

Applied Mechanics
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Lecture No. 04
Analysis of Equilibrium

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EQUILIBRIUM

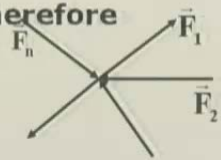
The state of rest (in an appropriate inertial frame) of a system particles and/or rigid bodies is called Equilibrium.

Conditions Of Equilibrium:

1). Equilibrium of a particle:
All forces are concurrent. Therefore

$$\vec{F}_R = \sum_1^n \vec{F}_i = 0$$

In component form

$$\sum_1^n (F_x)_i = 0; \sum_1^n (F_y)_i = 0; \sum_1^n (F_z)_i = 0$$


The diagram illustrates a particle at the center of four concurrent force vectors. Vector \vec{F}_1 points horizontally to the right, \vec{F}_2 points vertically downwards, \vec{F}_3 points diagonally down and to the left, and \vec{F}_n points diagonally up and to the left. All vectors originate from the same central point.

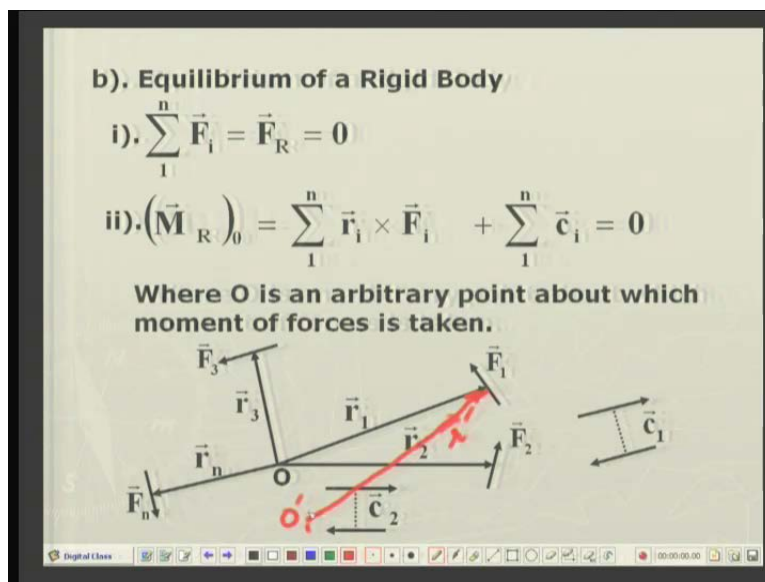
Let us start with today's lecture. The title of this lecture is Analysis of Equilibrium. The state of the Equilibrium is defined as the state of rest in an appropriate inertial frame of a system of particles and/or rigid bodies. You may recall that inertial frames are a set of observers or reference frames, which are at rest with respect to each other or at most they are in uniform motion along a straight line as seen from each other. When a system consisting of particles, or particles and rigid bodies, or only rigid bodies is at rest in a suitable initial frame, then we call that, the system is in equilibrium. What are the conditions which ensure this state of equilibrium? This can be followed in a very systematic manner. First of all we will consider equilibrium of a single particle. By definition, a particle is a zero dimensional body i.e. it has no length, breadth, thickness, diameter, etc. So geometrically it is a single point. When this single point or particle is subjected to several forces, naturally all the forces must meet at the particle. They are

concurrent forces. We can say that the conditions of equilibrium of a particle are same as the conditions of equilibrium of concurrent forces.

Suppose F_1, F_2, F_3 up to F_n are force vectors which are passing through the same point as shown in figure, then the resultant force F_R is the vector sum of all these individual forces. This resultant force should be equal to zero to maintain the state of equilibrium.

In the component form, the same condition can be written as the x components of all the forces when added together equals to zero, the y components of all the forces when added together equals to zero, the z components of all the forces when added together equals to zero.

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Here the forces are no longer concurrent forces and they are passing through different points of the body. Then the conditions of equilibrium are that The sum of all these individual forces is equal to the resultant force F_R which is also equal to zero. The moments of all these applied forces taken about any arbitrary point O, i.e. the resultant moment is also equal to zero.

