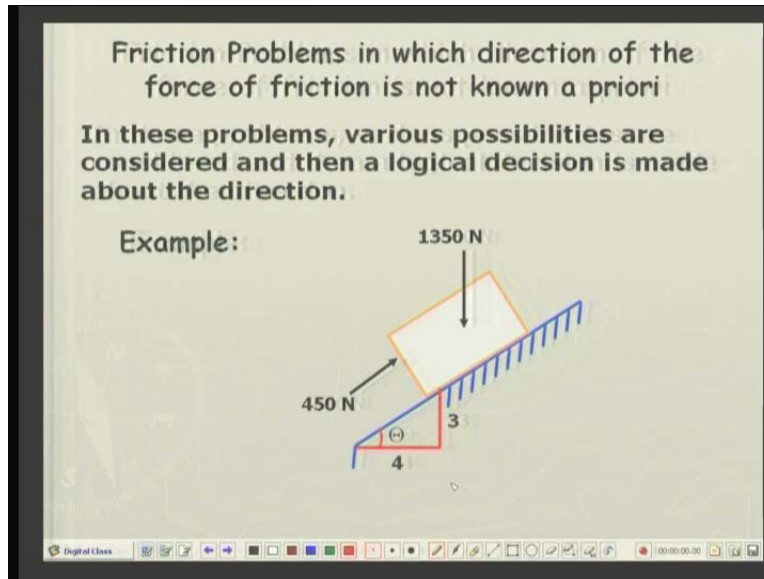


Applied Mechanics
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Lecture No. 8
Friction and its application (Contd.)

Let us start with our next lecture, namely lecture eight, which is a continuation of our previous lecture on friction and today, in this lecture we will be continuing further with friction and its application. You can recall that last time when we were defining friction, it was said that friction is a distribution of force which tries to prevent motion and if motion is already existing, then it tries to impede that motion and it was due to the presence of asperities on the surface. We also discussed two types of friction and mainly our emphasis was on dry friction or Coulomb friction. We discussed, also, the laws governing dry friction and then we looked into some examples on friction. In most of those examples, we knew which way the direction of motion is likely to take place or it was already taking place. So the force of friction was applied in the opposite direction. When we discussed the equations of equilibrium, in most of the examples or problems, we discussed two equations of motion or two equations of equilibrium were sufficient, namely, sum of the forces in the x direction is equal to zero, sum of the forces in y direction was equal to zero. The third equation for plane forces for the two dimensional case namely the sum of moments about an axis perpendicular to the plane of forces is equal to zero was very rarely used because we generally don't know a priori, the direction or line of action of the normal reaction. In the end, we took up an example, namely the toppling or tipping of a crate, in which we could fix the point through which the normal reaction was passing and then the equation of equilibrium. The third equation equilibrium, namely $\sum M_z$ about that point, was taken to be zero and now we will continue with few more examples.

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We will be discussing friction problems in which the direction of force is not known, a priori. That is, we do not know in which direction the friction force will be acting. So we have to make a judicious choice depending upon the circumstances of the problem. Let us look at this problem. Here is a block which is lying on a inclined plane. The slope of the inclined plane is also given. Now a force of four hundred fifty Newton is trying to push the block up the inclined. We have to decide whether this force will cause the motion on the block up the incline or it will stay in equilibrium or, in spite of applying this force, the box will slide down. So we have to make a decision. We will examine various possibilities, one by one, and then come to a logical conclusion.

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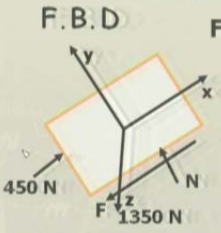
Case 1: Block tends to move up the incline.
Solving equilibrium equations:

$N = 1080 \text{ N}$
 $F = 360 \text{ N}$

Force of friction at impending slip:

$F_{\max} = \mu_s N = 0.25 \times 1080$
 $= 270 \text{ N} < 360 \text{ N}$

F.B.D



Therefore, the block will not stay in equilibrium and it slides down the incline.

First, we will consider that the block tends to move up the incline, that it is going up so and we are considering the case of impending motion. The box is still in equilibrium but a slight increase in the force can push the box upward. So if we look at this problem since the impending motion is upward. Therefore, this static force of friction will be down the incline. So that is why I have taken it in this direction. So now the box is in equilibrium due to these forces. The weight is already given as thirteen fifty Newton's which is vertically downward. So you can see that it will be weight of thirteen fifty, as we have already shown is over here. This weight can be broken into or resolved into two components, one is in the normal direction and the other is in the parallel to the incline. So if we consider the local coordinates, x parallel to the incline and y is the perpendicular to the incline. Then sum of the forces in x direction is equal to zero, sum of the force in the y direction is equal to zero.

So from that we can calculate from two equations. We can find out the two unknowns, namely the normal reaction required and the secondly, the force of friction. The force of friction F which is down the incline comes out to be three hundred sixty Newton's. That is, the force required at the time of impending slip, that is, it is about to go up the incline. Maximum force of friction which can provided because the coefficient of static friction is

already given as point two five. So according to Coulombs third law friction, this is mu s times N. So this will be equal to point two five into the normal reaction ten eighty and this comes out to be two hundred seventy Newton's which is less than the force of friction which comes into play at the instant of impending slip. That is the maximum frictional force which can come into play. It means that the system or the box cannot stay in equilibrium because sufficient force of friction is not available. Hence the box will slide down. Therefore the block will not stay in equilibrium and it slides down the incline. That is the conclusion from case one. Let us move to the second case then.

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Case 2. Block moves down the incline:

$$\sum F_y = 0 \quad \therefore N = 1080\text{N}$$

$$\sum F_x = 450 + F - 1350 \times 3/5$$

$$F = 0.2N = 216\text{N}$$

Hence $F_x = 450 + 216 - 810 = -144\text{ N}$

Thus the block experience a net downward Force of 144 N, therefore it slides down.

