

Applied Mechanics
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Lecture No. # 06
Structural Mechanics

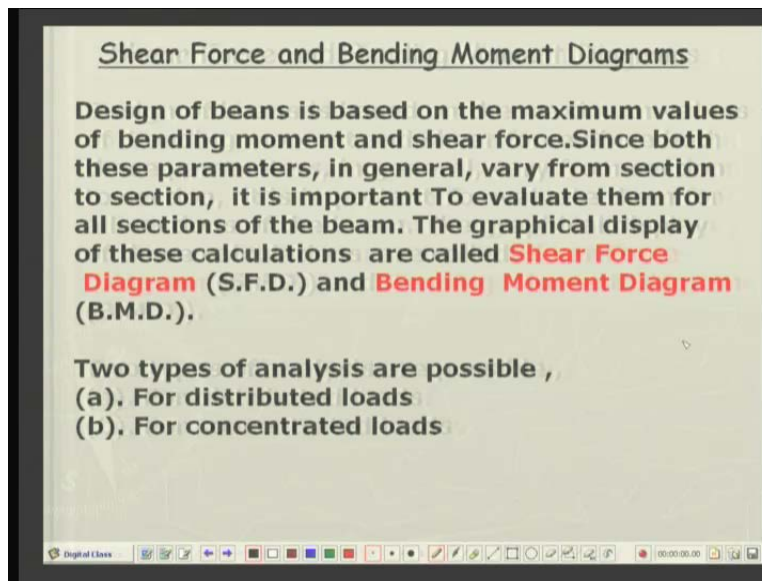
In today's lecture we will continue with structural mechanics. Let us recapitulate what we have already learned about structural mechanics. In the first case, we have analyzed trusses, which consists of two force members i.e. the forces are applied at only two points along the member and as such, there were only tensile or compressive forces in each member of a truss. The second type of structure which we started discussing last time was a beam. A beam is meant to take up transfer loads and forces as well as moments in the plane of the beam. Now even if there is an oblique load, we can decompose that load into a transverse component and an axial component. The axial component will produce according to its direction, either tension or compression along the beam. Whereas transfer component produces two types of reactions: one is the shear force and the other is the bending moment. This we have seen from consideration of equilibrium.

Now towards the end of last lecture, we also studied the properties of a section that whenever a beam is cut into by a plane section, two surfaces are produced. One of them is a positive surface and the other is a negative surface. Positive surface is one on which, the outer normal points towards a positive x axis whereas the negative surface has its outer normal pointing in the negative x direction. On a positive surface, a vertically upward shear force is a positive shear force whereas on a negative surface the vertically downward shear force is a downward and positive shear force

So two positives or two negatives make a positive shear force. However if the surface is positive but the shear force is downward then it is a negative shear force. Similarly on a negative surface, an upward force will be a negative shear force. Similar nomenclature is applicable for bending moments. On a positive surface, an anticlockwise moment will be a positive bending moment whereas on a negative surface a clockwise moment will be a positive bending moment. Also on a positive surface a clockwise moment i.e. a negative

moment is a negative bending moment and we can say for the negative surface an anti-clockwise moment will produce a negative bending moment. So again remember two positives and two negatives make a positives bending moment whereas one positive and one negative makes a negative bending moment. So let us continue our discussion from this point onward.

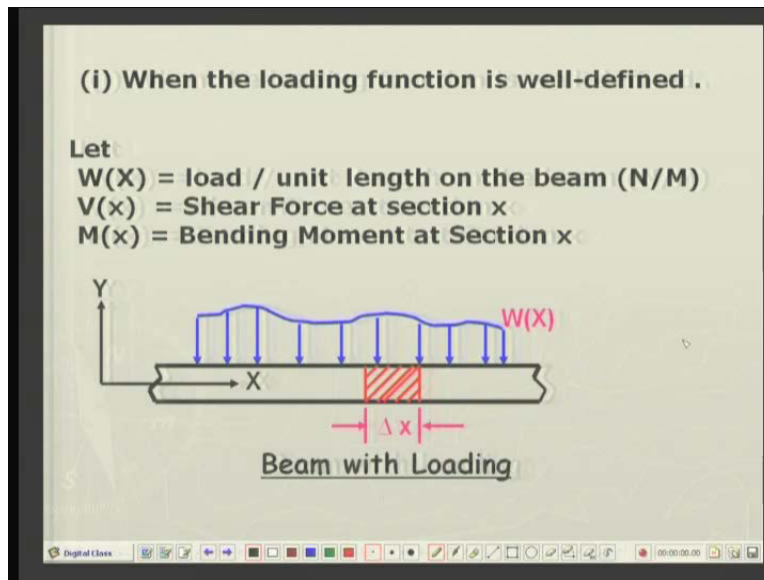
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In the design of beams which you will be learning in later courses like solid mechanics or strength of materials, there are two important considerations. One is the maximum bending moment and the other is the maximum shear force, because the size or shape of the cross section of the beam is very much dependent upon these two parameters. So it is very important for a designer to find out the magnitude of the maximum bending moment and where it occurs. Similarly the magnitude of the maximum shear force and where does it occur is also found out. In order to determine these or to get this information we must calculate or graphically display bending moment at every section as well as shear force at every section. A graphical display of this information is called a bending moment diagram or shear force, the diagram which depicts bending moments is the bending moment diagram, etc.

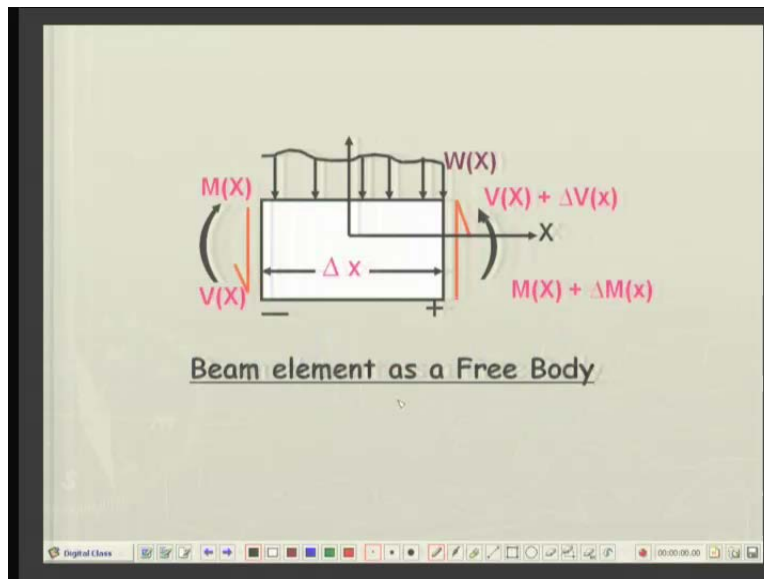
Let us start with the procedure to obtain or to draw these diagrams. Now for this we will study the equilibrium of an arbitrary beam loaded by transverse load. There are two types of analysis which are available and useful. One is for distributed loads and the other is for concentrated loads.