Consider different forces acting on the body. Choose an arbitrary point O in space. The position vectors of the arbitrary point O on the force F_1, F_2, F_3 up to F_n are vectors r_1, r_2, r_3 up to r_n . Hence the moment of the force F_1 about O is r_1 crossed with F_1 . Similarly the moment of the force F_2 about O is r_2 crossed with F_2 , etc. up to the moment of the force F_n about O is r_n crossed with F_n . These are the first set of terms or the moments of forces about point O . The second set of terms is the moments of the couples. Couples are free vectors, so the rigid body may also be subjected to some individual couple. The moments of these couples when added together are given by the second set of terms i.e. the summation of c_i . The resultant moment consisting of moment of forces and moment of couples should be equal to zero. Hence for the equilibrium of a rigid body these two conditions must be simultaneously satisfied. These vector conditions can also be easily written in the component form. For example, the sum of all the x components of the resultant forces is equal to zero, the sum of all y components of the resultant forces is equal to zero, etc. and similarly for the resultant moment, the x component to resultant moment is individually zero, the y component to resultant moment is individually zero, etc. For example, choose another point O' . Now join O' with force F_1 . So this will be vector r' and let the vector joining O and O' be vector ϕ .

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For any other choice of O , say O' the above result is still valid. Let vector joining OO' be $\vec{\varphi}$. Then

$$\vec{r}_i = \vec{r}'_i + \vec{\varphi}$$
$$\therefore \vec{r}_i \times \vec{F}_i = (\vec{r}'_i + \vec{\varphi}) \times \vec{F}_i = \vec{r}'_i \times \vec{F}_i + \vec{\varphi} \times \vec{F}_i$$

Hence

$$\sum_1^n \vec{r}_i \times \vec{F}_i = \sum_1^n \vec{r}'_i \times \vec{F}_i + \sum_1^n \vec{\varphi} \times \vec{F}_i$$
$$= \sum_1^n \vec{r}'_i \times \vec{F}_i + \vec{\varphi} \times \sum_1^n \vec{F}_i$$

But $\sum_1^n \vec{F}_i = 0$

$$\therefore \sum_1^n \vec{r}'_i \times \vec{F}_i = \sum_1^n \vec{r}_i \times \vec{F}_i = 0$$

Note that the position vector, r_i , is equal to r_i' plus φ' i.e. the original position vector is equal to the new position vector plus the position vector of O' with respect to O . Taking the moment all these forces, r_i crossed with F_i is equal to $(r_i'$ plus $\varphi)$ crossed with F_i and this simplifies into $(r_i'$ crossed with $F_i)$ plus $(\varphi$ crossed with $F_i)$. Now taking sum on all these forces one can easily see that moment of all these forces is equal to the new moment plus, φ crossed with the sum of all the forces F i.e. summation F_i .

This is equal to the resultant force and according to the first condition of equilibrium i.e. the resultant forces is equal to zero, it means that the second term on the right hand side vanishes and leads us to an interesting result that, the sum of moments of all the forces about the new point O' is equal to the sum of moments of all these forces about the old point O and this is equal to zero. The conclusion is that, it is immaterial whether you choose O or O' or any other point in space.

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Equilibrium of a System of Rigid Bodies
Rigid bodies in contact/interconnected.

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FREE BODIES:
Isolate individual rigid bodies or subsystems.

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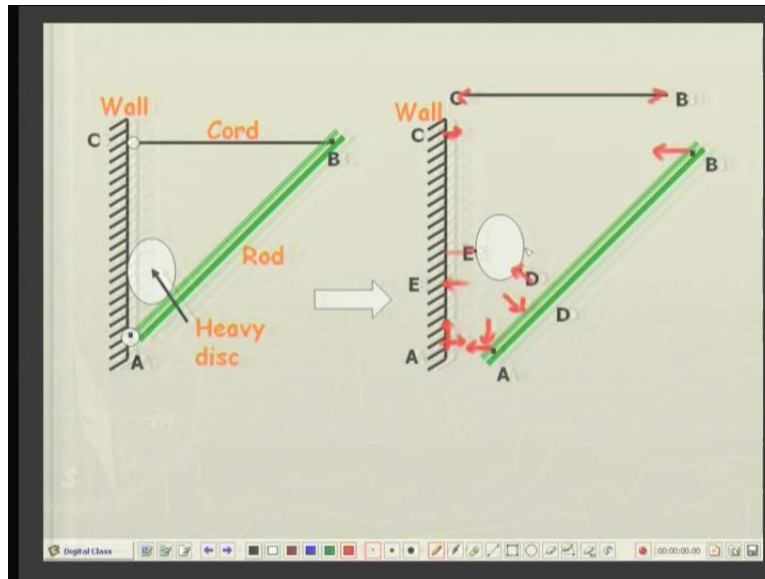
Replace contacts by suitable reactions on both sides.

