Applied Mechanics

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Lecture No. 5

Structural Mechanics

In our last lecture, we discussed the concept of free body diagrams and the conditions of equilibrium while analyzing the equilibrium of a system of rigid bodies. In today's lecture we will make use of these concepts in a very important area called structural mechanics. So today's lecture is titled structural mechanics. What is structural mechanics?

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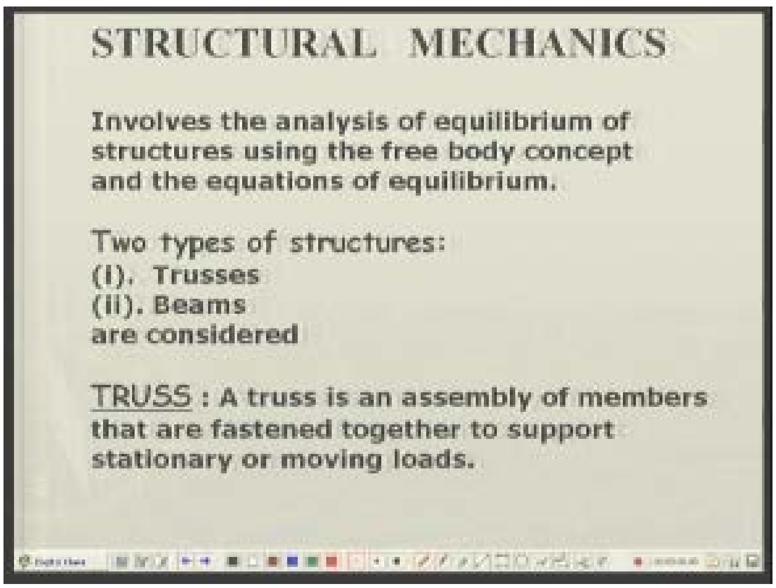

This branch of mechanics involves the analysis of equilibrium of structures using, as I said, the concepts of free body and equations of equilibrium. We will be studying two types of structures, namely, number one trusses, number two beams. First, we will start with the analysis of trusses. What is a truss? Truss is an assembly of members or elements that are fastened together to support stationary or moving loads. They may be fastened through riveted joints welded joints or pin joints.

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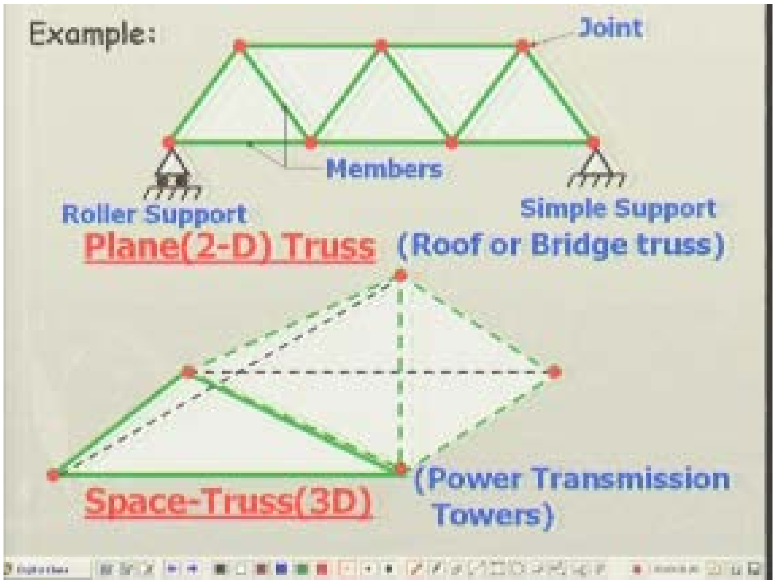

I will give you some examples of trusses. First of all, two dimensional trusses or plane trusses. Here is an example, the member. These green lines are representing the members or elements of a truss and the whole assembly is a plane or two dimensional truss. You must have seen such structures or such trusses while crossing a railway bridge. On the both sides of the train, you can see a series of such structures which support the load of train as well as other members of the structure. The other example of plane truss is when you visit a work shop or a godown and you look up, you will find that the roof and other loads are supported by a series of plane structures and they are also examples of trusses. The three dimensional truss or space truss is a similar structure but it is in all the three dimension, that is, length, breadth and height and the example of such truss you will find in power transmission tower for the distribution of electricity, etcetera.

Now the basic element in the plane truss is a triangle. You can see that each one of them is connected to the others like this. Whereas, for a three dimensional space truss, the basic element is a tetrahedron. We will be mostly concentrating on the analysis of two dimensional or plane trusses and before we take up the analysis of the structures, let us make some simplifications or idealizations.

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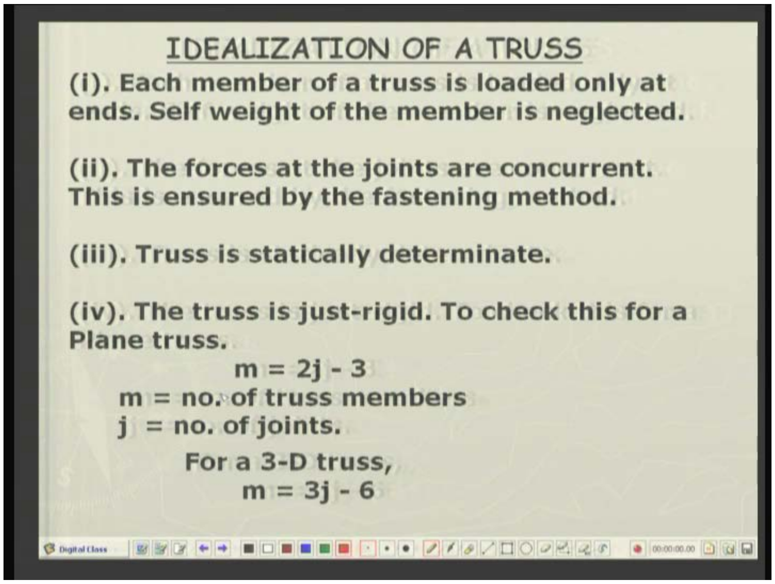

Each member of a truss is loaded at the two ends of the member. That is, we are neglecting the self-weight of the truss or any other transverse load which may be acting on the members. You may recall that when there are forces only at the ends of a member, let us say, two force member and from the equilibrium consideration, the force has to be collinear, passing through the two ends or two pins of the member. So in that sense, the forces in the members of the truss will be either tensile forces or compressive forces.

The forces at the joints are concurrent. That is, joints where various numbers come together are called joints. For example, going back to the picture, this is one joint, another joint, etcetera. So you can see that over here, one, two, three, four members are coming together. They are meeting at a point and although we are representing these members as lines, that is, they are represented by their axis. So all these four axis meet at a point and since the forces are along the axis of each of the member, it means that the forces on the truss members are concurrent forces. Truss is statically determinate. That is, we will consider only those trusses, which can be solved with the help of equations of equilibrium. No other concepts or no other equations are needed, although in more complex civil engineering structures sometimes we make use of statically indeterminate trusses but we will not be considering them.

