Applied Mechanics
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Lecture No. 7
Friction and its Applications

Today, we will have lecture seven of statics. The title of today's lecture is friction and its application. Intuitively, you are all aware of friction. This is when two bodies are rubbing against each other. For example, our hands. You can experience some heating up of the hands or some impediment to relative motion between the hands.

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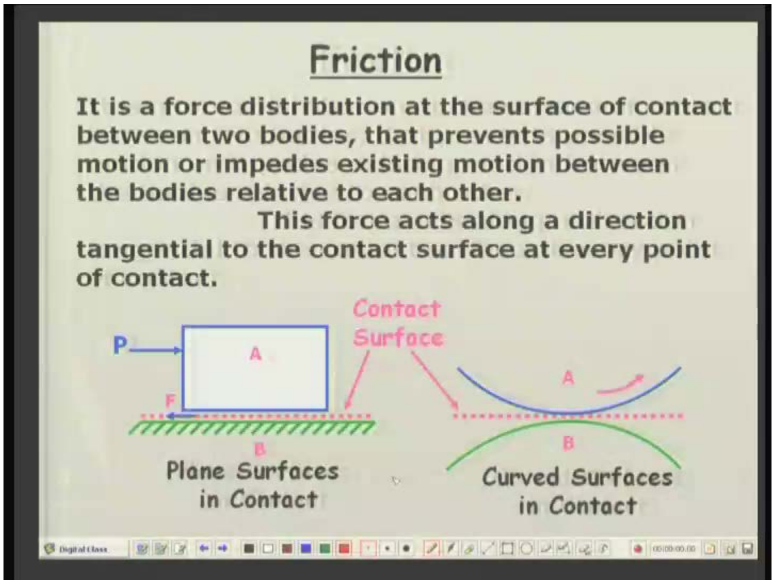

We will define friction as a force of distribution. It is a force distribution at the surface of contact between two bodies that prevents possible motion or impedes existing motion between the bodies relative to each other. So it has two types of effects. First of all, it will try to prevent the relative motion between two bodies and secondly, if the motion is already existing, that is, the bodies are moving pass each other with some relative velocity, then the role of friction is to slow down that motion or impede that motion.

Let us see how this force and where it acts. For example, I take this case of a box lying on a surface. Here is a box A and it is a lying on a plane surface. So this bottom surface of the box is in contact with this floor and this is the plane of contact. This dotted pink line shows the plane of contact. Now if I try to move the box towards right, then the force of friction will be opposing that motion and this force will try to impede that motion and hence this force will be acting towards left. So it is a resisting force. On the other hand, if two bodies are curved bodies or at least one of them is curved, then the contact between the two bodies is just a point contact or a line contact. In the previous case, there were infinitely many points of contact. Here there will be much less number of points of contact. Essentially, if these are two spheres, there will be one point of contact. If one is a cylindrical surface and other is a plane surface, then there will be a line of contact. In the example one, when the surfaces are both plane, there is total surface of contact. So in the second case, we will consider the tangent plane as the contact plane. So suppose wheel A is moving in an anticlockwise direction, then the force of friction will be in the clockwise direction. It is always opposed to the direction of the relative motion between the two surfaces.

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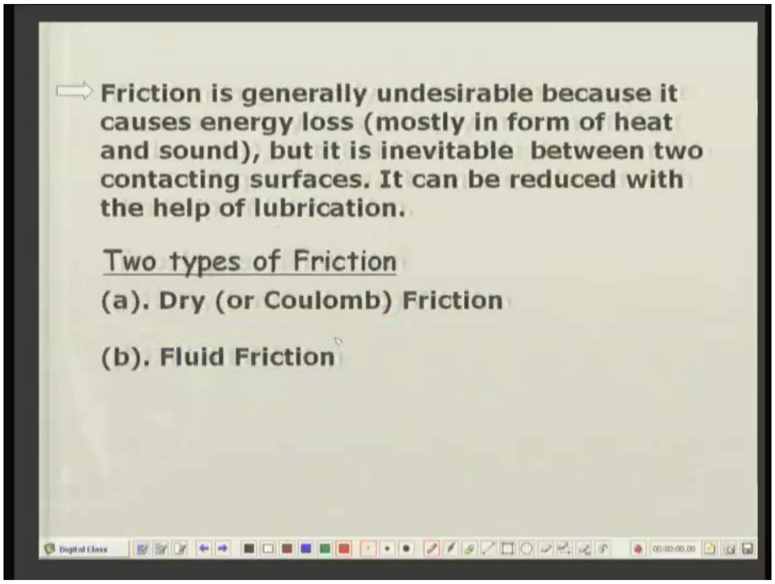

Well, friction is generally undesirable because it causes loss of energy. As you will be learning later on, when there is force and the point of application goes through a small displacement, then some work will be done. That is, the work loss due to friction. But in some cases, this force of friction is a useful entity. As we will see that it will be very much useful in transmitting power from one point to another point through friction between belt and a pulley or in truss bearings, in clutch, in breaks. All these are some of the useful applications of force of friction but generally it is work lost in the form of heat or sound. It is a quite well-known experience that when two bodies are rubbing against each other, some screeching sound is produced, as well as, if you are rubbing your hands against each other, the hands become warm. Sometimes heat can be very large.

There are two types of friction: Number one, dry or Coulomb friction and second is fluid friction. In this course, in this lecture, we will be concentrating on dry friction. Fluid friction is involved in case of lubrication. That is, when we try to reduce dry friction by spreading a layer of some fluid or viscous material in the form of oil or a lubricant.

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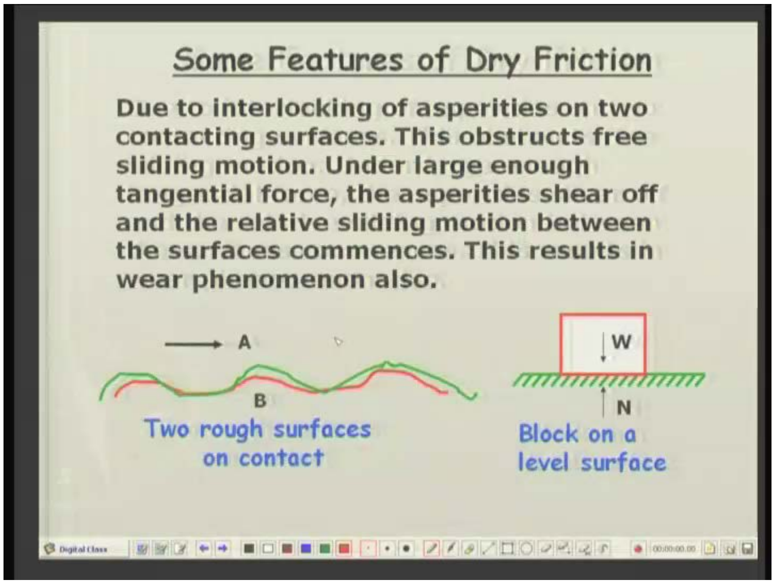

So now let us see what are the salient features of dry friction? How it is produced what causes this dry friction well the two contacting surfaces although they may look to be very smooth and plain, actually if you look at those surfaces under proper magnification, large enough magnification, then you will find that there is some undulation or roughness on each of the surfaces. Here, I have shown two bodies A and B and near the contact point, we have enlarged the profiles of the surfaces and let us see one profile shown. Green is given over here and the other profile of the surface is B. Now due to this roughness, there are asperities. The undulations and these asperities interlock with the each other. Okay. If the surfaces are absolutely plane, there are no undulation. Then there are no asperities but in general case, there will be some asperities, large or small. Now when we try to move, let's say, surface A relative surface B, these asperities will interlock. They will try to prevent the relative motion unless the force is large enough, that the surface A can shear cut of those asperities and move forward. Sometimes, due to these asperities, the contact stresses can be so large that some heating is produced. So there will be local melting at the contact point.

