Material Science
Professor S. K. Gupta
Department of Applied Mechanics
Indian Institute of Technology Delhi
Lecture No 27
Mechanical Behaviour of Materials

While so far in this course we have seen three levels of structure, crystal structure, (())(1:08) and the microstructure. Then we saw the movements of atoms starting with diffusion and then we saw what are the phase, transformation and mechanism and whether we are in a position to control it and what are the possibilities when we can control this. Now we shall look at the relationship between the properties and the structure and then try to see if I require a particular property where I can get that kind of a structure in a material, if it is not there can I develop it that is something which we have already learned. We will have to comprehend all this information. Once we understand that is the relationship of the property to the structure.

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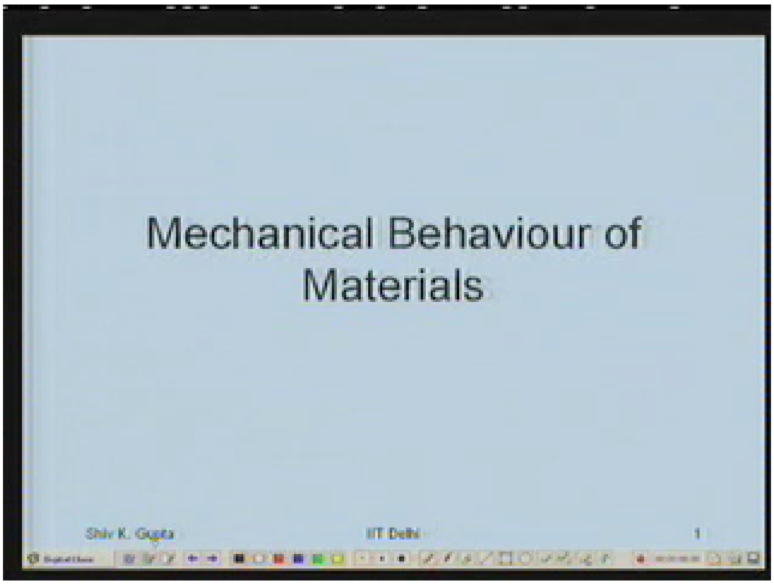

Mechanical Behaviour of Materials

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We start with the first property "Mechanical behavior of materials."

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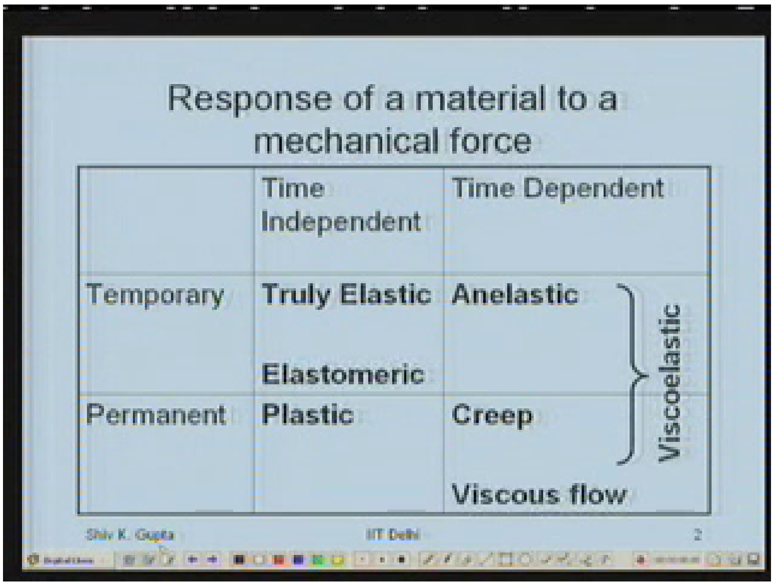

Response of a material to a mechanical force

	Time Independent	Time Dependent	
Temporary	Truly Elastic Elastomeric	Anelastic	Viscoelastic
Permanent	Plastic	Creep Viscous flow	

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Mechanical behavior I can say is the response of the material to a force, mechanical force. I hope you understand the mechanical force like that (())(2:20) weight, we apply the weight and that is the gravitational force. The property of the behavior of the material can be divided on the basis of 2 things, whether behavior is temporary for the period you apply the force or it is permanent even after the force is removed. Second thing whether the behavior is going to be dependent on time or it is independent on time.

So on that basis I have made this small table. These columns I have made, time independent and time dependent columns and similarly I made them the rows temporary and permanent behavior. First of all by time independent what I mean is the moment the force is applied whatever response of the material in this case has to be in the form of deformation or change in shape of the material. That has been brought up instantly is time independent, right? And then when the deformation goes on as a function of time and the force is applied is becomes time dependent.

Similarly a temporary deformation is the one, you applied the force, you saw the deformation, you remove the force, and material comes back to its original shape and size. Permanent is when you apply the force deformation takes place, shape changes of the material, when you remove the force it doesn't recover back to its original shape and size that is the permanent shape in change. We have given them some names to these. The deformation which is temporary as well as independent of time that is instantaneous is called truly elastic behavior. That is what we shall see today and we have also seen already an elastomeric behavior, the rubber like elasticity that is also time independent and temporary.

But this one is linear and obeys the Hook's law while this one is non-linear, it does not obey the Hook's law. Similarly, when the time independent deformation goes on it is permanent and we call it plastic deformation. We have all seen this while doing a tensile test on steel and aluminum. After the yield point, after the yielding there is a permanent change in shape of the material. Similarly when there is a temporary deformation which is dependent on time that is, you apply the first, you apply for one hour, deformation goes on for one hour it keeps on increasing and keep it for more time and it will still go on and that is what an elastic behavior.

In such a situation when the force is removed there is a recovery, it come back to original shape but again comes back as a function of time, we have spent one hour in deforming it, it may not take less time to comes back. It has recovered as a function of time. Okay, that is an N elastic behavior. And this permanent deformation which is dependent on time is called Creep or it is called the viscous flow. You have seen this probably in Physics, viscous fluid which obeys the Newton's law. And Creep is what is in crystalline solid we shall look at takes place is a function of time, is a permanent change in shape.

But the phenomenon which is dependent on time, which temporary and permanent can go on together, both of them can go on together, such a deformation is called viscoelastic behavior. This viscoelastic behavior is mostly we see in polymeric materials, okay. While I will not much time to talk about Anelastic and Viscoelastic behavior but I shall be able to devote time to other, elastomeric behavior we did see when we talked about the rubber like structure in polymers and then we saw the mechanical behavior and thermodynamics of rubber elasticity. Today we shall talk about the Hook's law or the truly elastic behavior of materials.