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Let us start with the case where the load is distributed along the length of the beam. The distribution function is very well defined. Let us say, $W(x)$ is the load per unit length of the beam and its units will be Newton per meter. $V(x)$ is the shear force distribution from section to section as a function of x and similarly $M(x)$ is the bending moment at any section x . This shown over in the figure is the beam and the XY coordinates are shown perpendicular to the plane of the screen. There is an arbitrary loading function $W(x)$. Let us focus our attention on a small segment of the beam whose length is delta x .

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Blowing up that segment of width, delta x and remembering that $W(x)$ is the loading on this segment, then on the right hand surface the outer normal is in the positive x direction. Outer normal is drawn like this so it is a positive surface and accordingly the left hand section is the negative surface. On a positive surface, we have a positive shear force $V(x)$ going upward and similarly on a negative surface, we have a downward shear force which will be positive. Now let us say on this surface, if $V(x)$ is the shear force then since the shear force is varying along the length of the beam on a surface which is at a distance delta x from this surface, let the shear force be $V(x)$ plus its variation delta $V(x)$. Similarly $M(x)$, the bending moment on a negative surface clockwise moment will be a positive bending moment and on a positive surface you have $M(x)$ plus its variation over length delta x that is that is delta $M(x)$. So now this segment of the beam of its delta x is in equilibrium because the entire beam is in equilibrium. Hence any internal free body, whether it is a small segment or a large segment, should also be in equilibrium. So we focus our attention on this segment loaded with a direct load and on the faces you have bending moment and shear force.

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Equilibrium Of Free Body :

i) $\sum F_y = 0 \therefore -W(x)\Delta x + V(x) + \Delta V - V(x) = 0$
or $\frac{\Delta V}{\Delta x} = W(x)$
As $\Delta x \rightarrow 0$, $\frac{dV}{dx} = W(x)$ (1)

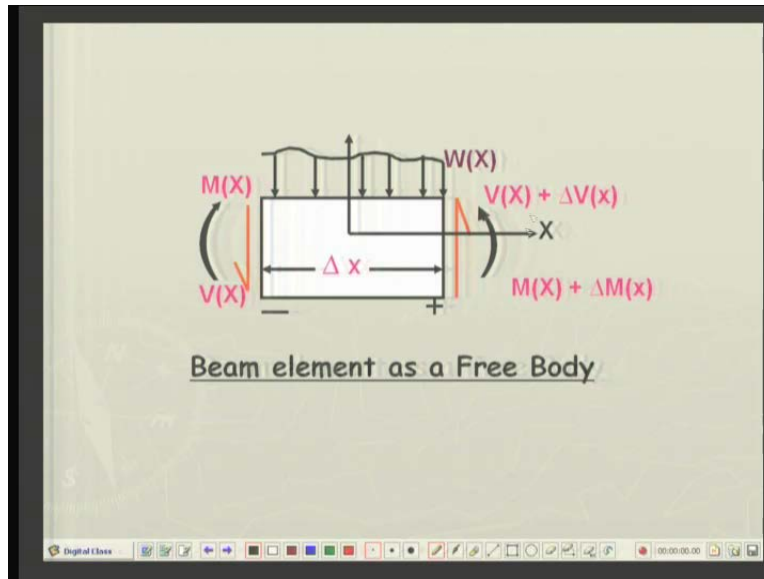
ii) $\sum M_z = 0 \therefore M + \Delta M - M + V(x)\Delta x + W(x)\Delta x \cdot \frac{\Delta x}{2} = 0$

Simplifying and then letting $\Delta x \rightarrow 0$,
 $\frac{dM}{dx} = -V(x)$ (2)

Writing down the equations of equilibrium for this free body, sum of all the vertical forces i.e. $\sum F_y$ is equal to zero. Going back to the figure there is the $W(x)$ which is the load per unit length and Δx being the length because it is a very small length. Then you can consider it to be an average load times Δx , so that it is total load on this segment then this is vertically downward. Vertically upward loads are $V(x)$ plus $dV(x)$ and downward load is again $V(x)$. So looking at this equation, minus $W(x)dx$ downward load, plus Vx , plus dV upward, minus $V(x)$ downward. This is equal to zero.

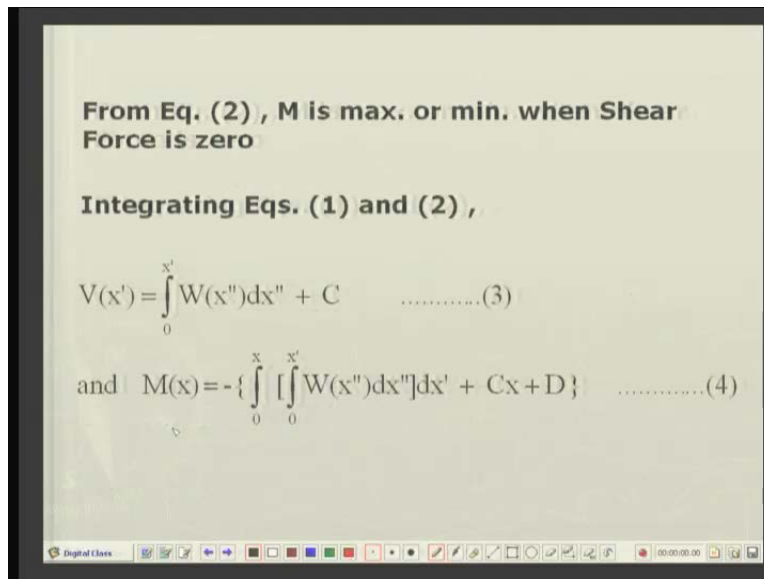
Now $V(x)$ and $V(x)$ will cancel out and you can easily see if I divide both sides by Δx , then we get $dV/\Delta x$ is equal to $W(x)$. And since the segment is infinitesimally small, as Δx goes to zero, the quotient becomes the derivative. So dV/dx is equal to $W(x)$. So this is the equation number 1 which relates the slope of the variation of shear force with the distributed load $W(x)$. So dV/dx is equal to Wx . Now again carrying out the free body analysis, the sum of the moments must also be equal to zero. Therefore $\sum M_z$ is equal to zero.