So again, when we apply force at the contact points, the material will be removed or sheared off and that is why, when there are two rubbing bodies, rubbing surfaces, some loss of material is always there. This is called wear of the bodies. So that is the material although it may be very less quantity but some material loss is there called wear and tear. So now, how does this force of friction come into play? First of all, let us say, here is a block of a W which is lying on a level surface and we considered the state of equilibrium. There is a downward force W and an upward reaction from this surface on to the block, will be force N W is equal to N. Both are in the same line of action. W is same as that of N. So the body is in equilibrium. There is no motion.

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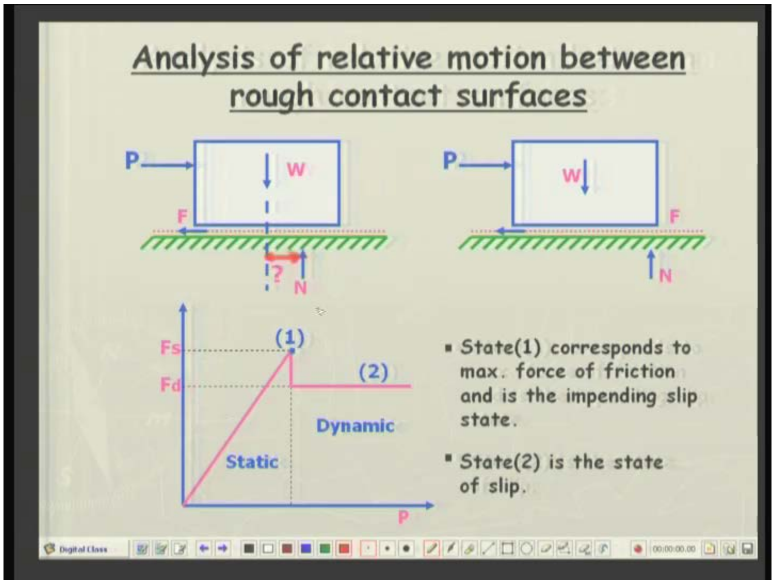

Now the same block we try to push with a force P from the left hand. So it is trying to push the block towards right and to maintain the body in equilibrium because the body has not yet started moving. There must be generated an equal and opposite force F which is at the contact surfaces between the block and the floor. So this force F is the force of friction. Now look here, P is equal to F for equilibrium but the lines of actions of P and F are different. So they produce a couple. So this couple will be a clockwise couple. Now in the vertical direction W is the weight of the block. It is acting vertically downward. So there has to be a vertically upward force to maintain the body in equilibrium, sigma Fy is equal to zero. So normal reaction N exactly equal to W will be produced but the line of action of this normal reaction N will be such that, the distance between the line of action of W and that of N should exactly balance the couple due to the P and F. The couple due to P and F is a clockwise couple. So the counter balancing couple between W and N must be anticlockwise. So that the normal reaction will lie on to the right of the line of action of W. So this distance between the two should be such that it exactly balances the pervious couple. So as we keep on increasing P, the pushing force P which is trying to move the block, the couple is increasing, the value of W and N are same. So to balance the increased couple the point of or line of action of N should also keep on moving towards right. So at any stage, the normal reaction will adjust at the distance from W in such a manner, that the body is maintained in equilibrium till the critical state will come, that N is passing through the edge of the box because if it goes outside this, then it is no longer acting on the block.

So these are the gradual stages of this simple thought experiment. Now, if we look at the graph between the applied force P and the force of friction, as P is increasing, the force of friction will also be increasing in order to maintain the body in equilibrium. So the relationship between P and F will be a straight line as shown over here, till a point is reached that the force of friction cannot exceed that state. So that is the state number one. Just below it, the bodies about to have a relative motion with respect to the floor on which it is sitting. So starting from zero friction, a friction force is gradually increasing to the maximum. This state leveled as state one is the state of impending motion. If you increase the force P just beyond this state, the body will start to move towards right in this experiment. As soon as the motion is started, the force of friction will immediately go through a decrease. That is, there will be jump discontinuity. So earlier, the force of friction was Fs, that is, static friction, the state of impending motion, and once the motion has commence, the force of friction will be the dynamic friction Fd and always remember that this dynamic friction is less than static friction and this state will be called the state of slip. So remember, state of impending motion, state of slip.

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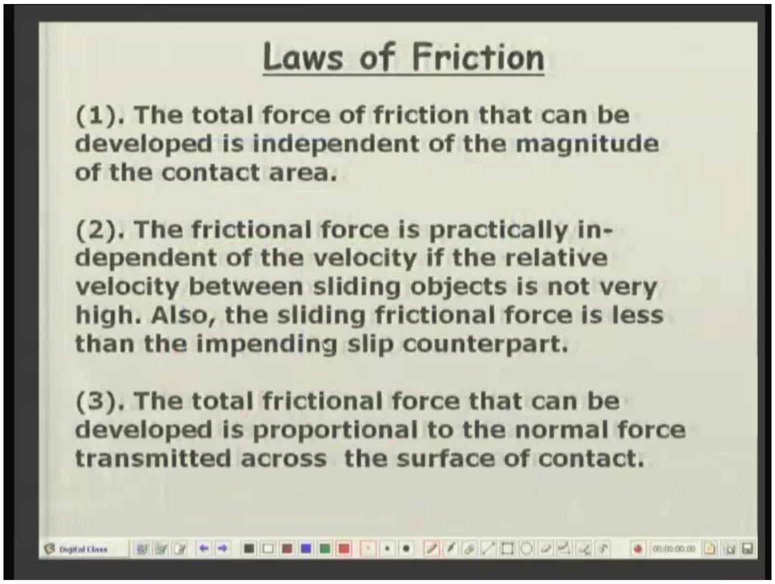

Now, within the context of Coulomb friction, let us examine, what are the various laws governing this friction? First of all, the total force of friction that can be is independent of the magnitude of contact area. You remember, in earlier diagram, I showed gave you two examples.

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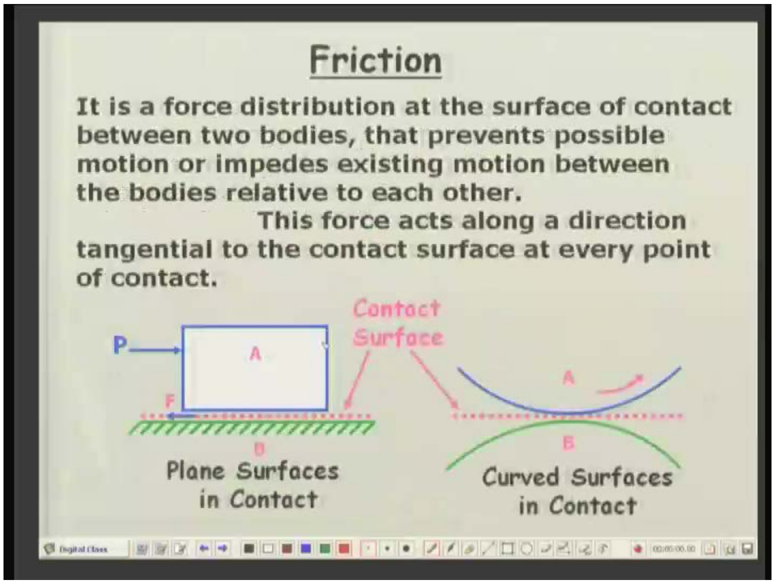

In one case, there was a finite area of contact between two plane surfaces A and B. In this case, the area of contact is very small or theoretically it is zero area of contact, even if it may be line of contact but it does not mean that the force of friction here will be zero. So it is independent of the contact area. It depends on so many other factors. Number two, the frictional force is practically independent of the velocity. If the relative velocity between the sliding bodies is not very high unless it is very high, then there is some dependence on the magnitude of the velocity. For moderate velocities, force of friction is independent of the magnitude. Also the sliding friction force is less than the impending slip condition, force, as we have already seen because Fs is less than Fd and thirdly, the total frictional force that can be, is proportional to the normal force transmitted across the surface of contact. That is, the normal reaction. So the force of friction is proportional to the normal reaction, as we have seen in the previous discussion. This is proportional. So this is a straight line relationship.