↓

Consider the equilibrium of each free body.
i.e. $\vec{F}_R = 0, (\vec{M}_R)_0 = 0$

It is a system consisting of several bodies which are in touch with or in contact with each other or they are interconnected through linkages with each other, which leads us to a very interesting concept of free bodies. Let us consider a system of rigid bodies. Now isolate each individual rigid body from the other bodies or its subsystem of rigid bodies from the surrounding bodies and wherever the bodies were in contact with each other, at that point or on those surfaces, introduce some forces and moments depending upon the type of contact or interconnection. On the other side of the contact also we will have equal and opposite forces and moments. The individual bodies or subsystems of bodies are replaced by suitable reactions on both sides to identify the free bodies. Now taking each free body as a separate entity, consider the equilibrium and the conditions of equilibrium i.e. 1) resultant of all the forces is zero, 2) resultant of all the moments is equal to zero.

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Consider an example to illustrate the above mentioned point. Suppose the system of rigid bodies consists of a rod which is held by a pin towards a wall. The rod is also connected to the wall with the help of a chord shown in the above figure. Also between the wall and the rod, there is a heavy spherical or oval disk, which is simultaneously in contact with the wall as well as with the rod. Now break the system into four individual bodies. One is the wall as shown in the above figure; the next is the chord separate, then the rod separate and finally the disk separate.

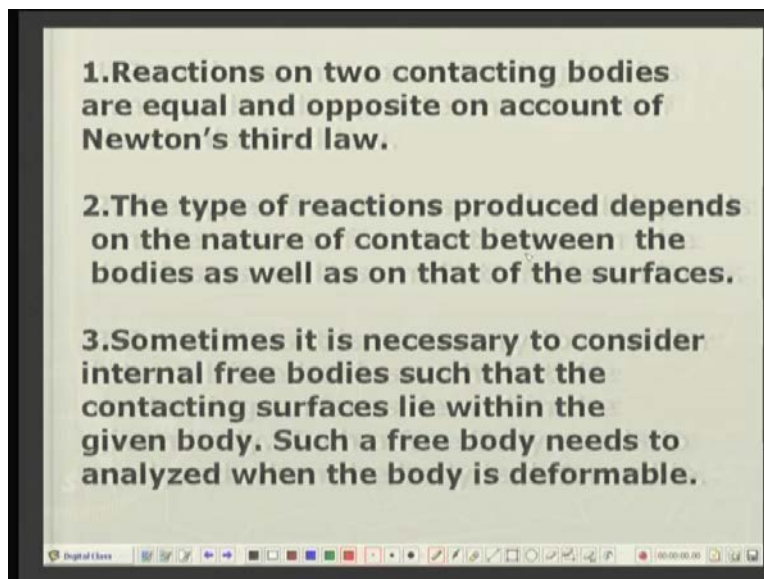
Now mark the points of contact based on the figure. Let the contact point be E and D. As a result there is a point E on the disk separate and corresponding point E on the wall, also point D on the disk separate and a corresponding point D on the wall. Similarly for the chord point C, there exists a point C on the wall and a corresponding point C on the chord, also for the chord point B, there exists a point B on the rod separate and a corresponding point C on the chord.

Now at all these contacts points, whichever marked, there will be forces and moments. It will be noticed that for different types of contacts the reactions will be different. For example, the chord will be under tension at point C on the chord whereas it will be an

axial force at point B on the chord. Hence there will be equal and opposite force on the wall as well as on the rod. Similarly there exists a pin joint at A. At this point there is a horizontal force and a vertical force but on the rod there will be equal and opposite forces. The force on the wall is towards the right direction and the corresponding force on the rod is towards the left direction. Also the force on the wall is upward and the corresponding one on the rod is downward.

Similarly between the wall and the disk, the contact can either be smooth or rough. Assume that the contact is very smooth and hence there is no friction. As a result, on the wall there will be a compressive force. Also on the disk there will be a compressive force and normal to it. Similarly on the contact point D, there is a force on the point D on the rod, and assuming the contact surface to be smooth, then there will be a force equal and opposite on the point D on the disk. Thus the bodies are isolated and the contacts are replaced with suitable forces and moments.

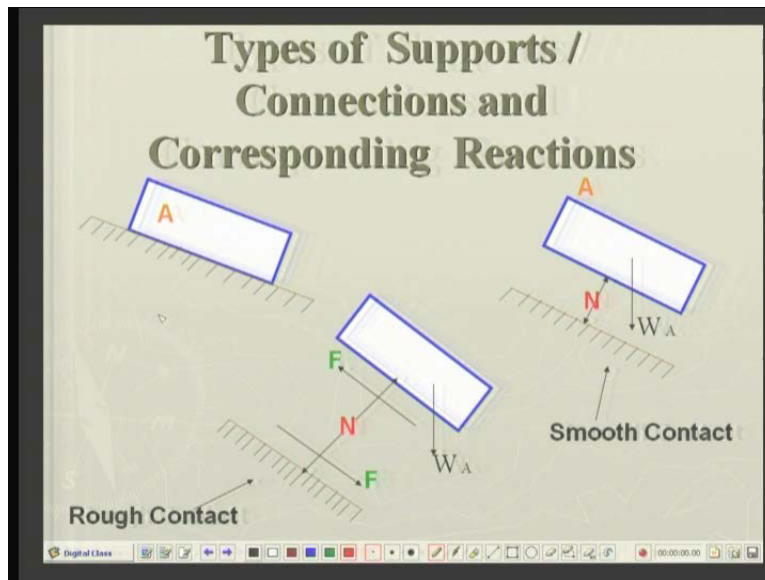
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Summarize about the free bodies, reactions on two contacting bodies are equal and opposite on account of Newton's third law. The type of reactions produced depends on the nature of contact between the bodies as well as on that of the surfaces. I.e. the contact