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So again going back to the figure, what are the moments applicable? There is a moment of M_x which is clockwise, an anti-clockwise moment of M_x plus dM_x . So you will have M plus ΔM minus M . Now this is the moment due to the shear force, $V(x)$ into Δx and this is the moment due to the applied load $W(x)$. So these four terms when they are simplified, M and M will cancel out and when Δx goes to zero it becomes very small. See the remaining terms are first order infinitesimal i.e. they have Δx only but the last term has two Δx . So it is proportional to Δx square and hence it is a second order in infinitesimal. So in the limit, this term will contribute very small and hence we neglect it. So dM by dx is equal to minus $V(x)$. This means that the slope of the variation of bending moment, dM by dx , is equal to the shear force with the negative sign.

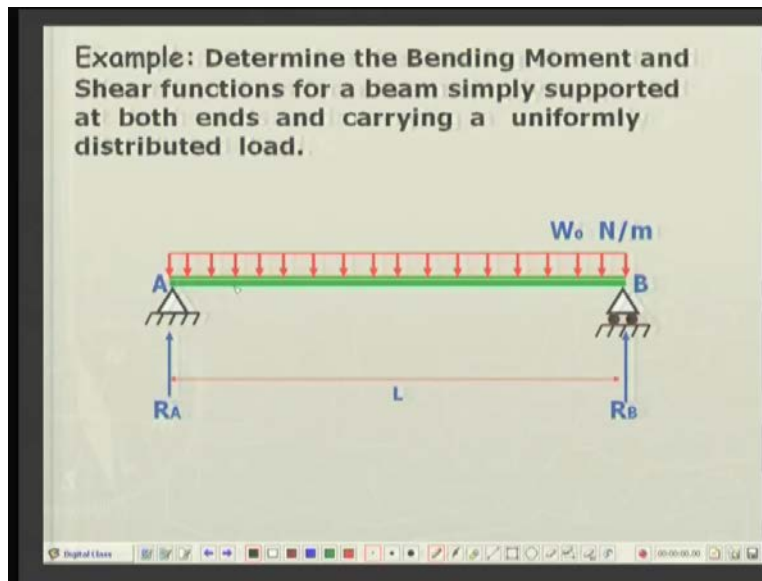
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Equation (1) and (2) gives us the relationship between the loading function, shear force distribution and bending moment distribution. We can integrate these two equations to get the shear force function and the bending moment function. From the equation number, you can integrate it and you get the equation number (3) i.e. $V(x)$ or x prime is obtained by integration of $W(x)$ and that is the dV by dx . This is the constant of integration and secondly you integrate this once again because the derivative of bending moment is equal to negative of the shear force. So from here you can find out $M(x)$ is equal to negative. So this is $V(x)$. Once we integrate it then C will be also integrated to get Cx and D is the constant of a second integration.

So equation (4) is the expression for bending moment in terms of the load distribution function $W(x)$ and equation (3) is the expression for the shear force in terms of the load distribution function $W(x)$. C and D are two constants of integration which must be determined from the boundary conditions. So to summarize when the loading on a beam is given by a well-defined function, it may be a constant function or a linear function or a sine function or an exponential function, (any well-defined function) then through differentiation and integration, we can get the bending moment distribution as well as the shear force distribution. To illustrate this procedure let us consider a very simple example

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Suppose there is a beam of length L . Incidentally the length L is called the span of the beam. So we will say a beam of span L meters and it is supported at two ends. At one end there is a simple support or a pin joint where theoretically both vertical and horizontal reactions are possible and at the other end is a roller support where only vertical reaction is possible. So the total number of reactions which can be solved for R_3 is in general. Although in this case, loading is only vertical. So there is no horizontal reaction and three reactions can be determinant from three equations of equilibrium. The entire beam is considered as a single free body. So the numbers of equations are three and unknown variables are three. So it is a statically determinate problem. Now we want to determine for this problem, the bending moment diagram as well as shear force diagram. First we will determine the analytic functions and then we will plot the diagrams.

Well going back to the picture, you can see that everything is uniform. Loading is uniform, geometry is uniform and there are only vertical reactions. So the problem is a symmetric problem. Consider that through the middle section of the beam there is a mirror, then the right hand and then the left hand beam, and loading is a mirror image of the right hand beam and the corresponding load. So whenever you have a symmetric problem then automatically the reactions will also be symmetric i.e. R_A is and R_B both

are equal. You can do it otherwise also by considering $\sum F_y$ is equal to zero and $\sum M_z$ is equal to zero. Let us say if I take the moment about the point B, then you will get R_A easily and similarly if I take the moment about the point A, then you will get R_B easily.

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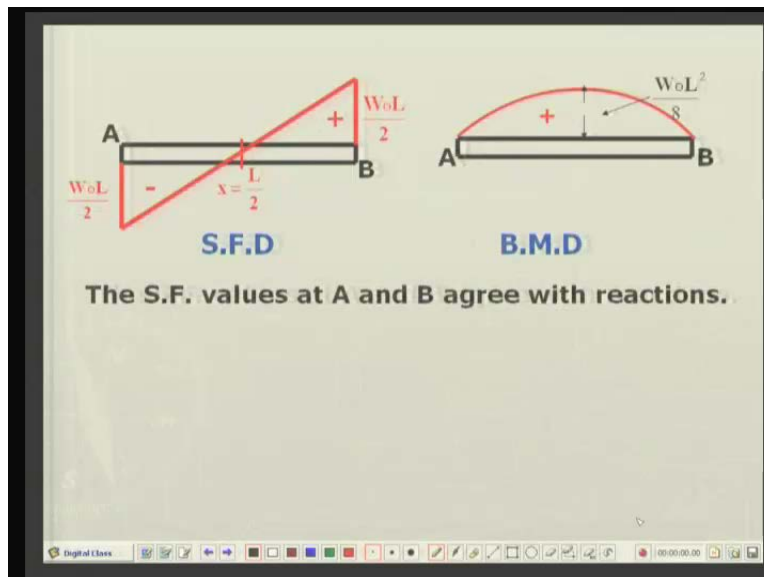
By symmetry of loading, $R_A = R_B = W_0 L / 2$
 Substituting $W(x) = W_0$ in above equations and simplifying:
 $M(x) = -W_0 x^2 / 2 - Cx + D$
 Boundary condition: Ends are simply supported
 $\therefore M(0) = M(L) = 0$
 Then $D = 0$ and $C = -W_0 \frac{L}{2}$
 Hence $M(x) = -W_0 \frac{x}{2} [x - L]$
 Now $V(x) = -\frac{dM}{dx} = \frac{W_0}{2} (2x - L) \dots\dots(4)$
 At Beam ends, $V(0) = -W_0 \frac{L}{2}$, $V(L) = W_0 \frac{L}{2}$