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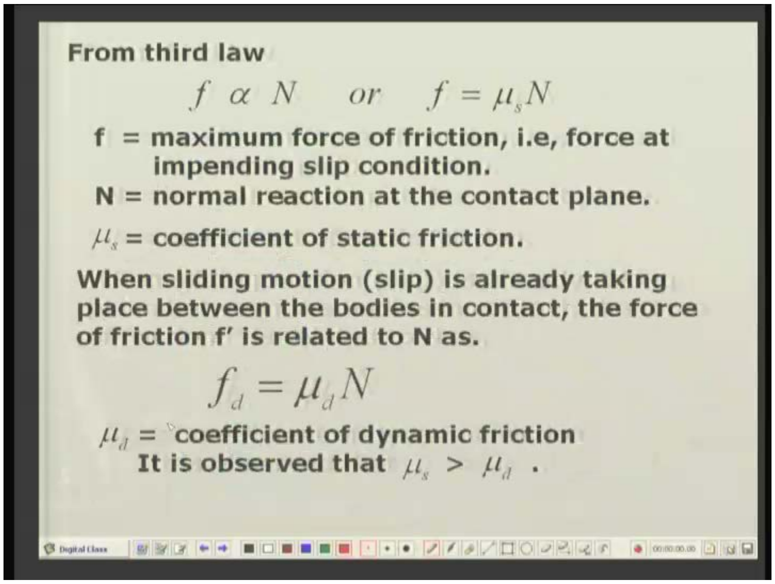

Well, the third law is put in a mathematical form. f is proportional to N, N is the normal reaction. So we can write it as, f is equal to mu s times N where mu s is the constant of proportionality and called coefficient of static friction. f is the maximum force of friction as we have seen, that is, the force at the impending slip condition and N is the normal reaction across the contact plane. When sliding is already taking place, that is, the slip condition is achieved, then same law is still applicable but the constant of proportionality is now different. Now, this f is static fs and this is fd. The dynamic friction is equal to a new constant mu d which is the coefficient of dynamic friction into N and since force of friction at the state of impending motion is greater, the coefficient mu s is greater than the coefficient mu d.

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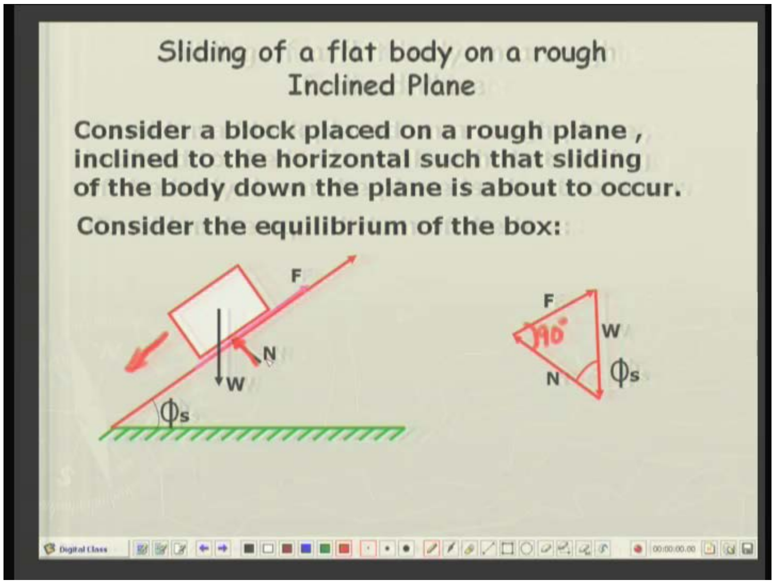

Now, how to visualize this coefficient of static friction or the state of impending slip? A very simple experiment is helpful in understanding this. Suppose you have a plane, a level surface, which can be made more and more inclined. That is, you start from the horizontal, as we have here a horizontal surface and you imagine there is a smooth ply board plank and there is a hinge.

So when the plank is gradually lifted, the angle of inclination of this plank becomes more and more. So this is the phi, the angle of inclination to the horizontal on this plane board. You have a given box or given surface whose coefficient of friction is to be measured. Always remember, the coefficient of friction is a characteristic of two contacting surfaces. It is not a property of one surface but it is a property of two contacting surfaces. Let us say, one surface is wood, the other is steel or rubber and steel or wood and aluminum, etcetera, etcetera. So here is a block of weight W and it is lying on an inclined plane whose inclination is gradually increased and a stage will come when the block will start sliding down on to this surface. So at that stage, when it is just about to slip down or slide down, the angle of inclination is phi s. Let us examine, you can say just before that state is reached, the state of equilibrium of the box, that is, the block or box has not yet started to slide down but it is about to, that is, the state of impending slip. So at that stage, let us see the triangle law of forces for equilibrium. There is a downward force of W, there is a force N, normal reaction which is normal to the surface over here and then there is a force of friction F which is along the plane of contact. So drawing the vertical W normal to the surface and along the surface, we can complete the triangle and then the inclination between W and N is the angle of friction or static friction and it is designated as phi s. We can see by the triangle law of forces, F divided by N is equal to tangent phi s. You can easily verify it because F and N are mutually perpendicular. So this is ninety degrees. So you can easily see that F divided by N, the tangent of the angle phi s or phi is tan inverse of F divided by N and by third law of Coulomb, friction F divided by N is the coefficient of friction mu s.

So the phi s which is called the angle of static friction is nothing but tan inverse of the coefficient of static friction and similarly we can examine when the body or block is sliding down the inclined plane, same problem but now you can say that this block is going down this inclined plane at a uniform velocity. Then the angle here which will be able to maintain a uniform velocity will be slightly less than phi s. It will be angle phi d and same equations of equilibrium.