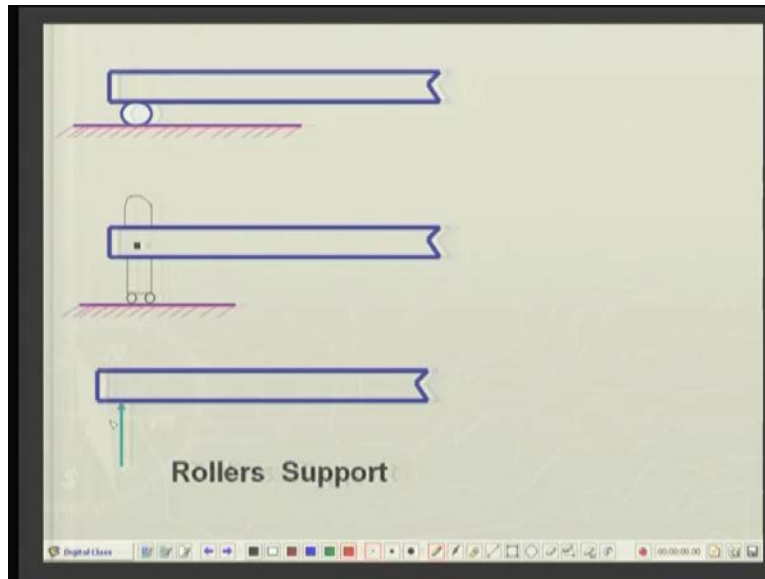
surfaces are smooth and friction less or they are rough and there exists a friction. Sometimes it is necessary to consider internal free bodies such that the contacting surfaces lie within the given body. This issue will be dealt with later as this is not immediately relevant for rigid body mechanics.

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Various types of supports and connections and the corresponding reactions. Consider a block which is lying on an inclined surface. Also consider a smooth contact and hence there is no friction. As a result the block is having a weight W_A and due to this gravitational force it is pressing against the surface and this action and reaction is normal to the contacting surface if there is no friction. So an upward arrow is the reaction from the surface to the block and a downward arrow is the reaction from the block on to the surface. When the contact is a rough contact, i.e. there is friction between the two, then the reactions are normal reactions as before but in addition there will be a force of friction parallel to the contacting surface and an equivalent opposite force on the surface. Taking the resultant of two forces F and N on the surface this resultant will be passing through the junction of these two forces as shown in the figure.

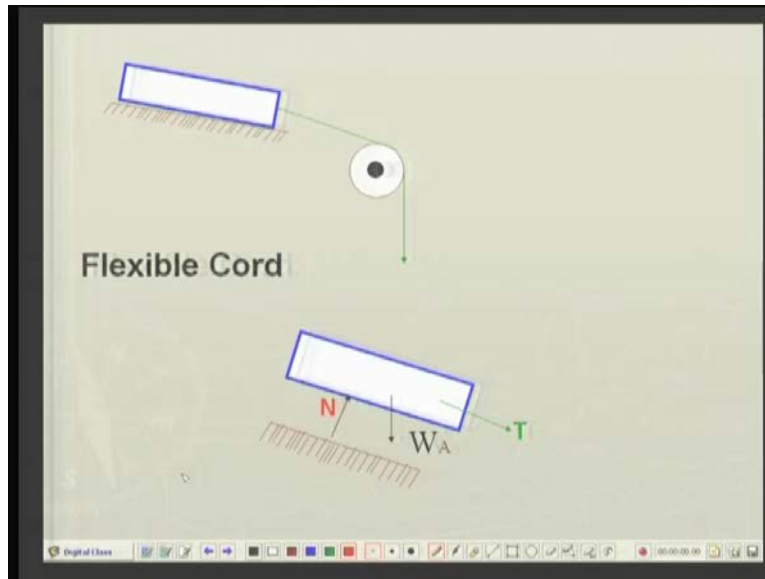
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Another type of contact is a beam which is resting on a surface with the help of some roller bearing or some ball or there may be a rod which is pinned with the beam and this rod itself is resting on rollers, thus allowing the rod to move on the uh given surface.