The truss is just rigid. Well what is a rigid truss? There are two types of trusses: just rigid and over rigid. In the case of over rigid truss, if we remove one member, the truss does not collapse. It can still maintain its shape and take up loads but in the case of just rigid truss, if you move one member, the whole structure can collapse as soon as you apply the load. For example, again going back to the example of the truss which I have shown for the two dimensional case, suppose there is an additional member here. Well without this member also the truss was maintaining its integrity, that is, it doesn't collapse. So it means that by putting an additional member here or a member here, we are making the truss over rigid.

Now what are the conditions to ensure that our truss is just rigid for a two dimensional case? The following condition can be or should be checked. m is equal to two j minus three where m is the number of truss members, j is the number of joints. For a three dimensional truss, m is equal to three j minus six. Let us go back to our example. Here is a two dimensional or a plane truss. Now let us count the number of joints: One, two, three, four, five, six, seven. So seven joints. It means number of members should be two j minus three fourteen minus three eleven. So if you count them, one, two, three, four, five, six, seven, eight, nine, ten, eleven, it is just rigid. Similarly, you can make a simple calculation for a three dimensional truss. One thing more. When you look at the plane truss, the basic element of the truss is a triangle. Now if I start with this triangle, I will add two members here and one joint. So I get a truss of this type. I add a further member and one joint here. So in this way, by adding two members and correspondingly one joint, you can build up the whole truss. So those trusses which are built up from simple triangles in this manner are called simple trusses.

So now after having understood what the requirements for a just rigid simple truss are and restricting our self to the plane case, we will now look at how to analyze the forces in a truss?
So we start with the procedure for analysis of a truss under applied loads. Mind you, the loads are only to be applied through the joints. No in-between forces can be considered here.

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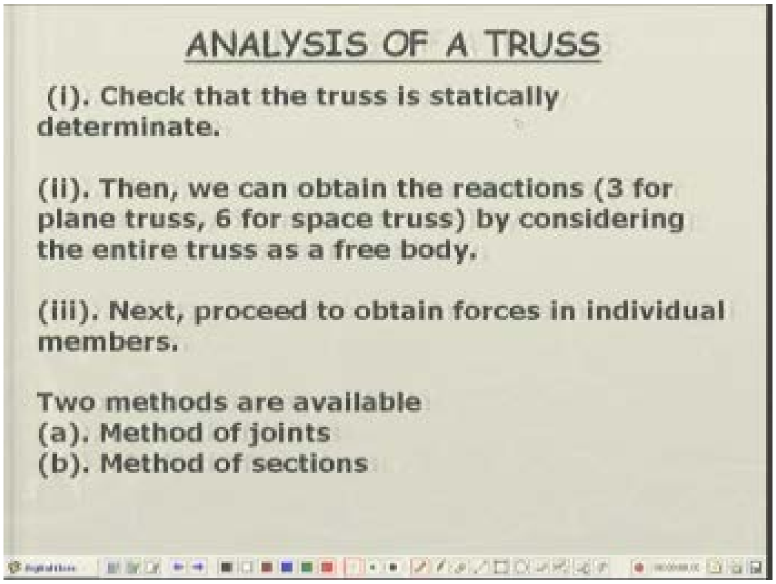

First of all, you check that the truss is statically determinate. That is, the number of unknown Forces given, as produced by the supporting conditions, should be equal to the number of equations available for the equilibrium. For a plane case, you remember, we have three conditions of equilibrium. It means, if there are three unknown reactions from the supporting structures foundations, etcetera or supporting pillar, then the truss is statically determinate. After having ensured that it is statically determinate, we go ahead and obtain the three reactions for the case of plane truss and six reactions for the case of space truss or three dimensional truss and after getting the reactions from the supports and conditions. Then we solve for the forces in each and every member or as required in the given number of members of the truss. There are two methods available for determining the forces in various members or elements of truss: number one, method of joints and the second is method of sections. So we will start with method of joints.

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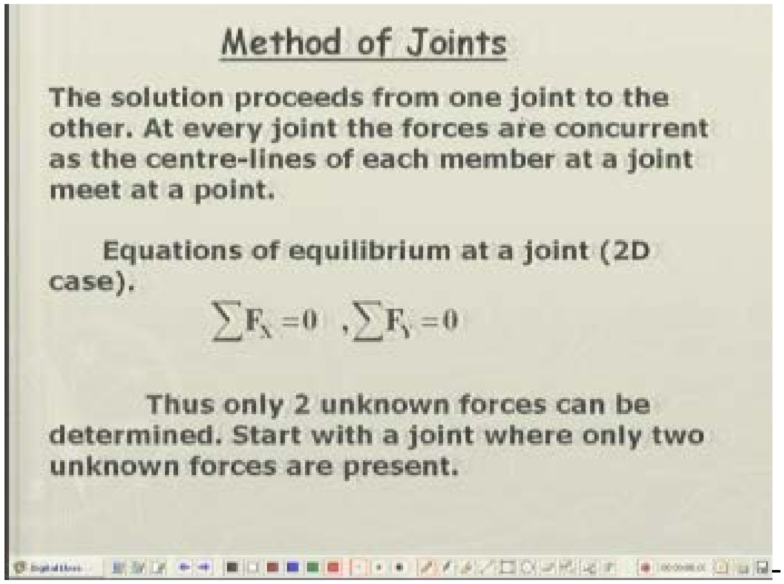

Well, in the method of joints, the solution proceeds from one joint to the other at every joint since the forces are concurrent forces. Therefore only two equations of equilibrium are available. So it means, I can solve for only two unknown forces. What are those two equations of equilibrium? Sigma Fx is equal to zero. That is, sum of all the horizontal forces acting at that joint equal to zero. Similarly, sum of all the vertical forces at that point should be equal to zero.

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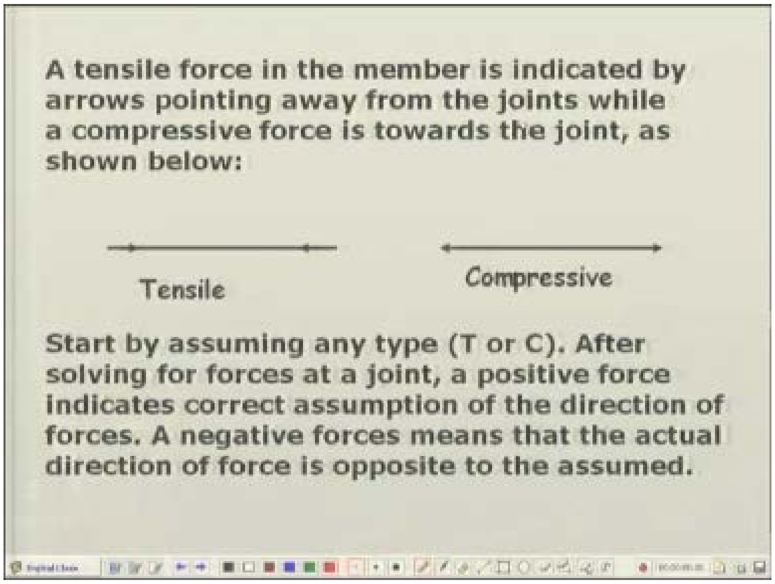

Now how to label these forces? How to identify whether these forces are tensile forces on the member or compressing forces on the member? Generally, we identify the forces as to how they are acting on the joints. For example, suppose there is a joint over here and a joint over here. If the force is such that it is pulling the joint like this or this joint in this direction, then by equal and opposite reaction it means, the force in the member will be like in this direction. That is, these are the forces as they act on the joints and in the case of compressive forces, it means they are pushing the joints like this. So the tensile force in the member is identified by the arrows invert along the member and similarly, the compressive forces are identified by arrows outward along the members. So please remember the arrows are according to the direction in which the forces are acting on the joints.