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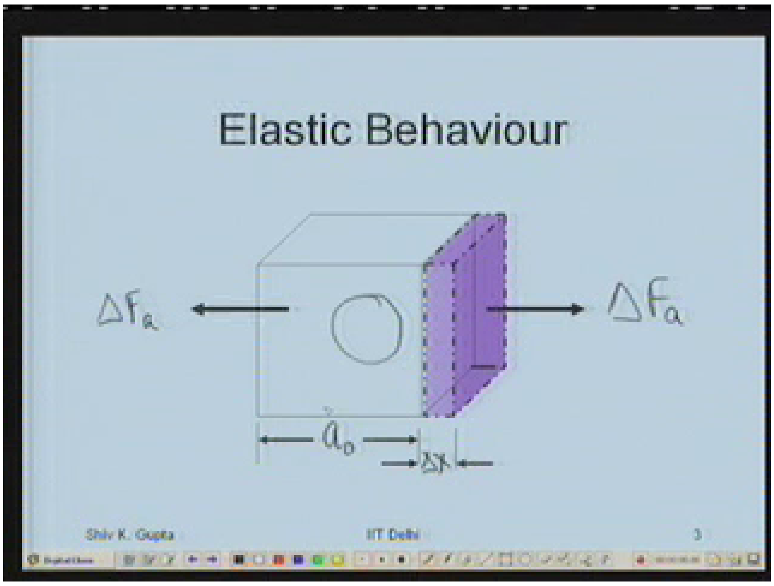

Elastic Behaviour

Now, while looking at the elastic behavior of materials, I consider again a very simple structure, simple cubic structure, unit cell, there is one atom sitting in the middle which is bonded on the 6 sides to other neighbors and I apply a small force in tension, this bonds will extend in this direction and let us say this unit cell has dimension is 0. And this extends in this direction by an amount let us say delta X, right?

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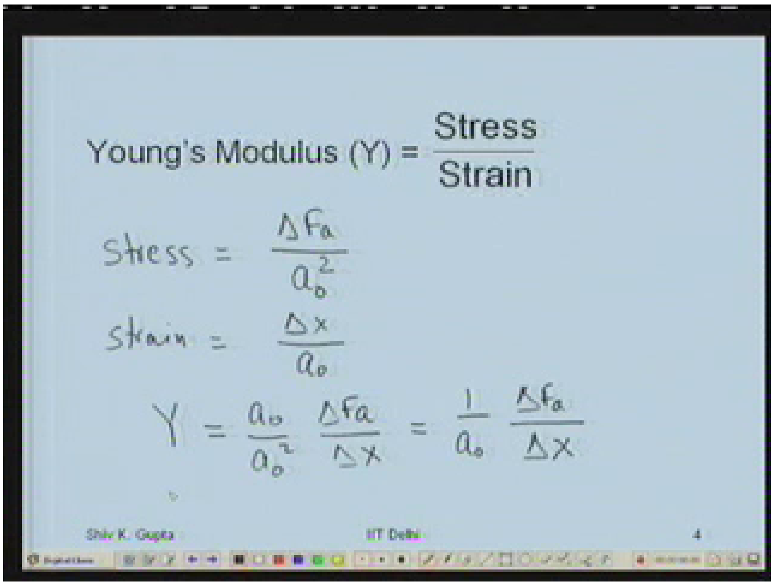

$$\text{Young's Modulus } (Y) = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = \frac{\Delta F_a}{a_0^2}$$

$$\text{strain} = \frac{\Delta x}{a_0}$$

$$Y = \frac{a_0}{a_0^2}\,\frac{\Delta F_a}{\Delta x} = \frac{1}{a_0}\,\frac{\Delta F_a}{\Delta x}$$

When Hook’s law is obeyed we write as stress divided by strain, what is the stress we have created. Force we have applied and divided by the area of cross section, and the strain we have is elongation divided by initial length and if I to use this I can write the Young’s modulus as… Which can be written as, right? Is it correct? So I have written the definition of the Young's modulus from the Hook's law stress divided by strain or stress is proportionate to strain and proportionally constant is called the Young's Modulus. And about that Young's Modulus is 1 upon A0 delta F A divided by delta X.

We notice that when I apply a force to the material they develops a force which is a tensile force, they develops a force which is compressive force in the material, the atom tries to pull each other towards them. It means if applied force is negative of the force generated in the material.

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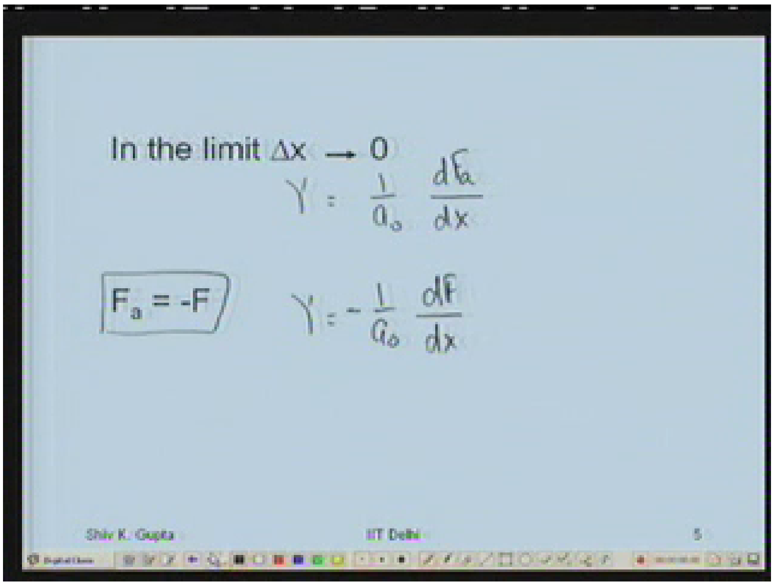

In the limit Δx → 0

$$Y = \frac{1}{a_0}\frac{dF_a}{dx}$$

$$\boxed{F_a = -F} \qquad Y = -\frac{1}{a_0}\frac{dF}{dx}$$

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So I can write the Young's Modulus in the limit delta X standing to 0 as dFa divided Dx, if I use this I can write this as 1 upon a0 the minus sign dF by dx. F is the force between atoms and this is the applied force.

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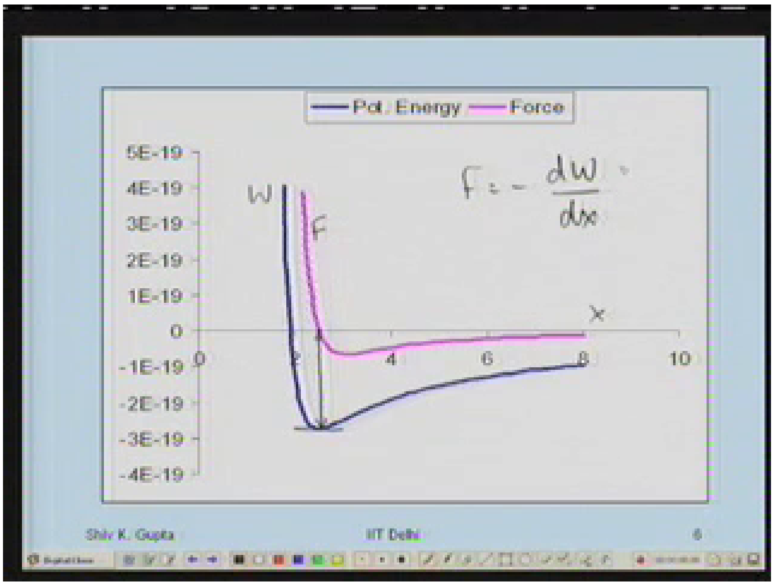

Now let us look at the force between the atoms what we know about. When we bring two atoms from infinity, a potential energy, this is the potential energy versus distance and this is the one, and comes and stays at the minimum, this is what we called the bond energy. At this point the force between atoms is 0 that is a advance. F is minus dW by dx and that is a negative scope here going to infinity so it becomes a positive force 0 according to infinity and the slope here is 0, the force becomes 0. This is positive the force becomes negative. So this is the derivative production energy versus distance, the minus sign is the force, that is the definition of the force.

