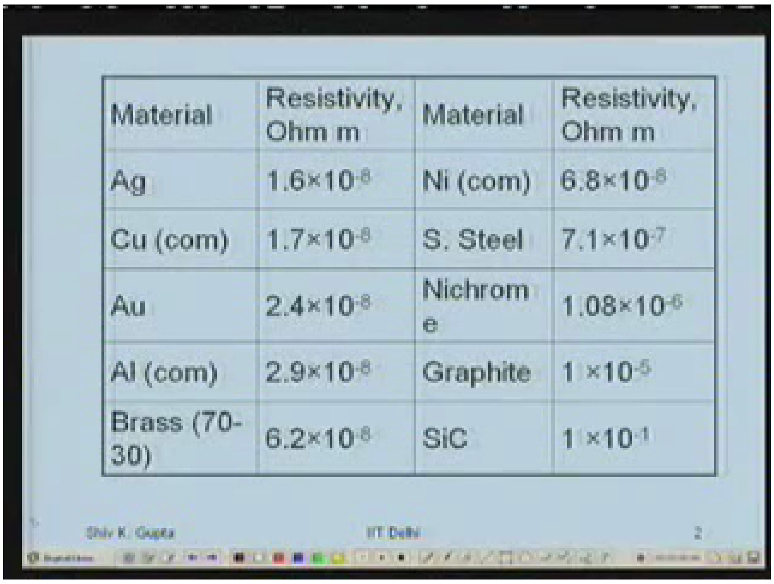

Material	Resistivity, Ohm m	Material	Resistivity, Ohm m
Ag	1.6×10^{-8}	Ni (com)	6.8×10^{-8}
Cu (com)	1.7×10^{-8}	S. Steel	7.1×10^{-7}
Au	2.4×10^{-8}	Nichrome	1.08×10^{-6}
Al (com)	2.9×10^{-8}	Graphite	1×10^{-5}
Brass (70-30)	6.2×10^{-8}	SiC	1×10^{-1}

Here resistivity of the material or the electrical conductivity of the material is what I am showing you for different materials and you will notice surprisingly this is one property among materials which has of the order of 25 orders of magnitude difference in values, 25 orders of magnitude is not a small number (())(2:08) it goes electrical resistivity ranges in the range of 10 to the power minus 8 to 10 to the power 16 or more than that. So this 25 orders of magnitude difference makes it a very special property.

In this table I am showing you these conductors and resistors here you see silver, copper and gold these are the least value of resistivity means electrical conductivity is the best for these materials, aluminium is slightly less, butt is also a good conductor of electricity then I have some alloys brass is a copper zinc alloy, then I have nickel here which is again a metal commercial this means commercial purity material, okay.

If I take high purity material it might be little better conductivity, these the impurities decrease the conductivity or increase the resistivity, while conductivity and resistivity I am using the two words one is reciprocal of the other let me just remind you that and stainless steel is then a pure conductor as compared to the others, then Nichrome is a resistor which you use for heating elements the other materials in this category, then Graphite is also can be used as a resistor for heating purposes and for other elements, then Silicon carbide is the one which has a very high resistivity and these values of resistivity I have listed at room temperature there is a dependence of this resistivities on temperature as well.

Another important thing about these materials which I have listed here as I increase the impurity content in these materials the resistivity increases or the conductivity goes down. Secondly all these materials resistivity increases with increase in temperature this is one category of materials which I have called conductors and resistors we shall look into that behaviour after we look at the other category of materials in next table.

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Material	Resistivity, Ohm m	Material	Resistivity, Ohm m
SiC	1×10^{-1}	Bakelite	10^{7} - 10^{11}
Ge, pure	4.5×10^{-1}	Window glass	$>10^{10}$
Si, pure	2.3×10^{3}	Al_2O_3	10^{10}-10^{12}
		Mica	10^{11}-10^{15}
		Diamond	$>10^{14}$
		SiO_2	$>10^{16}$

Silicon carbide I again listed here, so far I showed you some materials in the range of 10 to the power minus 8 to 10 to the power minus 2 or minus 3 but there was one silicon carbide which 10 to power minus 1 I have listed it here as well along with other two elements Germanium which is pure state as a silicon and then in the pure state, purity again semiconductor grade purity these materials are semiconductors, their electrical resistivity lies in the range of 10 to the power you can say minus 3 to 3 because I have not listed all the materials there are some materials which will have the resistivity less than what is in silicon carbide.

So we will be asking why you have listed silicon carbide in two places, silicon carbide upto about 710 degrees centigrade acts as semiconducting material but when I increase the temperature beyond 700 it starts behave like a conductor and can be used for heating elements as one of these heating elements in industrial furnaces silicon carbide is used and it can go upto easily 14, 1500 degree centigrade.

So that is why it is listed two places but these are semiconducting materials, these semiconducting materials is that conduct but conduct less because resistivity is high.

However, the conductivity is a strongly dependent on the impurity content as the impurity is increased, conductivity increases, resistivity decreases (())(6:53) unlike the conductors and resistors it is happening in other (())(6:58).

And as I increase the temperature for these materials it is the conductivity which increases not the resistivity. So it is other way round is happening that is why I have put this in different tables these are semiconducting materials. The last I have another set of materials where the I have very high resistivity ranging from 10 to the power 6 and goes upto 10 to the power 16 may be 10 to the power 18 in some of the cases.

It cannot conduct any electricity or any current these materials these are called insulators because you will not be able to do any conduction while you can do some conduction here you cannot do any conduction here. But here also whatever the resistivity is going to decrease with increase in temperature unlike the conductors but they are for all practical purposes they do not conduct.

