Applied Mechanics

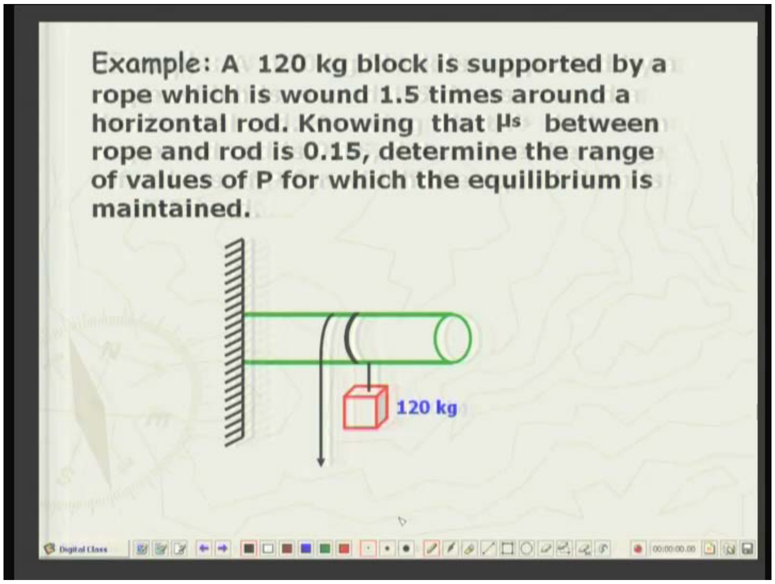
Prof. R. K. Mittal
Department of Applied Mechanics
Indian Institute of Technology, Delhi

Lecture No. 9

Friction and its Application (Contd.)

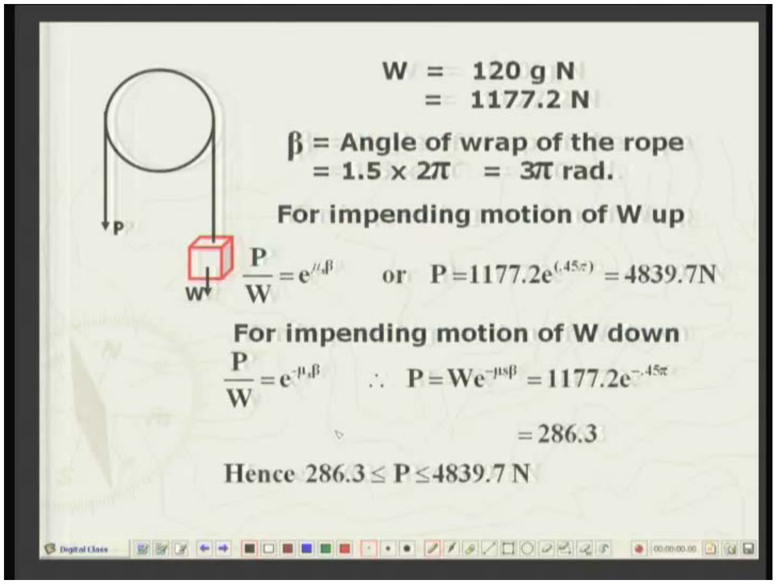
Today we begin with lecture nine. This is continuation of the previous two lectures on the topic of friction and its applications. In the last lecture, that is lecture eight, we started our discussion on belt friction. This is an important application of the concept of friction. With the help of belt and pulley system, we can transmit power from one point to another point. In the last lecture, we derived an important relationship for the tension in the two sides of the belt as it goes over the pulley or drum. The angle of wrap, that is the angle through which the belt is in contact with the drum, is beta. Then, at the instant of impending slip between the belt and the pulley, the ratio of the tensions on the two sides of the pulley is given by exponential of mu s, that is the coefficient of static friction times the angle of wrap. If on the other hand the belt is slipping, that is, there is relative motion between the pulley or drum and the belt, then this ratio of the two tensions on the two sides of the pulley is given by the exponential of coefficient of dynamic friction times the angle of wrap. Now after having established that relationship, we also saw how to calculate the torque being transmitted by the belt when it has tension T one and T two and what is the force exerted or reaction exerted by the bearings on which the pulley is mounted. So we should be able to calculate the necessary force to maintain the tension in the belts. Today, we will take up few applications or examples of these concepts and let us begin with example one. This example is shown here.

(Refer Slide Time: 3:55 min)



A weight of a block of hundred twenty kilo grams is supported by a rope which is wound one and a half times around a horizontal rod. So it can be easily seen that this is the block of mass hundred twenty kilo grams and it is being held with the help of a rope which is wound on this horizontal rod. Knowing that the coefficient of static frication mu s between the rope and the rod is point one five, determine the range of values of P. P is the applied force at the free end of the rope for which the equilibrium is maintained. So, when will the equilibrium be disturbed? When the mass either goes up or goes down. So there are two cases to be examined. First, we will consider when the mass is going down.

(Refer Slide Time: 5:17 min)

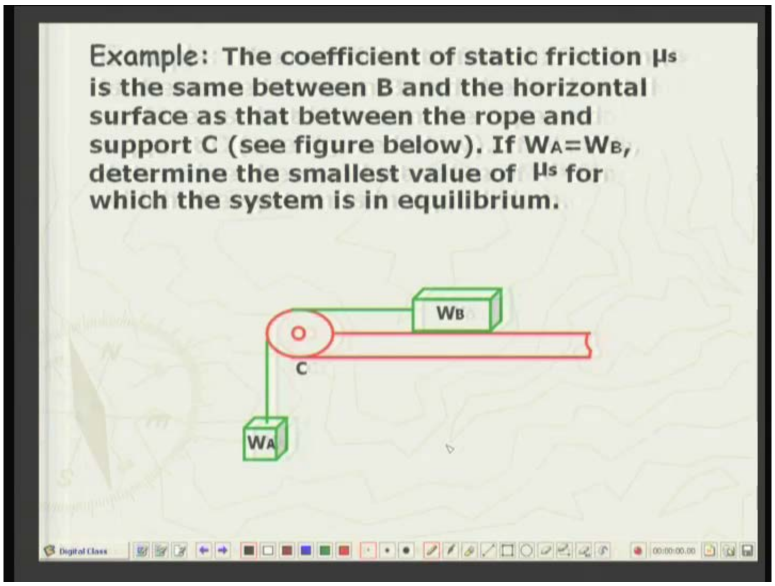


So the force at this end, mass is given as hundred twenty kilogram. So mg, that is, the force due to gravity, is mass times the acceleration due to gravity, that is nine point eight one meters per second. So you can see that this gravitational force W is equal to hundred twenty g Newton's which is eleven seventy-seven point two Newton's. Now the angle of wrap is between the rope and the rod because it is going over one and a half tons. So in each ton the angle subtended is two pi. So one point five times two pi is three radians. Now, when the impending slip is, such that the mass is about to go up, that is, this end where the pull P is being applied is going down. So in that case, the tension P will be larger than that of W. So P over W, that is T one over T two, is replaced by P one. P over W is equal to exponential of coefficient of static friction times the angle of wrap which we have already calculated. So in other words, P is equal to eleven seventy-seven point two as calculated here into exponential of three pi radians into point one five. So it will be point four five pi. The angle is always to be given in radians. So this calculates out to be four thousand eight hundred thirty-nine point seven Newton. So when this much downward force is applied at this end, then the weight will go up.