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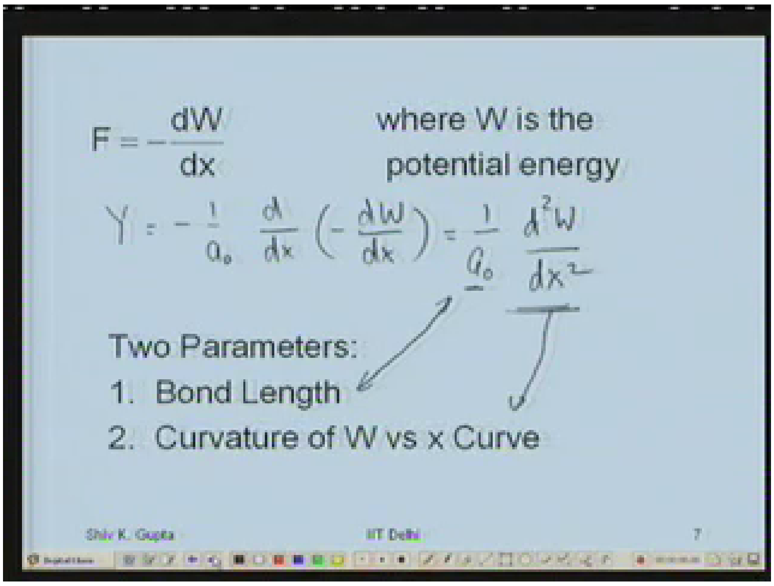

$F = -\frac{dW}{dx}$ where W is the potential energy

$Y = -\frac{1}{a_0}\frac{d}{dx}\left(-\frac{dW}{dx}\right) = \frac{1}{a_0}\frac{d^2W}{dx^2}$

Two Parameters:

1. Bond Length
2. Curvature of W vs x Curve

So knowing this you can put it down in the Young's Modulus which is minus 1 upon A0, dF by dx which come d by dx of minus dW by dx, minus and minus would become plus, it will be 1 upon A0, d square W by d x square. So basically the Young's Modulus what we see now depends upon two things, one is this number A0 what is nothing but the bond length. I am at very elementary estimate of the Young's Modulus and d square W by d x square can work it out as the curvature of the potential energy versus distance curved at minimum potential energy.

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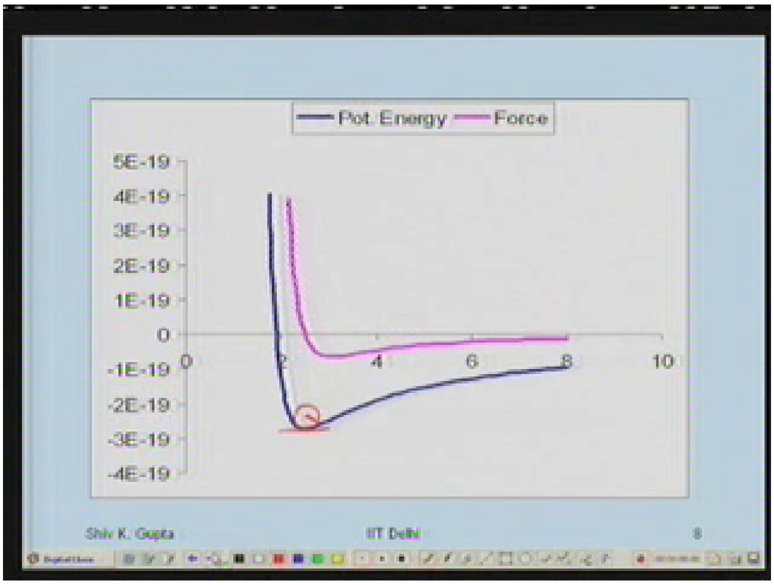

By curvature what I mean is that is if I draw a circle which is tangent here, this tangent is common to both of them, the radius of this is called the radius of the curvature and curvature is a reciprocal of the radius. So a curve which is quite flatter here shall have a very high radius, a small curvature. The one which is deep shall have a small circle, small radius and high curvature.

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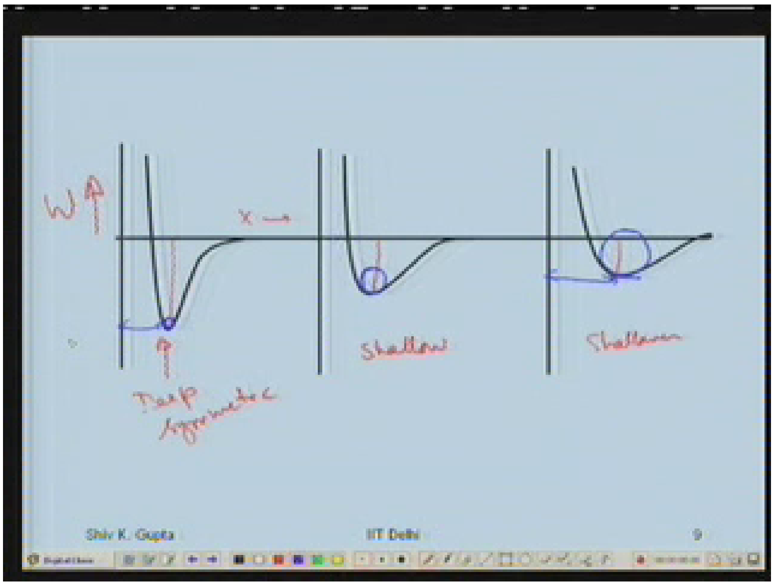

All these I will show you here. There are three kinds of shapes I have shown, very deep and a bit symmetric curve, this is all potential energy on this axis and distance on that axis and all the three curves. So this kind of one is very deep and a bit symmetric though by nature this curve is asymmetric. It is more flatter to the right as compared to the left it is very steep. This one is shallow as compared to the first one, and this is shallower. This curve says the potential energy of the bond energy at the equilibrium spacing is large it is not so large here, it is very small there relatively. Very strong bond, not so strong bond, a weak bond, so bond energy is small as a weak bond.

And here what you can put in as a circle is a much bigger one what you can put in here is a smaller one or you can put here is a very small one. Very small radius, larger radius at a very large radius. So curvature here is the least, curvature here is the most. Such a bond material if it has will give rise to high modulus, this one shall give me low modulus because curvature is small, d square w by d x square is small. Similarly when a bong is strong usually bond length is smaller.

1 upon A0, means the bond length is smaller modulus will be higher and a bond length is larger usually the weaker bond sets the situation. You can see that this distance is larger as compared to this distance, so stronger bonds are smaller, weak bonds are (())(16:47), secondary bonds you know that in the range of about 5 to 10 Angstrom, but primary bonds are in the range of 1.5 to 2 Angstrom. So that is the thing that we have fine that stronger bonds shall high modulus and weak bonds shall give me low modulus and the two things

involved are the shape of the potential energy well , that is the curvature and the bond length, they are the two things.