And some materials like window gross which has some ions and basically it is silica and silica you can see there is a large difference in the resistivity or the conductivity this is a very poor conductor while this is better conductor than that but they are all insulators is because there are ions like sodium ions and calcium ions which can make some movement around. See they are not strongly bounded like the silicon and in the case of SiO2 alright.

So these are the categories of materials which gets a three categories materials usually when we discuss this electrical properties and materials we talked the first category is conductors and resistors, second one as semiconductors and third one as dielectrics. But I will not be able to devote time for the dialectic and I shall talk about conductors and semiconductors only.

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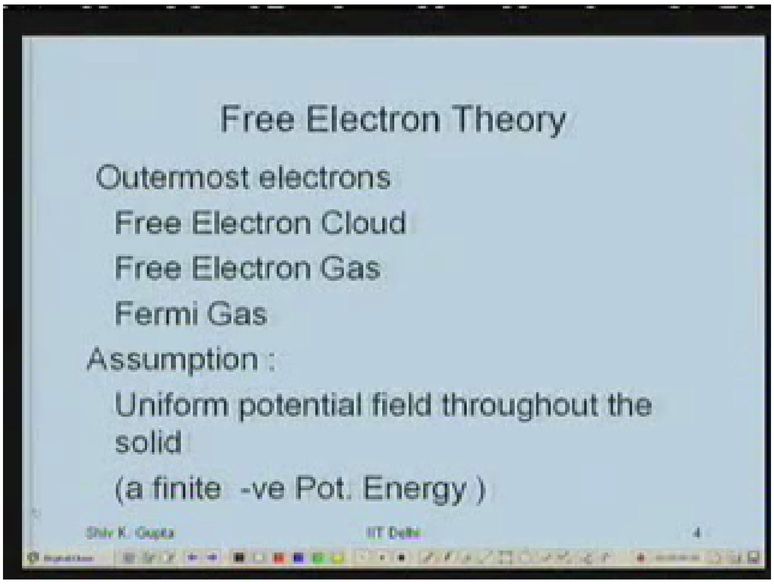

Alright to start with the conductors we make use of the free electron theory to understand the behaviour of these materials, these free electrons are the outermost electrons in an atom and in a solid they all form a pool of electrons because when these atoms or these conductors are brought together to form a solid whether FCC or VCC or whatever solid crystalline solid is formed the outermost electrons which are present they form a pool and they surround the entire solid with that is what is called the free electron gas, free electron cloud or the Fermi gas.

So here we are there is a reference to outermost electrons which are least strongly bounded to the nucleus. Inner line electrons are more strongly bounded and they are called free electron while you know in metals the basically conductors are listed all the metals and alloys. The number of electrons per atom the outermost orbit is much less than 8 even by sharing they cannot make it 8. Therefore, they share these electrons and that is the reason this is the outermost electrons become free they are shared by all all the atoms or positive ion cores in the solid they are all sharing these so they are available to all of them they are moving all around that is why it is called the free and it is not so free that it can escape the material it is being (())(11:21) together.

Another thing before we start talking about after knowing what these electrons we are referring to we are not talking about the inner line electrons. The assumption made in the free electron theory there is a uniform potential field throughout the solid uniform which I will show you when I will talk about semiconductors it is not so really but since I have free electron theory is some kind of a model which works very well and that assumes that there is a uniform potential field and for these outermost electrons it is an attractive field so this is small or a finite negative potential energy.

But in this assumption we make this to be 0, if it is small you can see that this to be 0, alright that is what this free electron theory assumes that assumes uniform potential field it is going to be small,l finite negative potential which is keeping these electrons together to the tyoe to the solid they are not escaping the solid and this is considered 0 in the free electron theory.

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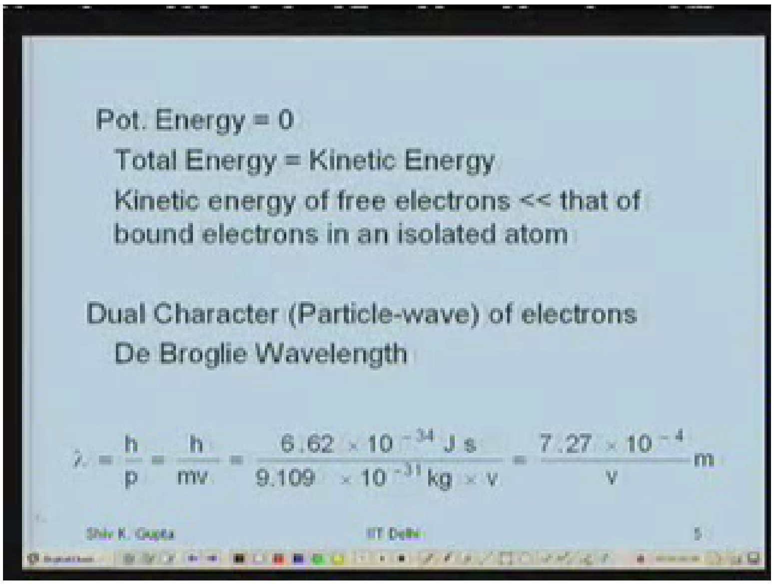

So (())(12:43) potential energy to be 0, if so any particle which or any object which does not have potential energy all the energy will be then kinetic energy that there are only two forms of energy potential energy and the kinetic energy. So total energy of these electrons which are free electrons is considered to be the kinetic energy we shall show you that its kinetic energy of these free electrons which are the outermost electrons is much less than that those electrons which are inner line or the bound electrons bound to the nucleus is much less than that I will show you that.

Next this is a dual character of the electron that is it behaves like a particle and it also behaves like a wave and this gives us a De Broglie wavelength for these electrons, De Broglie wavelength is defined as h is Planck's constant divided by p the momentum of the electron, okay Planck's constant and momentum can be written as mass into velocity, mass of the electron, velocity of the electron but the Planck’s constant is 6.62 10 to the power minus 34 Joules second and the mass of the electron is 9.109 10 to the power minus 31 Kilo Grams into the velocity.