Now, let us examine this case when the weight is going down. The other extreme case is that this slip is such that the weight is just about to go down. Then we will have W over P is equal to the coefficient of static friction times the angle of wrap or P over W is equal to minus of e to power minus mu s into angle of wrap. So doing the same calculation with only differences, that is there is a minus sign here in the exponent, we will get the force equal to two hundred eighty-six point three Newton's.

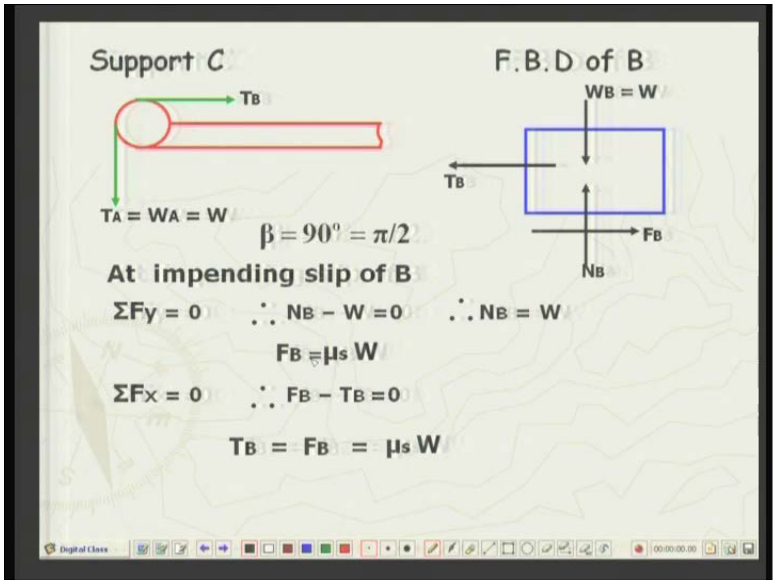
So in these two extreme cases, when once the mass is going up, the other time mass is going down, the forces are two hundred eighty-six point three and four thousand eight thirty-nine point seven. So these are the two extreme cases. Any force in between the two, the weight will be neither moving up nor down. So the equilibrium of the weight will be maintained. So these are the limits of the forces.

(Refer Slide Time: 8:57 min)



Let us take up another example in which we have weight WB and another weight WA. The example reads like this: the coefficient of static friction mu s is the same between B and the horizontal surface. So this is the weight WB and this is the horizontal surface. So this coefficient of friction is same as that between this green rope and the corner support C. What it means is that, this mu s over here and mu s over here of the contact angle is same. If the two weights WA and WB are same, determine the smallest value of the coefficient of static friction, mu s, for which the system is in equilibrium. So we have to find out that coefficient of friction which will be able to just maintain, that is, it will be the case of impending slip. That is very essential to note down, when the slip is likely to take place. That is the critical case and anything beyond that the slip will actually take place. So let us have a look at the free body diagrams.

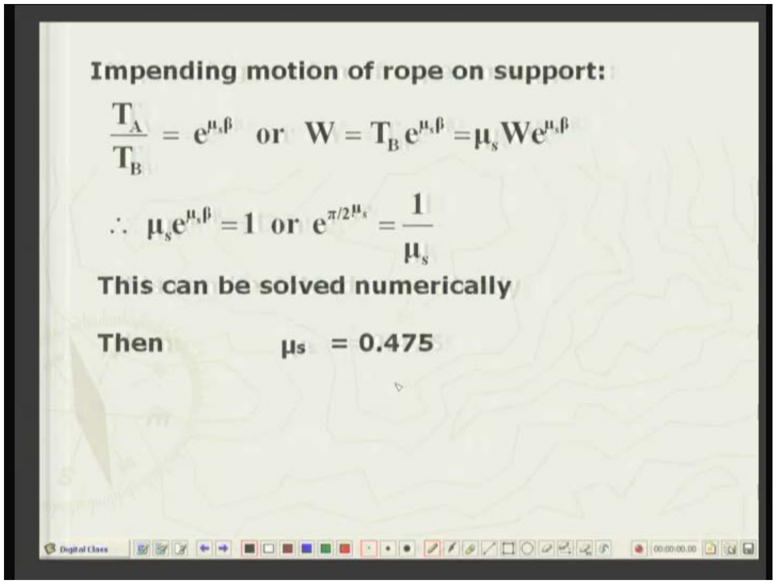
(Refer Slide Time: 10:37 min)



Now there are two free body diagrams here, one side there is a tension TA and on the other side the tension TB. Now on this side naturally, this tension will be due to the weight WA. So TA will be automatically equal to WA and on this side, the tension is TB. So we can see a tension TB on this side. This we will calculate from the free body diagram of the block B. Now the angle of wrap. First we will take up the forces around the support C. So the angle of wrap here is pi by two. So first, we will take a free body diagram B. So let us see what the forces on the block B are. There is a downward weight W because WA is equal to WB is equal to W. So we know that and the normal reaction upward from the horizontal surface on to the block. So if we consider the summation of forces in the y direction equal to zero, then it is easily seen, that NB is equal to W. From third law of Coulomb for dry friction, we know that at impending slip, the force of fiction FB on the block B is equal to mu times the normal reaction. So mu s times W.

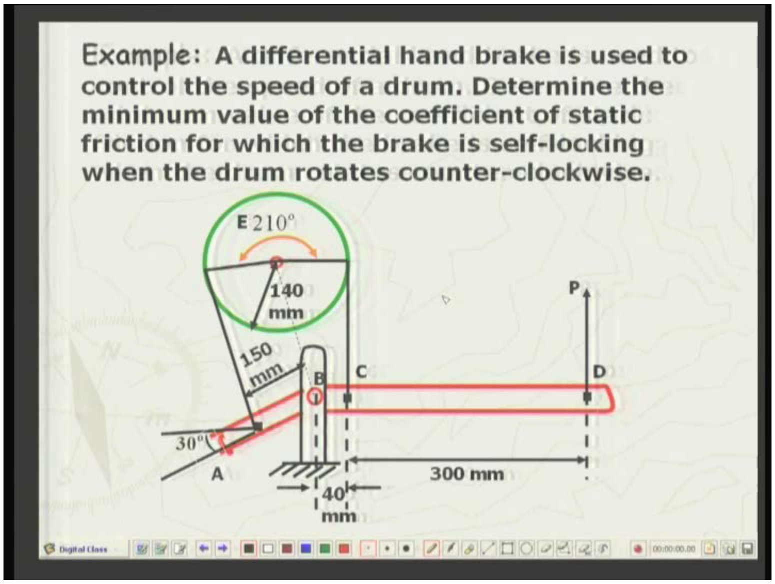
Now look at the equilibrium in the x direction and summation of forces in the x direction equal to zero. What are the forces TB and FB? So obliviously B is equal to TB and FB. We have already found out mu s times W.