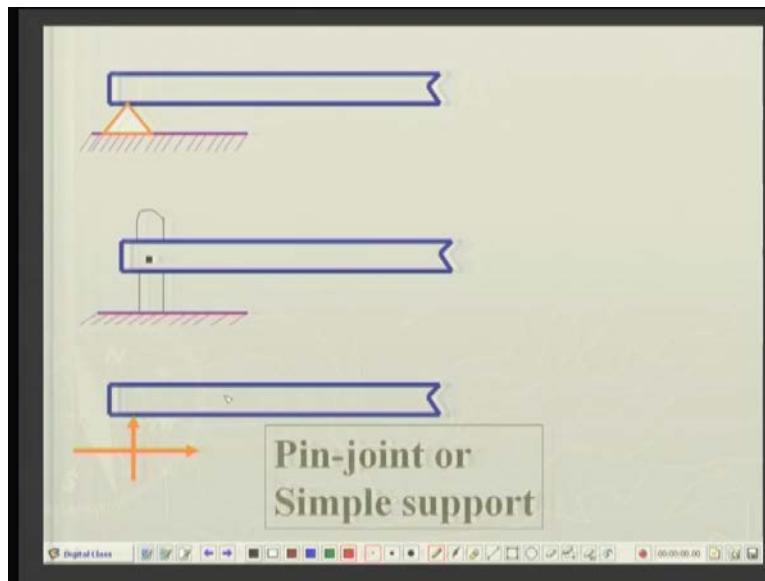
Now it is obvious that through this roller bearing or balls the vertical forces can be transmitted but any horizontal force will not be sustained because it will lead to the motion of the beam. So any force trying to push or pull the beam will be causing only motion. Hence there is only one reaction i.e. the vertical reaction between the beam and surface.

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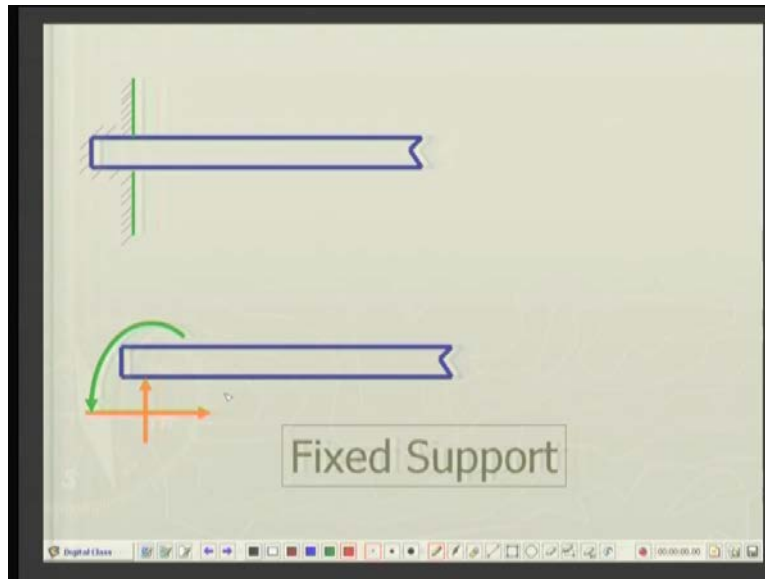
Suppose the block is being pulled with the help of a flexible cord string or chord, then this is schematically shown in the above figure. Taking a look at the free body of the block this string is replaced by a tension along the direction of the string and it cannot sustain any normal component because of the flexibility. So it can have only tensile force on the block and between the block and the surface there exists only the normal reaction, if there is smooth contact, otherwise there will be friction also. Hence according to the nature of the surface the reaction can be chosen.