If you simplify this turns out to be 7.27 10 to the power minus 4 divided by the velocity of the electron and that is the wavelength m which this electron represents that means more the velocity of the electron less will be its wavelength smaller will be its wavelength more the velocity means more kinetic energy more energy more energy less wavelength, okay.

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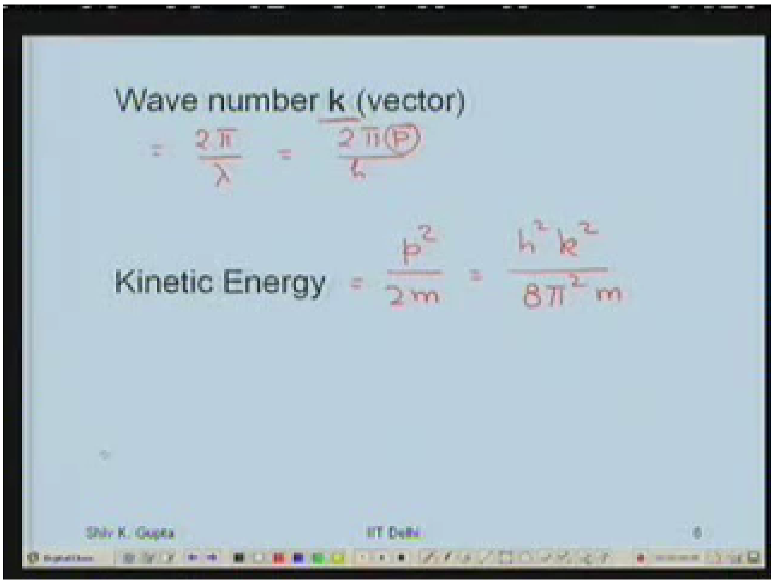

Now we define wave number k 2Pi by lambda and I say this is a vector lambda wavelength is a scalar or is it a vector? Lambda I just define as h divided by p, so p is a momentum and momentum is a vector. Therefore, this k which is equal to 2Pi into p divided by h is a vector, right and kinetic energy you right as half m v square which you can write as p square divided by 2m and I can substitute the p from here, h square, k square divided by 2Pi whole square which will be 4Pi square into 2 will become 8Pi square, alright.

And k as you can see is inversely proportional to the wavelength, right this is given here. Now the electrons which are free in the solid they can have large wavelengths which will be which can be restricted by the size of the solid and when there a large wavelength the energy will be small, okay and similarly the inner line the bound electrons they have small wavelengths was the small wavelengths they shall have a high kinetic energy that is what the statement I made, right.

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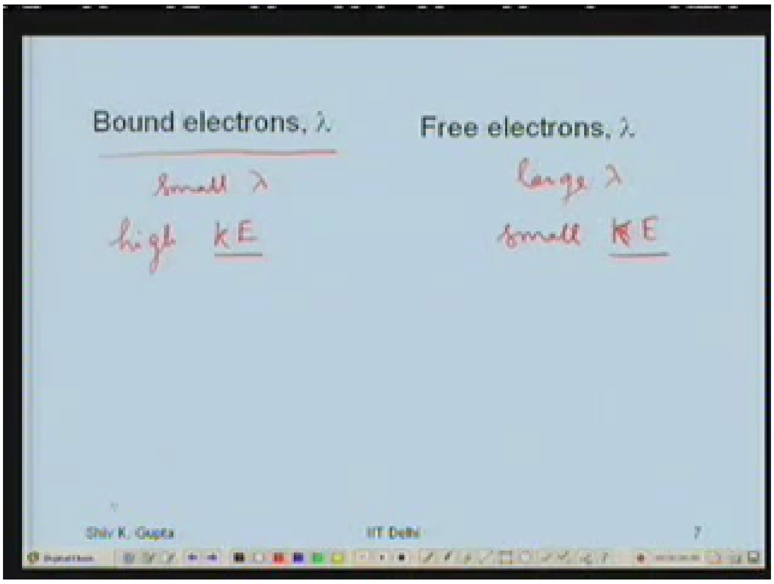

And that is what is it I talked about the bound electrons has small value of lambda and therefore high kinetic energy, free electrons have large lambda and therefore they have small kinetic energy.

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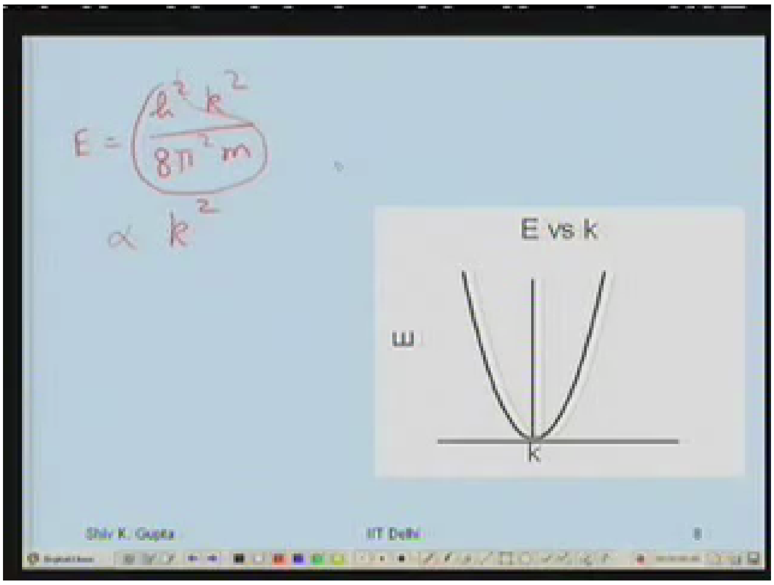

Now if you recall what I have written E as m is a constant the mass of the electron, Pi is a constant, 8 is a constant, h Planck's constant so E is proportional to k square, k is the magnitude of the wave number and this or you can talk about k as a momentum of the electron, E versus k forms a parabola, k can be positive, k can be negative once it is a vector, these electrons could be moving in the positive of the x direction, or electrons could be moving in negative of the x direction in the solid.