Case two: Block moves down the incline. When the block is moving down the incline, the force of friction will be up and again, taking x axis parallel to the incline and y normal to the incline. Again, there is a four hundred fifty Newton force and a vertical downward load of thirteen fifty is already there. Again, sigma Fy is equal to zero, which will give me the normal reaction as ten eighty. As in the case one, ten eighty is incidentally thirteen fifty into cosine of angle theta. So once you get this, then the force of friction F which comes into play from the equilibrium consideration is four hundred fifty up the incline force of friction F and then thirteen fifty into sin of theta, which is three by five. So F comes out to be two hundred sixteen Newton's. So this force of

friction which is generated, when it is about to slide down is equal to when it is sliding down is, that is, point two, the coefficient of dynamic friction, is point two times the normal reaction. That is $\tan 10^\circ$. So it is two hundred sixteen. So, when F_x is calculated with this value of F , it comes out to be minus one forty-four Newton's. What does it mean? Positive x is in up the incline. So negative means down the incline. So there is net downward force of one forty-four Newton's in case two.

So the box will not only slide down but it will slide down with the net force of hundred forty-four Newton. In dynamics, you will learn that this will result in a downward acceleration. So in the presence of four hundred fifty Newton's upward force up the inclined force, the box will still slide down the incline with a force of one forty four Newton and hence if I divide by the mass of this box, then it will be equal to the acceleration of the box. So in this problem, we have seen that we examine various possibilities because we did not know which way the force friction is to act. We examine the two possibilities, force of friction downward and force of friction upward and then come to a logical conclusion.

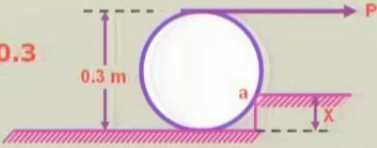
Well, very often the friction concepts and the law's of friction are helpful in understanding some of the phenomena or some of the actions which we perform in our daily lives and I will take up one or two simple examples and you might be able to correlate these examples with your experience.

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Very Often the concepts of friction and the equations equilibrium provide good understanding of simple devices or actions we perform in our daily lives. This is illustrated by the following examples:

(1) Pulling a wheel Over a step.
What is the maximum height x of a step so that the force P will roll a 25 kg wheel over the step without slipping at the corner (a) of the step.

Take $\mu = 0.3$

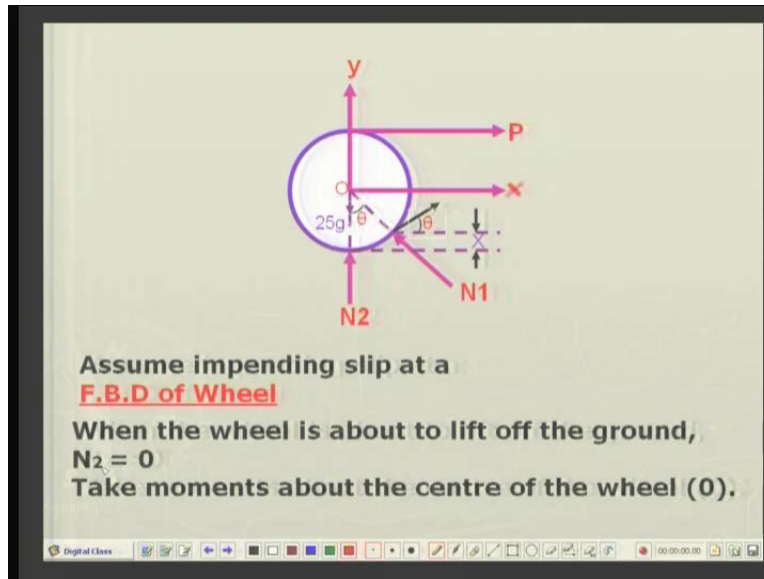


The diagram shows a circular wheel of diameter 0.3 m on a horizontal floor. A horizontal force P is applied to the top of the wheel, pointing to the right. The wheel is in contact with a step of height x . The contact point between the wheel and the step is labeled 'a'. A vertical dimension line on the left indicates the diameter of the wheel is 0.3 m. The step is shown as a horizontal surface of height x above the floor level.

Example one is pulling a wheel over a step. Suppose you are looking at this wheel which is rolling on this floor and then suddenly it encounters the step. You can even take the example of an automobile which is to climb up a small break or a stone. The wheel of the automobile. So what happens at that instant? Sometimes, you may find that the wheel starts slipping down. It is not able to climb. It slips. So we want to examine what is the maximum height of the step x . So that the force P will roll a twenty-five kilo gram wheel over the step without slipping at the corner of the step.

So this wheel of diameter point three meter is being pulled by a force P and the attempt is to raise the wheel up the step, that is, it goes to the higher level but without slipping. So we can allow only this state of impending slip. The coefficient of friction is static friction, of course, is given to the point three.

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Let us look at the free body of the wheel. Now, there is a force P and we take the coordinate x in the horizontal direction y is already given. There is normal reaction from the ground and to and at the corner, when the wheel is in touch with corner, there is a normal reaction N one. Now just at the instant of impending slip, the point in contact at the corner of the step, that is, this point will try to slip down, roll down and therefore, force of friction will be tangential upward at that point a . So now let us consider the equilibrium when the wheel is about to climb. It will leave contact with the ground. So it means N two will not be provided and we will take N two to be equal to zero at that instant.

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$\Sigma M_o = 0$ $P \times 0.15 = 0.15 \times \mu N_1$
 $\therefore \mu N_1 = P$ (i)
 Also $\Sigma F_x = 0$ $\mu N_1 \cos \theta - N_1 \sin \theta + P = 0$
 $P = N_1 (\sin \theta - \mu \cos \theta)$ (ii)
 From (i) and (ii)
 $\mu = \sin \theta - \mu \cos \theta$
 or $0.3 = \sin \theta - 0.3 \cos \theta$
This equation is independent of Wt. of the wheel.

Now, let us take the moment of all the forces acting on the wheel about point O. So from equilibrium consideration, sum of all the moments about point O is equal to zero. So there are two forces, one force P and the other force, the force of friction over here F. These two forces will produce moment about point O. All other forces N one will be passing through O and hence it will not produce any moment. So P is providing a clockwise moment whereas force of friction is providing an anticlockwise moment. So you can see P into radius diameter of the wheel is point three. So radius is point one five is equal to point five one five into mu times N, that is, the force of friction at impending slip. So canceling out point one five and both sides, we get mu times N one is equal to N two is equal to P. So this pulling force is equal to the force of friction at that instant and then sigma Fx is equal to zero.