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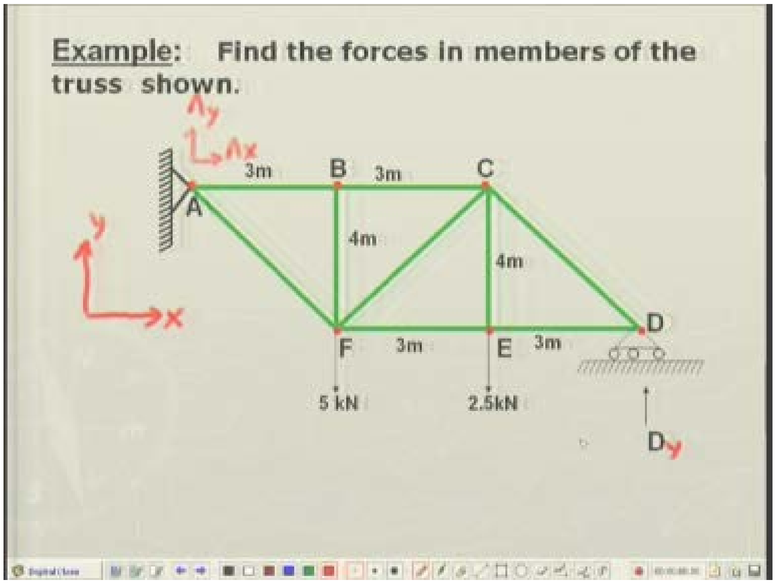

Let us see one example and then solve it with the help of method of joints. Here is a simple truss. You can easily see that it is just rigid by counting the number of joints and then the numbers of members that the condition m is equal to two j minus three will be satisfied and it is built up from simple triangle. So it is a simple, just rigid truss and also suppose if I had put an additional member along BE, then it will become an over rigid truss but fortunately that member is not there. We are having a just rigid truss. Now here are two supports on the truss A and D. D is a roller support. A is a simple or pin jointed support. So you can see that there are three unknown forces from the supports. Two reactions at point A, one in horizontal direction and one in the vertical direction, and one reaction at D, that is, only vertical reaction. It cannot sustain any horizontal reaction because of the rollers. It will start translating. So there are two at A and one at D. So three reactions and the conditions of equilibrium available in a plane for such a truss are also three. So the truss is soluble hence we can say that it is statically determinate.

Now, first of all, we will consider the entire truss as a free body. That is, we will isolate it from its supports by putting two reactions. In all the problems, first, you should identify your coordinate axis. Let us say, horizontal is our x axis and the vertical direction is our y axis. So at A, there will be two reactions, Ax in this direction and Ay in the vertical direction, and at D there will be a reaction, Dy. So these are the three reactions. So let us consider the truss as a free body. The three conditions of equilibrium.

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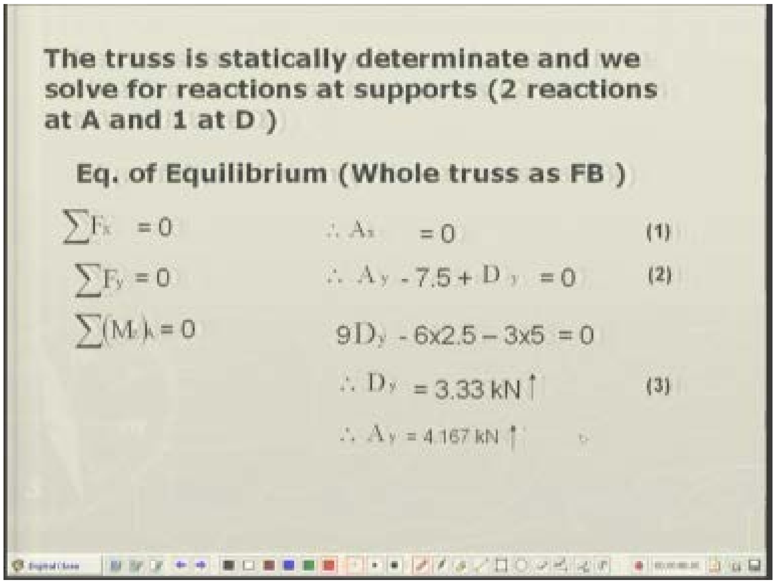

Sigma Fx is equal to zero sigma, Fy is equal to zero and sigma Mz about any point A. Now why we have chosen point A is because at point A, there are two unknown reactions. So both Ax and Ay will have zero moment about point A because they pass through point A. So our equations will be very easily solved. First of all, Ax is equal to zero. So in the horizontal direction, there is only one force Ax and on the other horizontal force. So immediately we know that Ax is equal to zero in the vertical direction. Though, what are the forces? There is a five kilo Newton and two point five kilo Newton, downward both of them, and there are two vertically upward forces Ay and Dy. So you can easily see that here is Ay and plus Dy minus seven point five, two downward forces equal to zero and then you take the moments about point A. So this whole length is three plus three six nine. So this will be Dy into nine, that is, in the anticlockwise. So it is positive and this will be two point five into momentum is six. That will be clockwise. So minus six into two point five minus three into five. So you can say, nine Dy minus six into two point five minus three into five equal to zero. So these three equations immediately give us the Da. Dy, the vertical reaction at D, is equal to three point three three kilo Newton's and Ay is equal to four point one six seven kilo Newton and Ax is zero. So we have solved the three unknown reactions. Next, we start with joints. We will start with a joint, where there are only two unknown forces because only two equations are available. So if I look at the truss once again, joint A has two members. So it means, there are two unknown forces. Now, start with joint A. Draw a free body diagram of all the forces at A and then for the time being, you assume the forces in member AB or AF, either away from the joint or towards the joint. We cannot presuppose that it has the tension or compression. We arbitrarily make a choice.