So wave number can be positive and negative therefore it is plotted both ways and say it is a parabola that we get here and in this parabola in this model it is a cut inverse energy levels that we know there is a poly exclusion principle working for the outer most electrons to and these energy states are quantized, alright. So we shall try to look at that so poly exclusion principle as I said thus and what is the poly exclusion principle same energy more than two electrons do not has the same energy only two electrons can be at the same energy level and that to one with the positive momentum other with the negative momentum energy level is the same.

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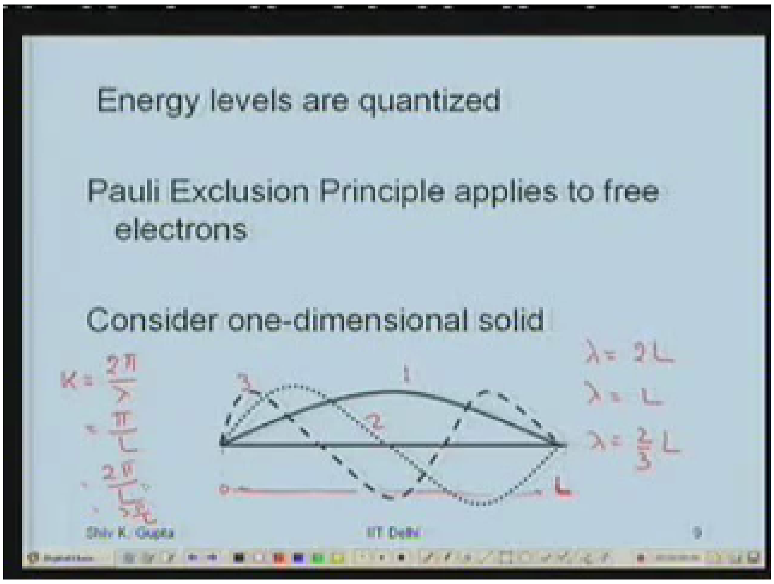

Now I make a very simple model and the very simple thing is in solids we know are 3 dimensional but I am making 1 dimensional solid which I can extrapolate to 2 dimensions and 3 dimensions because vectors can be added in 3 dimensions. Let us say this is my solid and I define its length 0 here and this is the length L here so my solid is a length from here to there L.

It is possible for this free electron to be treated as a wave but the extremities of the solid shall have the nodes, alright if I do that it is possible to have 1 wavelength like this that means wavelength of the solid is or with the electron is 2L double the length of the solid this is how the wavelength. And if so I get the K equal to 2Pi by lambda and lambda is 2L so it becomes Pi by L.

Let us second possibility denotes being at the extremities is that the wavelength becomes becomes half of that there is another node in the middle. So the second one the lambda is L and when I put this lambda is equal to L, K becomes 2Pi by L or it could be that these are again the extremities but there are two nodes in the middle. So lambda is 2 by 3 of L and I substitute this here K becomes 3Pi by L, see I already (())(22:56) it wave number is Pi by L, 2Pi by L, 3Pi by L and wave number can also be negative this is (())(23:09) or it is 1, 2, 3, 4, 5 to go on like this.

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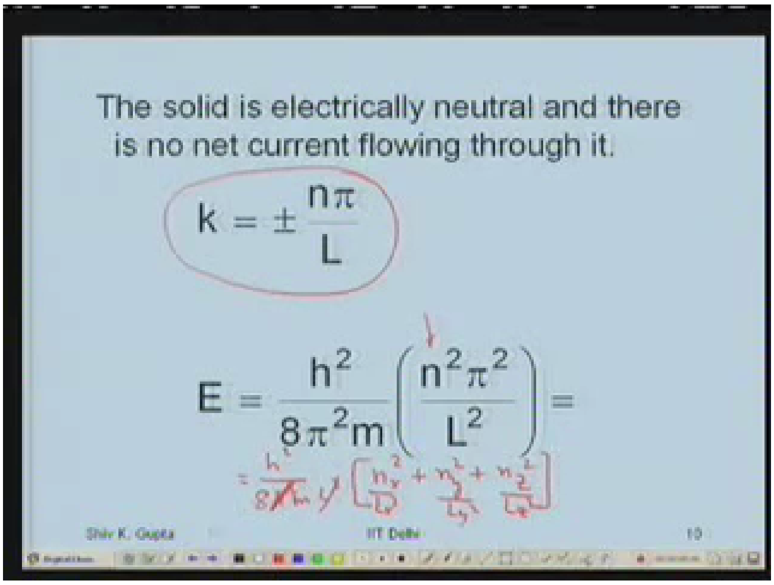

So wave number can be plus minus n Pi by L and that minus so as I said is taken because in a current electron is all flowing in the positive direction in any solid should have some current all the time flowing through it. The net current when you measure turns out to be 0, so electron must be traveling the opposite directions as well. So wave numbers are both plus and minus and it is plus minus n Pi by L.

When I substitute this K as plus minus n Pi by L in the formula for kinetic energy which is h square k square upon 8Pi square m, k square becomes n square, plus minus will become only plus, Pi square and L square this is for the 1 dimensional solid we have written. If I want to write it for the 3 dimensional solid I will have to worry about this n in second direction, the third direction and the quantum number may not be the same in the 3 directions.