(Refer Slide Time: 13:15 min)



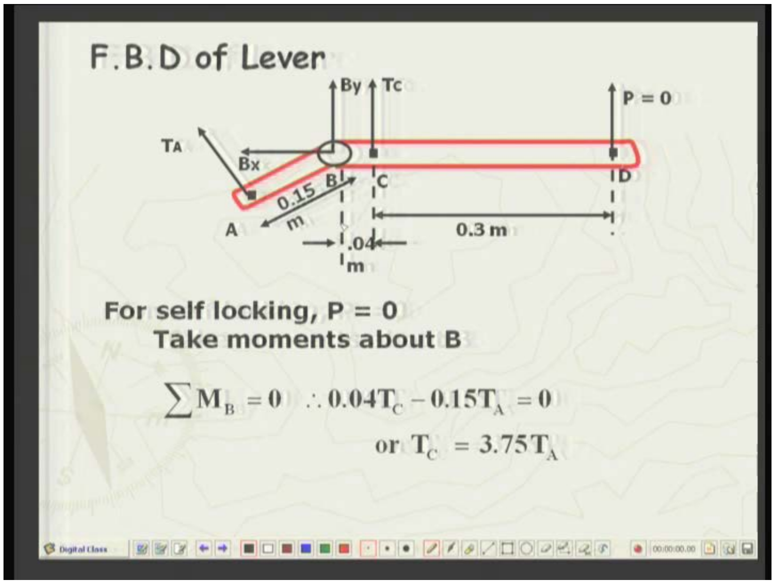
So now, we come to the support C, where the rope is going through an angle of wrap of ninety degrees. So TA over TB, that is, look here when the slip takes place between the rope and the support C, at that time the block will also just begin to slip. So the slip of rope and the support is happening at the same instant as the slip between the block and the horizontal surface. So it means, we will use the coefficient of static friction for this exponent at that impending slip case. So substituting for TA which is equal to W and then TB, etcetera, which is mu times s, mu s times W into exponential of mu s beta. So simplifying W and W will cancel out. So we have the simple equation coefficient of static friction mu s times exponential of mu s times beta is equal to one right and it means that beta is pi by two. So exponential of pi by two times coefficient of static friction is equal to reciprocal of coefficient of static frication. This equation can be solved numerically. There is no closed form solution. So solving it, we find that the coefficient of static friction comes out to be point four seven five. So otherwise the slip will take place, if the coefficient of friction is less than that.

(Refer Slide Time: 15:38 min)



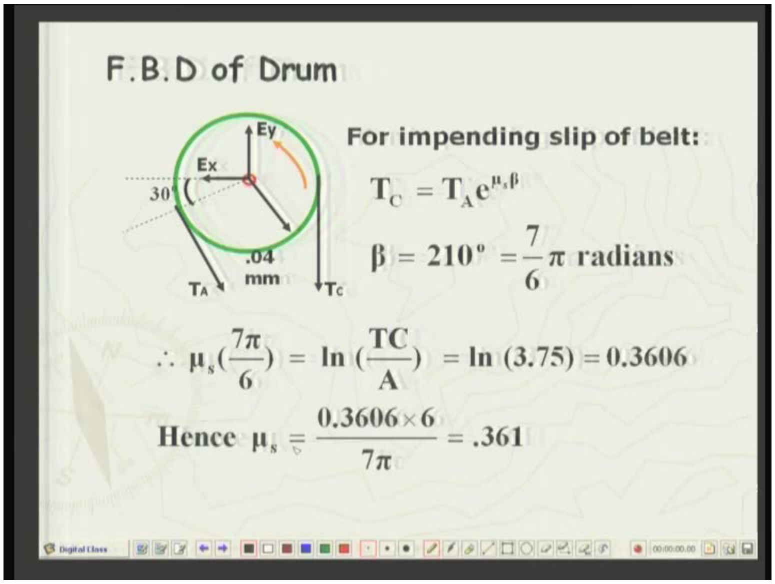
We will take up another interesting, practical example. This is about the differential hand brake. Suppose in a workshop or in your automobile, there is a flywheel and we want to bring it to rest. Then we use this hand brake or deferential hand brake which consists of a belt going over the flywheel or the drum and the force is applied through a lever system as shown over here. So the example reads as such: A differential hand brake is used to control the speed of a drum. Determine the minimum value of the coefficient of static friction for which the brake is self-locking. That is, we do not have to apply any external force when the drum rotates counter clockwise. So the drum is rotating counter clockwise. Well, this system is shown over here. Various dimensions are given.

(Refer Slide Time: 16:56 min)



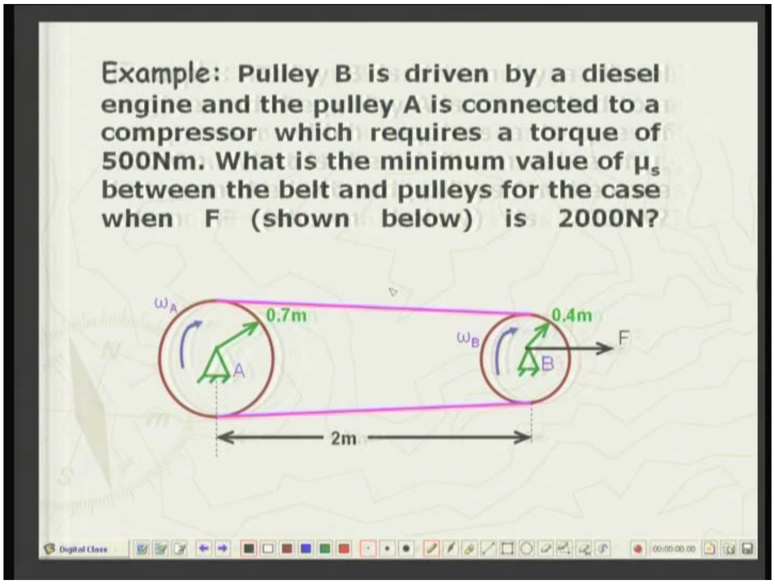
So let us go on with the free body diagram of the lever. Well, this is the lever, this is the pin joint about which the lever can rotate. So at the pin, there are the reactions Bx and By. Now, this is the applied force P but since the system is self-locking, that is, without application of external force, the breaking action has to take place. So we will set P is equal to zero. These are the ends of the belt which are going over the drum. So TA on one side, TC on the other side. So let us say, take moments of all the forces about point B. Why we have chosen point B? Because two of the unknown reactions Bx and By pass through that point and hence their moment about B will be zero. So, we will do so and P is already the applied load. P is already known to be zero. Then we are left with TA and TC. So taking moments about B and setting equal to zero and taking the dimensions as given over here, point four meters into TC, that is, anticlockwise and this is TA into point one five, that is clockwise. So in this way, point zero four TC minus point one five TA is equal to zero. Hence TC comes out to be three point seven five TA. So the ratio of the two tensions TC over TA, comes out to be three point seven five.