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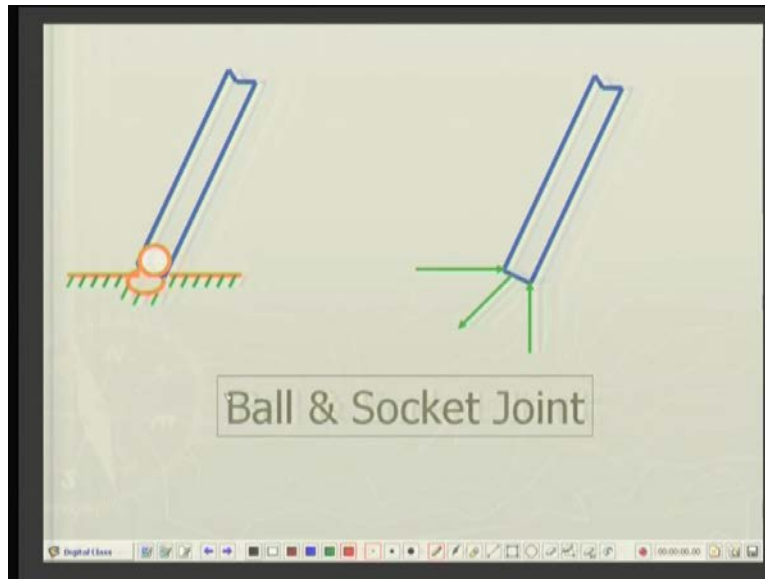
Suppose there is a pin joint or it is supported on a knife edge as schematically represented in the above figure, then in such a case both horizontal and vertical reactions are possible, but no moment is possible. Any moment will lead to the rotation of the beam and this is not sustained. So for a simple joint or pin joint, horizontal and vertical reactions are permissible.

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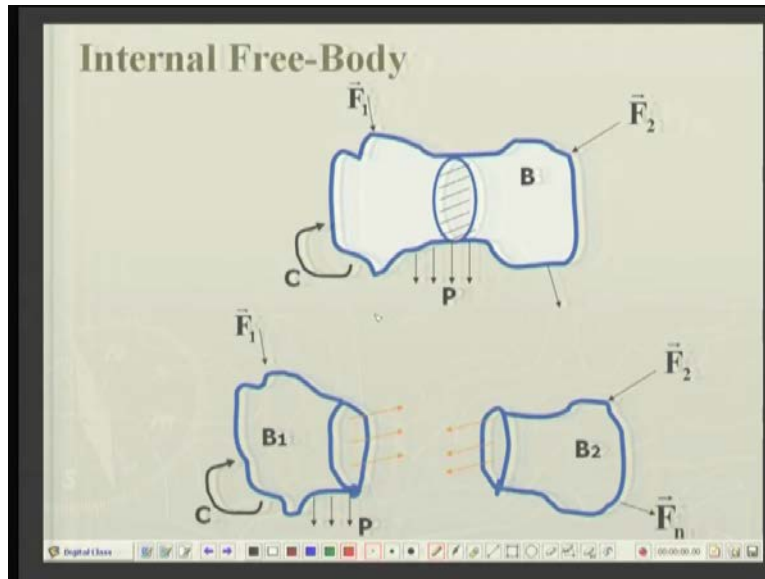
On the other hand, if the beam is rigidly fixed in to the wall or it is welded to a heavy structure or it is grouted in a suitable floor or a concrete beam then three reactions are possible; horizontal force , vertical force, moment to resist any rotation of the beam.

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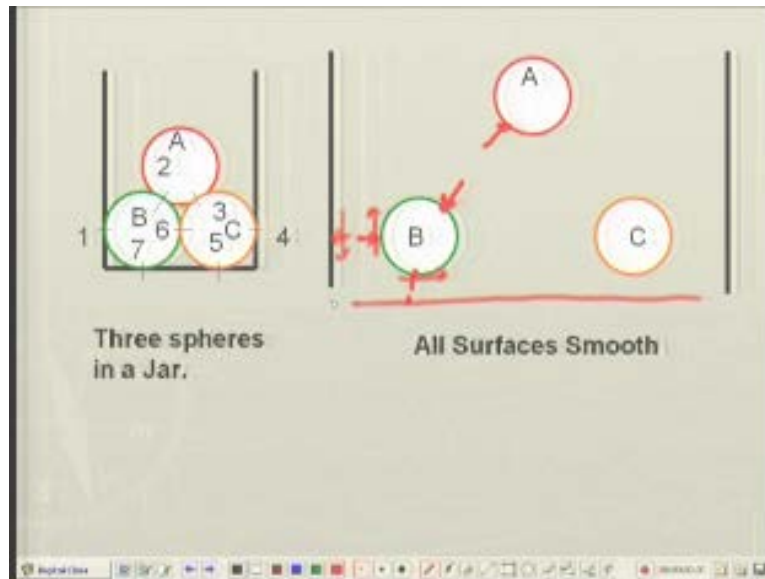
All the previous examples were in a plane. Consider now a three dimensional contact i.e. there is a rod which is having a ball end and which can move in a suitable socket in to the wall or floor. This is called ball and socket joint. In this case all the three reactions i.e. along 1) x axis, 2) y axis and 3) z axis are possible but no rotation can be resisted. Hence any moment m_x , m_y or m_z will simply lead to the rotation of the wall and therefore it cannot be sustained.