So that it may forms it a vector and it can be written as h square 8Pi square m or Pi square cancels that is alright it is not needed 8 m L square then inside I can write x square and y square plus n z square. So this is how from 1 dimension I have gone to the 3 dimensions, somebody may ask me when 3 dimensions is my length in the solid why should it be same L, let it not be then what I can do is I can take this L from here and put it down here L x square L y square L z square.

So this is how from one dimensions I can go to the 3 dimensions, when a 3 dimensional solid (())(25:43) the energy of the electron which is quantized in this fashion. Important thing is to be to be seening what is the difference between two quantum states is the energy level between the two of them and x and y and z can have the values 1, 2, 3, 4, 5 right.

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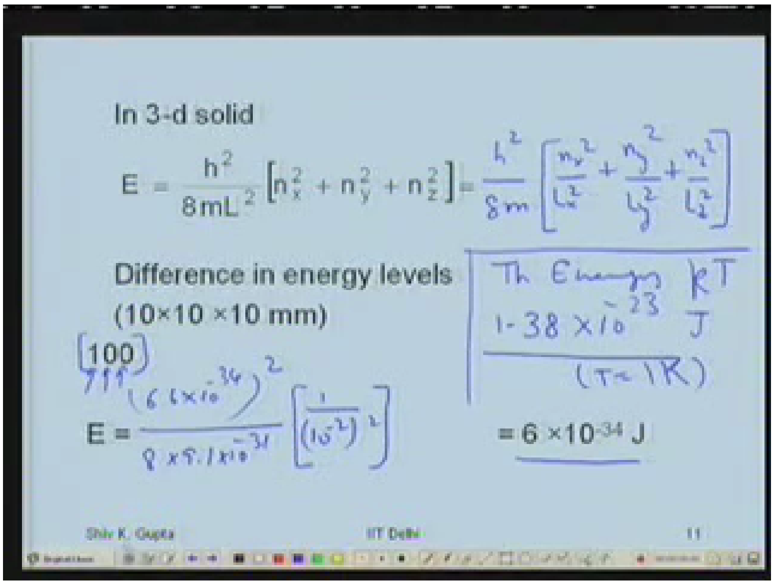

Let us consider this in the 3 dimensional solids or I I put the L inside also and then showed you that this thing which you want to write you can write it like that also let me treat a simple example of a solid which is a cube of 1 centimetre size 10 millimetre by 10 millimetre by 10 millimetre. Let us take the quantum state 100 and x is 1 and y is 0 and n z is 0 all three cannot be 0 that is not travelling, alright.

So this is E will be then you can substitute the value of h whatever it is put a square of that 8 into 9.1 10 to the power minus 31 multiplied by 1 upon or other term will be 0 and 10 millimetre can be written as 10 by 1000 10 to the power minus 2 meters and let put a square of that. So simplify this this number turns out to be 6 10 to the power minus 34 joules, I just wanted to compare this number (())(28:13) energy state could be 110 so that shall become double the difference is going to be only this next state I can go to 111 is just like what you have h, k and L in the middle indices, right.

I can do that and the difference is 6 into 10 to power minus 34 joule between two energy levels. How much is this energy? I consider an average thermal energy even a very small temperature like 1 kelvin let us say, what is that thermal energy kT, k is 1.38 10 to the power minus 23 joules at 1 kelvin if T is 1 kelvin, okay room temperature it will be much more 300 times of that compare this with this that means average thermal energy available to these electrons is much more than this quantized energy difference between two quantum states.

And this is a very negligible energy as compared to that therefore we can consider it to be a continuous function E versus k can be considered as continuous function, alright. We have put different values I have put taken a value which is reasonable but you do not use these solids ya ya I can understand what you are trying to say, right if you reduce that it will but this is (())(30:19) kind of dimensions I have taken 1 centimetre is not normally if you have a conductor will be running in meters. So this energy level is so small and we treat it as to be a continuous function.

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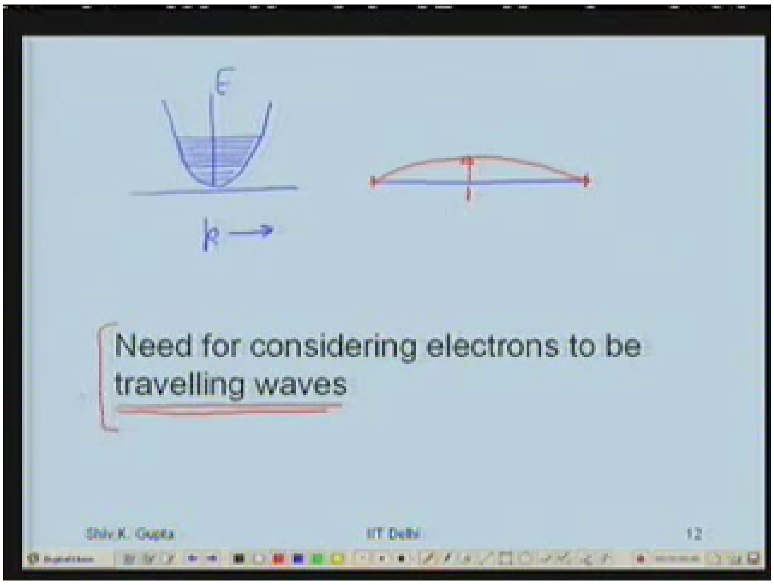

Now if that is what it is that I consider it to be a continuous function but then in the solid means E versus k I can take it like this and the energy levels are quantized to the difference between them is so small there is no we are taking them to be a continuous can take any value of the energy electrons can take. The problem we considered in the 1 dimensional solid electron I consider to be a wave like this this was not essential I do not want to have electron outside the solid, the amplitude well if I want to find out the way electron is in the solid probability of is given by the square of the amplitude and you will find that it is localize somewhere in the centre where is the maximum probability here and it is 0 there.