(Refer Slide Time: 19:03 min)



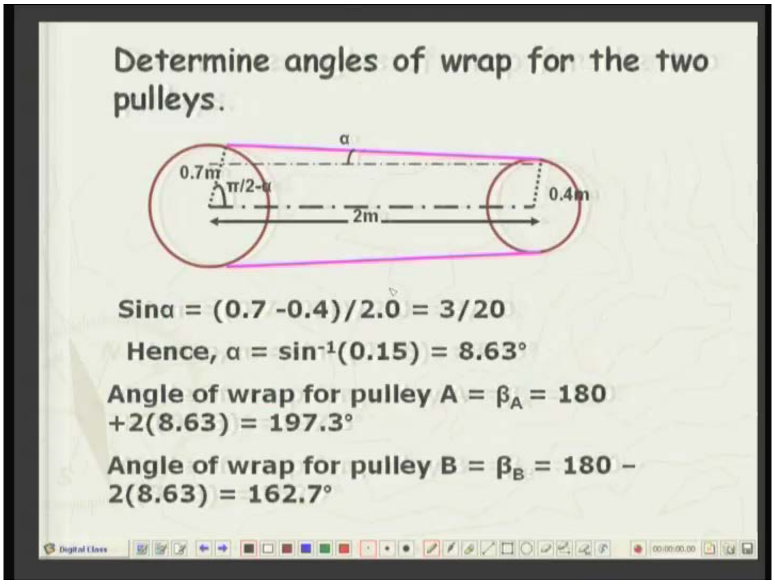
Now we look at the free body of the free body diagram of the drum or the belt or the pulley. Well, for impending slip condition, since the applied torque is counter clockwise, to break it, that is, to bring it to halt, the torque due to the tensions of the belt should be clockwise. So TC over TA is equal to e to power coefficient of static friction times the angle of wrap. Now, what is the angle of wrap? It is given as two hundred ten degrees which is equal to seven by six pi radians. So take logs on both sides. It means log of exponential will be mu s times seven pi over six. This is equal to TC over TA. Since we have already determined this ratio as three point seven five, log of three point seven five which comes out to be the point three six zero six and from here we get the coefficient of static friction. To break the motion of the drum, to bring it to stop that comes out to be point three six one. So that is a very practical example of friction. Let us continue with few more examples.

(Refer Slide Time: 20:59 min)



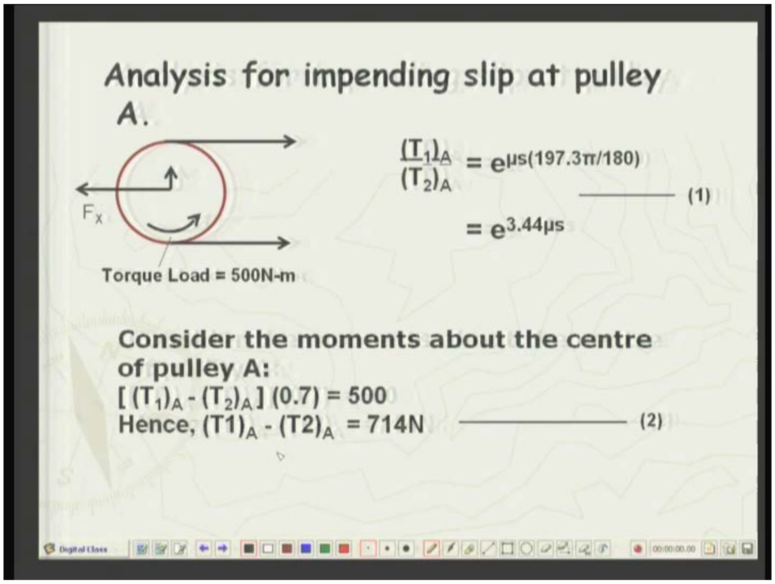
So that we understand various aspects of friction, here is a case of two pulleys, pulley B is driven. So the B is the driven pulley by a diesel engine and the pulley A is connected to a compressor which requires a torque of five hundred Newton meters. What is the minimum value of mu s between the belt and the pulleys for the case when F, that is, this tension at the bearings, is two thousand Newton's. So what is the minimum value of mu s if we have slightly less than that slip. So it is the case of impending slip. Various dimensions, the diameters, etcetera, are given.

(Refer Slide Time: 22:01 min)



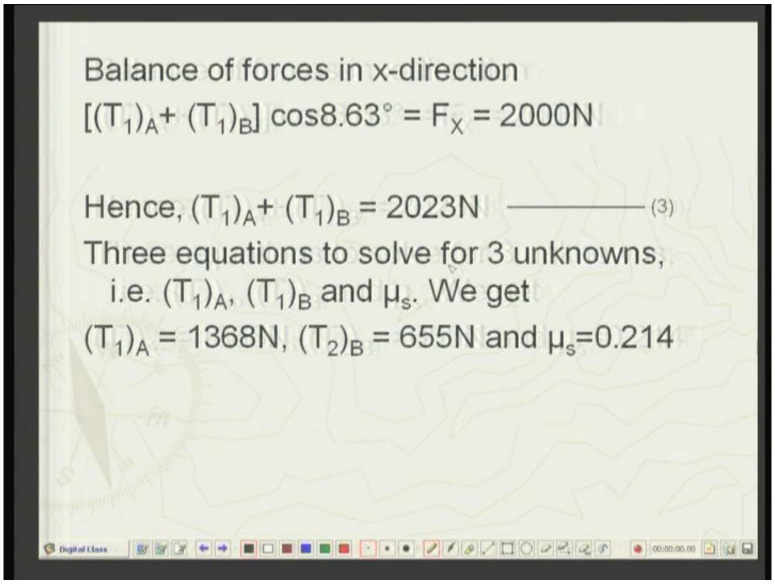
So let us begin with the solution of this problem. Well first of all, we have to determine the angle of wrap on both the pulleys. For this, we make use of the geometry of the problem. The distance between the centers of the pulleys is given as two meters, the radii are point seven meters and of the smaller pulley, it is point four meter. If I complete the parallelogram here and this angle alpha can be easily obtained. Sin of alpha is, you can easily see, is ninety degrees. So this makes a right angle triangle. So point seven minus point four. That is, this dimension divided by point two is over here divided by two, that is, the distance. So you look at this parallelogram. If this side is two meter, this side is also two meter. So this is sin alpha. So this is equal to three four seven minus four. So three by twenty. Therefore alpha is sin inverse of point one five which is eight point six three degrees. Now this is pulley A and this is pulley B. So for pulley A, the total angle of wrap will be hundred eighty degrees and one alpha here and one alpha over here. So plus two alpha. So beta A is equal to hundred eighty plus two times eight point six three which comes out to be one ninety-seven point three degrees. On the other hand, when I go to pulley B, then the angle of wrap beta b is equal to hundred eighty minus one alpha over here and one alpha over here. So that will come out to be hundred sixty-two point seven degrees. So after having calculated both the angles of wraps, then we will analyze.

(Refer Slide Time: 24:39 min)



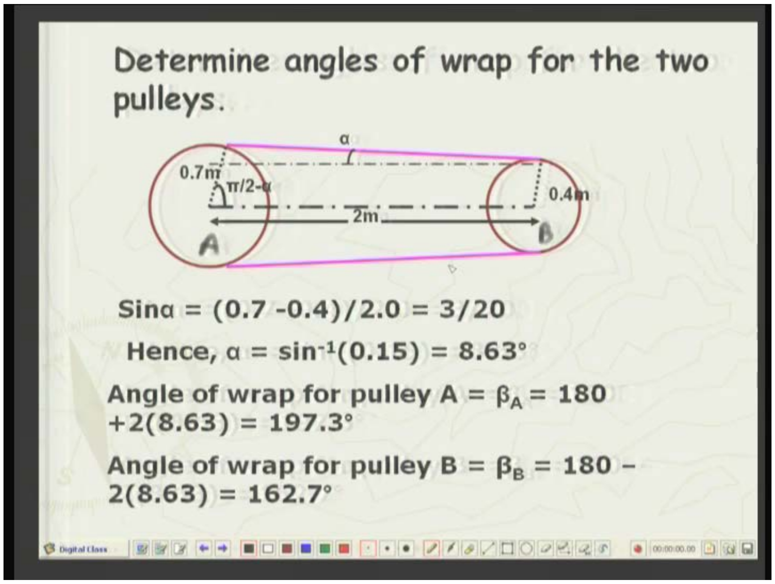
Now what is the information available to us? The angle of wrap was found out to be one ninety-seven point three and also, there is a total horizontal force from the bearing which is given as two thousand Newton's and then the total torque is also known. So there are three equations: first the pulley equation, that is T one over T two. First of all, we will consider the case, let us say, at the pulley A there is slip just about to take place. It is the case of impending slip at pulley A. So T one A over T two A, that is, T one A over T two A is equal to T two power mu s. Now, one ninety-seven point three is to be converted into radians. So pi into pi over hundred eighty because hundred eighty degrees is equal to pi radians. So one ninety-seven point three into mu s which is unknown. So we will have this ratio which is equal to exponential of three point four four mu s and then we will consider the moments about the center of the pulley A. T one A minus T two A. So into point seven. This is given as five hundred Newton meters and solving this, we will have the difference between the two tensions, T one and T two for the impending slip at A. A is equal to seven hundred fourteen Newton's and then the horizontal force component at the bearings is known.