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Although internal free body will not be used in this course, for the completion of the concepts let us suppose that there is an arbitrary body which is subjected to arbitrary forces pressure, couple, etc. imagine that the body is cut in to two portions B_1 and B_2 , then due to the molecular forces or atomic forces, each part of the body will be acting on the other part and vice versa. B_1 will produce a system of forces on B_2 and by Newton's third law B_2 will be producing equal and opposite forces and moments also in general on B_2 . But only the forces are shown here. So B_2 will also have equivalent and opposite of forces on B_1 . So there is one free body which is individually in equilibrium and also there is the second free body which is also in equilibrium. Hence this is an internal free body. The concept of such free body is crucial in the mechanics of deformable media such as solids, fluids, etc. All these concepts can be illustrated with the help a very of simple example.

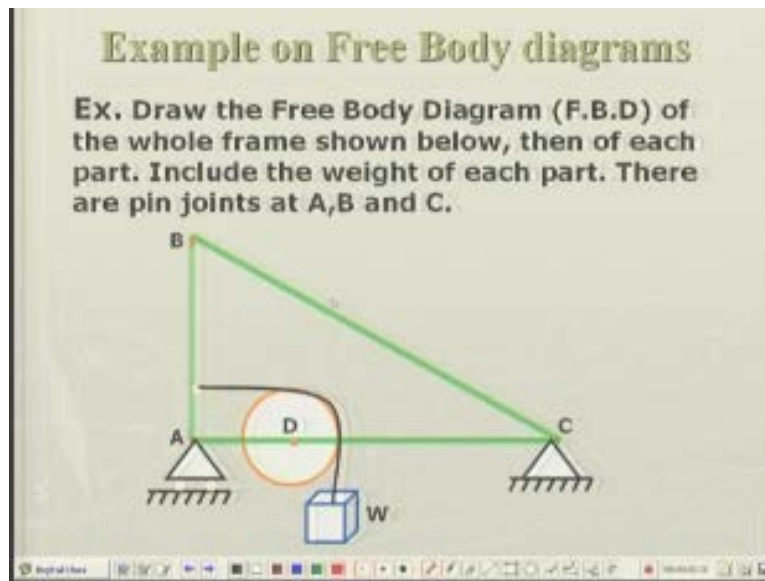
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Consider a jar in which three spheres or three balls (steel balls or wooden balls) are put and they are in contact with each other. One can enumerate various points of contacts. Let us say between sphere A and B there is point of contact at 2, between A and C there is point of contact at 3 and so on as shown in the figure. Now separate out the walls and the three spheres A, B and C.

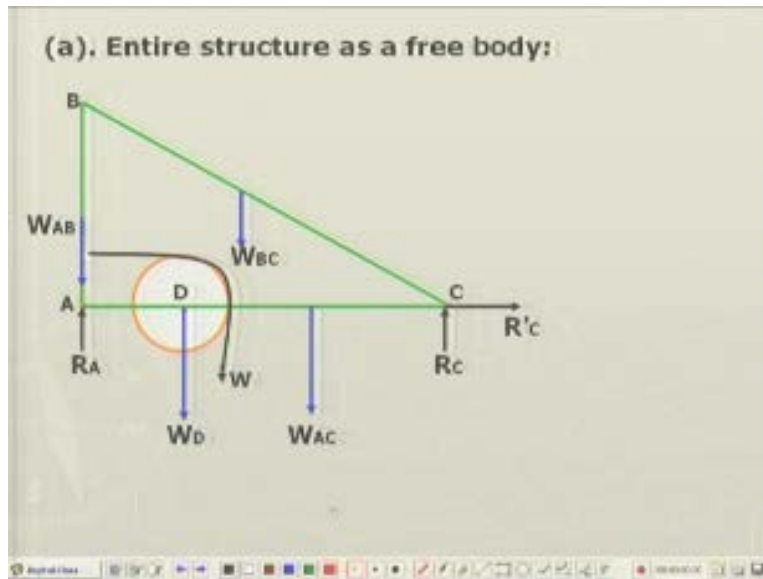
Consider the contact point on point B then there is a corresponding contact point on sphere A. And if this contact is a smooth contact, then A will act on B through a force and there will be equal and opposite force acting on A. Let us say between the wall and the sphere B at point 1, there is a rough contact, then it means that at point 1 there will be a normal reaction and therefore force of friction on the wall and an equal and opposite force (normal reaction and a force of friction) downward. In this way all the contact surfaces can be completed. For example, consider point 7 between the sphere B and the floor of the jar. Then in the separated figure there is a contact point at the bottom of the sphere B and corresponding point on the floor of the jar. There exists a vertical force and the force of friction on the sphere B and an equal and opposite force on the floor. Hence this is the method to draw various forces between the contacting bodies.