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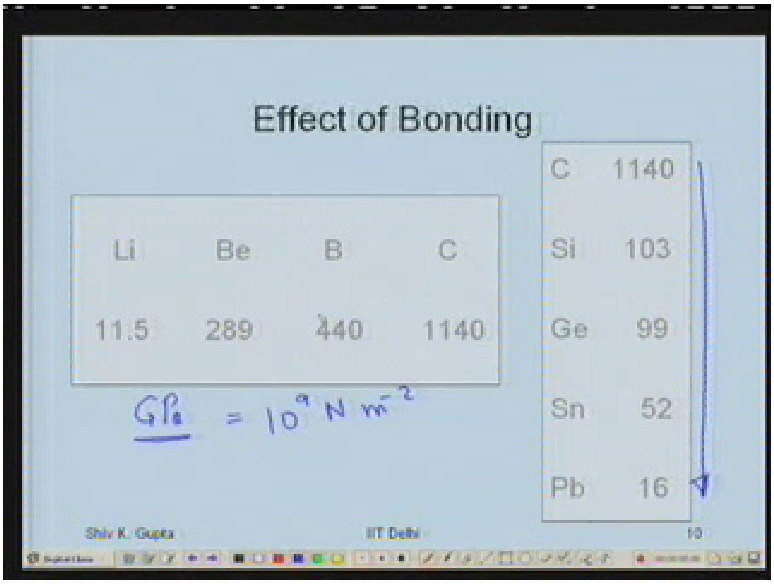

Effect of Bonding

Li	Be	B	C
11.5	289	440	1140

GPa = 10^9 N m^{-2}

C	1140
Si	103
Ge	99
Sn	52
Pb	16

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In here I show this effect in terms of the elastic modulus or Young's modulus of the different materials. Some of them are taken from the periodic table. This is the same period, lithium, beryllium, boron and carbon. And the modulus values we always keep in terms of gegaPascal. 1 Pascal is 1 Newton per square meter therefore it becomes 10 to the power 9 Newton per square meter. That is the unit in which we give the value of all elastic moduli whether it is Young's Modulus, Shear Modulus or Bulk Modulus. So you see the values, the order or magnitude, for lithium it is 11.5, it is a typical metal, not so strong bond.

Go to beryllium covalency increases 286, bond become little stronger. Boron it is still stronger 440 and as I go to diamond SP3 bond 1140 gegaPascal, it increases. I am not shown to group 5, what happens then now I start to get along with the covalent bonds I start to get the secondary bonds because in group 5 I have sheets, (())(18:55) sheets which are bonding (())(18:57) bonds and in such situation the property is dictated by the weaker bonds, the modulus becomes very strong for such materials. So that increases when all the primary bonds are present and metallic to typical covalent bonds it goes on.

Similarly I look at one particular period I shown the group 4, one particular group is the group 4 here, as I go down in the group it is the metallic nature which increases and covalent nature decreases and you see that modulus is falling, from 1140 it goes to down 103, for silicon and germanium it remains more or less the same, around 100 gegaPascal and then suddenly falls for Tin 52 and Lead goes down to 16 gegaPascal.

So metallic nature is increasing, the bond length is increasing and we see that bond is not so, the curvature is also decreasing, falls rapidly. On the other hand when I go from typical metallic bond to a covalent nature it goes up from 11 to 1140, means this is a 2 orders of magnitude increase in the value from here to there. So that is the effect of nature of the bond, bond length at the curvature at the equilibrium spacing.

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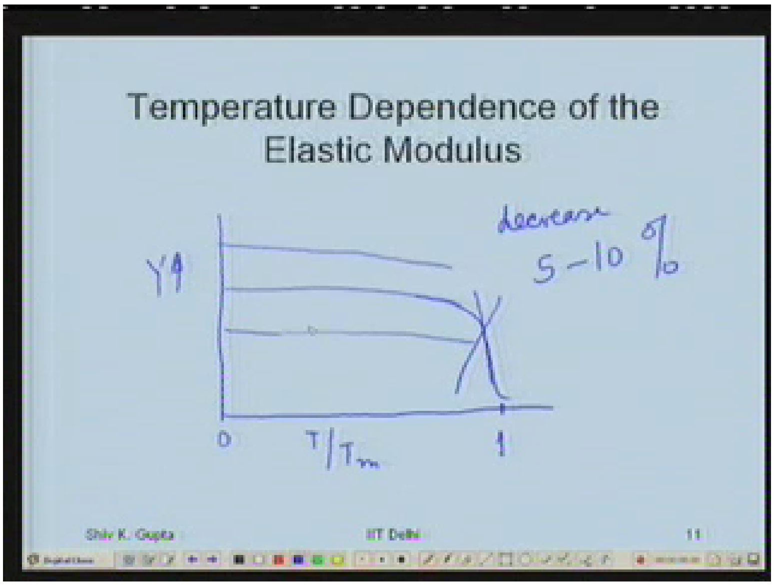

Temperature Dependence of the Elastic Modulus

Elastic modulus has been experimentally studied as a function of temperature and we find that if experiment is done very carefully so that the effect of temperature does not make it time dependent or the time fact we do not look into because elastic deformation is time independent deformation. What we find is that there is only about 5 – 10% decrease starting from the 0 Kelvin, I plot here T upon Tm the melting point, up to the melting point there is hardly any decrease of the modulus of the material.

This is the different materials, this is how it behaves, up to the melting point it doesn’t decrease, there is hardly a decrease of 5 – 10% but if I do not do the experiment carefully and I allow the time to play its role when the temperature exceeds 40% of the melting point time dependent deformation begins in materials and you will find more deformation is less modulus. Some of the books might be showing this, some decrease like that and that is an incorrect experiment. Time independent deformation what is elastic deformation does not decrease with the increase in temperature, the modulus remains more or less the same, right. That is the temperature dependence of the elastic modulus.

Student: So even in and a higher melting then the T melting point.

Professor: Then it is a liquid; there is no question of modulus, then no question of elastic modulus. Elastic modulus we are talking in solids only.

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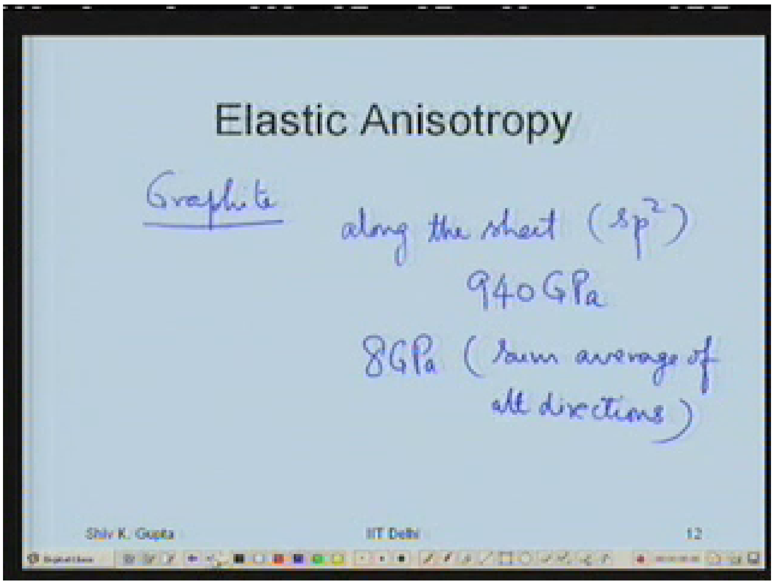

Elastic Anisotropy

Graphite

along the sheet (sp^2)

940 GPa

8 GPa (some average of all directions)

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Now elastic and isotropy, and isotropy I mean the direction and the crystalline structure. If the property remains same in all directions I called is isotropic, if the property does not remain same in all directions it becomes Anisotropic. You would notice that if I talk about even a simple cubic crystal along the A, B, C3 axis, bond lengths are same alright, at a certain value of the bond length A0 let us say, but if I look at the 110 directions which are the phase diagonals. Bond length becomes under root 2 of A0.