So by symmetry of loading as well as the beam R_A is equal to R_B , total load on the beam is $W_0 L$ where W_0 is the load per unit length and L is the length of the beam. So the total load is $W_0 L$ and hence each of the reactions will be equal to half of it. Now since the loading function is a constant function W_0 it is independent of x . So you can substitute into the equations which we have derived earlier. For $M(x)$, if I substitute for this W_0 then integrate it and we will see that we will get this simple equation $M(x)$, the bending moment at any section x is equal to minus W_0 into x square by two parabolic plus Cx plus D . C and D are to be determined from the boundary conditions i.e. (1) both ends are simply supported, (2) they cannot sustain any rotation or any moment therefore the moment at x is equal to zero as well as x is equal to L has to be equal to zero, because if there is a non-zero moment it will cause the rigid body rotation of the beam. So it means that first I put x is equal to zero then all the terms will be zero and say D must be equal to zero. Then I put x is equal to L . So that will determine again M at L is equal to zero.

These are already found out to be zero, so this term will be minus W_0L square by two and this term will be equal to CL . So you can easily see that C will come out to be minus W_0L by two. This is the expression.

So substituting back these values of D and C we will we can easily know that the variation of the bending moment i.e. function $M(x)$ is equal to minus W_0 into x by two into x minus L in the brackets. Once we differentiate it with a negative sign we will get the shear force and in doing so we will get the shear force $V(x)$ is equal to W_0 by two into two x minus L . As a check when we put V is equal to zero and at $V(x)$ is equal to zero and x is equal to L , we will get the shear force as W minus W_0 over L by two and W into L by W_0 into L plus W_0 into L by two. So these are precisely the two end reactions and it means our analysis is proceeding on the correct lines.

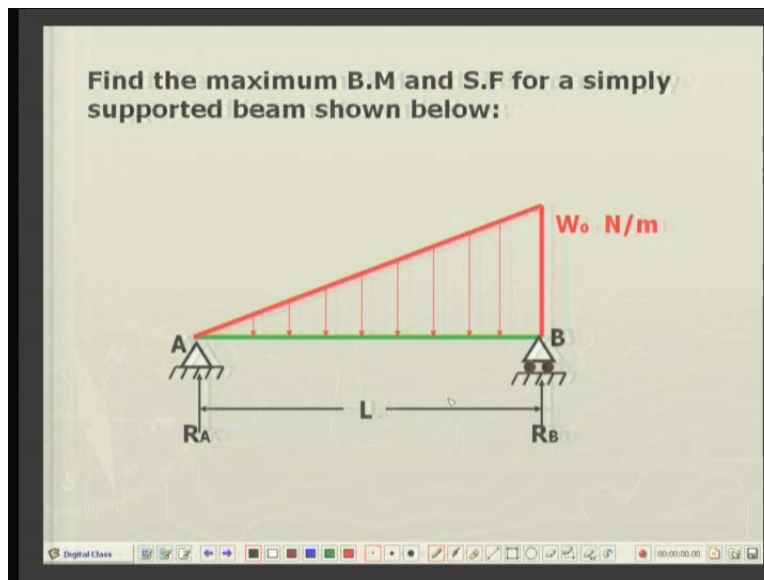
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And if I plot at the end A, the shear force is equal to negative of W_0L by two. So minus W_0L by two and the function is shear force function which is a linear function. So it means it varies in a straight line and at the end B the same shear force is positive. When we plot the bending moment we have seen that at both the ends bending moment is zero. So it starts from zero and you can see that the bending moment is zero at both these ends.

This term will be x^2 by two and this will be a parabolic term, so the function will be the shape of the bending moment diagram and it will be parabolic. Between A and B this is the parabola and the maximum shear force, bending moment occurs. Well you can see as I had mentioned, for maximum or minimum, the dM by dx has to be equal to zero and at that point the shear force must be zero because dM by dx is equal to minus $V(x)$. So it means since the shear force is zero at the center of the beam, the maximum bending moment will also occur at the center of the beam. It can be easily verified that this is occurring at the center of the beam, at x is equal to L by two. So its value at that point is $W_0 L^2$ by eight. For well-defined loading functions, we can quickly and simply get the bending moment and shear force diagrams and they are consistent with the end conditions. I will take up another example.

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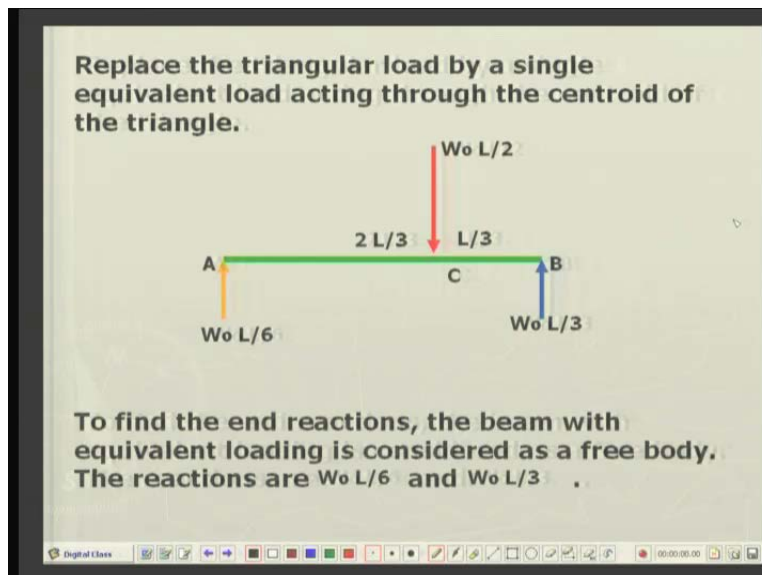


Here the loading is a triangular loading i.e. there is a zero loading at one end and gradually rises to a maximum loading of W_0 which is load per unit length Newton per meter. This is the loading rate, so you can imagine that on this beam a triangular wall has been built. The height of the wall is increasing load and this is also increasing. So in this way you can visualize this problem. Again one end is simply supported while the other end is roller supported. No horizontal load and hence no reactions are possible. No

horizontal reactions are needed, so only vertical reactions will be needed. Now to analyze this problem you can replace this triangular load by equivalent loading. You have learnt in lecture three and four, how to replace a given loading by its equivalent loading.