(Refer Slide Time: 26:50 min)



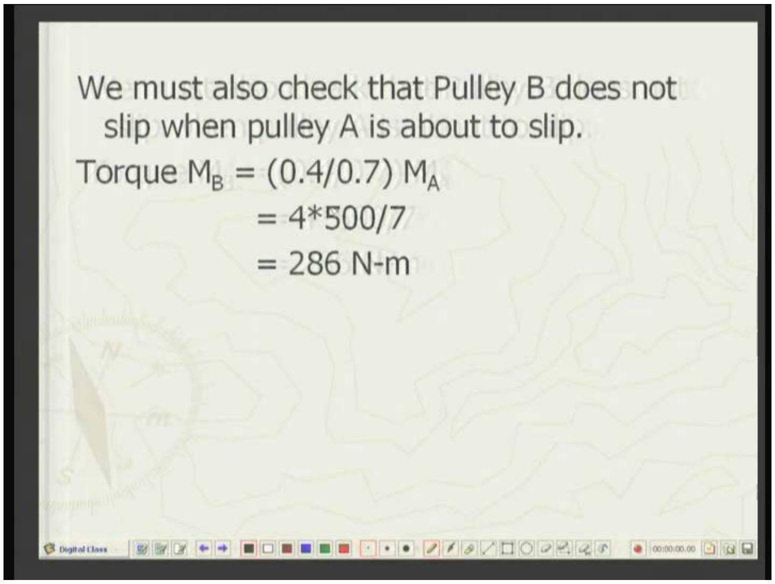
So we will take the components of the belt tensions in the horizontal direction. For example, on one side it will be T one A cosine of eight point six three degrees, on the other side also it will be T one B uh cosine of eight point six three degrees.

(Refer Slide Time: 27:12 min)



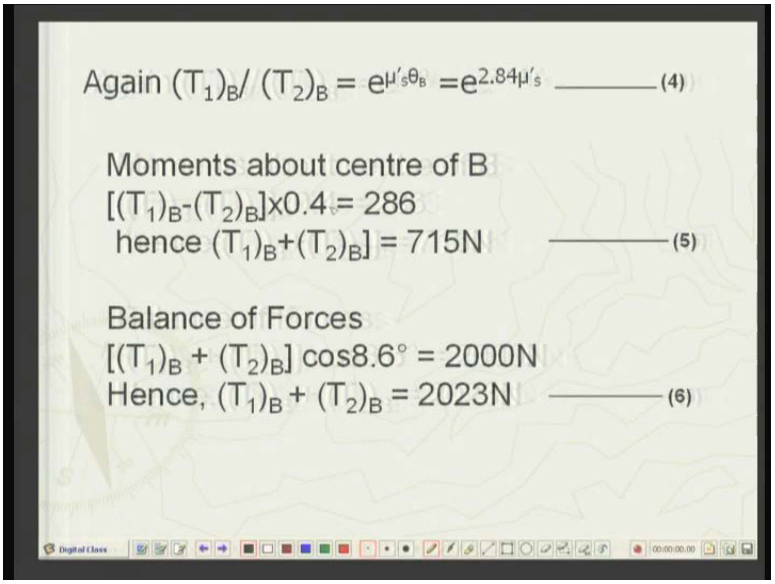
Just to show you, let us say, this is T one A. So we will take the horizontal components. Adding up the two, we will get T one A plus T one B is equal to two thousand and twenty-three Newton's. So we have three equations, three unknowns, that is, T one A, T one B and mu s. So the system of equations can be solved. The tensions T one A comes out to be one thousand three hundred sixty-eight Newton, T two B comes out to be six hundred fifty-five Newton and mu s is point two one four. Well you may be wondering that at the pulley A there is no slip but at the same time there may be slip at pulley B. So we have to see whether there is slip at pulley B or is this coefficient of static friction point two one four preventing the slip or not. We will do the similar analysis for pulley B as we have done for pulley A.

(Refer Slide Time: 28:44 min)



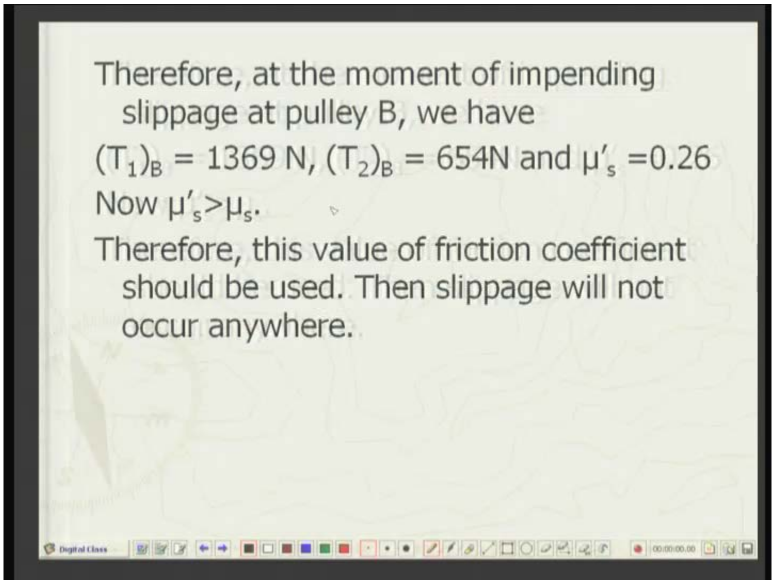
So let us do that. Well first of all, five hundred Newton's meter was the torque which was being supplied by the uh pulley A. So what will be the corresponding torque at the pulley B? That, you can easily say, the MB over MA. That is, the torque at B over torque at A is equal to the point four divided by point seven in the same ratios as the radii or the diameter. So you will have the corresponding torque is two eighty-six Newton meters. Well this equation comes from the work principle, that is, the work done, if there is no friction loss or any other loss, then the work done by pulley A should be equal to work done by pulley B.