I don’t want to expect the modulus to be same in 100 directions on the 110 direction. So it is crystalline structure whether it is simple cubic, body centered cubic, face centered cubic as long as I am talking about a single crystal the properties are different in different direction. How normally we use polycrystalline solids, and in polycrystalline solids I have millions and billions of crystals, each one differently oriented in space so any direction in the 3 dimensional solid will be the some average of all possible directions and therefore we will find property to be same in any direction and that will be an isotropic material, it is not a single crystal, it is a polycrystalline material.

But there are other materials which will definitely show me different properties in different directions because of the nature of the bond. Like if I talk about graphite, graphite I talk of crystalline graphite. If I look the directions which are along the sheet, along the sheet I have only SP2 hybridization, all primary covalent bonds. Any direction you try to pull you are pulling the primary bonds. Modulus is very high to the order of 940 gegaPascal. But when you do across the sheets there are only secondary bonds between the sheets and you are pulling the secondary bonds, the bond modulus is very small.

To the extent that if you take an average of all possible directions where many of them are lying along the sheet the value of the modulus 4 falls down to 8 gegaPascal. Similarly when we have polymeric materials, when you are stretching along the chains very strong, modulus would be higher but if across the chain it would be very weak, they are only secondary bonds. So we find elastic Anisotropy in materials and there could be single crystals crystalline solids it could be polymers, it could be material like graphite where we have primary bonds in one direction and the directions which are on the sheet are much less directions than the direction across the sheet. The modulus could be every small value.

Student: For the isotropic material it would be same.

Professor: Yes, in all directions it will be same. Like you are measuring the modulus of steel which could be anywhere between 200 – 210 gegaPascal, any direction you take it will be the same value because we have the polycrystalline solid and all the grains, neighboring grains are differently oriented. Orientation is not the same in different grains.

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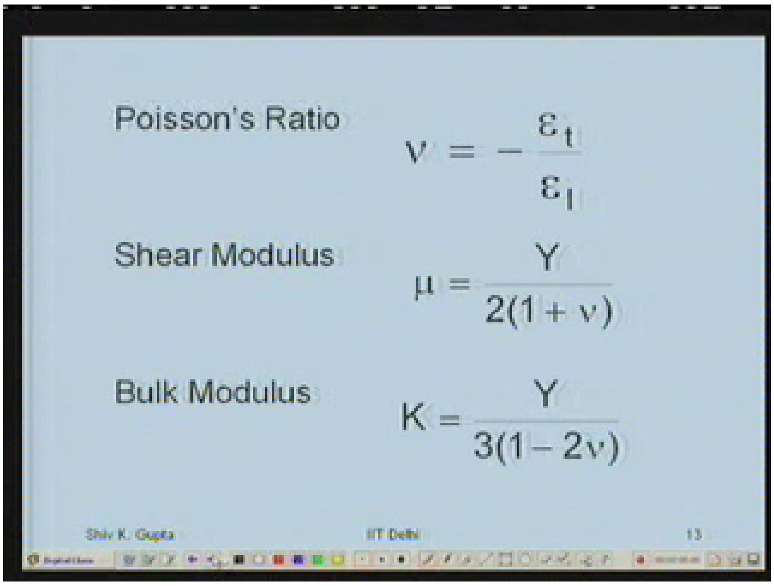

Poisson's Ratio $\nu = -\frac{\varepsilon_t}{\varepsilon_l}$

Shear Modulus $\mu = \frac{Y}{2(1+\nu)}$

Bulk Modulus $K = \frac{Y}{3(1-2\nu)}$

Here are the some different some relationships between shear modulus, bulk modulus with the Young's modulus. Before that we define the Poisson's Ratio. That is when I stretch a material, I should you a unit cell being stretched but I didn't show you what is happening in the lateral direction. In the transfer direction there is some strain there is contraction, slight contraction that is why I put a minus sign there. New Poisson's Ratio is positive, this is the contraction and the transfer's direction, this is the elongation in the direction of the applied force, longitudinal direction and the minus sign that is called the Poisson's Ratio.

And shear modulus is defined as Young's Modulus divided by 2 into 1 plus the Poisson's Ratio mu. The bulk modulus K is defined as Young's Modulus divided by 3 to 1 minus 2mu. It is merely the definitions and I don't want to get into the shear modulus bulk modulus properties, basically I am interested in structure property relationship. Whatever happens to the Young's Modulus happens to these two other modulus also.

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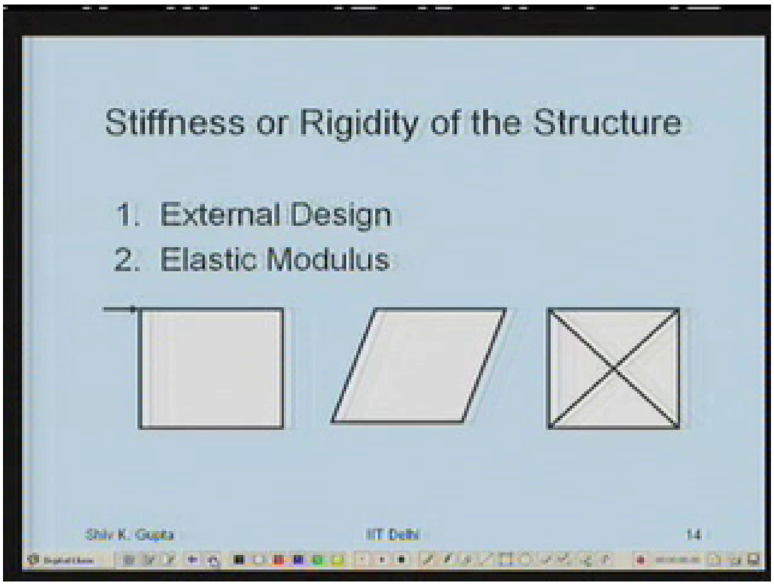

Stiffness or Rigidity of the Structure

1. External Design
2. Elastic Modulus

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As related with this as engineers are concerned it is the property called the stiffness of the material or the rigidity of the material. When I use a material I am using it in some form, some structural form of the material and depending upon the design of the material or design of the structure its rigidity or stiffness can be increased or decreased. That is external design is the one responsible for defining the stiffness or rigidity. But if I take same design made of two different materials, let us say one made up of aluminum, other one made up of copper and third one made up of steel then if the same design, same structure is there it will be depend upon the modulus of the material that is why two things I have written.

External design and elastic modulus. Here I show you a frame made of 4 bars. If I just apply a force on this it will deform in the fraction. However I can put two tie rods in the structure right here and it becomes stiff. Now apply if the force it doesn't deform, right. So that is what I mean by external design. We increase the stiffness of the rigidity of the structure by shear design but if my design is constant or the same then it depends upon the property of the material elastic modulus, right. Therefore when we want a stiff structure or rigid structure I want the modulus of the material to be higher.