What are the horizontal forces? There is force P here and the horizontal components of N one and the force of friction. So sigma Fx is equal to zero means mu N one, that is, force of friction times cosine theta minus N one sin theta plus P is equal to zero. So P comes out to be N one into sin theta minus mu times cosine theta. From solving from equation one and two we get the final result as mu is equal to sin theta minus mu times cosine theta. mu is known to us point three.

So we get to the final result that point three is equal to sin theta minus point three times cosine theta. Now you can see from this equation that this equation is independent of the weight of the wheel. Whether it is a light wheel or a heavy wheel, it does not matter. Well square this both sides of the equation and rearrange the terms.

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Squaring the equation on both sides and simplifying

$$.09 + .18 \cos \theta + 0.09 \cos^2 \theta = \sin^2 \theta = 1 - \cos^2 \theta$$

Solving :

$$\cos \theta = .834$$

$$\theta = 33.4^\circ$$

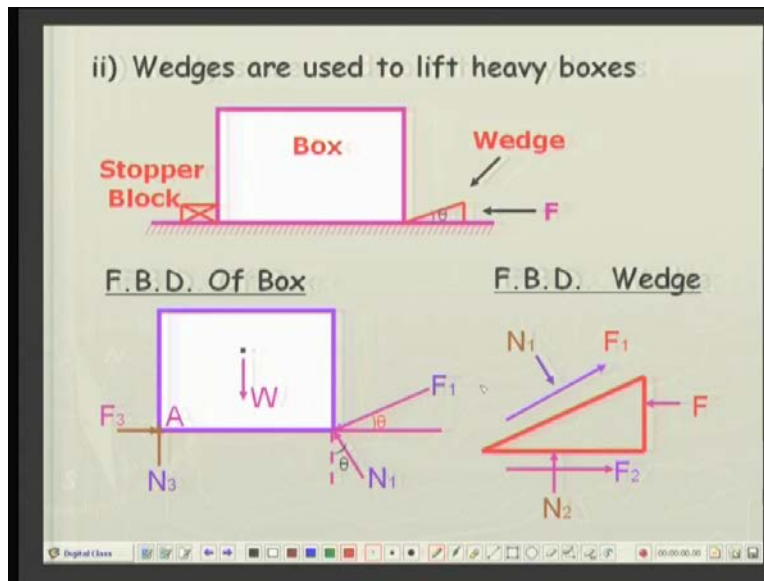
$$X = 0.15 - 0.15 \cos \theta = 0.15(1 - .834)$$

$$= .0249\text{m} = 24.9 \text{ mm}$$

You can come to this equation. It is a quadratic equation in cosine theta. So since cosine theta has to be less or equal to one. One solution will be a realistic solution, other will not be so. You will get cosine theta is equal to point eight three four which means that theta is equal to thirty-three point four degrees. So, the angle theta, that is, this angle theta is equal to thirty-three point four degrees and from the geometry of the problem, I can find out the height x, namely, this will be equal to radius minus r cosine theta. So the height X, capital X of the step, is equal to the radius point one five into one minus cosine theta, already known as point eight three four.

So it is point zero two four nine meter or twenty-four point nine millimeters. So this height of the step which the wheel can tackle without slipping down and this height is independent of the weight whether it is the weight of the heavy wheel or a light wheel.

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Another day to day example which we can try to understand with the help of concepts of friction and you might have seen very often that workers when they try to move a heavy box push a wedge underneath the box. So that it is slightly raised up and during that action, what are the various forces, which way the friction is likely to act and so on and so forth, we will try to understand. So wedges are used to lift heavy boxes. So let us look at this problem. There is a heavy box which is stopped and a wedge is used to push it between the box and the ground. So that the box gets slightly lifted up and why do not we provide any support over here or a stopper here? During that pushing action the box will move. It will slide in this direction. To prevent that, we have a rigid block, so that the box does not move. Let us examine the various forces which come into play in this section. So the angle of wedge is equal to theta.

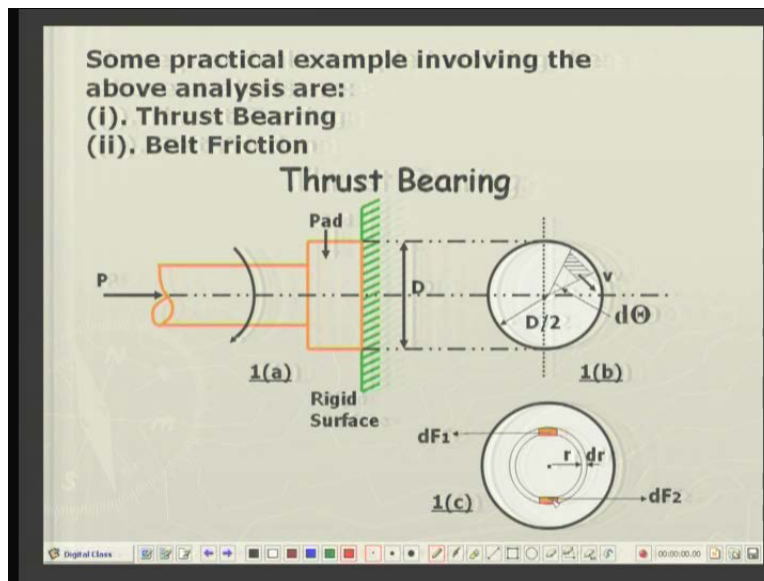
So let us examine the forces. This angle is very small. So we will still consider the system to be almost horizontal. So at that instant, there will be these forces reaction from the stopper F_3 which is trying to prevent the motion of the box to the left. So the reaction will be to the right and from the ground there is a normal upward reaction at this corner because otherwise the box is tilted. Only one point is in touch with the ground at that instant and then between the box and the wedge, you can see that the wedge is being

tried to push into the box. So there is a motion in this direction. So there will be an equal and opposite force of friction, that is, this force is actually parallel to the surface of the wedge. Something like this. So this force upward this inclined surface and then there is a friction force between wedge and the ground F_2 , normal reaction from the ground, normal reaction from the box on to the wedge. So the wedge is subjected to five forces: two normal reactions, two forces of friction and one pushing force F and the box is at this corner subjected to F_1 and N_1 , equal and opposite to the corresponding forces on the wedge. Now we can examine, you can do it as an exercise yourself, we can examine the equilibrium of the wedge. Also, we can examine the equilibrium of the box. So you can easily see that there will be number of equations namely the two equations from the equilibrium of the wedge, $\sum F_x$ is equal to zero, $\sum F_y$ is equal to zero but when I come to the box, you will see, since the line of actions of the normal's are normal reactions are known, there will be three equations of equilibrium. So total will be five equations and how many unknowns are there? We have two forces of friction F_1 and F_2 and the stopping force F_3 and then you will have the reactions also. So you can solve the equations of equilibrium, to find out what the various components of the reactions are. Now, so far, in all our friction analysis we had contact surfaces which were plane, that is, box is on a level ground or etcetera, etcetera or wedge, etcetera.