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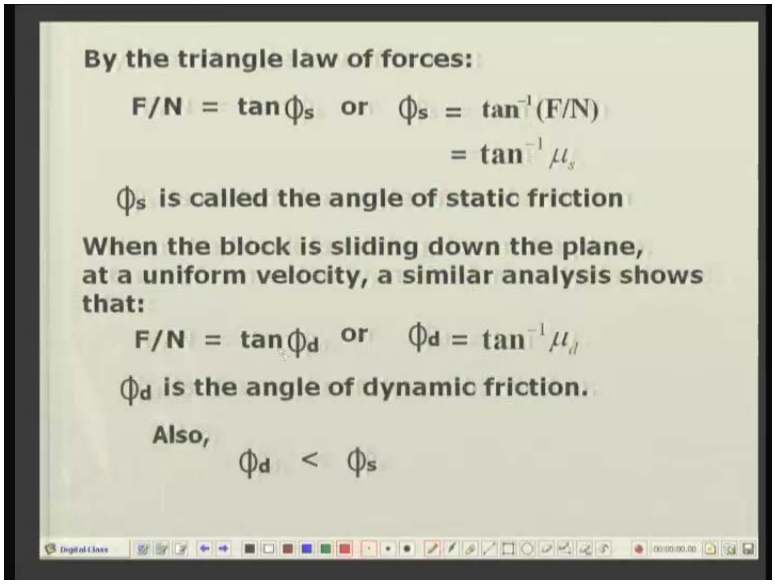

And the triangle law of forces will give us the result that F divided by N is equal to tangent of phi d. That is, the angle of dynamic friction. So phi d is equal to tan inverse of mu d and obviously the angle phi d is less then phi s because mu d is less than mu s.

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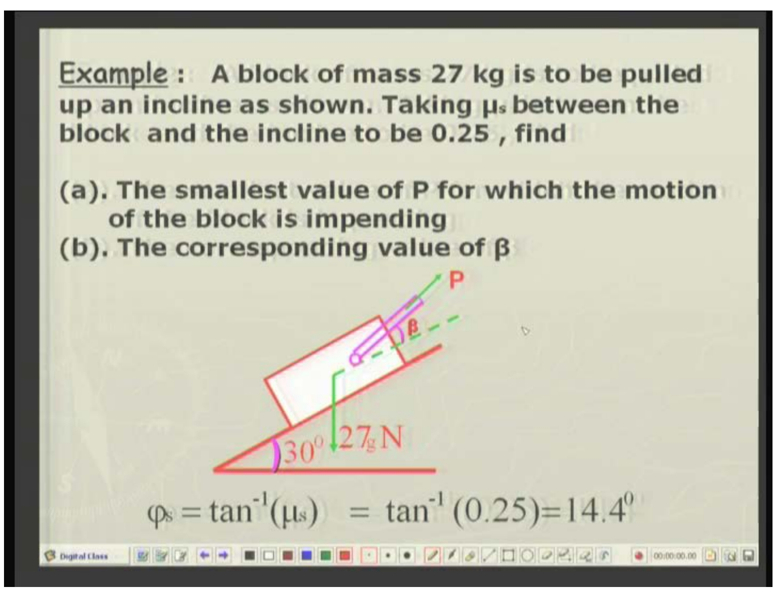

Let us fix off our ideas more firmly by looking at few of examples. The first examples I am considering is, a block of mass twenty-seven kilograms is to be pulled up an inclined plane as shown in this figure. Here is an inclined plane. This is the horizontal plane. The angle between the two plains or the inclination is thirty degrees. This block is to be pulled up the inclined plane through a lever shown here with the force P and the inclination of the lever is angle beta to the direction of the moment. Taking the coefficient of static friction between the block and inclined as point two phi, find the smallest value of P for which the motion of the block is impending. That is, the block will just start to move up and secondly what is the angle required, that is, angle beta which will pull the block up the incline?

First of all, let us see, the coefficient of friction. mu phi s is given and this is equal to coefficient of friction. mu s is given and mu s, you know, is tangent of the angle of static friction. That is, phi s or in another word, phi is tan inverse mu s. So this angle will be fourteen point four degrees.

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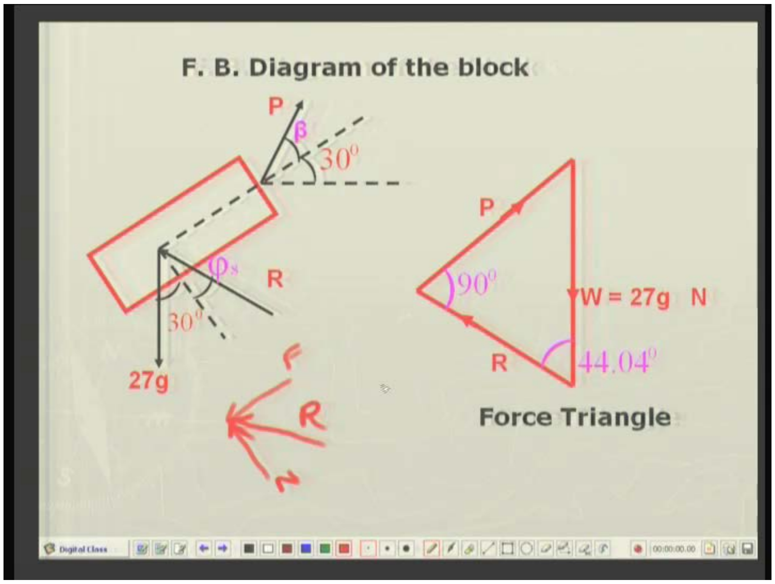

Now as we have done in the previous analysis, we have again a rectangular block shown here. The weight of this block is mass times acceleration due to gravity. So twenty-seven g is the vertically downward force due to weight and then there is a normal reaction and the force of friction. So since the block is to move up, the direction of impending slip is upward. So the force of friction is downward. So the normal reaction and force of friction being two vector forces, here is normal reaction, here is force of friction. So the resultant of these two forces will be resultant R. So this resultant R is shown and the angle between the two, as we can see from the triangle law is the angle phi s and the force P which is trying to pull the block up is shown here which is inclined at an angle beta to the direction of motion as shown here and this direction of motion is at thirty degrees to the horizontal. So completing the triangle law we will have the vertical force W and the normal reaction R and since the problem says what the smallest value of P which will cause the impending motion is. So, from this point, what will be the shortest distance? Thinking geometrically, there are two vectors W and R and from this point what will be the shortest distance so that this corresponds to the minimum force that will be along the normal direction, that is, a line which is at ninety degrees to the resultant reaction R resultant of force of friction and normal reaction?

So from this point, we will draw a normal to the direction of R and that will complete the triangle of forces and this will give us the minimum force which will be capable of pulling the block up. This is the geometrical solution. You can do it analytically also by swimming an angle alpha and then minimizing the force.

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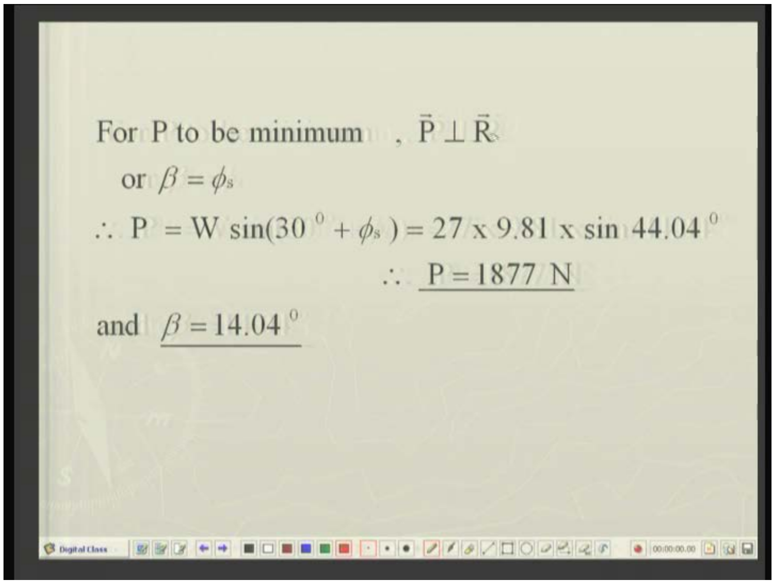

So for P to be minimum, P is perpendicular to the resultant R and it will be perpendicular when beta is equal to phi s because the angle between two lines is equal to angle between the normal's. This twenty-seven g force is normal to this and this is normal to P. So whatever the angle here, same is the angle over here and at that stage you can calculate what the force P by sin law of triangles is. That is, this is the sin log, we have to find out P. So P over this angle of forty-four point zero four degrees is equal to W over sin of ninety degrees right. So you will have P is equal to eighteen seventy-seven.