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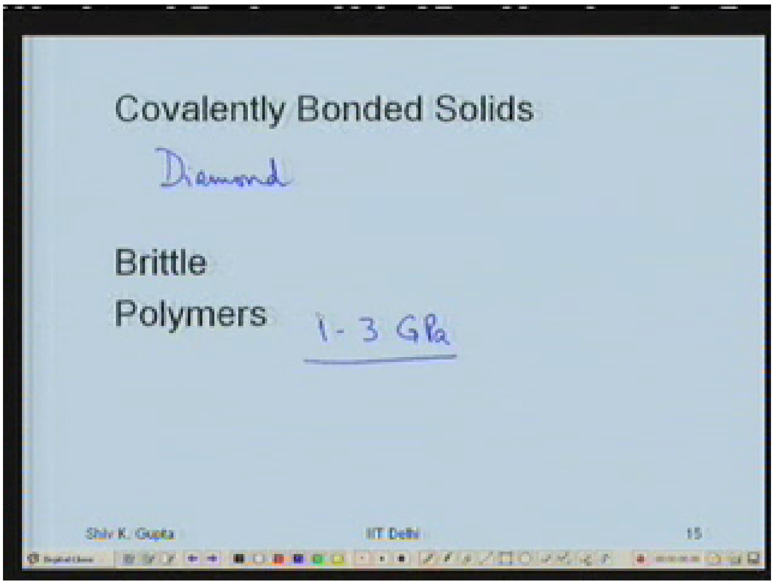

Covalently Bonded Solids

Diamond

Brittle

Polymers 1-3 GPa

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High modulus we find amongst covalently bonded solids, we can also find this in ionically bonded solids we will come to that. And highest we find in a material what is called diamond as I just showed you. But I cannot use the diamond though it is a very expensive material, not available in great abundance. I cannot build a structure even if it is available because it is a very brittle material. It fails within its elastic limits only, it is a brittle material, I will show you the difference between brittle material and ductile material. Today it should be good enough for you to feel a material is brittle if it doesn't show the plastic deformation.

A material is ductile if it shows the plastic deformation before it fails. So brittle material fails within the elastic limits, they don't even reach their yielding and diamond is one such material, it is a brittle material. We cannot exploit covalently bonded solids like this or glasses for that matter where the covalent bonds are found the materials are brittle, we cannot use for structural applications. Then there are covalently bonded solids polymers. Usually between the bonds are formed in polymers along the chain are primary bonds, SP3 bonds but across the change they are (())(32:46) forces. The result these polymers have a very low modulus value, its range is usually between 1 – 3 GegaPascal. It is a very small range.

Student: (())(33:06)

Professor: Yes, it will be if the chains are (())(33:11), along the chain it will be little higher and across the chain it will be, and we cannot have 100% crystalline solids and long chain polymers, so there will be some non-crystalline regions along the direction and therefore it would not be a very high modulus in that sense.

However if I have a network polymer it will be little more because I have all three dimensions, in three dimensions I have primary bonds, it will be more but even there it is also not very high, it may go up to maybe 5 GegaPascal, that is the kind of values or the order of magnitude for the modulus values we have in these materials and it is not possible for me to use them for any structural application unless if it is possible for me to increase the modulus.

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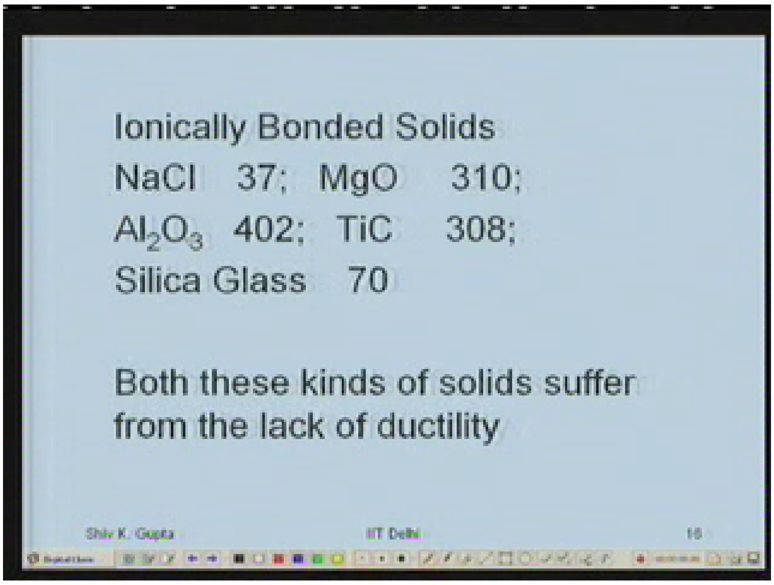

Ionically Bonded Solids

NaCl 37; MgO 310;

Al_2O_3 402; TiC 308;

Silica Glass 70

Both these kinds of solids suffer from the lack of ductility

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Then there are ionically bonded solids. In here this 10th of the bond or how well it is going to be deeper depends upon the charges on the ion. The charges on the ions are small (())(34:23) 1it may not be very deep well but if its charge is more it can be a deeper well. That is what I have tried to show here. In sodium chloride charges are 1 and 1, it is only 37 gegaPascal. In all these modulus values and gegaPascal as I said magnesium Oxide 2 into to 310. Al2O3 charges are 3 and 2, 402, Titanium Carbide it is 4, this 308, Silica Glass, silica is 4 oxygen is 4, 70.

So charges on the ions are ones which are responsible for affecting the nature of the bond however it is stronger, the well is going to be deeper and the modulus, the curvature is going to be smaller or larger, it shall depend upon this therefore this can be there. But even these materials tend to become brittle materials whether it is sodium chloride, magnesium oxide, Al2O3, Titanium carbide, or silica glass these are all very-very brittle materials that is they do not show the plastic deformation before they fail. So ionically bonded solids as well as covalently bonded solids do not have ductility, they lack ductility, I cannot use them or exploit them for structural applications.

Student: (())(35:49)

Professor: Titanium is 4 plus.

Student: (())(35:55)

Professor: Well this depends upon not merely the charges, shape of the potential energy well that may not have relationship to the charger that depends upon the constant which are there.

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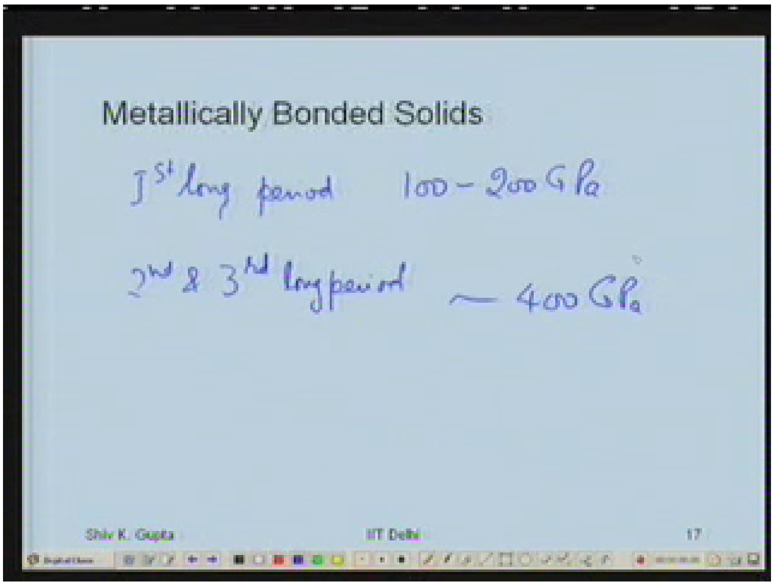

Metallically Bonded Solids

1st long period 100 - 200 GPa

2nd & 3rd long period ~ 400 GPa

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Now among the metallically bonded solids we get good ductility and the modulus values are not very high usually. In the first long period where I have lot os metals, common metals which we use, it could be in the range of 100 to 200 gegaPascal, but only when I go the second and third long period I can go up to 400 gegaPascal but not more. So the range is in the first long period it could be 100 to 200 gigaPascal but when you go to second and third long period there is a transition element that you can go up to 400 gigaPascal.