Now each and every point of the plane surface has the same motion, whether it actually slips when the bodies are actually slipping past each other or whether it is a case of impending slip. So the direction of motion of each and every point is same. So the force of friction which is opposite to the direction of motion or impending slip is also same. So that is a very simple or a an easy situation where the magnitude and the direction of friction force are same for all the points but in many cases what happens is that the impending slip or actually happening slip is the direction of that slip is varying from point to point and also the normal reaction may also vary from point to point. So as a result of these two things the force of friction is variable, both in magnitude and direction on the entire contact surface.

So how do we then analyze such problems? There is a simple technique. We will consider a small, infinitesimal area and we assume that on this small area the force of friction, both in direction and magnitude, is uniform. Then we will integrate the force of friction over the entire contact area to get the net effect and this whole procedure can be illustrated with the help of some examples.

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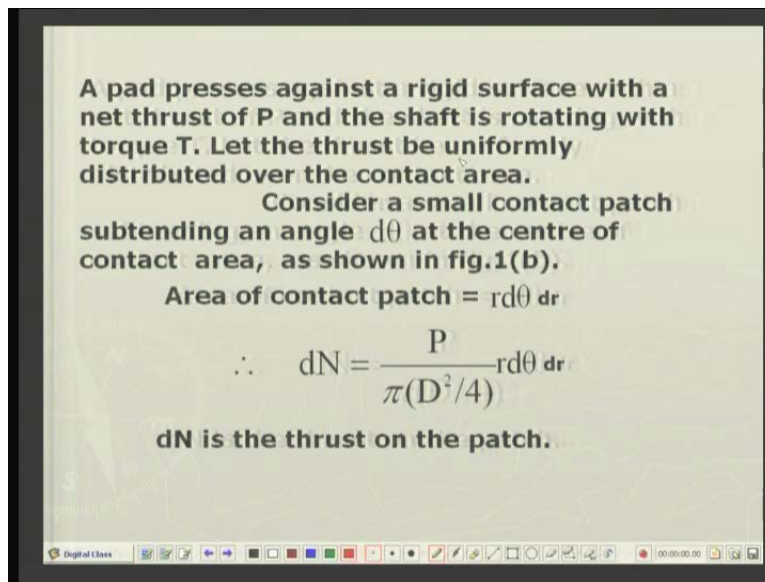


I will take up two examples to illustrate this fact, that is, variable fact. The situation of variable force of friction namely, the first example deals with truss bearing and the second example with belt friction. Now, first, I will take up truss bearing. It is like this. Suppose there is a rotating shaft which is pushed against a rigid surface through a kind of friction pad and the shaft is rotating. So if I look it on the side view, I will see that this contact surface between pad and the rigid surface is a circular area, whose diameter is D or radius is $D/2$. If I consider a small element of this contact area, subtending a small angle of $d\theta$ at the centre of this surface and we will assume that the direction of rotation is clockwise, then the motion is tangent at that point. That is, over here the direction of motion is like this. On the vertical diameter it will be like this.

So at every point, the direction of slip is different and accordingly, the force of dynamic friction which opposes the direction slip will be also varying along the whole surface.

I am depicting a small ring out of that surface and if I see the top most point of the ring, the direction of friction dF_1 will be to the left horizontal. If I see the lower most point, it will be to the right dF_2 . At these two points, it will be upward and at this point, it will be downward. So you can see that at every point, direction of friction is changing.

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So let us see, how we can analyze this problem. First of all, we will assume that the thrust, that is, the pushing force between the pad and the rigid surface, is uniformly distributed. Each and every point receives the same pressure, force per unit area. We consider a small contact patch, subtending an angle $d\theta$, illustrated earlier at the center of contact area. Then the pressure acting on this patch is the total force P . This total force P which is distributed on this area uniformly, then P divided by the area of the contact. That is, P divided by $\pi D^2/4$. This is the pressure acting on the entire area times the area of the infinitesimal patch. Well, $r d\theta$ is the arc length, dr is the radial length. So arc length into radial length will be the area of the contact patch. So this is the total normal reaction

on this contact patch. Once again, if I show it, this is the area $r d\theta$, the $d\theta$ into dr times the pressure on this pressure is total force divided by the entire area.

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The frictional force on this patch:

$$df = \mu_d dN = \mu_d \left(\frac{P}{\pi \times D^2 / 4} \right) r d\theta dr$$

μ_d = dynamic friction coefficient.

This force acts on tangential direction (as Shown at 2 places in fig.1(c)). It causes a torque dM at the centre of the shaft.

$$dM = r^2 \mu_d \left(\frac{P}{\pi \times D^2 / 4} \right) d\theta dr$$

Integrating over the total contact patch:

$$T = \int_0^{D/2} \int_0^{2\pi} \mu_d \frac{P}{\pi D^2 / 4} r^2 dr d\theta$$

$$= \mu_d \frac{8P}{D^2} \int_0^{D/2} r^2 dr = \frac{PD\mu_d}{3}$$

So that will give me the normal reaction acting perpendicular to this patch and the force of friction because the slip is already taking place. So dynamic coefficient of friction times the normal reaction will give me the force of friction acting on this patch which is equal to μ_d times P divided by $\pi D^2 / 4$ into the area $r d\theta$ into dr . Now this force of friction is producing a moment at the axis of the shaft, that is, the force times the moment arm moment arm is simply the radius. So this will be dM into moment term. So $r^2 \mu_d$ into rest of the expression is as such. So this is the moment generated by the force of friction acting on this small infinitesimal patch. Now I integrate this infinitesimal patch to the moment generated at the axis. If I integrate it over the entire surface of the contact, that is, θ is varying from zero to two π to cover the entire three hundred sixty degrees and r is varying from the center to the outer periphery which is at a distance $D/2$ from the center, the total torque, which is generated by the friction force, is integral of this expression from zero to $D/2$ overall and zero to two π over θ . Well, first of all, the θ integration is very easy. There is no θ term in the integrand. So this will be two π and then the r integration. So if you do this

integration, I will get P, that is, total truss times the diameter of the contact pad times the dynamic coefficient of friction divided by three. So very simple expression. That is the torque due to frictional torque, equal to the truss times diameters time the dynamic coefficient of friction divided by three.