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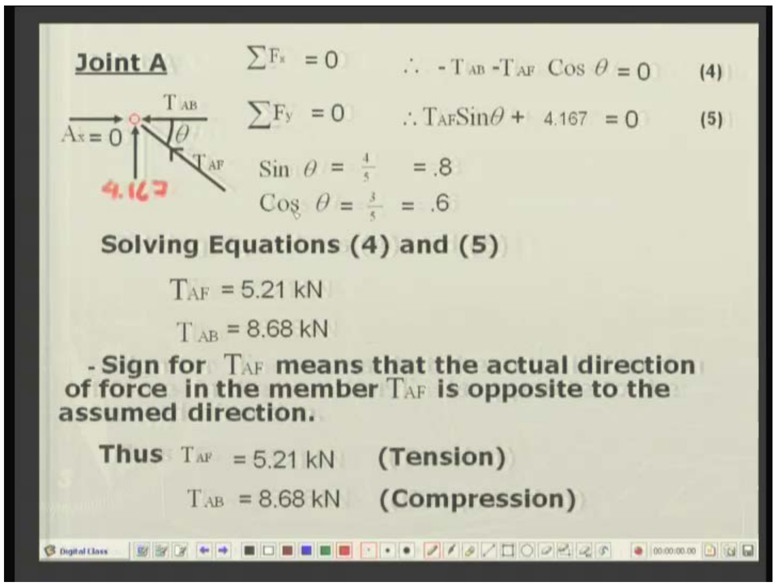

So what we have done here is, I have taken the force along AF towards the joint and force along AB also towards the joint and Ax is equal to zero and this is Ay over here. So we have found out Ay has four point one six seven kilo Newton. So sigma Fx is equal to zero sigma, Fy is equal to zero. Sigma Fx is equal to zero means minus because positive x direction is towards the right. This force is towards left. So minus TAB minus the component of TAF along horizontal. So minus TAF into cosine theta is equal to zero. Similarly, we can write for Fy that there is a vertically upward force of four point one six seven and this will have a vertically upward component of TAF sin theta.

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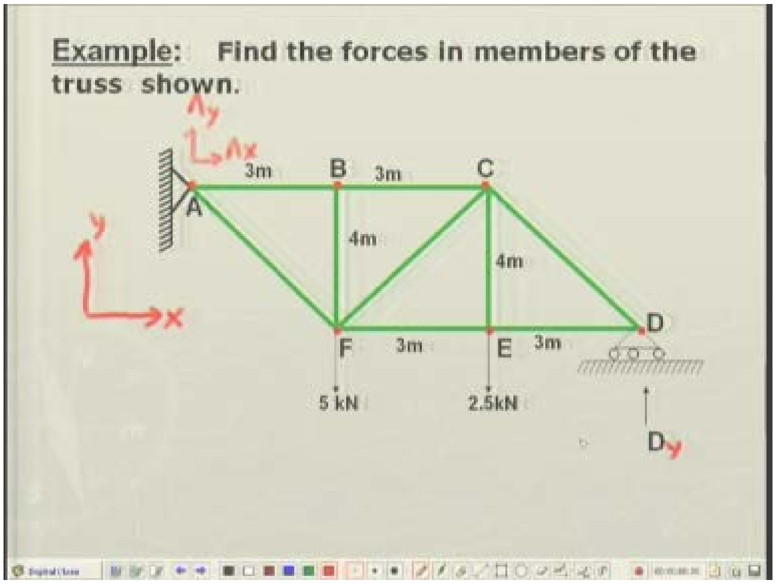

Now from the geometry of the truss, you can find out what angle theta is. Over here, this is angle theta. So this is three meters, four meters sin theta and cosine. You can easily see that by Pythagoras theorem, the side AF will be five meters. So sin theta will be equal to four by five, cosine theta will be three by five. So four by five sin theta is equal to point eight, cosine theta is equal to point six. Then substitute sin theta and cosine theta and solve from equation four and five for TAF and this will be in negative sign here, TAF and TAB. Now, this negative sign in the evaluation of TAF means that the actual direction of force in the member TAF is opposite to the assumed direction. If we had assumed the TAF to be towards the joint, that is, it is pushing the joint compressive force but because of the solution which shows that it is negative five point one, it means actually that the force is away from the joint. So it is a tensile force. So TAF is five point two one kilo Newton tension, TAB is equal to eight point six eight compression.

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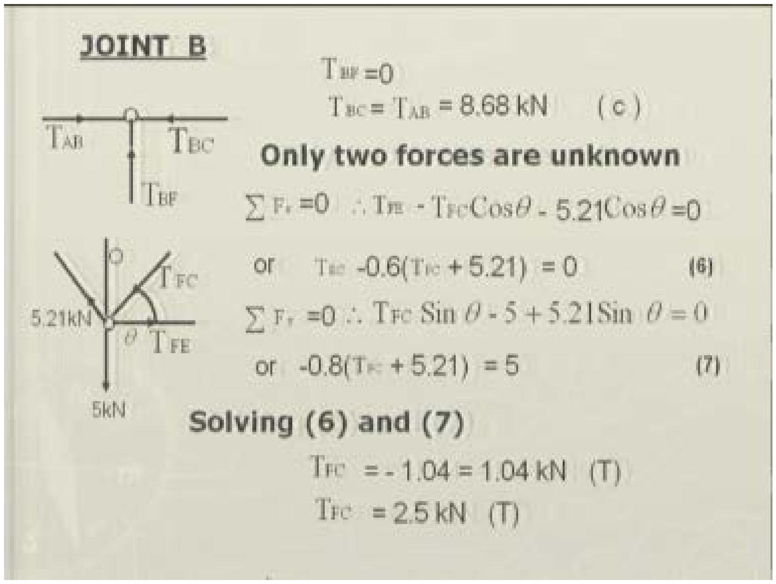

Now we should proceed to the next joint B. In joint B, there are three members. This joint is very simple to actually solve, for there is no vertical force. So it means TBF has to be equal to zero. We will consider, sum of all the vertical forces is equal to zero, no applied vertical force here. So it means the number BF does not require any force. So TBF and in the horizontal direction TAB is already known and it means TBC will be equal and opposite to AB, for force in TBC and BC will be equal to that in AB. So it will be eight point six eight kilo Newton. So again it will be a compressive force. TBC towards the joint means, it is a compressive force. Now next we will take the joint F. So this is how we proceed it. A to B, B to F. Now, in this case, it appears that there are five forces: one, two, three, four, five. This force we have already found to be zero. This is a given force. Force TAF has been already found. So the unknown forces are only force in the FC member and force in the FE member. So again there are two unknowns and hence two equations of equilibrium will be sufficient. This is joint F. So sum of horizontal forces. By now you are very well versed with this and sum of the vertical forces is equal to zero. Just write down the equation, substitute of sin thetas and cosine theta. You will find two equations and solving for two unknowns, you will find that TFC is minus one point zero four. Now minus implies that our choice of arrow was wrong. The arrow should have been opposite. So it means, we have chosen it as a compressive force but actually it is a tensile force. TFC is positive. It means our choice of arrow was correct, so it is a tensile force. So from joint F we proceed to joint E. Joint E is very easy to evaluate. Now once again, if we go back to joint E, there are two members: one member over here and this member TFC has been already found out. So only two unknowns. Out of these two unknowns, this force will be exactly equal to the force in this CE member and this force should be exactly equal to the force in the FE member.