But what I assumed the free electron and it is pervading the whole volume of the solid free electron gas is enveloping the whole solids is going everywhere free electron cloud we said we call it fermi gas gases goes everywhere. So probability of finding the electron anywhere and everywhere should be same and that possibly I can take care if I treat this to be as travelling waves rather than standing waves like this, with these amplitudes and the wavelengths I treat them as travelling waves then the probability of locating the electron anywhere in the solid shall be the same. So that is overcome by considering this as travelling waves, alright.

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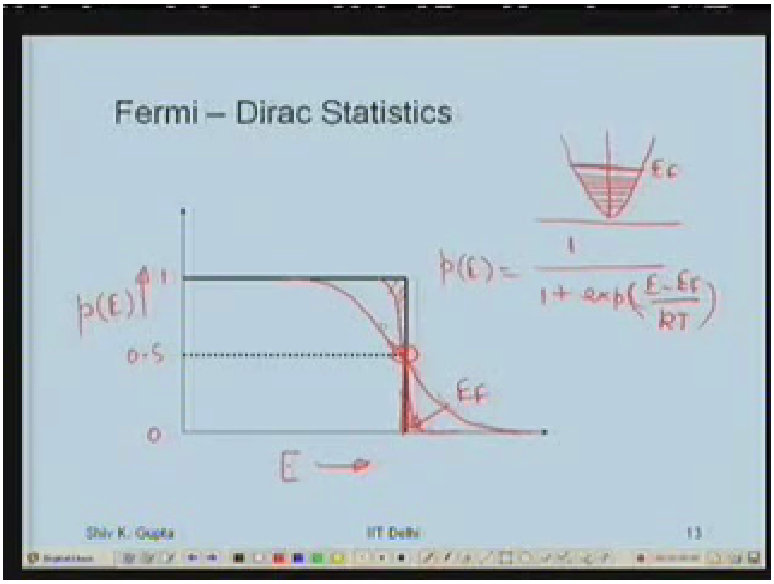

Next we talk about this energies taken up by these electrons which are present in the solids, here is the probability on the y axis and this is the energy level. If I consider at 0 kelvin the conductor now we go back to this there is no thermal exaltation available to the electrons I am talking about the free electrons and these are the energy levels they are occupying and whatever number of electrons I have there in the solid let us say they fill all the energy levels upto here that is what I refer to as a Fermi energy and that is the Fermi energy here.

So probability is one of occupying energy level at 0 kelvin upto Fermi level because all electrons have been exhausted I have no more electrons and beyond Fermi level the probability of occupation is 0. But when I increase the temperature some electrons which are very close to the Fermi level they get excited take higher level of energy and they move from here the probability here becomes less and it increases it here beyond 0. So these electrons from here go in this side they got excited.

As the temperature rises this goes on at a higher temperature more of these go on to the other side means the higher energies. So they occupy higher energy levels that is what is happening and this Fermi-Dirac Statistics gives us the probability at any temperature of occupation and energy level is 1 divided by 1 plus exponential of E minus E F by kT. See when the temperature T is 0 kelvin and E is less than E F this number is exponential of minus infinity and exponential of minus infinity is 0 so it becomes 1 that is what it is.

And at temperature T equal to 0 kelvin when E becomes greater than E F, E minus E F divided by 0 becomes plus infinity and that is infinity 1 upon infinity becomes 0 that is 0 step function. When the temperature becomes greater than 0 kelvin and now I am talking about E becoming equal to E F, once E becomes equal to E F this is exponential of 0 denominator is not 0 this exponential of 0 is 1, 1 upon 1 plus 1 is 1 upon 2 is 0.5 that is 0.5 here. So at Fermi level the probability of occupation the energy is 0.5 at any temperature greater than 0 kelvin, right.

I can define the Fermi level like this as long as Fermi energy is not a function of temperature with some solids I will show you it is a function of temperature later we will talk about it. So I can say the probability of occupation is 50 percent when the energy is equal to the Fermi energy by electrons, electrons will occupy that energy levels 50 percent of course we are using them not at 0 kelvin at any temperature greater than 0 kelvin.

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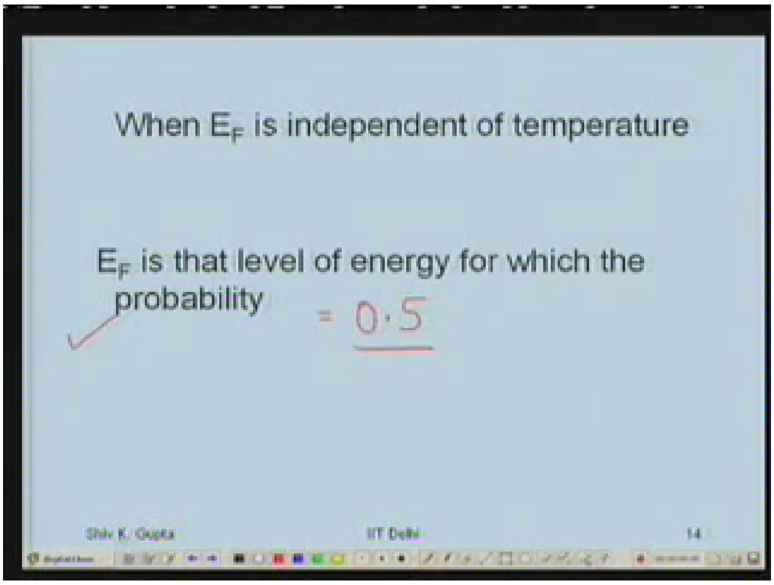

That is what I am saying when E F is independent temperature, the E F is that level of energy for which the probability of occupation is 50 percent but in a conductor I can always say as I started with Fermi energy is that highest level of energy which is occupied by an electron at 0 kelvin I have used to all electrons the one which has the highest energy is the Fermi energy, all energy levels below that above failed 100 percent, right. So that can be said about the conductors but this is going to be true for all solids.