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The assumption of uniform pressure is valid when the contacting surfaces (pad and plate) are new and flat. When the surfaces are worn out, The pressure is non-uniform.
Let the pressure

$$p \propto \frac{1}{r} \quad \text{or} \quad p = \frac{a}{r}$$

This constant (a) can be obtained in terms of the total thrust P:

$$P = \int_0^{D/2} \int_0^{2\pi} p \, r \, dr \, d\theta = \int_0^{D/2} \int_0^{2\pi} \frac{a}{r} \, r \, dr \, d\theta$$

$$= 2\pi a \left(\frac{D}{2}\right) \quad \therefore a = \frac{P}{\pi D}$$

Well, this problem we did on the assumption that the pressure between the contact pad and the rigid surface is uniform. That is quite justified when the surfaces are new, that is, there is no wear and tear of the pad but after some usage and wear and tear has taken place, the pad is no longer exactly flat. It may have worn out from the edges and less wearing out at the center. So let us assume that the pressure instead of being uniformly distributed is distributed in such way that less pressure as we go away from the center linearly. That is, P is proportional to one by r. Or introducing a constant of proportionality P is equal to sum constant of proportionality divided by r. So again, of course at center, it is very large pressure infinite. Theoretically speaking, infinite pressure but what we want to say is that pressure is decreasing as we are going away further and further from the center. First of all, what is the total truss corresponding to such a pressure? Integrating $r \, dr$ into $d\theta$ is the small infinite area of the small patch times the pressure. When integrated gives me the total truss and carrying out this integration we

can easily find that the total thrust is equal to two pie a into D by two and from there. Since P is known, the total truss is known. We can find out a is equal to P total truss divided by pie times D.

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Normal reaction dN on a small patch is:

$$dN = p r dr d\theta = \frac{P}{\pi D} (1/r) r d\theta dr$$

The force of friction on this patch,

$$df = \mu_d dN$$

\therefore The torque T transmitted

$$= \int_0^{D/2} \int_0^{2\pi} r \mu_d \frac{P}{\pi D} dr d\theta$$

or

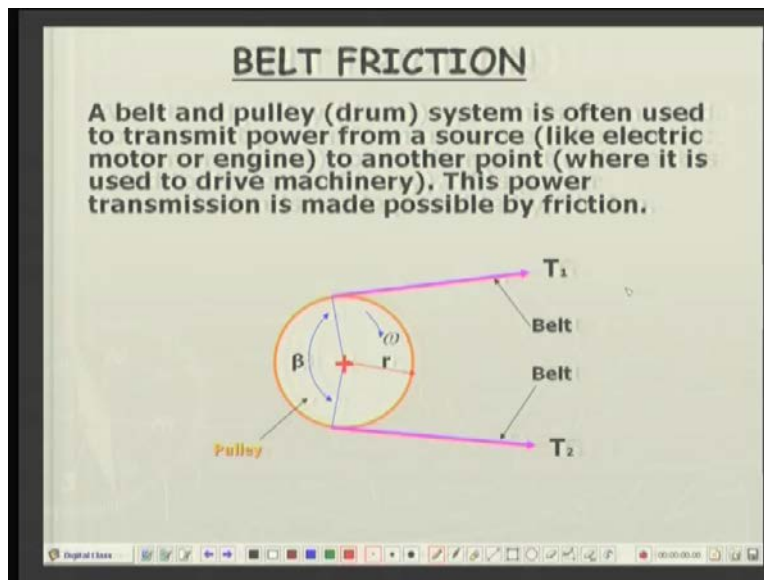
$$T = \int_0^{D/2} r \mu_d \frac{P}{\pi D} 2\pi dr = \frac{\mu_d P D}{4}$$

Therefore, the torque capacity of an old thrust bearing is less than that of a new bearing.

Once we have got the pressure, rest of the analysis is exactly same as in the previous case, that is, uniform distribution of pressure. So dN , that is, the normal reaction on an infinitesimal patch is equal to P times the area of the patch and this P is already found out. So we substituted again. We apply the coefficient of dynamic friction and we get the force of friction on this infinitesimal patch and then again, we take the moment arm as the radial distance of the patch and then we integrate it to get the friction torque transmitted. So this integration is exactly on the same lines and after integrating and putting the limits, we will get that this friction torque is equal to μ_d times P times D over four. In the case of uniform distribution, we had in the denomination three. In the case of linear distribution, maximum at the center, minimum at the edges, we get one by four. So it means that the torque carrying or torque transmitting capacity of a worn out pad is less than that of the new pad, which is obvious. But here is a full-fledged explanation based upon laws of friction. Well in some problems, conical thrust bearings are also used instead of a flat surface of contact. We have a conical surface. Well, the

pressure will be normal to the conical surface, that is, the slant surface and again you can consider a ring of contact and you can then find out what the force of friction around that is and what the normal reaction is and what the contribution to the torque is because you can you know the moment arm, namely, the a radius at that point. So again, integrating over the entire slant surface of the cone, we can find out the total torque transmitted for the conical truss bearing.