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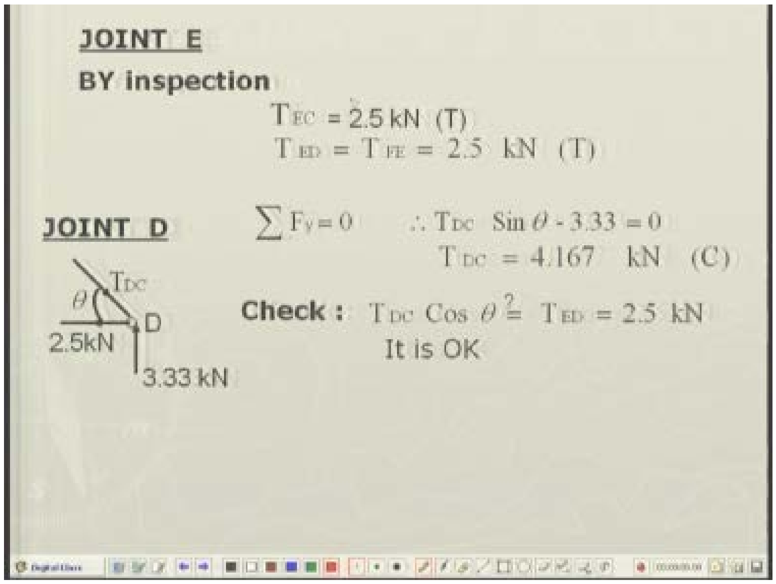

So at joint E, simply by inspection, we can write down TEC is two point five kilo Newton. Tension TED is TF equal to TFE two point five kilo Newton's in tension and finally we come to the joint B which is the supporting joint. Well, just one unknown force is needed to be calculated, TDC, that is, we will have one equation, sigma of Fy. Sum of the vertical forces is equal to zero which will give us TDC is equal to a four point one six seven kilo Newton's. If I go back to this joint, we have already found out, Dy. Now, as a check, do we get this condition, that the component of the force in TDC is equal to the applied reaction and the calculated reaction Dy? The check is okay. It means our whole analysis was okay. No error was committed. So let us see if TDC cosine theta, that is, the vertical component of TDC, which we have found out to be one four point one six seven. This is the horizontal component. Sorry. This is equal to the TD which we have already found out here and it is two point five kilo Newton's. So it means our analysis was perfectly okay. So this was the method of joints.

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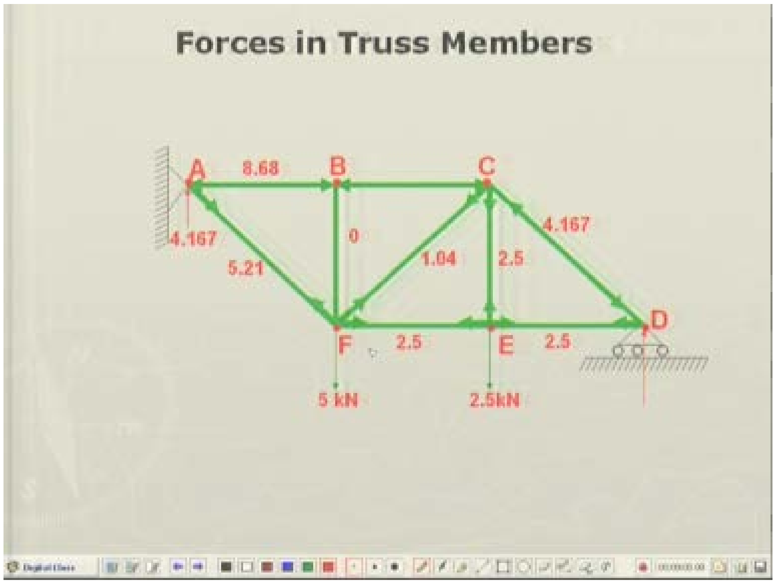

Next, before I close up this analysis I have drawn the truss, once again, with all the force members. This is the complete solution of the truss problem and here are the reactions and you can see that these are compressive tension, etcetera. So you can immediately recognize which member is in tension, which member is in compression. So this completes the solution by method of joints for this simple truss.

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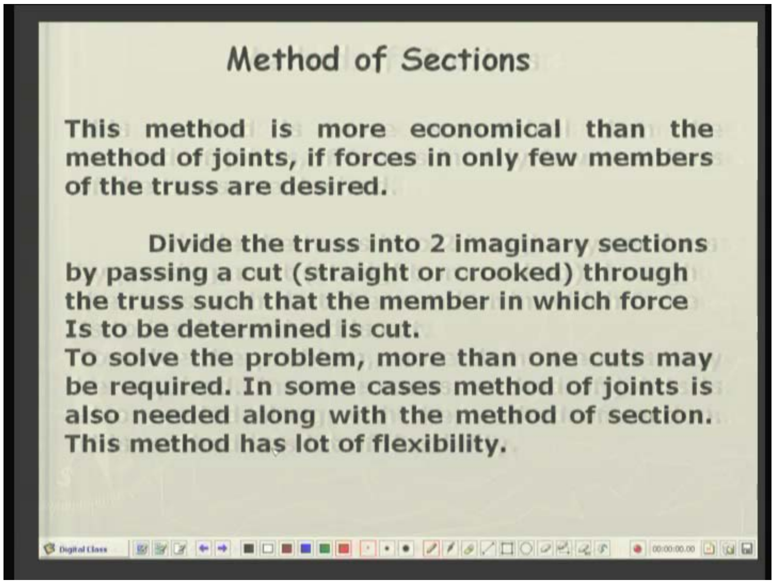

Now the second method which we can use very effectively is the method of section and mind you, this method is also very economical, when you want to solve for forces only in few of the members and not in all the members. If it is to be done for the all the members then perhaps method of joints is easier but when you want to calculate or determine the forces in only few of the members, not in all the members, then you can do it by method of section. Now how does this solution proceed? In this solution, we will divide the whole truss into two sections. That is, we will think or imagine that the truss has been cut into by a section. This cut may be a plain cut or very crooked cut. So depending upon your ingenuity and the type of members to be solved for, you can devise a suitable cut and once the truss is cut into two parts, either you can use the left part for analysis or the right part and then again by making use of the equations of equilibrium, very judiciously, you can solve for the unknown member. So this type of solution is very economical, very flexible and sometimes, in some of the problems, you use method of sections plus method of joints. That is, together and this will make your solution very easy and simple. So flexibility is a very important characteristic of method of sections. Let me take up a problem to illustrate this method.

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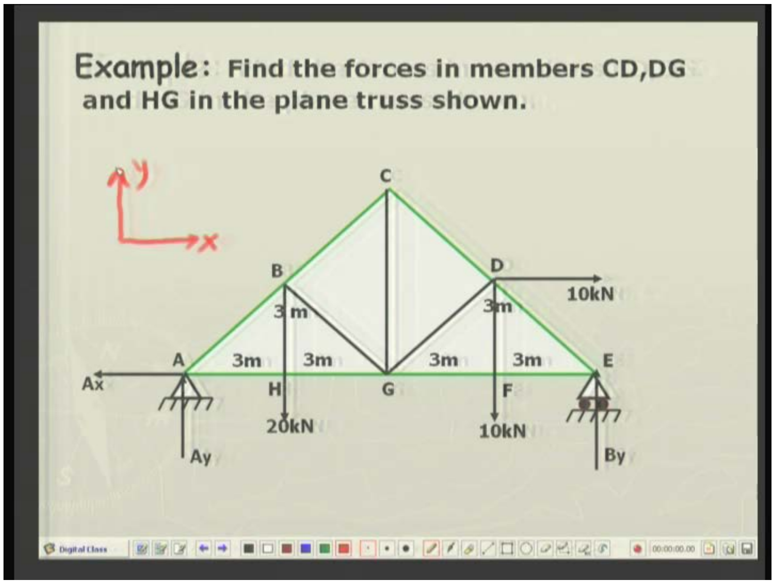

Here is a truss. Again I leave it to you to check that this plane truss is just rigid and obviously this is a simple truss. These are various joints and various forces over here. The supports are a simple support on this side and the roller support on this and the loads are a twenty kilo Newton at joint h ten kilo Newton joint F, both are vertical whereas there is a horizontal load. Also, sometimes wind loads on roof trusses. So this is of the magnitude ten kilo Newton at point D.