For a triangular load, we can find out the total load which will be the area under the triangle. The area under the triangle will be base times altitude divided by two, so W_0 into L by two is the total load and it will be acting through the center of gravity of the triangle i.e. centroid of the triangle. And this occurs at somewhere along a line which divides this total span into two parts, two L by three and L by three as I am showing in the next slide.

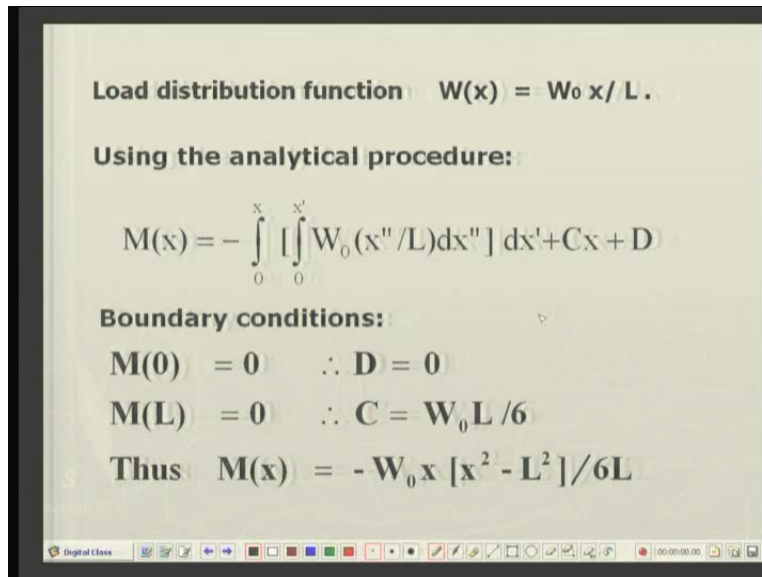
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The total triangular load is replaced by a single concentrated load $W_0 L$ by two, which is area under the triangle and its location is such that the span is divided into two parts: two L by three and L by three. If you consider this entire beam, with this loading as a free body then you can find the reactions from the equilibrium equation. I will not do it now; you can do it by yourself. It is done by simply taking the summation of vertical force is equal to zero and summation of the moments about any point A or B. Both points are equally good, you can find out one reaction at point A will be W_0 into L by six while the

other action at B is a W_0L by three. So having obtained these two reactions, we go back to the analytic solution.

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Load distribution function $W(x) = W_0 x / L$.

Using the analytical procedure:

$$M(x) = - \int_0^x \left[\int_0^{x'} W_0 (x''/L) dx'' \right] dx' + Cx + D$$

Boundary conditions:

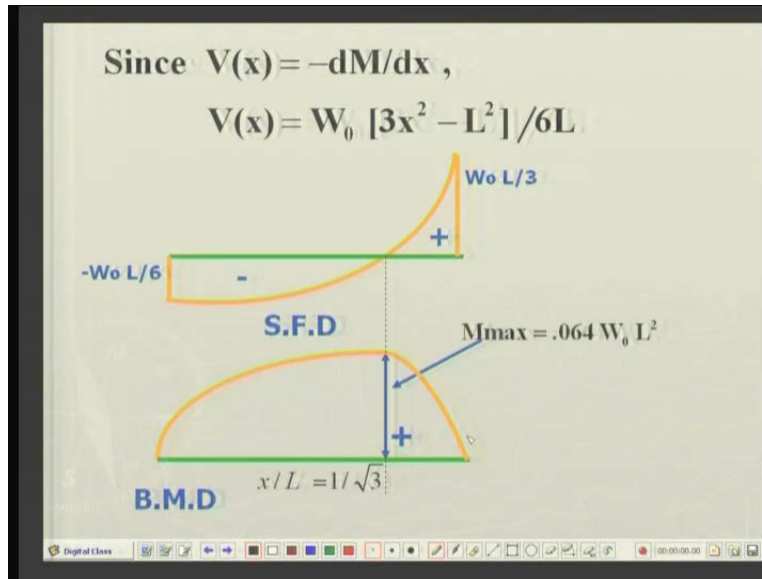
$M(0) = 0 \quad \therefore D = 0$

$M(L) = 0 \quad \therefore C = W_0 L / 6$

Thus $M(x) = - W_0 x [x^2 - L^2] / 6L$

Load distribution function is a triangular function. Linear function so it means it will vary from zero to maximum of W_0 . So you can easily see that this will be equal to the load distribution W_0x divided by L . Now the formula which we have already derived connecting $M(x)$ i.e. the bending moment distribution with the loading distribution. So this is the direct substitution of the formula and the loading function $W(x)$ is replaced by W_0x divided by L . Integrated to once, twice you will and then since both the supports are simple the bending moments at the supports must be equal to zero. So double M at x is equal to zero which will give this constant of integration D also equal to zero. M at x is equal to L which is equal to zero. This gives easily the constant of integration as W_0L by six. Once we have obtained these two constants I substitute back into this equation to get the final expression for bending moment. $M(x)$ is equal to minus W_0x taken common outside into x square minus L square whole divided by six L . So this is the expression for the bending moment.