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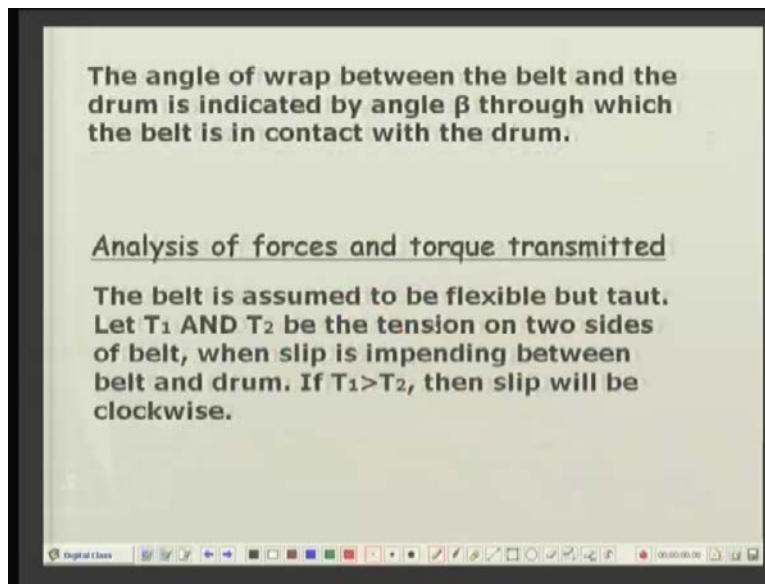


Now we come to a very important application of friction, namely, belt friction. Well, you must have gone to a work shop or some factory where several lathe machines or other work shop machinery are being driven from an overhead shaft with the help of belts or you can look under the bonnet of your car. You will find that there are two or three pulleys which are driven by the power from the engine and these pulleys are needed to drive the dynamo or the alternator of your automobile cooling system and other components of your car. So what is being done is that, the power or useful mechanical work is being transmitted from the point where it is being produced or generated to several different locations with the help of belts and pulleys and drums. So a schematic diagram of this arrangement is that, here is a pulley or a drum of radius r and a flexible but torque, that is in tension, belt or a rope is going over this drum. It is in contact with

the drum over a part of the drum and the angle up to which it is in contact is called the angle of rack. So beta is the angle of rack, that is, the angle between the normal, where the belt leaves off and tangentially it leaves off here as well as over here. These are the two points. So the angle between these two points is the angle of rack.

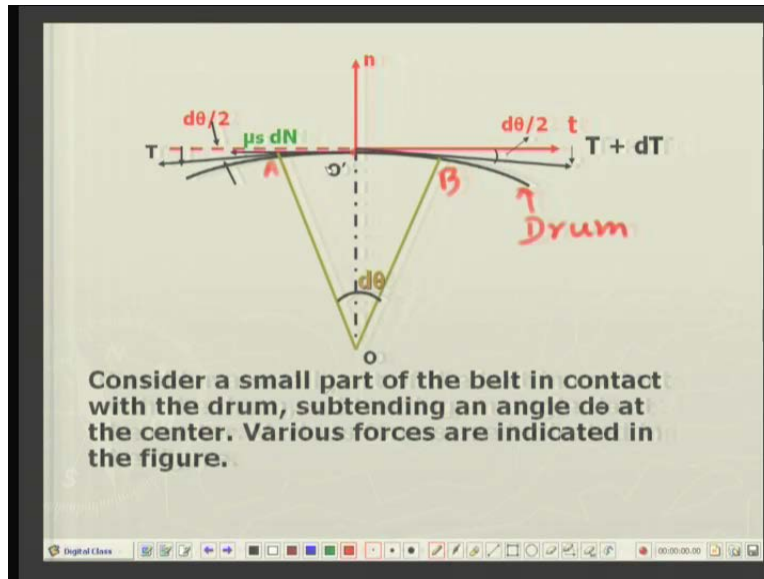
That is way the belt is rotating clockwise with an angular speed of omega radiance per second although we will not be going into the dynamics of it but we will examine what the various forces are and we will assume that the belt is not slipping.

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So let us see how we can analyze the forces and torque transmitted. Remember the tension on one side of the belt is T one, on the other side is T two and if the impending slip is to take place in the clockwise direction, then T one will be greater than T two, that is, the belt will be trying to slip like in this direction.

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Let us look at a small segment of the belt. It is in contact with the drum surface. So this you can say drum or pulley and the drum in contact with the belt. We are considering only a small part of the angle of rap namely the angle subtended at the center of the drum is $d\theta$. Well, if I consider the center point of this and let us say, this is the center point of this contact $d\theta$. So half of it is $d\theta/2$ on the left side and $d\theta/2$ on the right side. The belt is having the tangent. So the force of tension on one side will be T and on the other side it will be $T + dT$ because the force is varying throughout the belt from T to $T + dT$. So the variation on this small angle is from T to $T + dT$ and this is tangential at this $T + dT$. dT is tangential at this point and T is tangential at this point, you can say point A . So T is tangential at point A and $T + dT$ is tangential at point B . We extrapolate this tangent back. So you can see that this will make an angle $d\theta/2$ and an equal angle $d\theta/2$ on the other side. Consider the component of forces in the normal direction and the tangent direction at the midpoint O .

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Consider the equilibrium equation in normal (\hat{n}) and tangential (\hat{t}) directions of the middle of the belt segment.

$$\Sigma F_t = 0$$

$$\therefore -T \cos(d\theta/2) + (T + dT) \cos(d\theta/2) - \mu_s dN = 0 \quad \text{--(i)}$$

$$\Sigma F_n = 0$$

$$\therefore -T \sin(d\theta/2) - (T + dT) \sin(d\theta/2) + dN = 0 \quad \text{--(ii)}$$

using small angle approximation, the above equation become:

$$dT = \mu_s dN \quad \text{--(iii)}$$

$$-T d\theta - dT(d\theta/2) + dN = 0 \quad \text{--(iv)}$$

So if we do so, let us examine the equilibrium and we consider the case when the slip is just about to take place, that is, at the instant of impending slip. So sum of all the forces in the tangential direction T direction, in this horizontal direction, is equal to T cosine d theta by two. That is the horizontal component of the left hand tension force and this T plus delta T cosine d theta by two. That is the horizontal component or tangential component of the right hand force minus the force of friction. The slip is going to take place in this direction. So the force of friction will be opposing that. So it will be towards left and this is equal to mu S, that is static coefficient of friction times the normal reaction Dn. Right. So you have once again this component of force. You take the horizontal component, this force. Take again the horizontal component and you get the net force equal to zero that is T minus T cosine d theta by two T plus dt cosine d theta by two minus mu s dN is equal to zero. Now you come to the normal components sigma F Fn is equal to zero. Again, the normal components, this will have a normal component upward, this will also have a normal component upward and then there is a normal reaction. So this will be normal downward and normal upward. So you will have minus T sign d theta by two minus T plus dt sign d theta. These are the two downward normal components of tension and this is the normal reaction component upward. This is equal to zero. Now these two equations can be simplified. Remember that the angle d theta is

very very small. So $d^2\theta$ is also very small. So for very small angles, you remember from your trigonometry, cosine of θ is equal to very close to one when θ is very very small. Similarly $\sin \theta$ is very close to θ itself when expressed in radians. So when θ is small, it means cosine of $d^2\theta$ is replaced by one.