So again the starting point is the truss as the whole truss is a free body. Well, first you start with identifying your x axis and y axis. Then you say that the reactions at the simple support, both horizontal and vertical, are possible. So Ay and Ax. You might be wondering why I have taken Ax towards left whereas normally I have been taking the horizontal reaction towards right to make it positive. You can easily see that the load in the horizontal direction is ten kilo Newton towards right. So without doing much effort, I can say that the horizontal reaction at A will be towards left and not only that its magnitude will be equal to ten kilo Newton and at E there is only vertical reaction.

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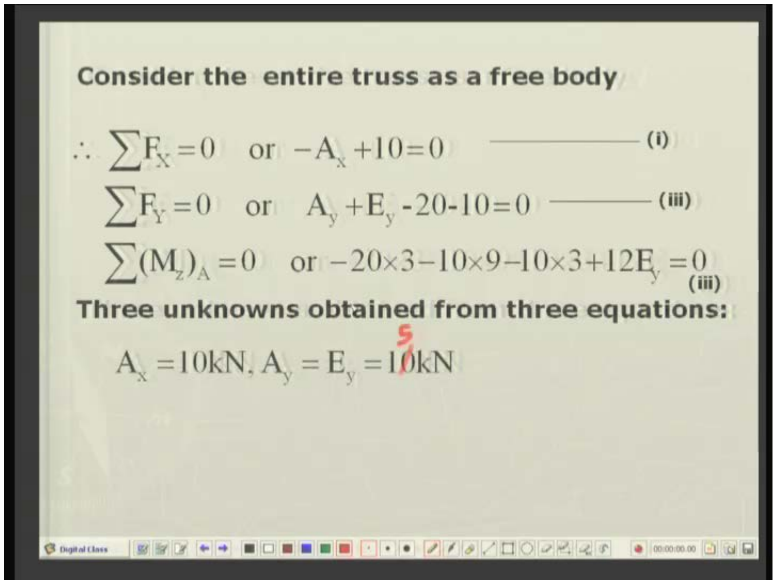

So these three equations, one sigma Fx is equal to zero, sigma Fy is equal to zero, sum of the vertical forces. Again vertical reaction at A, vertical reaction at E and two downward forces and then moment about A is more convenient because there are two reactions passing through it. So very easily you know you can say that the horizontal reaction at X is ten kilo Newton, vertical reaction at A and E both are equal to ten kilo Newton. So Ax is ten kilo Newton, Ay and Ey are ten kilo Newton. Now let us see, we were asked to find out the forces in members CD DG and HG. Not in every member but only selected few, three of them. CD member, where does it lie here? CD and DG here. So looking at these two members, why not take a section over here? So this will cut CD as well as DG. So let us see.

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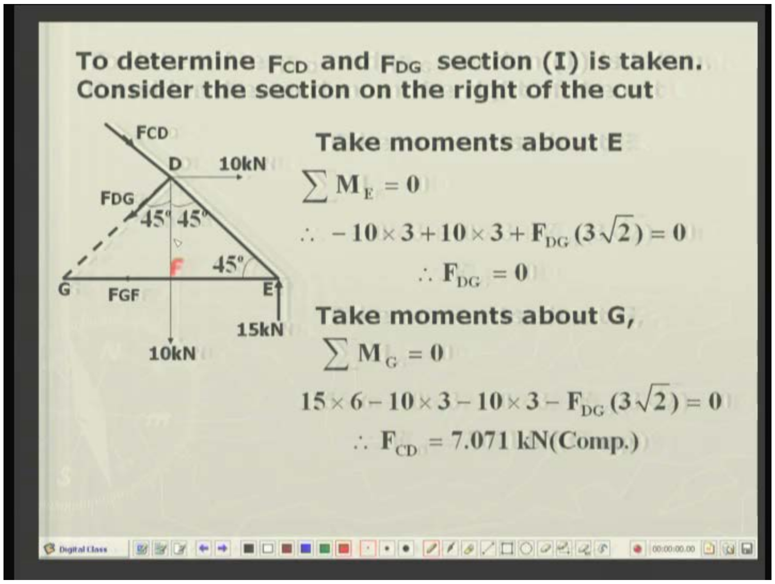

I will take a section over here and see that once the cut has been taken I will consider the truss on the right side of the cut. So this is the cut truss on the right. Just a minute. There is an error. Here this should be fifteen kilo Newton's. Ay plus Ey is thirty and both will come out from here. Equal. So it is fifteen kilo Newton's. So this is fifteen kilo Newton vertical reaction. So looking here, these are the given forces, ten kilo Newton's horizontal force and at this point we have point F. So this is point F. Okay. Now, first of all, we will solve for DG. Why? I will tell you. So the forces FC FCD and the force FGF which are produced due to this cut. This is force FGF. They both pass through point E. Okay. So if I take moments about point E, the moment of these two forces and this fifteen kilo Newton force will have zero moment. So only the moment which will be produced by FDG and a known ten kilo Newton force over here. So it means I can easily find out the unknown force FDG in just one step. That is, by taking moments about point E which should be equal to zero. Remember in consideration of equilibrium, the choice of point O about which we take the moment is arbitrary. We can always select a suitable point about which we want to take the moment. So whichever point can give me the solution most conveniently, I will chose that. So I am choosing the point E to take moments because all the unknown forces except one will have zero moment over here. So by taking this moments, there is a ten kilo Newton and this will produce a clockwise moment about E. So clockwise are negative ten into three. So minus ten into three plus. This will produce an anticlockwise ten into three plus FDG.

Now this is angle forty-five. So this total angle will be ninety degrees. It means DG is perpendicular to this. So you can calculate this distance because this is three meters, three meters, six meters. So six into cosine of forty-five degrees. So you will get FDG is equal to zero. Then I will take the moment about point G. What will happen?

This force FGF will have zero moment. FDG will have zero moment. Only FCD will have a non-zero moment and other two forces or rather other three forces are known forces. So it means in one step, in one go, I can find out FCD. This is exactly what we have done. That is, we have taken moment about point G and so it means in just two steps, that is, taking moments about just two points E and G, I have solved for two of the three unknown member. So it is very economical. Next we want to determine the force in the HG member.