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Once we take the derivative of this bending moment and put a negative sign, I get the shear force. Substituting back we will get $V(x)$ is equal to W_0 into, three x square minus L square, whole divided by six L . Both bending moment and shear force have been obtained as analytic functions and rest we have to only plot them. First plot the shear force at x is equal to zero; you will easily get W_0 minus L square divided by six L . So $W_0 L$ by six with a negative sign. This is ah when x is equal to L , we will get $W_0 L$ by three and as I check that our procedure has given correct results these are precisely the two reactions. This is a parabolic equation, so you join these two end points with a smooth parabola and it will intersect the x axis at this point. This is the point where shear force is zero and at precisely this point it will be maximum and when we plot the bending moment diagrams there will be a cubic parabola. Here is x and x square term so it means it's a x cube term. It is a cubic parabola and again at both the extremities at point A and B the bending moment is zero. So we get a cubic parabola and then after it goes through the maximum, it falls again and the maximum value can be determined from the previous expression. You can determine at that point where the shear force is zero. This is zero at x over L equal to one by root three, so at x is equal to L by root three or L divided by 1.732, the maximum bending moment occurs and its value is given over here.

So through these two examples we have seen that when we have well defined loading functions, we can easily get both the bending moment distribution and shear force distribution. One very important conclusion was when the shear force is zero, the bending moment is maximum or minimum.

Now the second type of analysis for bending moment and shear force is needed when the loading is concentrated. The load function may be itself concentrated i.e. at a point a non-zero load is being applied. Over almost zero length, you have a non-zero load and similarly at a point there may be a non-zero moment applied due to some twisting or some brackets as we will see. The problem with this concentrated load or moment is that they cannot be defined by regular functions. They need singular functions and the mathematical analysis, although it can be taken up but it becomes very complicated. So it needs functions like delta functions, heavy side function, etc. which will not be covering in this course but there is a simpler procedure which can be resorted to whenever you have concentrated force or concentrated moment applied to a beam. Wherever there is a concentrated load we will separate that section. At that point we will first consider a section to the left and then a section to the right and we will proceed from section to section and cover the entire beam. So in this manner I think the best way is to illustrate this procedure with the help of an example. Here is an example. A simply supported beam is a loaded beam which is AB. It is loaded by a concentrated load of two kilo Newton at point C.

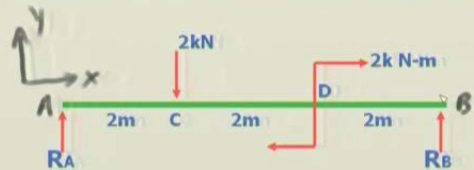
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Example: A simply supported beam is loaded as shown below by a concentrated load of 2kN (vertical) and 4kN (horizontal). Draw the B.M. and S.F. diagrams.

Solution:
First consider the entire beam as a free body.
Replace the off-axis horizontal forces by an equivalent axial force and moments.

And there is a bracket at ninety degree which is at 0.5 meters offset from the axis of the beam and this bracket is being pulled with the force of four kilo Newton. So now this is quite obvious. This is a singular force and here if I consider the free body of this bracket, it means there will be equal forces to maintain this in equilibrium. There will be an equal and opposite force of four kilo Newton and also due to these two there will be couple. The couple has to be maintained, so there will be a bracket which is in equilibrium and at this junction between the bracket and the beam there will be equal and opposite reaction. It means at that point there will be horizontal force of four kilo Newton towards right and there will be a clockwise moment at D of four into momentum being 0.52 kilo Newton meters.

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Equilibrium of F.B. :

$$R_A + R_B = 2\text{kN}$$
$$\sum M_B = 0 \quad \therefore -6R_A + 4 \times 2 - 2 = 0$$

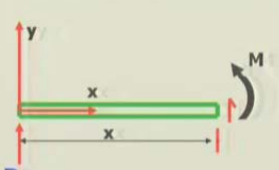
Hence $R_A = 1\text{kN}$, $R_B = 1\text{kN}$

So if I show it over here this beam with the bracket is simplified as a beam with a concentrated load of 2 kilo Newton at C. At D there is a couple whose moment is 2 kilo Newton meters and R_A and R_B are the two reactions. First part is you can determine the reactions by considering the entire beam as a single free body. R_A plus R_B is equal to 2 kilo Newton because sum of the vertical forces is equal to zero and similarly take moments about either point A or point B.

Here I have taken it about point B and from this equation taking moments let us say total length is 6 meters. So six into R_A , this is clockwise hence negative plus four into two, so two kilo Newton into four anticlockwise positive and then there is a negative moment due to couple of two kilo Newton meter equal to zero. So R_A comes out to be one kilo Newton and R_B also one kilo Newton. Overall equilibrium gives us the two reactions. Now I will consider in this case, three sections; one section anywhere between (consider the x direction and y direction) x is equal to zero and x is equal to two meters, then the second will be between C and D i.e. x must lie from two meters to four meter and then the third section will be from D to B. D to B means x going from four to six. So we will proceed from section to section. Let us do that. First section is anywhere between zero to two.

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(a). Consider section at various places:



Beam left of C as F.B. ($0 < x \leq 2$)

Equilibrium Conditions:

$$R_A + V = 0 \quad \therefore V = -R_A = -1 \text{ kN}$$
$$-R_A x + M = 0 \quad \therefore M = R_A x = x \text{ kN-m}$$

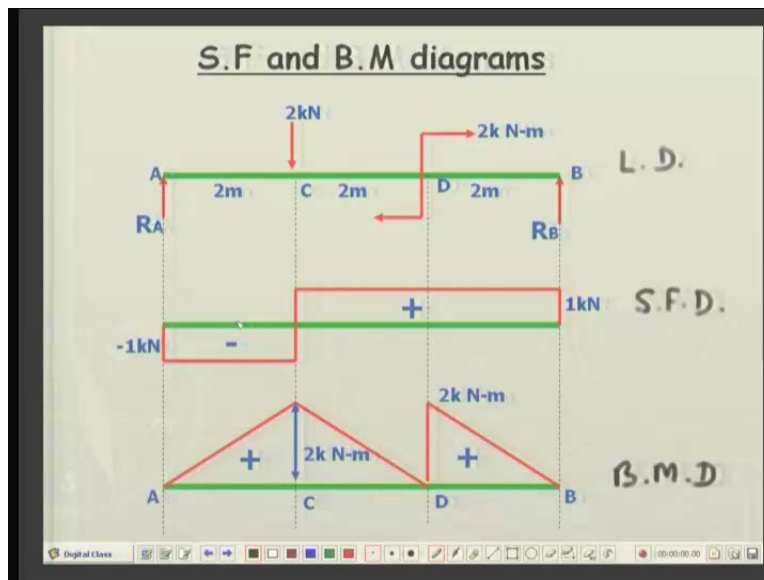
Consider the section but x is equal to zero, slightly more than zero. x is equal to or less than two. So in this first section let us note down all the forces. There is a reaction R_A which was determined as one kilo Newton and there is a moment M and the shear force on this section on the positive surface, a positive upward shear force of V . There is no other loading so consider the equilibrium of this part of the beam. Sum of the vertical forces R_A plus V , both upward is equal to zero. So it gives us V is equal to minus R_A which is equal to one kilo Newton. So it means in the section, the shear force is a constant value of minus one kilo Newton and then you take the moment about the middle of the section. First R_A will give you clock wise moment which is negative R_A into momentum is x . R_A into x plus M equal to zero. So M is equal to $R_A x$ which is equal to x since R_A is one kilo Newton. So simply M is equal to x kilo Newton meters and this a linear function.