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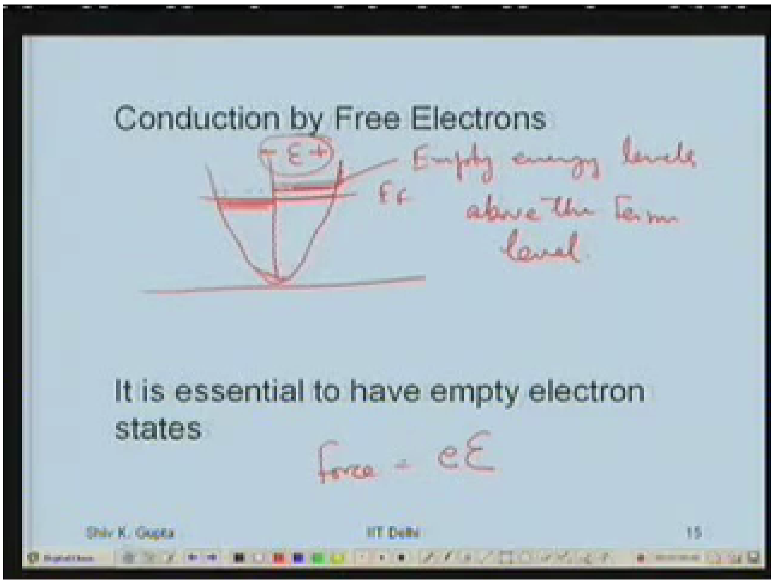

Now we shall look at the conduction by free electrons it is essential when they are conducting the free electrons that they should be able to occupy energy levels and that means there should be energy levels available above the Fermi level and the electron should be able to occupy, right that is what I mean is this is my Fermi energy the electrons which are here should be able to go to higher level of energy so that they can travel they can move.

If this energy level is not there it cannot. So this is another assumption that there are energy levels available above the Fermi level and electrons are in position to go and occupy them and these are empty levels. So it is essential to have empty electron states. Let us say I apply electric field like this so what would happen the electrons which are here they are the ones which are going to go and these states will become empty here, some more from here will go there.

Therefore I have less number of electrons with a negative momentum and more number of electrons with positive momentum, more electrons are travelling to the right and less electrons are travelling to the left, so there is a net current flowing to the right when I apply a electric field like this these electrons are flowing to the usual current will be flowing to the left only.

So those current normally what you call the current flows from positive to negative, electrons are going to go from negative to the positive so they will occupy these energy levels above the Fermi level here and they will be less in number here and more in number there then, okay and there will be conduction taking place in the solid. So the field we have applied means I applied some onto these electrons, force applied onto the electrons the charge on the electron times the electric field and when you apply a force on a particle it accelerates that is the Newton's law of motion.

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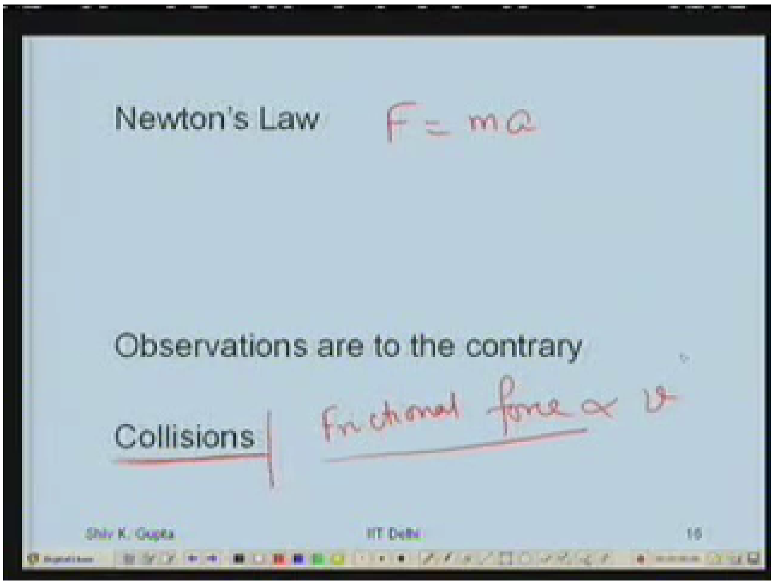

So when I apply the electric field electrons keep accelerating, the velocity keeps increasing, the kinetic energy keeps increasing what happens to the current it should keep increasing but what you have noticed we have made a circuit (())(43:42) you have seen that put diameter to find the moment you put the key on there is some current is shown and that remains there current does not change, right this is what the Newton's law is saying that it should keep accelerating and therefore the current should keep increasing does not happen that way. So our observation is contrary to the Newton's law.

And the second thing what you observe is Newton's law also says once they are moving with some velocity, right it should continue to move with the same velocity when you switch off current comes to 0 current does not remain in the circuit current becomes 0 they do not keep moving means there is something which you can call as a frictional force acting and this frictional force is the one the form of this frictional force is actually for a particular you can treat this to be collisions.

The particular which is travelling collides with something whatever it is during the collision it can lose its energy, right it can lose it can lose all the energy, it can lose part of the energy so for velocity will come down and this is happening to all the electrons. In a given solid you may have electrons to the tune of 10 to the power 23 in a cubic centimetre you may have that many (atoms) electrons and those electrons are moving around they are the free electrons I am talking about.