(Refer Slide Time: 29:56 min)



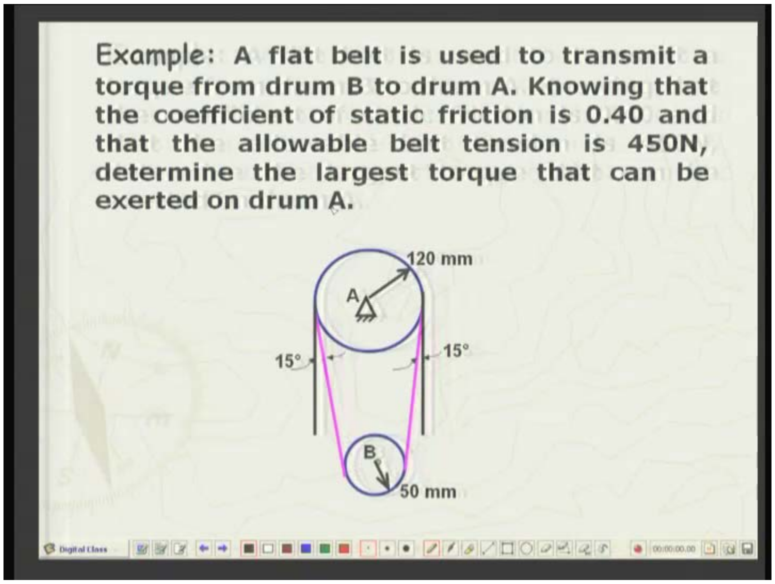
So once again, the ratio of the two tensions T one B divided by T two B, that is, when the slip is just impending at the pulley B. So suppose the corresponding coefficient of static friction is mu s dash then e to power mu s dash times the angle of wrap at pulley B angle of wrap is known to us, that is, it was calculated as one sixty-two point seven. So which converts into radians as two point eight four times the coefficient of static friction mu s dash? Again, take the moments about the center of the pulley B, then the sum of the two tensions is seven hundred fifteen. Then balance of the forces, that is, reaction at the bearings is again two thousand Newton's and this will again give me T one B plus T two B is equal to two zero two three. So again three equations, three unknown and if we solve it, we will get the corresponding tensions as six five four.

(Refer Slide Time: 31:22 min)



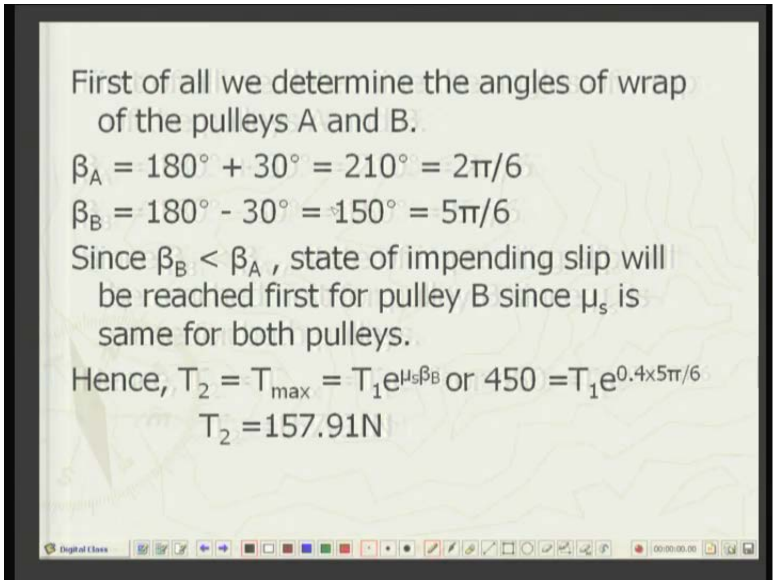
The most interesting of them all is mu s dash which is the coefficient of static friction at the pulley B which is point two six. So for the no slip condition at pulley A, it was point two one four. So the new coefficient of static friction is greater than the previous one. If we have this coefficient of friction, then there will be no slip condition at both pulley A as well as at pulley B. So this is the desired coefficient of friction which will eliminate the possibility of slip everywhere.

(Refer Slide Time: 32:24 min)



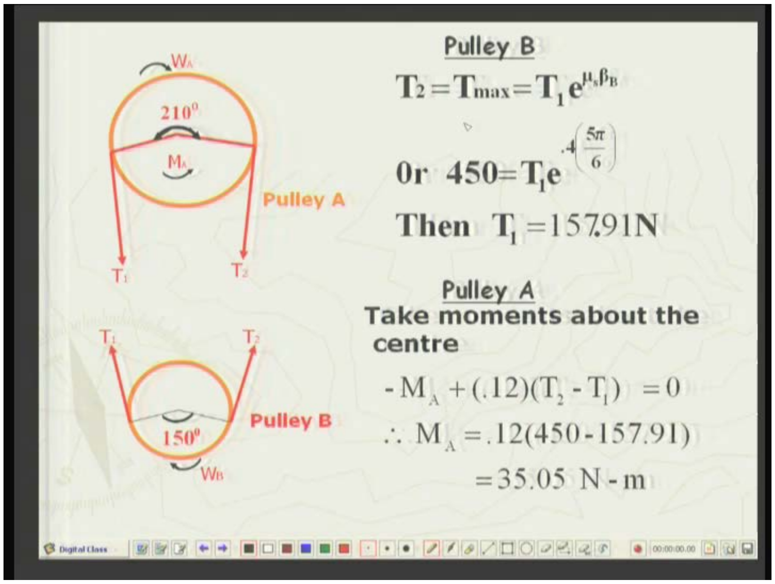
Just one more case. A flat belt is used to transmit a torque from drum B to drum A. Here in this problem, the coefficient of friction is already given. It is point four and the allowable belt tension of four hundred fifty. So the maximum tension at any point in the belt cannot be more than four hundred fifty Newton's. Determine the largest torque that can be exerted on drum A and here angle of wrap can be easily found out because the inclination on the belt on both sides to the vertical is known, namely fifteen degrees. So on one side, it will be hundred eighty plus fifteen plus fifteen, that is two hundred ten and on the other side it will be hundred eighty minus two times fifteen, that is hundred fifty degree. So angle of wraps are known. The coefficient of static friction is known. The torque to be transmitted is known. So all we have to find out is the largest torque.

(Refer Slide Time: 33:46 min)



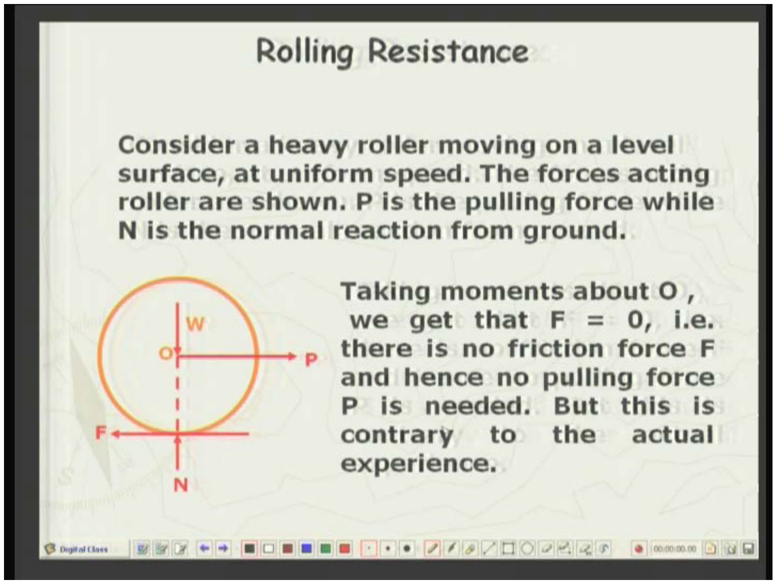
Well, as I said, the angle of wraps at belt A is two pi by hundred. This will be seven pi by six seven pi by six and five pi by six. Well, since the angle beta B, that is the angle of wrap at pulley B, is less than the angle of wrap at A, that is less portion of the belt is in contact with the pulley B, the chances of slip are higher at pulley B mu s because the exponent in one case will be, in case of pulley B will be less than in case of pulley A. Hence if the slip is to take place, it will first take place at B. So that is our critical situation. So the maximum tension on the pulley B, T max is calculated as T one. So T two over T one is equal to e to power mu s on to beta B. So maximum tension is known as four fifty. So T two is found out to be hundred fifty-seven point nine one Newton's.