Next we go to the section anywhere between x greater than or equal to two and x less than or equal to four. So in that section you can easily see the forces R_A down ward force or two kilo Newton and similarly V and M , so again $\sum F_y$ is equal to zero, $\sum M$ is equal to zero, etc. V plus R_A minus two is equal to zero. Hence V gives us

two minus R_A which is R_A which is one kilo Newton plus one kilo Newton and M will be equal to four minus x kilo Newton meter.

Then we come to the third section between D and B and at D there is a concentrated moment due to bracket. So we have a concentrated moment of two kilo Newton meters and you have one force, one moment and the end force due to reaction. Again $\sum F_i$ is equal to zero, $\sum M_z$ is equal to zero and these two equations are obtained. Now only thing to do is we have to depict these result graphically. Let us do that.

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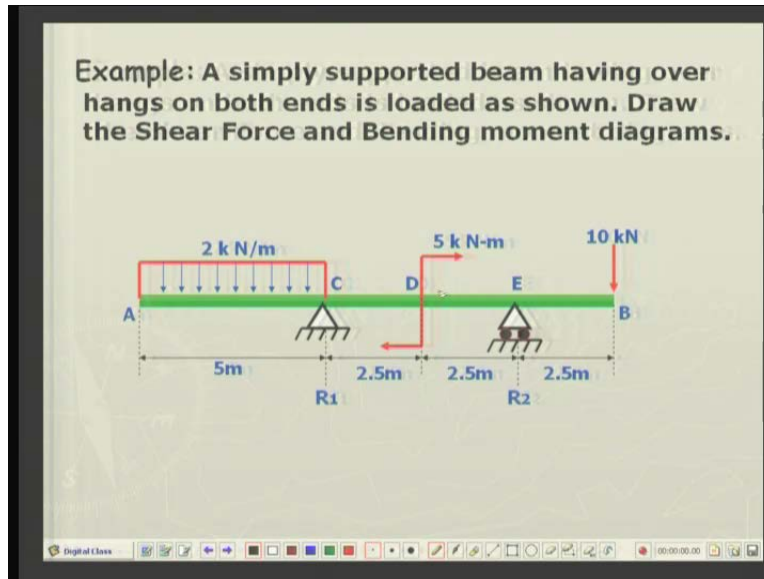


This is the beam as given to you. R_A and R_B are two reactions which are both found to be one kilo Newton each and then applied load of two kilo Newton, applied moment of two kilo Newton meters. The first one is the loading diagram LD, the next is the shear force diagram SFD and the last one is the bending moment diagram BMD. You can see that wherever there is a concentrated load or a moment or end reactions, I have drawn vertical line just to indicate the sections.

You have first between zero and two. If you look at the equations, the ones we had between zero and two one kilo Newton negative and a linear function x . So one kilo

Newton negative which has constant in that section and a linear function for a bending moment and then at this point of C, there is concentrated load of two kilo Newton. So there will be jump at the indicating two kilo Newton. So one kilo Newton and one kilo Newton and this discontinuity amongst two kilo Newton and after that there is no loading. Here only there is moment, so the shear force will be constant and it is equal to the reaction at B which is positive one kilo Newton. So we see that wherever there is concentrate load there is a jump in the shear force diagram. A concentrated moment will not produce any change in the shear force, on the other hand where there is a concentrated moment there is a jump in the bending moment. So this is again linear, if you see in the second section again it is a linear function and in the next section again it is a linear function. So it consists of three linear functions. BMD consists of three linear functions and one jump. So this is exactly what we get, three linear functions. All parallel to the slope is same and this is the jump. Well the shear force was going through zero over here and it is consistent with our results, that bending moment is maximum when the shear force is going through zero. So everything is timing up very well. This is how you will plot a bending moment and shear force diagram when the loadings are concentrated. Let me take up a very interesting problem with the mixed type of loading on some part of the beam.

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There is a distributor load, a well-defined uniform function as it is something called rectangular loading. Uniformly distributed load or rectangular load and there is a concentrated moment and a concentrated load. Now this beam is having double overhangs i.e. towards left of the support C, the beam is loaded and towards right also of support A, there is a load. So these two positions on either side of the supports they are called overhangs. First we will consider the entire beam as a single free body to determine the two reactions and then we will proceed from section to section. There will be one section from A to C, second section C to D and third section D to E and the last section E to B.

Now I will not go through the same section wise analysis because once you get enough practice you can almost do the analysis just by looking at the beam and the type of loading. We have done already a problem, analytic problem when the loading was a uniform loading. We saw in that case the shear force diagram was a linear function and the bending moment was a quadratic function. You can expect similar things to happen. From A to C, there will be a shear force which is linearly varying and a bending moment which will be quadratically or parabolically varying. Between C and D, there is no load so the influence of this uniform loading will be there. You can replace it by this uniform

load by single equivalent load at the middle of the rectangle and then you can determine again the shear force and bending moment, or you can also proceed from the end B. Again due to single concentrated load you will find that the bending, the shear force will be a constant function and the bending moment will be linear function. So if you remember these simple results you can do the bending moment shear force diagrams analysis very quickly. Let us do it well from reactions.