And they are the ones some of them are colliding, some of them are accelerating, so we can talk about the average velocity of an electron and when they are colliding what is actually the collision here? Collision here is the interaction of this negatively charged particular the electron with the positively charged nucleus, the positively charged nucleus is also not stationary we have already said that in solid atoms are oscillating about their mean position when they are oscillating about their mean position in any direction the randomly they are oscillating amplitudes are changing, they among themselves are colliding and their change shifting their energies.

This is the attractive potential is also moving it is not stationary and that is what can (()) (46:43) change in direction of the moving electron that is what I mean by collision, alright that is what I mean by collisions here is the effect caused by the potential field provided by the positive nucleus positive ion core essentially whether electrons are also bound to it. so that is what is going to affect this negatively charged particle and change its direction that is what happen in collision direction gets changed, two particles collide both start moving in different directions not the original direction.

And looking in the original direction the component or the velocity will be reduced. So current flowing will be reduced. Basically that is what is happening and there is a frictional force which I can call because of this try to take care of this the frictional force and which is proportional to the velocity of the electron, I can try to take care of this.

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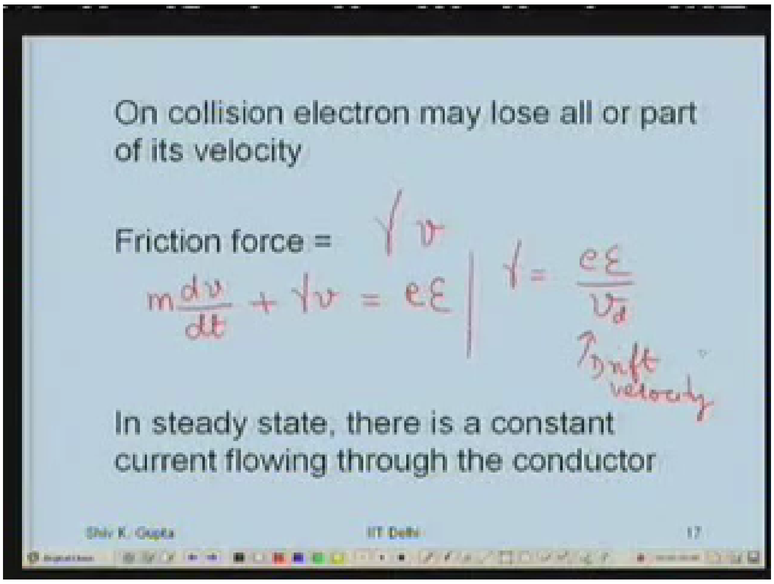

As I said on collision electron may lose all or part of its velocity means its energy and frictional force I say is some constant let us say gamma times the velocity of the electron. So if I write the Newton's law now mass into acceleration the electron plus the frictional force is what is equal to the applied force it is possible for us to work on what should be the gamma, you have seen that when you apply the potential e there is a constant current flowing through the circuit which you can measure on the emitter means there is a constant velocity of the electrons which I am talking about the average velocity so that is a steady state and therefore dv by dt should be 0 there, when dv by dt is 0 in the steady state then gamma v becomes or rather say gamma is what we are looking for becomes in steady state we call the velocity of the electron to be the drift velocity that we can substitute here and solve this equation, right is a simple equation first order equation we can solve this there is no problem.

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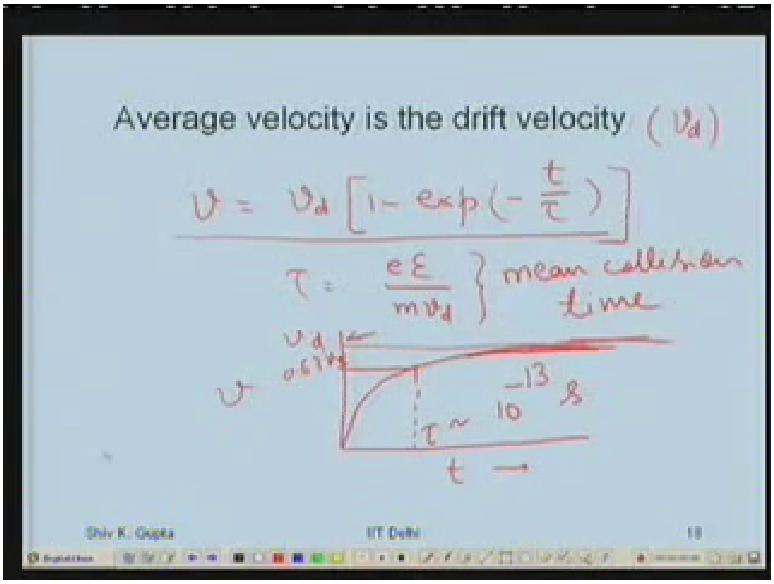

And this is the steady state said the average velocity is the drift velocity is what I just now said, solution we will have is the velocity is equal to drift velocity 1 minus exponential of minus if you work out you will get the Tau to be force divided by momentum ya this is the strength. If you look at this then the (Tau is) t is equal to Tau this is called the mean collision time, exponential of minus 1 exponential of minus 1 is 1 upon 2.7 is 0.37, 1 minus 0.37 is 0.63 so you have reached somewhere here and this is t equal to Tau. In the circuit when you switch it on immediately see the constant current that is the drift velocity is you reach this value steady state almost no time.

This time we will work out some problems also is of the order of 10 to power minus 13 seconds you cannot see it when you switch off the circuit you cannot see this whether it is changing, okay that is more than that is the delay in the response of the emitter, okay. So it is not possible for us to see that and that is the kind of time we will work on will be in that range.

So that is the what is happening and we are getting the drift velocity or standard steady state a constant velocity of the electrons and that is what I define is mean collision time or other definitions I think we will take in the next class.