Similarly over here $\sin d^2\theta$ will be replaced by simply $d^2\theta$ and similarly, $d^2\theta$ over here. So from the first equation we will have minus T and plus T canceling out. So dt is equal to $\mu s dN$. The first equation simplifies to the simple statement, dt is equal to $\mu s dN$ and the second slightly requires little more attention. Minus $T d^2\theta$ and then there again minus $T d^2\theta$ so it will be minus $T d^2\theta$ and second will be dt times $d^2\theta$ plus dN is equal to zero.

Now the second term. If you look at it there is an infinitesimal term dt and also $d^2\theta$ which is also very small. So these two are very small quantities and the product will be very very small extremely. So these are higher order infinitesimal. So in the limit suppose I let the $d^2\theta$ go to very very small zero, this term will be going to zero much faster. So I can neglect this term because second order term is infinitesimal and will be reducing much faster.

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The second term of the equation (iv) is very much smaller than the other two terms and hence neglected.

Therefore, $dN = Td\theta$ ---(v)

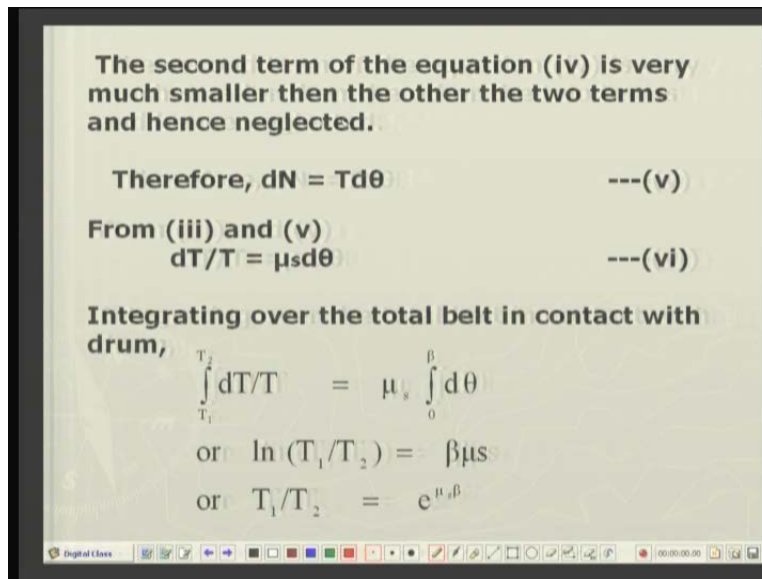
From (iii) and (v)
 $dT/T = \mu_s d\theta$ ---(vi)

Integrating over the total belt in contact with drum,

$$\int_{T_1}^{T_2} dT/T = \mu_s \int_0^\beta d\theta$$

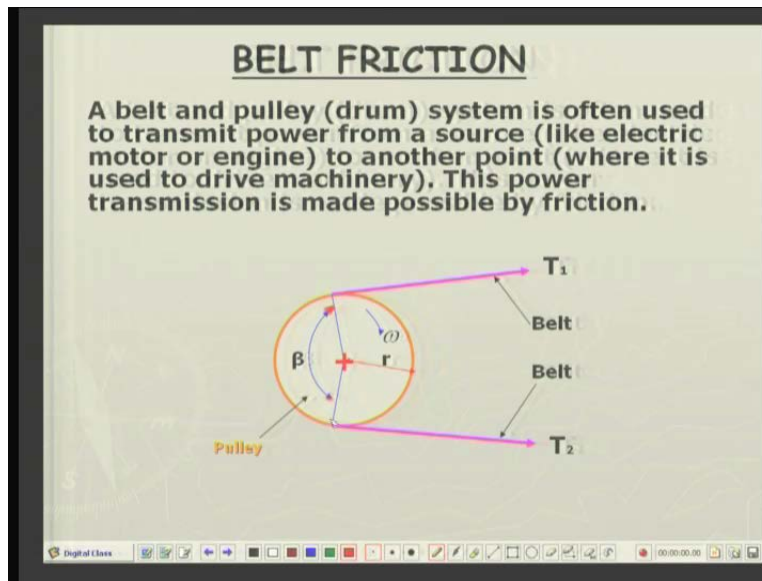
or $\ln(T_1/T_2) = \beta\mu_s$

or $T_1/T_2 = e^{\mu_s\beta}$



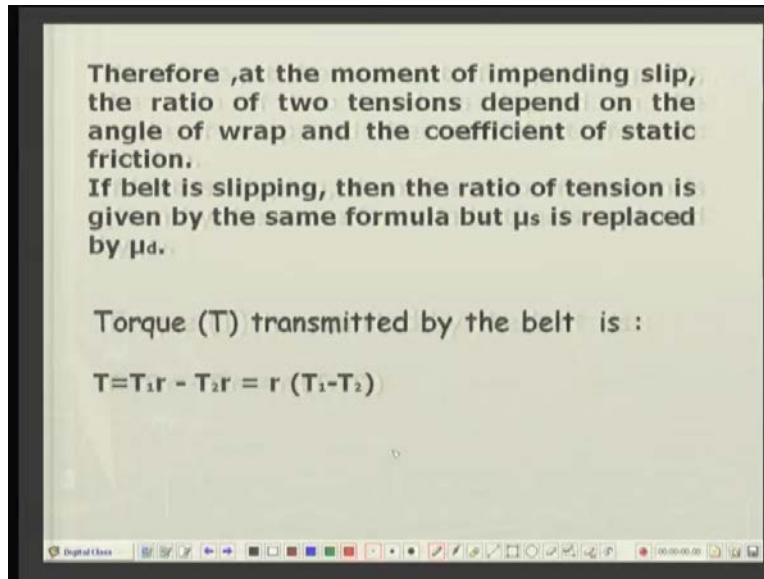
So it means, second term can be neglected in comparison to first and the third term and then the equation reduces to dN is equal to $Td\theta$ and combine the equation three and five. Five is over here and three equation is over here. So if you combine these two, then I have dt by T is equal to μ_s times $d\theta$, that is, the change in tension over the tension at that point is equal to μ_s times the angle over which this change is taking place.