(Refer Slide Time: 35:41 min)



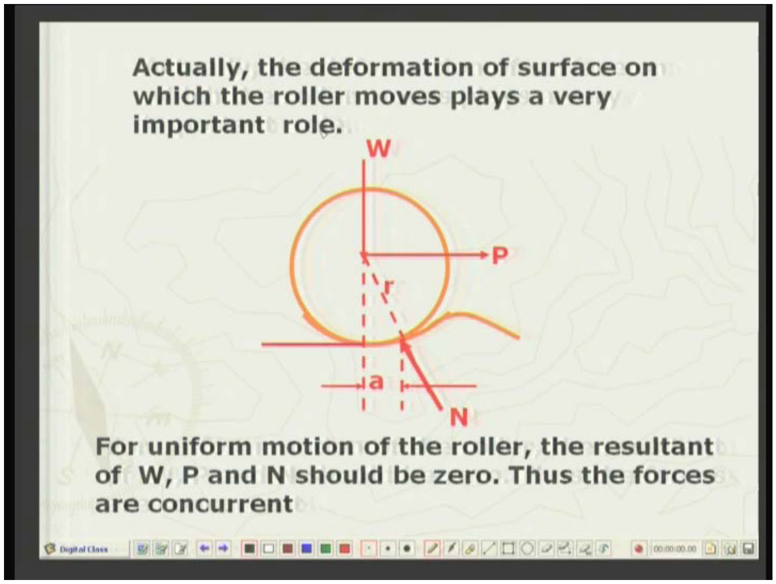
Let us examine pulley B. The angle is two hundred ten degrees. So you have T two which is the T max, is equal to T one e to power mu s beta B and this we have already found is hundred fifty-seven point nine one Newton and then we look at pulley A, the torque transmitted clockwise is to be found out. So, let us say, this MA is equal to the radius of this belt T into T two minus T one and this will be four hundred fifty minus one fifty-seven. So this will be thirty five point zero five Newton meters. With the help of these five or six examples, we have examined various types of problems which can be tackled for belt pulley systems. Now let us consider another aspect of friction, namely, the rolling friction or rolling resistance. You might have seen that suppose a heavy roller is used on a cricket pitch, to even sometimes more workers pull the roller on the pitch.

(Refer Slide Time: 37:48 min)



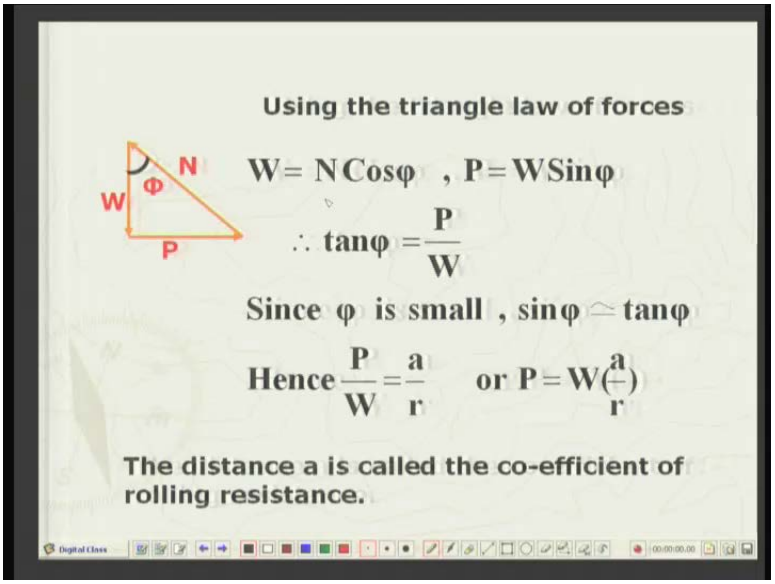
Now, if we examine that case, suppose there is a heavy roller weight W and it is rolling on a surface level surface. So the contact between the wheel and roller and the ground is a point contact through which a vertically upward normal reaction N is being applied. The pulling force is P. So at the point of contact between the roller and the ground, there is a force of friction F. Now, let us examine the equilibrium or state of equilibrium or state of uniform motion in the same straight line. That is also equivalent to a new inertial frame. That will be considered as a state of equilibrium. So to examine that, I will take moments about the center of the roller. Now W and N as well as P pass through the centre of the roller. So since the sum of all the moments is equal to zero, it means the force F is also having a moment equal to zero about the centre of the roller. It means that force F is zero. There is no force of friction and hence no pulling force is required to move or to maintain the uniform speed of the roller which is contrary to the experience after all the workers have to pull the roller to move it. Where does this anomaly lie? The principles of mechanics or equations of equilibrium dictate that there should not be a pulling force P but experiences show that there is a pulling force P.

(Refer Slide Time: 40:01 min)



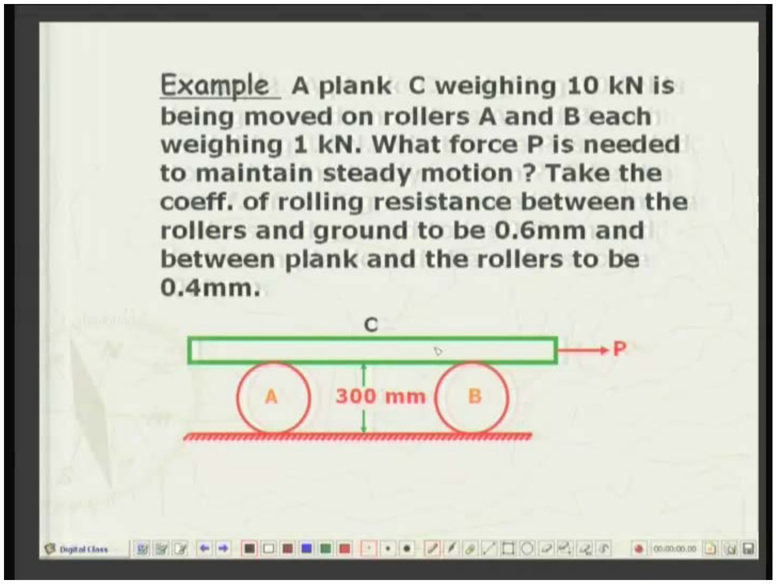
So to resolve this dilemma, let us look at what happens at the roller. At the contact point between the roller and the ground because of the heavy roller, the ground deforms, it will no longer remain a level ground. There will be a small deformation due to the weight of the roller. This can be examined by elasticity equations but that is not absolutely essential for us to do but let us say the ground has slightly raised up and then again it will go back to the level value. So this is under a microscope, you can say this is the contact position between the roller and the ground and the normal reaction instead of being through this point, now will be now acting through here. So now let us see, if all the three forces P, W and N pass through the centre, then the body will be in equilibrium and then we can use the triangle law of forces to examine the equilibrium of concurrent forces. So this is exactly what we have done.