And they have a partial covalent nature of the bond because the overlap of the D-shell, first long period and this partial covalent nature of the bond is responsible for them to have little higher typical aluminum as about 70 gigaPascal. A typical metal like copper has about 130 gigaPascal, right. Steel, the iron is about 210 gigaPascal so that is the range in which we have the values of the Young's Modulus and we do not have like the 1100 gigaPascal in diamond, that kind of value we do not have but these are the materials which we can exploit structural applications.

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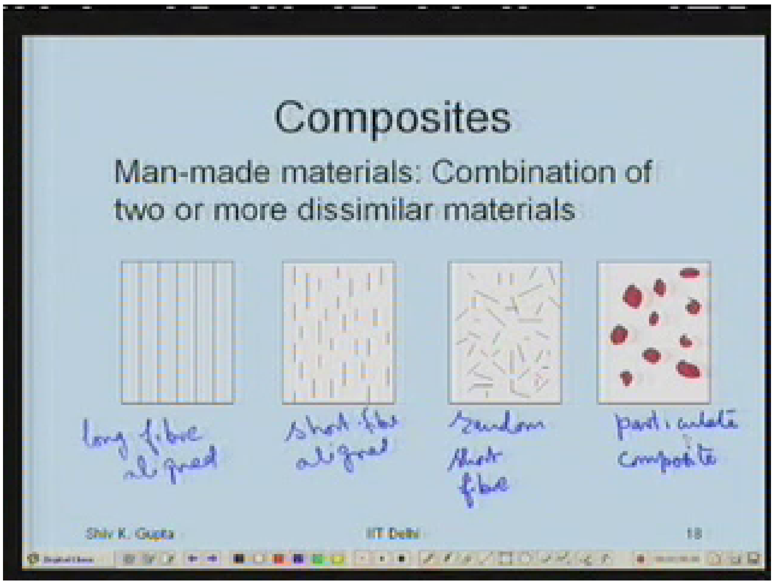

Composites

Man-made materials: Combination of two or more dissimilar materials

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Now this is a material property which is no dependent on the structure, depends upon the bond line, depends upon the curvature of the Condon-Morse curve at the equilibrium spacing, thereby what manipulation I do the microstructure it is not going to get affected. The bond between iron-iron will remain the same. What kind of modification I try to do or alloying I try to do that is why elastic modulus is called a structure insensitive property. Iron and steel is not very different, it is varying between 205 – 210 gigaPascal, right.

However we try to modify this value of this modulus by making composites and composites are manmade materials, you can simply define it as a combination of 2 or more different materials, 2 or more dissimilar materials rather I should say, okay. So 2 or more different or dissimilar material are used to make the composites. Combination it is, whether at the combination at the interface between the 2 materials, there is a chemical bond present like the grain boundaries is not guaranteed. There is only adheration between them.

And the goodness of the material, is how good is the adheration is at the interface, the words 'How good the interface is, how strong the interface is?' and that will tell you how good the material is but this is what we can probably do today to improve upon the modulus. Here I am showing the different kinds of composites which are possible, there are 2 materials here I have shown. One in the form of long fibers going from one end of the material to the other end and second is the matrix. So you can call it long fiber composite, long fiber aligned.

I can have long fiber non-aligned. It could be randomly placed in the 3D volume. Here I show this small short fibers which are also aligned but if I (())(41:07) do not go for one end to the other end, the short species and here I show the short fiber randomly aligned and here I have this second material is in the form of particles so it is called particulate composite.

Normally you people are using helmets which are having the fiber reinforced plastic as a material. There the fibers are placed in randomly in all orientation but they are long fibers. Long fibers plays randomly all over the volume, zig-zag, they are just lying there going from one end of the material to the other end. Then short fiber aligned you have to make effort to make this material but this is not so difficult to make, it can again randomly placed in and can just simply make the matrix, pour the matrix on top of it.

This is probably one of the earliest kind of exploitation made by the human beings. In villages they make huts of mud and mud is not purely mud, this is mixed with husk and the husk is a short fiber which is randomly oriented and that is the kind of used and the reinforced concrete cement if you have seen, usually rods are aligned, vertically they are placed parallel and they are also placed horizontally, so you are putting them in 2 directions, aligned fibers.

And we have also seen in the concrete, (())(43:30) which is stone particles and you have the matrix of sand-cement mixture, so the matrix of sand-cement mixture this is (())(43:41), the particulate composite, this is concrete but when I reinforced with steel rods it becomes reinforced concrete cement. These are the different kinds of varieties of composites I can have. We shall analyze the modulus behavior with reference to this first one, we have long fiber which is aligned.

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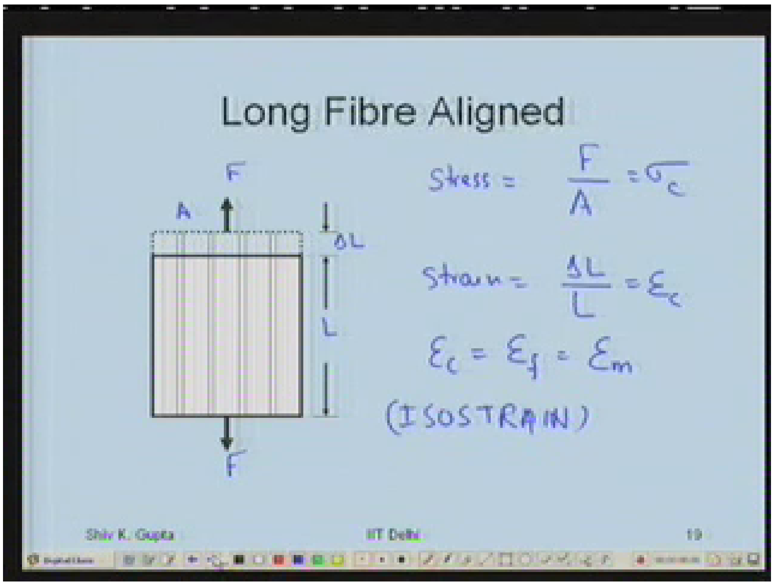

Long Fibre Aligned

Stress = $\frac{F}{A} = \sigma_c$

Strain = $\frac{\Delta L}{L} = \varepsilon_c$

$\varepsilon_c = \varepsilon_f = \varepsilon_m$

(ISOSTRAIN)

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This is the long fiber matrix aligned, I have just shown little diameter of the fiber, fiber diameter may not be as big, let us say this is the original length, let us call it L and this is the elongation, delt L, when you apply a force F and let us say the area of cross-section of the composite perpendicular to the force is A. So on the composites you have applied this stress and you have developed this strain. As I showed here the fiber elongates by the same amount as the matrix and when that is happening I say my interface is intact.

If suppose matrix elongates more and fiber does not elongate so much there is a slip at the interface, matrix is slipping with respect to the fiber. Interface has giving way, interface is not intact, (())(45:35) fail actually. So when I am talking about applying force I am not crossing that limit, I am within that, alright, that is important to understand or else if the fiber is getting elongated more than the matrix then the fibers will get pulled out of the matrix and once the interface gives way, right?