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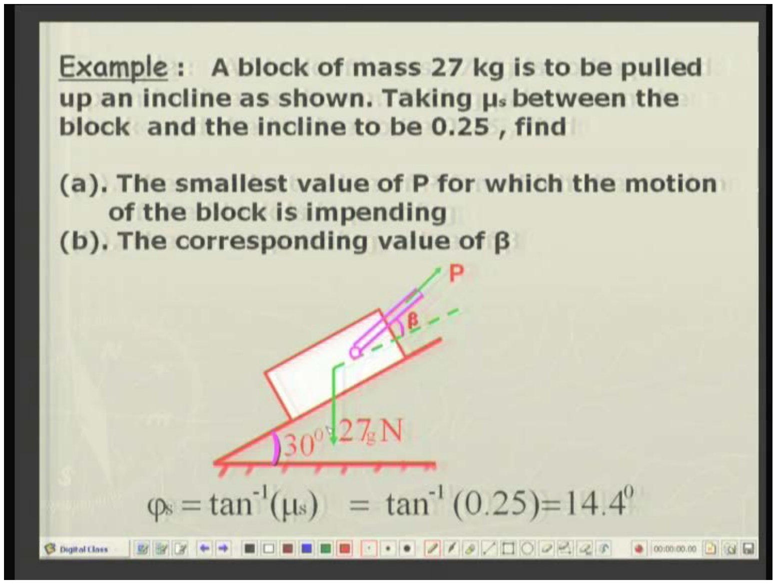

So when the lever in the pulling lever is inclined at the same angle as the angle of static friction, that is, fourteen point four degrees, at that stage the pulling force which will be required to move the block will be minimized and its value has also been obtained as eighteen seventy-seven new Newton's.

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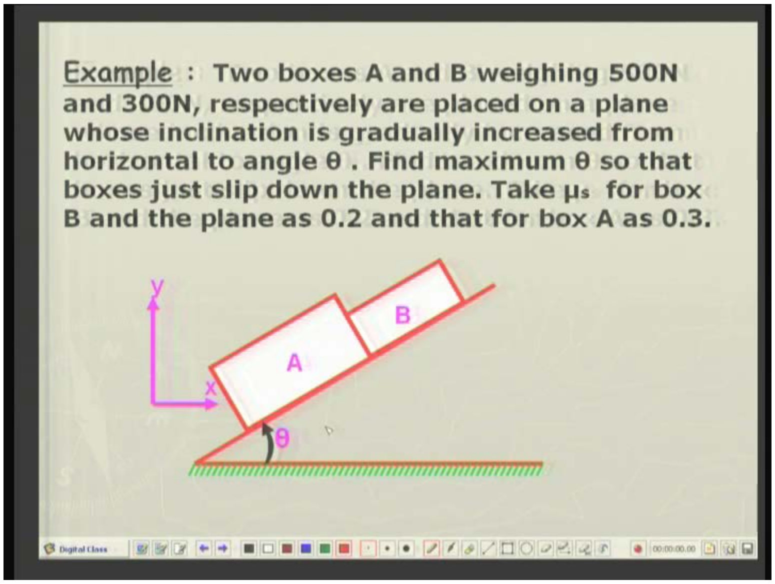

Now let us have a look at another problem in which you have more than one contacting surfaces. For example, there are two blocks A and B. They are both lying on this inclined surface and they are in contact with each other. So two boxes A and B, weighing five hundred Newton's and three hundred Newton's respectively are placed on a plane whose inclination is gradually increased, just like our mental experiment, from horizontal to angle theta. This is the angle theta. Find the maximum theta? So that box is just slip down the plane. Take the coefficient of static friction between box B and the plane as point two and that for box A and the inclined plane as point three. So different coefficient of frictions are applicable. Let us see. We want both the boxes to slip down. It cannot be only one box. Suppose we imagine that box A slips down but the box B does not slip down. As soon as box A slips down, B will also slip down because now the contact force between A and B is removed, when they are no longer in contact with each other. So there will be force when they are in contact. There is a contact force up the plane as box A moves. That force is no longer there. So the box B will be slipping down automatically. Similarly, when box B cannot slip down unless block A or box A also slips down, both these boxes will slip down simultaneously.

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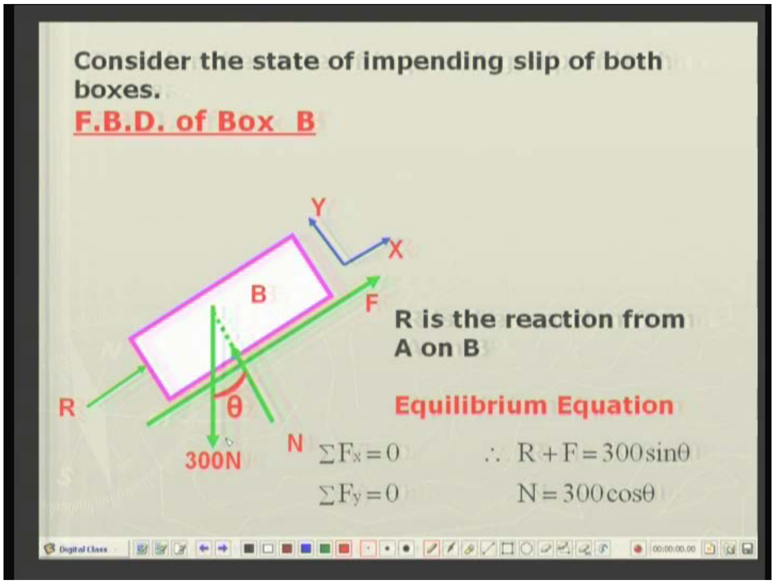

So consider the free body diagram of the upper box. That is, box B. Now what are the forces acting on this? There is a gravity force of the weight of the box, three hundred Newton's and then, there is a reaction R from box A on the left of it. So a normal reaction between A and B is R which is trying to push the box B up the incline. Now at the time of impending slip because the slip is to about to take place down the slope, the force of friction will be opposing it, that is, up the slope and this is the normal reaction. So these are the four forces acting on the box and if you consider the equilibrium of this box B sigma of all the axis parallel to the incline and Y is perpendicular so sigma FX is equal to zero. So R plus F is equal to the component of this force of three hundred Newton's in the X direction, which is three hundred sin theta down. Similarly sigma Fy is equal to zero. So this N is equal to three hundred cosine theta. So these two equations solve for two unknowns.