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Reactions:

$$R_1 + R_2 = 10 + 5 \times 2 = 20 \text{ kN} \quad \text{--- (i)}$$

$$\sum M_{R_1} = 0 \quad \text{--- (ii)}$$

$$\therefore 10 \times 2.5 - 5 + 5R_2 - 7.5 \times 10 = 0$$

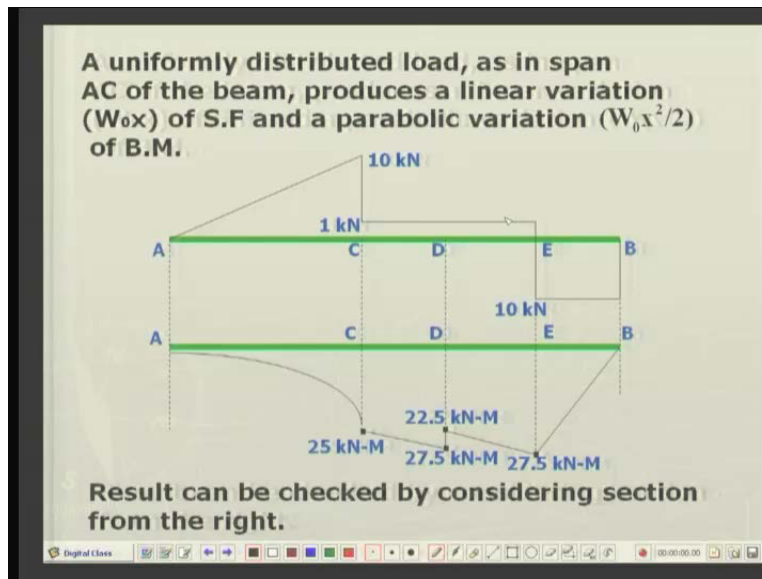
or $R_1 = 11 \text{ kN}$

$R_2 = 9 \text{ kN}$

Since R_1 and R_2 are concentrated, S.F.D. has jumps at C and E of magnitude 9kN and 11kN respectively. Similarly, B.M.D. has a jump of 5 kN-m at D.

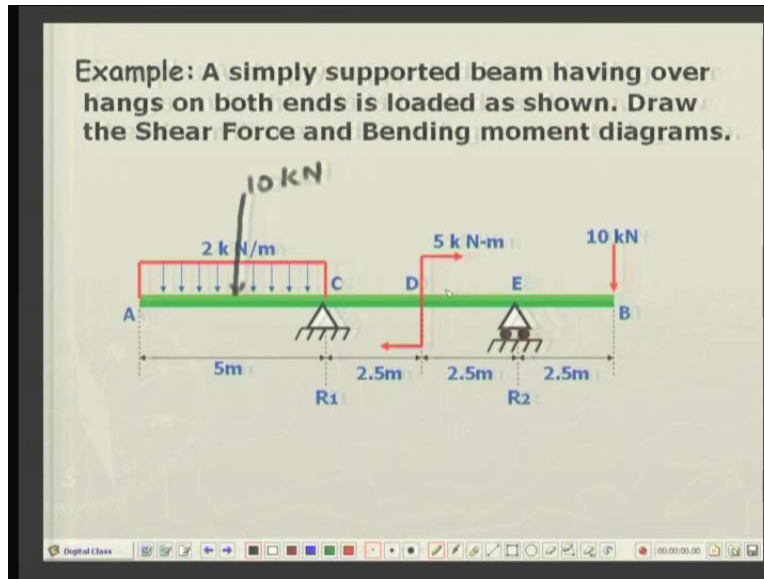
Considering the entire beam as a single free body, R_1 plus R_2 is equal to a load of ten kilo Newton on the overhang and a uniform load two into five that is ten kilo Newton. So ten plus ten is twenty and you can see that you have twenty kilo Newton. Now take the moments about the point C i.e. where the reaction R_1 is acting. You can quickly see that your R_2 , the reaction at the other end is obtained as nine kilo Newton and R_1 will be eleven kilo Newton because their sum is twenty kilo Newton. So by these two equation of equilibrium two reactions R_1 and R_2 are determined.

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Next we will see as I indicated in the first part overhang, that is from A to C, there is a uniform loading and the shear force will be linearly varying i.e. it is equal to two kilo Newton time x . It is a straight line function, so this will be a quadratic function that is starting from zero because this is an unsupported free beam and so no moment can be sustained there. So starting from zero it is increasing parabolically and the value can be obtained again. You replace it by the total load of ten into two that is ten kilo Newton at the center. We can say that this is equivalent load of ten kilo Newton and this will produce ten into two point five i.e. twenty-five kilo Newton meters. So this will be the bending moment at the point C and now if I proceed from here because that will be very easy proceeding from the end B. Again there is a load of ten kilo Newton over here and that will give me a constant shear force because there is no loading in between A and B. That will be a ten kilo Newton shear force and this ten kilo Newton times the distance here. So it is gradually rising and at point D you can see there is a concentrated moment so I can expect a jump in bending moment. At C there is a concentrated reaction, so there is a jump in the shear force diagram. See a jump in the shear force diagram here and a jump in the bending moment diagram here.

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You can recapitulate all the features of the bending moment shear force diagram without going through the total analysis. You should be able to have enough practice to do that, otherwise we can proceed in the usual way i.e. we will first consider a section between A and C. Consider a free body diagram, calculate the value of V and M. Then you consider a section between C and D, again you can consider the free body which will have the slow reaction here and shear force and bending moment on the cross section. Write down the equation equilibriums in this manner and you can precede with it. The third section will be over here, fourth will be in this part of the mean. If you proceed from section to section and analyze each of the internal free bodies, you can derive those equations for bending moment and shear force and then you plot them, or by looking at it, by recognizing what should be the variation linear, quadratic, cubic, etc. you can get some idea and wherever there is a concentrated load, whether it is an applied load or a reaction you will see a discontinuity or jump in the shear force. Wherever there is a concentrate moment you should see a jump in the bending moment diagram. So these are some of the automatic checks for you and finally, wherever the shear force goes through zero, you can expect a maximum, a local maximum or the minimum in the bending moment.

All the features, for example here it goes through a maximum, so shear force is zero at point E. You can see the bending moment is going through a maximum more here. To summarize we have seen that beams which constitute very important structural members, load bearing members, they need the information above the distribution or bending moment and shear force for designing them and this information can be obtained by plotting a bending moment and shear force diagrams there. Wherever the bending moment is maximum i.e. the point where the stresses as you will be learning latter on will be maximum, and accordingly the beam has to be designed on the basis of the maximum bending moment and also when the shear force is maximum again for certain types of beams. That information is very much needed particularly beams with webs, etc. So in our next lectures, we will start with further analysis in mechanics, namely Friction. Thank you very much for your attention.