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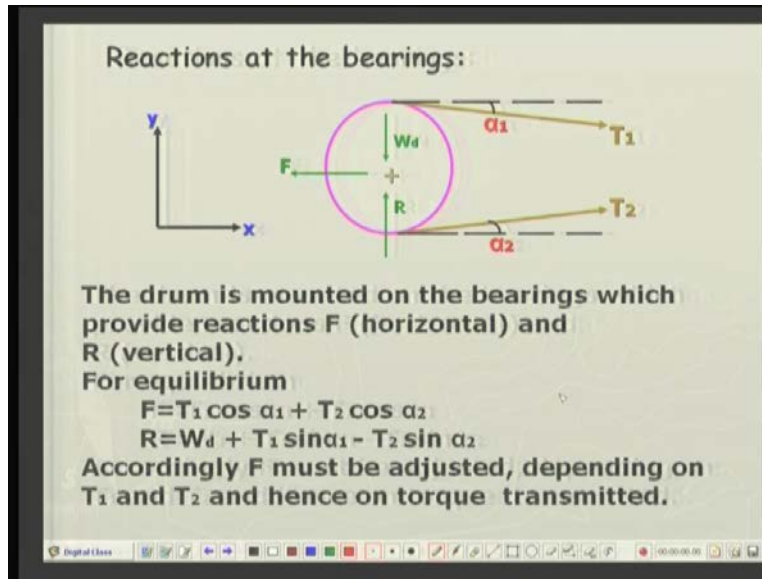
Then if we integrate it from end to end, that is, from the contact point, let us say from this point to this, over the entire angle of rap. At that point, the tension is T_2 . At this point, tension is T_1 . So it means $\int_{T_2}^{T_1} \frac{dT}{T}$ from T_2 to T_1 and correspondingly $d\theta$ is varying from zero to β the total angle of rap and μ_s being constant, that is, the same surface of the pulley and the same drum. So μ_s will be same. So μ_s can be taken out. This integral is very simple. This will give me natural logarithm of T_1 over T_2 and since $d\theta$ can be very easily integrated and, that is θ , in the limits it will be simply β . So this will be $\beta \times \mu_s$. Well, if I take the exponential on both sides \log natural \log and exponential, they will cancel. So T_1 over T_2 is equal to $e^{\mu_s \times \beta}$.

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Therefore, at the moment of impending slip, that is, when the slip is just about to take place, the ratio of two tensions depend on the angle of rap beta and the coefficient of static friction a mu s. Well, that was the case when the slip is just about to take place. Let us say when the slip has already taken place. That is, the belt is sliding over the drum or the pulley. Then the coefficient of friction mu s will be replaced by mu d coefficient of dynamic friction rest. Its expression is same, that is, T one over T two will be equal to exponential of mu d times angle of rap beta. Now, the torque transmitted by the system. Well, there is a force T one. There is force T two. Both are acting at the points, tangentially. So the distance from the center is radius r here, radius r over here. So there will be a clockwise torque T one into r. There will be an anticlockwise torque T two into r. So the net torque will be T one into r minus T two into r. So the net torque will be r times T one minus T two.

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Now look here. We have found out the tension T_1 and T_2 . If I look at the entire system, that is, this pulley which is mounted on some axial or a shaft through some bearings and the pulley is rotating over which the belt is going with these two tensions. Let us try to examine the equilibrium of this pulley or the drum. There is a self weight W_d of the drum and then there are two tensions in the belt which are, since the belt is rapped on the drum, these sources are essentially acting on the drum. So there is force T_1 and T_2 . These two forces can be resolved into two components. Let us say, the angle of inclination of this part of the belt to horizontal is α_1 , lower part of the belt to the horizontal is α_2 . So $T_1 \cos \alpha_1$ plus $T_2 \cos \alpha_2$ are the horizontal component of the belt forces on the drum. These have to be resisted by the reaction from the bearings which will ultimately be transmitted to the axial of the pulley. So this force reaction from the bearings F is equal to $T_1 \cos \alpha_1$ plus $T_2 \cos \alpha_2$.

Now, what about the vertical balance of forces? Again, downward force is W_d downward and you can again resolve horizontal and vertical downward. Similarly, this will be vertically upward. So the net vertical force, which is to be resisted, is W_d plus $T_1 \sin \alpha_1$ minus $T_2 \sin \alpha_2$ and this will also come from the reaction

of the bearings. So it means that the pulleys are in equilibrium or is being maintained in equilibrium by two reactions F and R . Now this F reaction can be adjusted depending upon the requirements, that is, tension T_1 and T_2 . You might have seen that. Again, taking example from automobile which many of you might have experienced yourself, when the belt becomes loose sometimes due to excessive usage after its rubber or some synthetic material, then the mechanic will push the pulley little bit. So that the distance increases and the tension in the belt is adjusted.

So in that case, the attempt is that the belt should not slip over the pulleys. It should move with the pulleys. No slipping because once slipping takes place, there will be lot of heat generated. The belt will deteriorate and it will ultimately break. So in a proper operation of the belt pulley system, the horizontal force at the reactions or the horizontal reaction at the bearings should be always adjusted in such a way that there is no slip.

Well, in today's lecture, we have seen the application of friction in many situations. Some of them are very relevant to our daily experiences. I have given examples from the wheel climbing up a step. I have given examples of automobiles in which belts and pulleys are used. Again, in industry or in a work shop, belts and pulleys are used. So the phenomena of friction is very relevant to our daily experiences. We will continue with more application of this phenomena in our next lecture. So thank you very much for your attention and I hope you will be able to correlate whatever you have learnt today with your future experiences.