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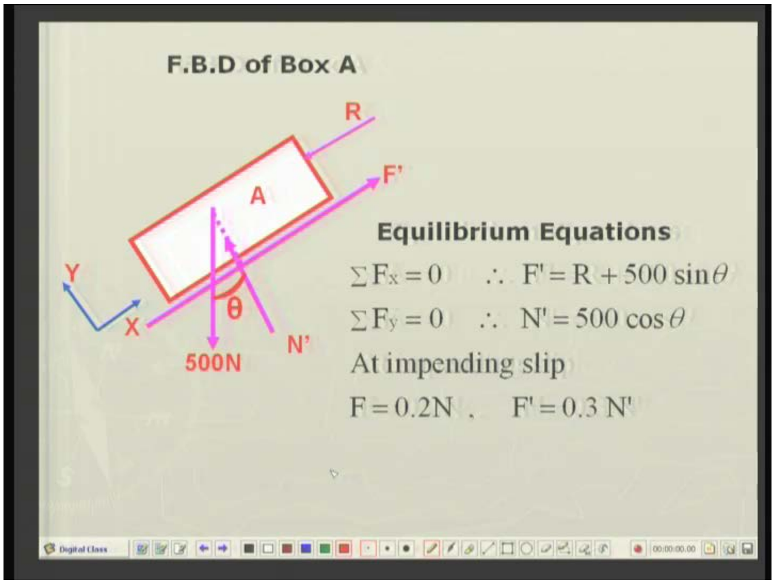

The free body diagram of the box A. By Newton's third law, there is equal and opposite reaction coming from the box B on to A. The reaction is R again. Force of friction between the two surfaces, that is, surface of the box A and the incline is F dash, the weight of the box is five hundred Newton. Again sigma FX is equal to zero sigma, Fy is equal to zero. So you write down similar equations. You have these two equations from the previous and also at the state of impending slip. When the motion is about to start by Coulombs third law, F is equal to the static coefficient of friction point two times the normal reaction. This is for box B and F dash, for, that is, force of friction acting on the box A is equal to the coefficient of friction which is point three times the normal reaction N dash.

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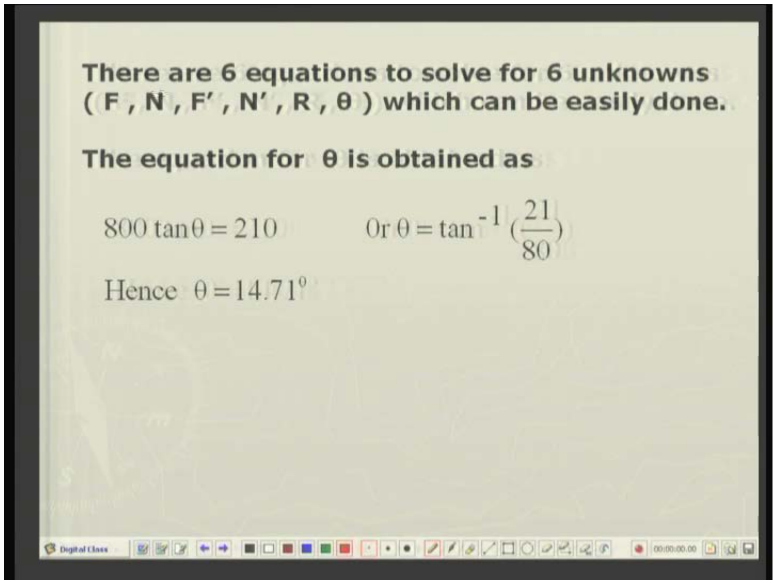

So total, we have six equations to solve for six unknowns. What are the unknowns? Two forces of frictions, two normal reactions, the normal reaction between A and B on the height surfaces and angle theta. Well, you can easily do it. That is, from these two equations, you can substitute for F dash and in the previous two equations, you can substitute for F. Then, you can eliminate N dash, etcetera and N and ultimately its very simple analysis to come to the final equation. After eliminating R also you will have the single equation, eight hundred times tan theta tangent of angle theta is equal to two hundred ten or theta is equal to tan inverse of twenty-one by eighty, that is, theta is fourteen point seven one degree. So, you see that by repeated use of Coulombs friction law, we can solve for the angle required which will be just sufficient to cause the slip. If we increase the angle slightly, the two boxes will start slipping down this plane. Well, now you might have observed one thing in the problems. You have taken up only two equations of equilibrium, that is, sigma FX is equal to zero sigma Fy equal to zero whereas we have learnt when we were discussing the conditions for equilibrium, that, if the forces are plane forces, then three equations of equilibrium are to be satisfied, sigma FX is equal to zero, sigma Fy is equal to zero and the moment of the forces lying in that plane about any point, that is, sigma MZ about point O, any arbitrary point O, is equal to zero.

So why only two were involved, why not the third equation? Because in all these problems, we did not know or need to know the exact line of action of the normal reaction N. It was not of our interest or concern to find out where the normal reaction was going through. It was going through between the two surfaces. Normal reaction is always in the contact area. It cannot go out of the contact area. That much is sure but otherwise whether it is at the centre, towards the right of the centre or left of centre we did not need to find out. Now, there are certain problems, very interesting problems, in which we need to know or we have to guess the line of action of the normal reaction.

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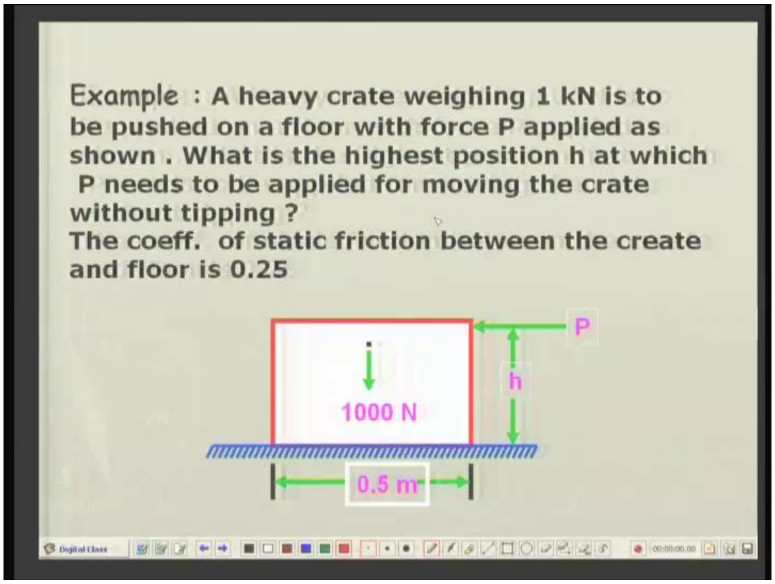

So I will take up analysis of one such problem, where a critical state of normal reaction will be required beyond which the body can topple over and this is illustrated with a simple example. This heavy crate, weighing one kilo Newton, is to be pushed on a floor with force P. So this is a box or a crate lying on a surface. The base of this box is point five meters. A force P is to be applied in such a way, that it does not topple the box. You might have experienced when you try to push a heavy table. If you push it too high, then the table or the chair tries to topple over.

So similarly, if I push it at a sufficiently high point, then the box may topple over around this corner. So, what is the highest position h at which P needs to be applied for moving the crate without tipping it or toppling it? So that, if I apply the force P slightly higher than that position, it will rotate around a corner? The coefficient of static friction between the crate and the floor is point two five. So let us look at this problem.