(Refer Slide Time: 41:27 min)



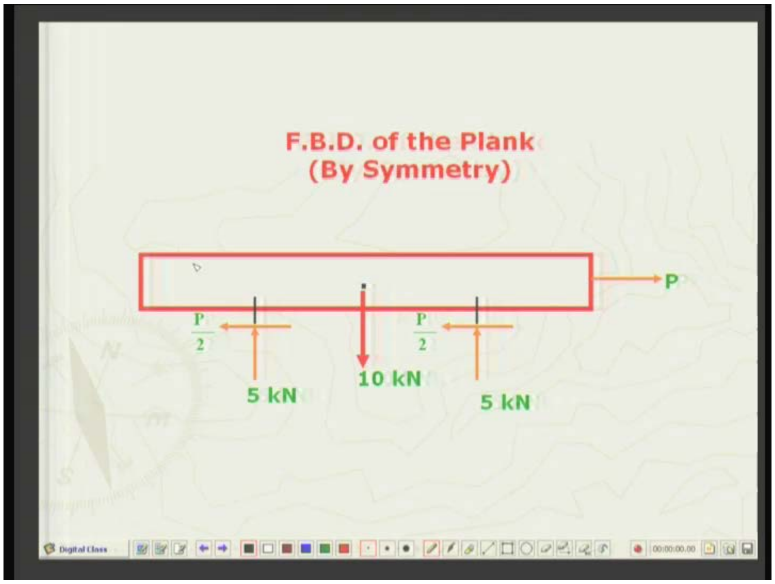
There is a normal reaction N, the pulling force P and the weight W. So completing this triangle, we can say that W is equal to N cosine phi. P is equal to W sin phi and tangent phi is equal to P over W. Now in small angle approximation, sin phi is very close to tangent phi and both of them are very close to angle phi itself expressed in radians. So we replace this tangent phi by sin phi then P over W is equal. If I go back to the picture here, this will be angle phi here. So you can see that if I complete the triangle, this will be a over r, that is sin phi. So P is equal to W a over r. This new constant a is called the coefficient of rolling resistance. Well it is different from coefficient of static friction mu which was a dimension less quantity. Here a will have in the dimensions of length millimeter or meter, etcetera. Then a over r will become dimension less and P will have the same dimensions as that of W.

(Refer Slide Time: 43:19 min)



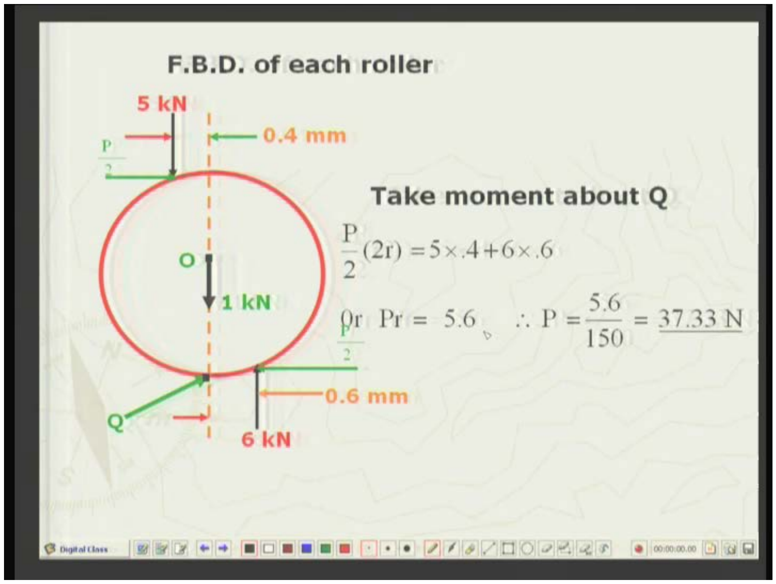
Now to fix our ideas about rolling resistance, let us consider an example. Here is a heavy plank weighing ten kilo Newton's which is resting on two rollers or two heavy wheels A and B and the distance from the ground is three hundred millimeters, that is point three meters, which is also the diameter of A and B and each of the wheels is weighing one kilo Newton. What force P is needed to maintain a steady motion to the right? Take the coefficient of rolling resistance between the rollers and the ground rollers and the ground to be point six millimeters. Once again note that the dimension of the coefficient is length dimension and that between the plank and the rollers to be point four. So at these two points, it is point four millimeters. Well the problem actually is very simple as well as very illustrated.

(Refer Slide Time: 44:39 min)



First of all, let us look at the free body of the plank. This is the plank which is being pulled by a force of P. Now here we will invoke the argument of symmetry. The two rollers are placed symmetrically or they are supporting the plank symmetrically. The total weight of the plank is acting through the center ten kilo Newton. So due to symmetry of the loading as well as symmetry of the geometry, it is quite obvious that the reactions will also be equal. Same argument we had in the case of beams also, you may recall. So it means the reaction here and reaction here will be same and essentially they will be both five kilo Newton because the total downward load is ten kilo Newton and since the coefficient of rolling resistance is also same between the wheels and the plank. So the friction force here will also be the same but the total force of friction is to be balanced by the puling force P. So essentially it will be P by two and P by two. So by symmetry arguments we have achieved some information, that is, the friction force will be P by two and a normal reaction will be five kilo Newton. Then we will look at the free body of each roller.

(Refer Slide Time: 46:17 min)



Now once again I am looking at one roller. The normal reaction is five kilo Newton's coming from the plank and this will be acting at a point whose distance is A, that is, coefficient of rolling resistance is point four millimeter. On the lower side, this point is in contact with the ground and the coefficient of volume resistance is point six millimeter. So the distance will be point six millimeter. The weight from the plank coming to the wheel is five kilo Newton's. One kilo Newton is the weight of the wheel. So the six kilo Newton downward has to be supported from the ground. So the vertical component of the reaction will be six kilo Newton. So we have found out all the forces acting on the wheel, five kilo Newton, six kilo Newton, one kilo Newton P by two and this will also be P by two because horizontal forces have to balance. Now you can take moments about this point or point O. Either point Q or point O will amount to be same and remember that these distances are very small. This dimension is three hundred millimeters. So essentially, what we will have is that, we will have P by two. I am taking moments about Q. So P by two into two r is equal to five into point four. Plus this is anticlockwise and this will also be anticlockwise plus six into point six six kilo Newton. So this equation, when simplified Pr is equal to five point six because this two and two cancels out. So two plus three point six or five point six. So P comes out to b five point six divided by hundred fifty which is radius R, that is, diameter is three hundred millimeter radius. So thirty-seven point three three Newton's.

So with these simple arguments, we have seen that the puling force required to move the plank as well as to let both the wheels roll around on the ground is thirty-seven point three three Newton's. So we have examined various types of friction problems. How the friction can manifest in various applications like a truss bearings belt and pulleys? Then the very important aspect is the rolling resistance that is between wheels, drums, etcetera and the rough ground, where the deformation of the ground plays an important role. So there are other possibilities of considering the applications of friction, namely, in screws, in screw jack or many other applications. Again the force of friction plays a very important role in supporting the load, etcetera. So we will close the chapter on friction and its application. We will start with the next topic in our next lecture.

Thank you very much.