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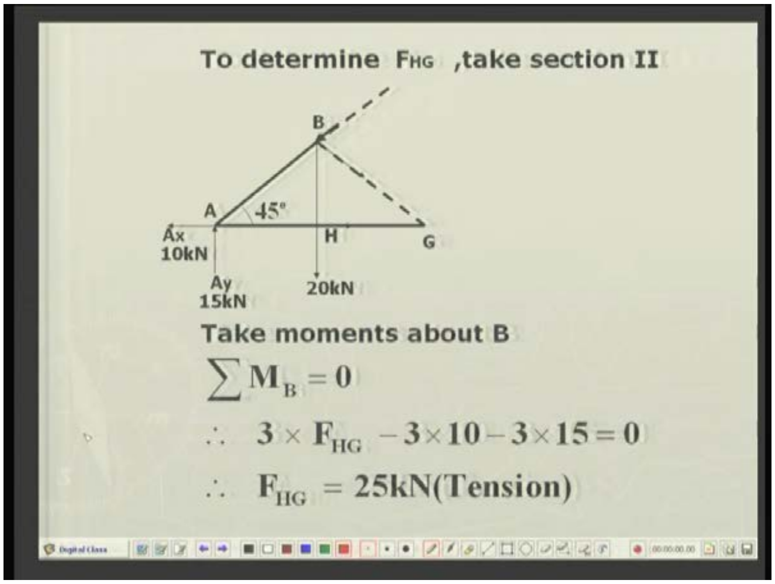

Here, we will proceed from the left hand end and take a second free body. That is, somewhere over here. HG.

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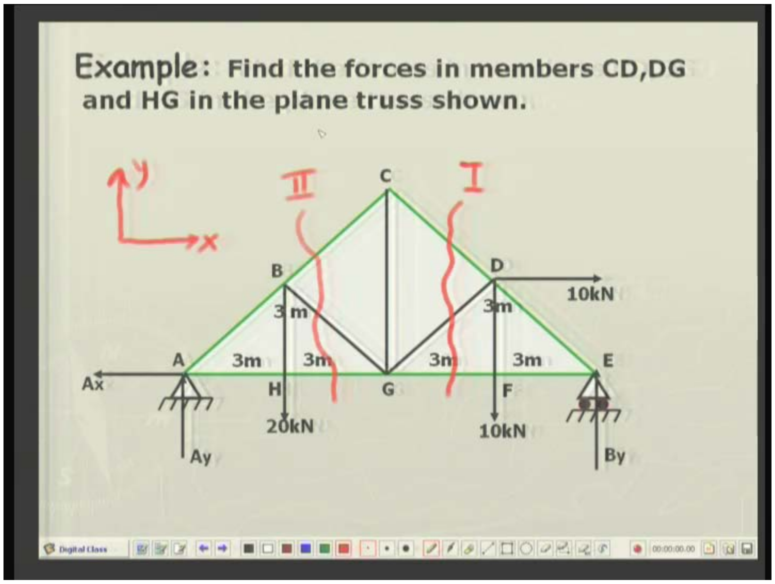

So I will be taking a free body like this. This is free body number one. Cut number one. This is cut number two. So if I do this cut, these are the members which are cut. Again, you can take moments about point B because at point B the two of the unknown members are passing through. They will have zero moment. Only the member in which we are interested in finding out the force will be there. Hence we can find out FHG is equal to twenty-five kilo Newton tensile load. So in this way, all the three forces in three members have been determined in just three steps by taking two cuts.

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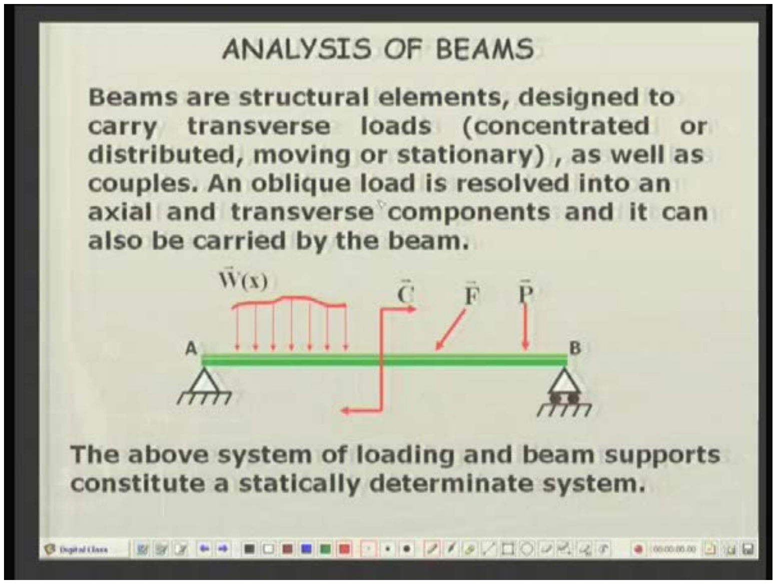

We have considered so far the analysis of trusses and one special feature of the trusses was that all the elements or members of a truss were supposed to take up only either a tensile load or a compressive load. That is, the forces are along the length of member or rather said, it more accurately about the line joining the two end joints of a member. So there are other types of structural members called beams, which can take up transverse loads. That is, the loads which are either normal or at least oblique to the member itself. So they are called beams. So let us start with the analysis of beams. Beams are structural elements designed to carry transverse loads. In contrast to truss members, which were taking up only axial loads, these loads can be concentrated or distributed. They can be moving loads or stationary loads, that is dead loads or loads due to some traffic trains or vehicles, etcetera or wind. Not only loads, that is, direct forces but moments can also be supported by beams. Whereas there was no such provision for the truss members. They were having only axial forces. Now if there is an oblique load, as I have shown over here, an oblique load can be resolved into two components: a horizontal component and a vertical component or a transverse component, axial component and a transverse component where the transverse component will produce tension or compression. Whereas the axial component will produce a tension or compression, transverse component will produce bending. As we will see later on, in somebody you can say beams are really one dimensional members. That is, one dimensional length is much bigger than the other two dimensions.

So, we will call them, for simplification, as one dimensional members subjected to either distributed loads or a concentrated load transverse or oblique or some couple having a moment which is acting on the beam. So the role of beams is quite different from that of trusses although both are structural member. Now in the analysis of beams again, we will be restricting ourselves to statically determinate system. That is, the number of reactions from these end supports should be just equal to the number of unknowns involved in the number of equation of equilibrium. So for example, here is a pin joint or a simple support which will have two reactions, horizontal reaction and a vertical reaction and the second is a roller support, which will have only one reaction, only vertical reactions. So three reactions and three equations of equilibrium in a plane. So this structure is a statically determinate structure. Now, in the analysis of beams we will introduce a very important concept namely the concept of shear force and bending moment.