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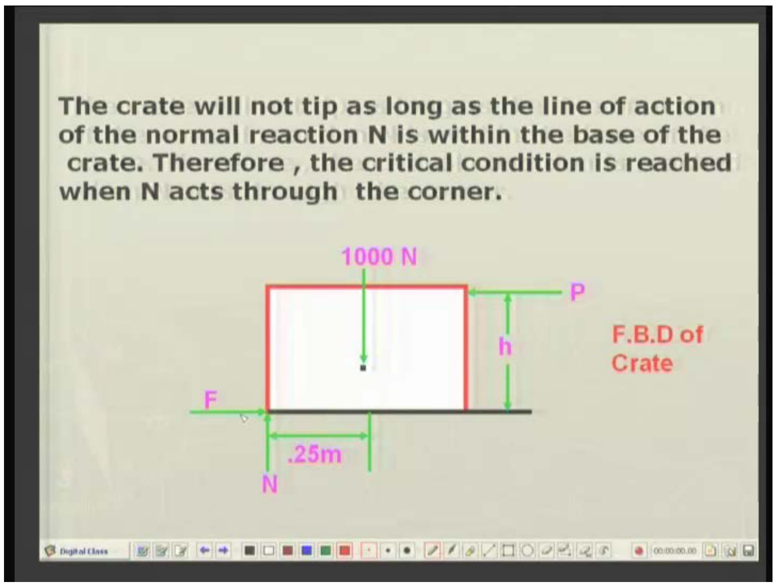

Now as I mentioned earlier, the crate will not topple or tip as long as the line of action of the normal reaction N lies within the base of the crate. So as long as it is because you can see here is force P and the opposing force of friction will be towards right, there creating a anticlockwise moment. Now, the weight downward is passing through the centre of gravity of, let's say, through this point and as the normal reaction is acting here or here or here, there will be clockwise moment. So, to balance this, this anticlockwise movement should be exactly equal to the clockwise moment and as the point P is higher and higher, more moment is created. It means the point of the line of action of N will move more and more to the left till a stage reaches when N is exactly passing through this tip, through this corner, because if I increase force even slightly more, it will go outside the base of the crate. So this is the critical state. So let us examine the state of equilibrium at this point. That is, a normal reaction passing through the corner force of friction force downward and the pushing force P.

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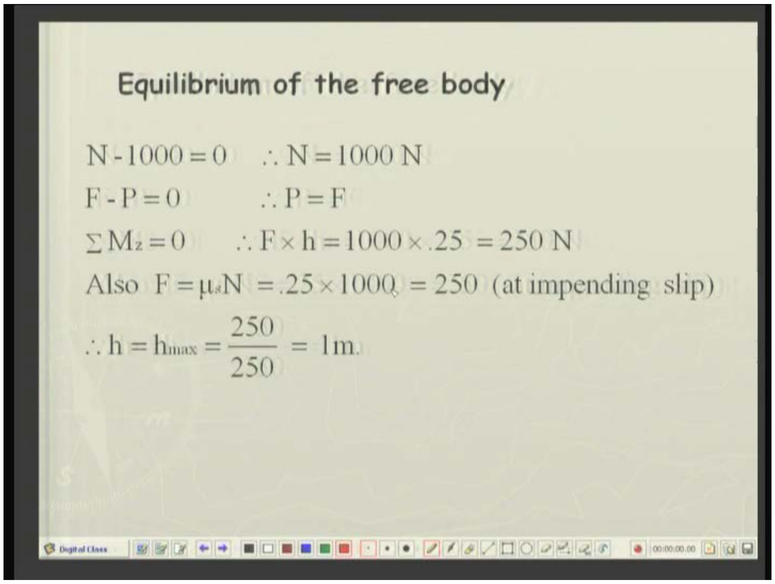

Again, the state of equilibrium. What are the conditions for equilibrium? Sigma Fy is equal to zero, that is, the normal reaction N minus downward load of thousand Newton's. So it gives us N is equal to thousand Newton's, sigma FX is equal to zero, that is, P is equal to F. That is obvious and then the moment about the corner point where we could have taken moment about any point but this corner point is most convenient because two forces are already passing through this point and the moment about this point will be zero. So I have need to take up moment of P and moment of thousand Newton's only. Sorry. This is force P. So this will be P into h, that is, the pushing force into h is equal to the moment due to the gravity force thousand into point two five two fifty and also the force of friction is equal to Newton's static coefficient of friction times the normal reactions. So this pushing force is two hundred fifty, of course. So both P and F are two hundred fifty Newton's at the impending slip and the highest point at this stage is two fifty by two fifty is equal to one meters.

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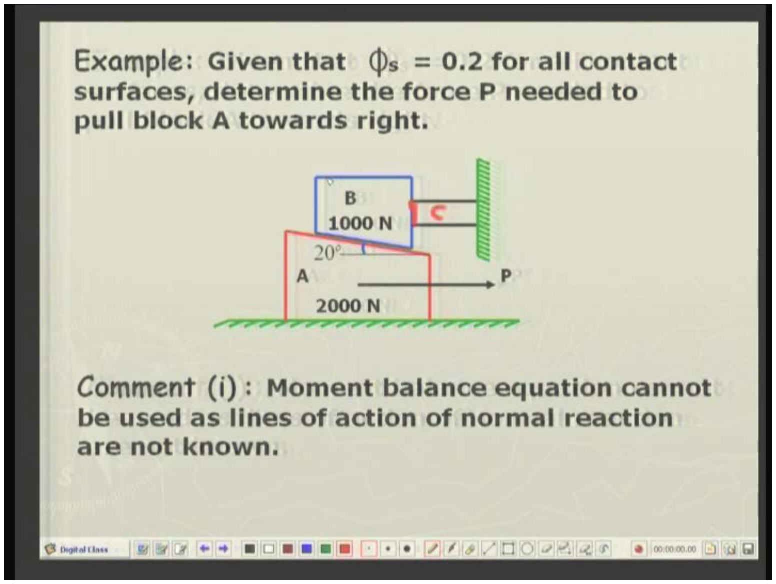

A third example where we have to imagine in what direction the relative motion is going to take place from looking at the geometry of the problem. We have to examine what the stage is, how the motion of various surfaces is going to take place. So here is a system of two blocks, block A and block B. There is a inclined, wedge like surface, at an angle, twenty degrees to the horizontal. So block B is resting on block A on this inclined surface and also block B is resting against a stopper on the surface. So there are three surfaces involved: the surface between block A and the floor, the surface between A and B, the surface between B and, you can say this rod, C. So the coefficient of friction between all contact surfaces is given as point two static friction and determine the force P needed to pull the block A towards right. So I want to pull it underneath this block B and you can easily imagine that if this block A is moved towards right, this block B will be lifted up because the contact point is moving like this. So it would be more and more height will be produced. So as the block B moves up, there will be a relative motion between B and C. So there are three relative motions involved.

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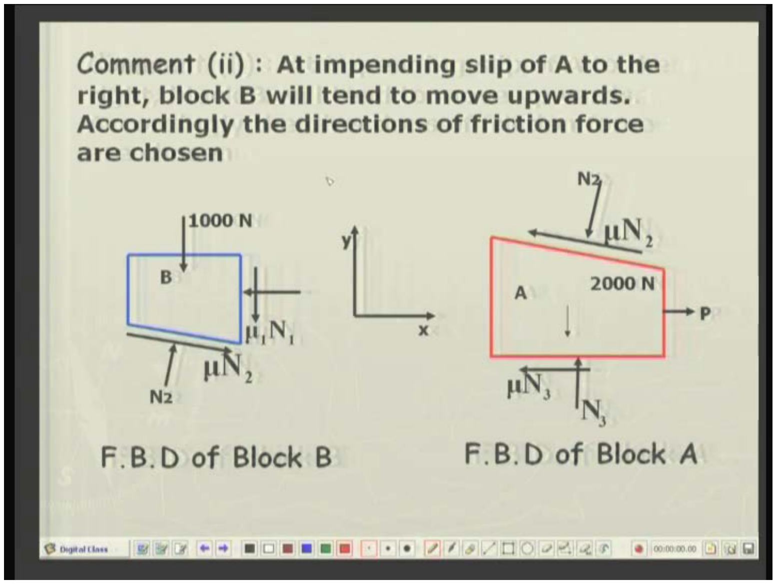

So at the impending slip, as we have said of block A to the right, block B will tend to move upwards and accordingly direction of frictions forces are chosen like this. So between block A and B, since the block B is going to move up and it means that a point on this will be moving in this direction so the force of friction will be along downward and towards the right and there will be equal and opposite force on the block A. So, you can imagine, that if the normal reaction between the block A and block B is N two, both are equal and opposite, the forces of friction will be mu and two and mu and two over here. Of course, the weights of the blocks, two thousand Newton are given and then I will look at the situation at the bottom surface of block A because this block is moving towards right. The force of the friction will be towards left and three is the reaction from the floor to the block. So, at the impending slip, the force of friction will be mu times N three and finally, on this surface against the rod C, since the block B is moving up, the force of friction will be downward and let us say, the normal reaction is N one. So it means, on both these blocks, the forces are now completely known. This is the free body diagram of block B, this is the free body diagram of block A and let us apply the conditions of equilibrium on first on block B.