So therefore I am assuming that strain in the fiber as well as the matrix is the same. So this is equal to let us say epsilon and this epsilon in the composite is equal to epsilon in the fiber is equal to epsilon in the matrix. Such a situation also called isostrain situation. The strain in the matrix as well as the fiber is the same, right?

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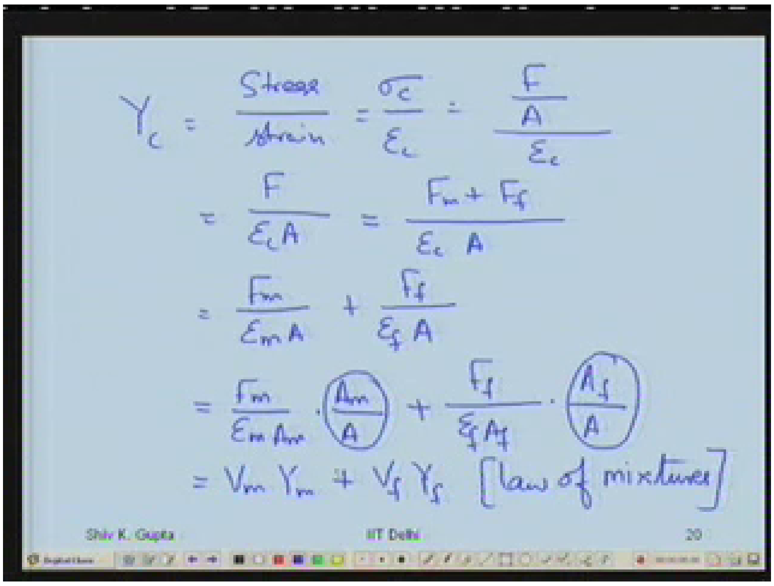

$$Y_c = \frac{\text{Stress}}{\text{strain}} = \frac{\sigma_c}{\varepsilon_c} = \frac{\frac{F}{A}}{\varepsilon_c}$$

$$= \frac{F}{\varepsilon_c A} = \frac{F_m + F_f}{\varepsilon_c A}$$

$$= \frac{F_m}{\varepsilon_m A} + \frac{F_f}{\varepsilon_f A}$$

$$= \frac{F_m}{\varepsilon_m A_m} \cdot \boxed{\frac{A_m}{A}} + \frac{F_f}{\varepsilon_f A_f} \cdot \boxed{\frac{A_f}{A}}$$

$$= V_m Y_m + V_f Y_f \quad [\text{law of mixtures}]$$

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So let us write down the Young’s modulus of the composite. Now we know the modulus of the matrix and the fiber are not the same, they are different materials. And strain is the same, the force taken by the matrix and the fiber is also not the same then and force I can then write as, force taken by the matrix plus the force taken by the fiber, this I can split.

This strain C I have taken as matrix here and strain of the fiber, because I know strain in the fiber is same as the strain in the matrix as the strain in the composite. So I replace that in these places, though actually I have only make it A plus B, right. Looking at this if I can have this in this area of the matrix it shall become the Young’s modulus of the matrix. Let us rewrite that way and then multiply by area of the matrix divided by A.

I don’t change the term, term remains the same aArea cross section of the fiber. Now in the composites I have shown the length of the fiber and the matrix is the same whether it is strained or not strained because strain is also the same. So the area fraction is as the same as the volume fraction. This is area fraction of the matrix. This is the area fraction of the fiber, so they become the volume fraction of the fiber and the volume fraction of the matrix.

While this is the Young’s modulus of the matrix, this is Young’s modulus of the fiber. So I can write volume fraction of the matrix times the Young’s modulus of the matrix plus volume fraction of the fiber times the Young’s modulus of the fiber, this is also called the law of mixtures. And in a composite like this where fibers are aligned, long fibers are aligned it gives me the upper bound of the modulus when fibers are not necessarily aligned, they are all randomly placed.

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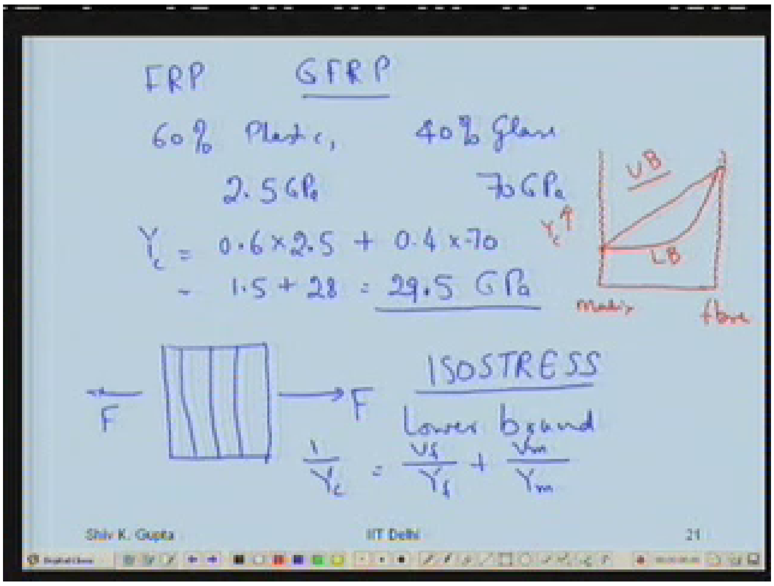

FRP GFRP

60% Plastic, 40% Glass

2.5 GPa 70 GPa

$Y_c = 0.6 \times 2.5 + 0.4 \times 70$

$= 1.5 + 28 = \underline{29.5 \text{ GPa}}$

ISOSTRESS

Lower bound

$$\frac{1}{Y_c} = \frac{V_f}{Y_f} + \frac{V_m}{Y_m}$$

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I cannot get more than this, is the maximum, modulus which I can attain, right? Say for example what do I get. If I talk about fiber reinforce glass, glass fibers reinforced plastic rather which is called FRP or sometimes which is called glass reinforced plastic. Let us say the plastic volume is 60% and glass fiber is 40% and the modulus of the plastic we know is, let us say I take very high value, something like 2.5 gigaPascal and glass is 70 gigaPascal, let us see what happens to the composite, is 0.6 into 2.5 plus 0.4 into 70 that makes it 1.5 plus 28, 29.5 gigaPascal.

You have not made it 70 but you have gone from 2.5 to 29.5 by adding 40% of glass fiber by volume, right? So that is the kind of improvement you can make to the upper bound. However if you take the same kind of composite but apply the force in this direction now, this is called isostress situation and isostress situation gives rise to lower bound. So Young's modulus of a composite with long fibers when they are randomly placed will be between the upper bound and the lower bound, right?

In the lower bound Young's modulus is volume fraction of fiber, Young's modulus of the fiber, you can work that out in the same as I did for the other one is very simple. So that is the lower bound and okay, alright let us use this part of the, let us say this is the matrix here, fiber here, Young's modulus composite, that is the 100% matrix, 100% fiber, alright. This is the upper bound and if you look at the lower bound it grow something like this.

So modulus of the composite of the man made material can be improved and that is of course the upper bound to the law of mixture, right? This is what maximum you can do to the modulus and not more, (())(54:05) structure insensitive property. Well we will stop here and will talk about the permanent deformation in the next class.