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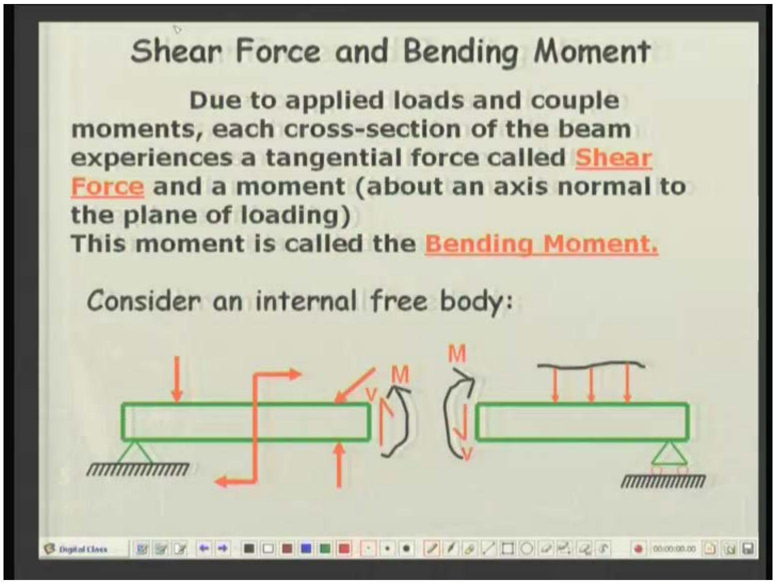

Now, since the forces are applied forces, are transverse, we have to see how they have to be withstood, that is, they will produce two types of reactions from the beam. One will be the shear force, a tangential force, and the other will be a moment called the bending moment. Now to illustrate this, let me consider, an internal free body. That is, this whole beam was arbitrarily cut. You imagine that with the help of a knife or a saw, we have cut the beam into two parts: one part on the left side and the other part on the right side. Since the beam is in equilibrium, each of the cut member or each of the section will also be independently or individually in equilibrium. So to maintain this equilibrium I consider the left hand section. These are the applied one force, second force and perhaps another force over here and this is the moment.

Now, since the cut is passing through this, as we have already done, there will be produced a tangential force acting through the section and the moment on that section. This is a moment and equal and opposite moment on the right hand section and a tangential force. This was on the left hand section. If it is vertically up on the right hand section, it will be vertically down. So by Newton's third law, actions and reaction being equal and opposite, this left hand section will be in equilibrium,due to the applied loads the shear force v and a moment M and secondly this right hand section is in equilibrium due to the reaction at the support applied load v and M. Now, this force v or tangential force v is called shear force and the moment M is called the bending moment.

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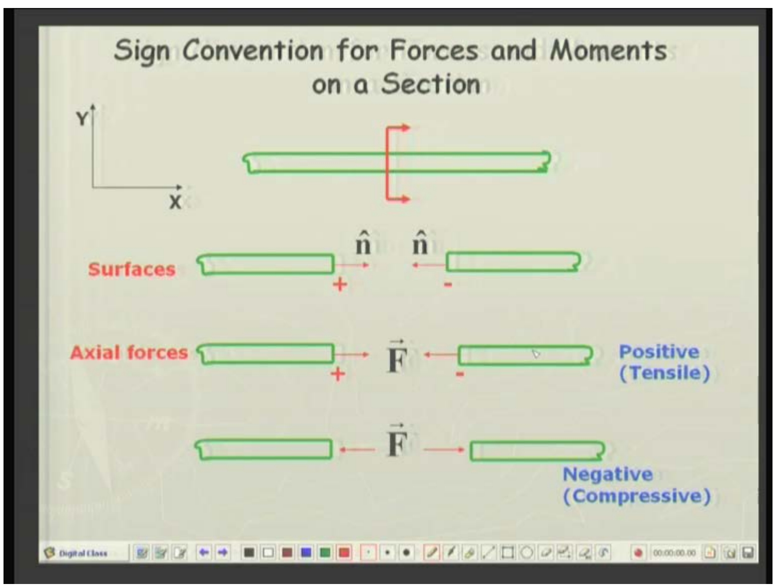

Now, a very important concept in the analysis of beams is the sign convention and I must advise you that this has to be carefully done. Otherwise errors are likely to happen. Suppose, this is my rectangular Cartesian coordinates x and y and of course z is out of the plane of the screen. Now, suppose I taken an arbitrary section over here. It means I have cut the beam into two parts and two surfaces are produced. I will consider the outer normal on each of the surfaces. These are unit vectors. Outer normal means pointing away from the body of the section. That is, this is the section. So outer will be like this. In this case, the outer normal will be towards the left. Here it is towards right. Now, if this outer normal is pointing in the positive x direction, then this surfaces is positive x. If this is pointing in the negative x, that is, towards left, then this is a negative surface.

So a single cut produces one positive surface, one negative surface and suppose there is a force over here, force F. If the force on a positive surface is in the positive x direction as shown over here, then the force is positive. If the force on the negative surface is also in the negative direction, that is, negative x direction, then also such a force is a positive force. But if on a positive surface the force is in the negative x direction or on a negative surface the force in positive x direction, then the force is negative. So this force is negative, this force is positive. So remember two positives or two negatives make a positive force, one positive one negative will make a negative force. Similarly, considering the vertical forces, the previous was for the axial forces.

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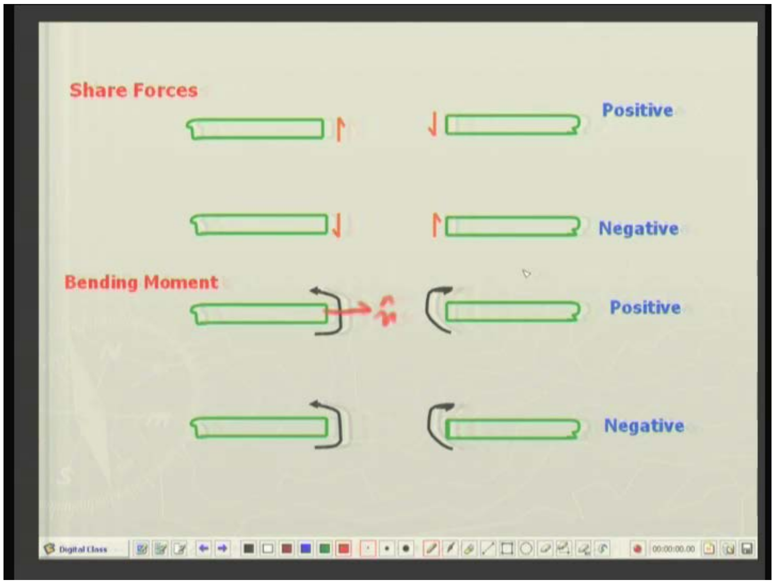

Now vertical forces on a positive surface, a positive upward force in the positive y direction or on a negative surface downward force, that will constitute a negative shear force. Similarly, on a positive surface negative downward force or on a negative surface positive upward force, that will constitute a negative shear force and same combination of signs will give me the positive bending moment or a negative bending moment. Again here is the unit normal in the positive x. This is the unit normal. So on a positive surface, an anticlockwise moment, that is, the positive moment that will give me a positive bending moment on the negative surface. A clockwise, that is, negative moment will give me a negative bending moment. So two negatives or two positives will give you a positive bending moment and similarly over here.

So this is a very systematic way of identifying positive shear force, negative shear force, positive bending moment and negative bending moment, positive axial force, negative axial force. So we will continue with the analysis of beams and I will certainly advice that you remember this simple but systematic way of identifying shear forces or bending moments, which are positive which are negative because in literature, different authors used different types of conventions but this is the most easily understood convention. So in the next lecture, we will consider how to analyze various types of beams and the loadings and draw the bending moment and shear force diagram. Thank you very much.