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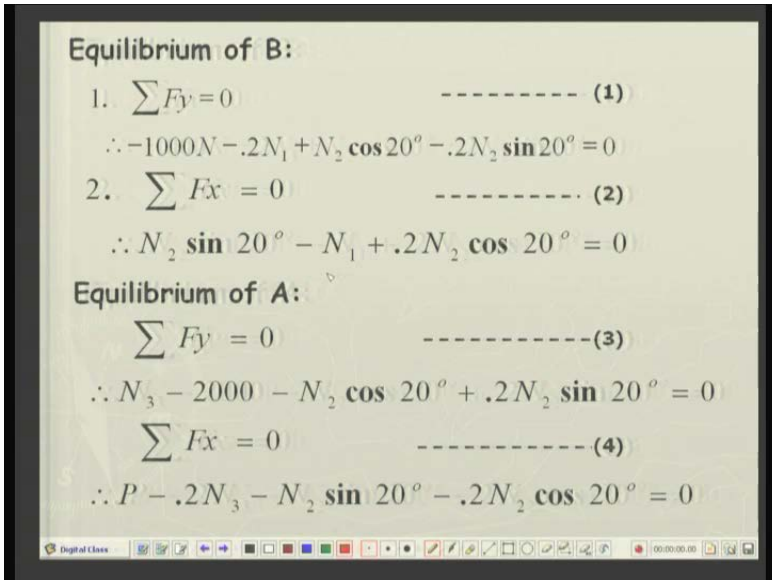

Sigma Fy is equal to zero. Well, you have enough practice with this. What are the vertical forces and what are the horizontal forces? Sigma on block B are vertical forces. A downward force of thousand downward force of mute mu one which is because all the coefficient static friction are equal to point two. So you have point two times N one. That is, downward. This is downward and then N two will have a vertically upward component, that is, N two times cosine of twenty degrees and similarly this will also have a vertical component. This inclined force, that is, mu times N two times sin of twenty degrees. This will be downward. So, if you look here, you will have minus thousand. This is thousand Newton's, not N, sorry. Point two times N one N two cosine twenty degrees minus point N two sin twenty degrees is equal to zero and this is sigma Fx then consider the horizontal components. There is N one here, mu N two times cosine twenty is the horizontal component. Again N two times sin twenty degrees may give me a horizontal component. So that is the equation number two. Similarly, you consider the equilibrium of block A. So again, vertical upward force, a downward force N two times cosine twenty degrees. Similarly, mu times N two times sin twenty degrees will be giving me vertically upward component and similarly, you can consider the horizontal forces. So by considering various vector components, you come to these four equations.

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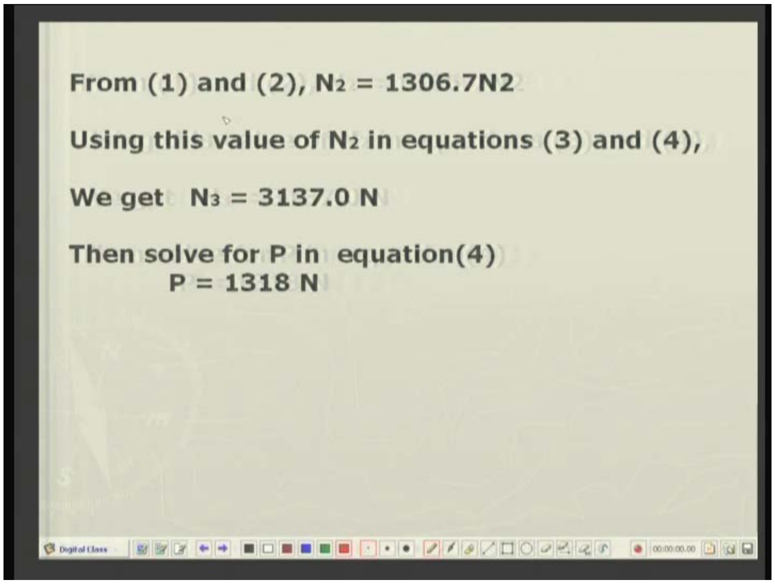

Now, there are four unknowns and you can solve for these four unknowns. From one and two, you can solve for N two and then the force required to move the block eight towards right comes out to be thirteen eighteen one thousand three hundred and eighteen. The normal reaction N three from the floor is also obtained, that is, three thousand one hundred thirty-seven Newton's and N two is no longer required.

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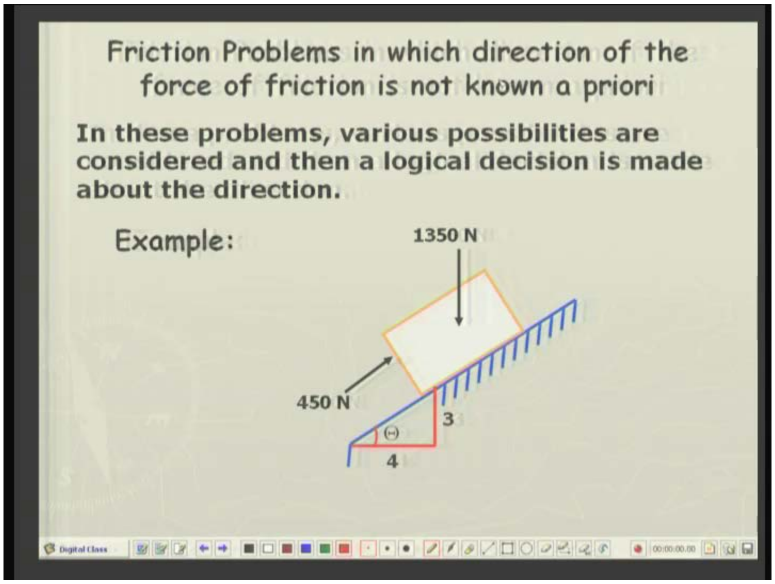

Now, there are certain class of problems in which you have to make a logical decision about whether the force of friction will be downward or upward or how the bodies are going to be moving with respect to each other. So various cases have to be examined one by one and then decided. So some alternative solutions have to be examined and some of the solutions will be feasible, some will not to be feasible. So what we mean to say is, you have to come to some logical choice. So such problems are very often encountered and in our next lecture, I will be taking up one such problem in which the direction of force is not known before-hand. Apriori means beforehand and then based upon equations of equilibrium, we will see what the appropriate direction of motion is and under that condition, what appropriate forces are. So, that, we will take up next time. Thank you very much for your attention in this lecture.