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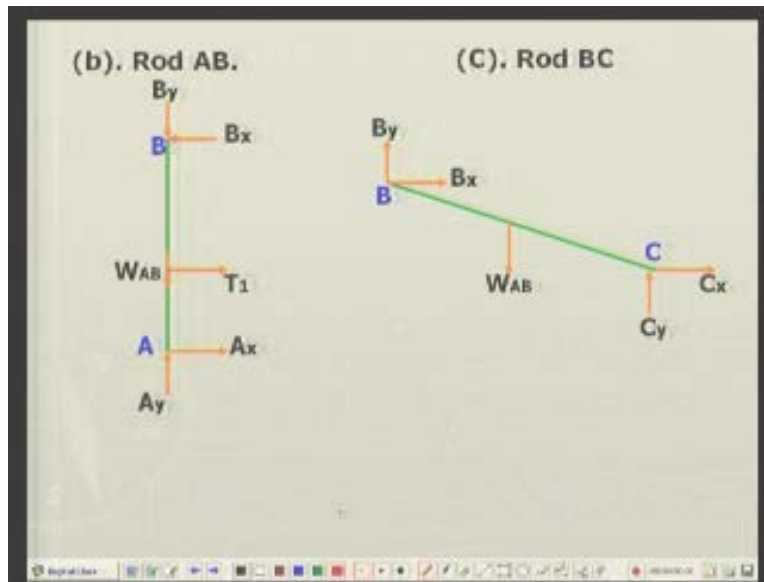
Consider another interesting example, a complete system where ABC is a triangular frame. A is a roller jointed support whereas C is a pin jointed or simple support and at point B the contact between the two rods is through a pin. On the rod AC there is a pulley through an axle or through a pin at D and on this pulley passes a flexible chord carrying a weight W. Further analysis of equilibrium will be seen later. Now divide the whole system of rods, pulleys, chord, etc. into various free bodies one by one.

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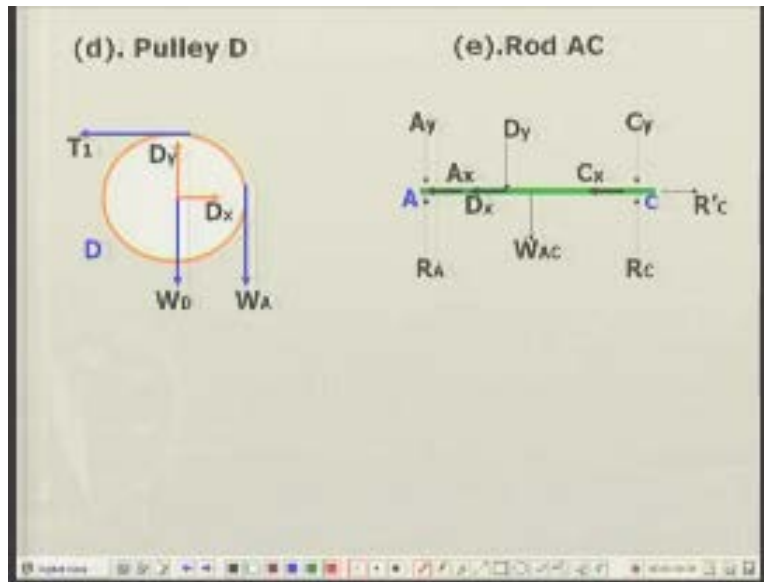
First of all consider the entire system as a free body which is isolated from its supports which were at A and C. Recalling, at A there was a roller support and at C there was a simple support. Roller support cannot sustain any horizontal reaction whereas simple support can sustain both vertical and horizontal reaction. Replacing the supports by vertical reaction at A, R_A and the horizontal and vertical reaction at C, R_C and R'_C respectively. The above figure is the entire structure as a single free body.

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Next consider the rod B's AB. Now the rods may be heavy rods and they would have their own weight also. B is a pin joint and so it means it can have both horizontal and vertical reaction. At A there is a normal reaction from the supports as well as the reaction at the pins, because at point A it is connected with the rod AC. Besides the reactions, there is a pin reaction at A_x and A_y , B_x and B_y . W_{AB} is the gravitational weight of the rod acting through its centre of gravity and T_1 is the tension due to the string or the chord. It is a flexible string so it can have only tensile force T_1 . At slant rod BC, reactions at B are B_x and B_y and reactions at C are C_x and C_y . W_{AB} in the figure of rod BC should be W_{BC} , vertical downward gravitational weight of the rod BC.

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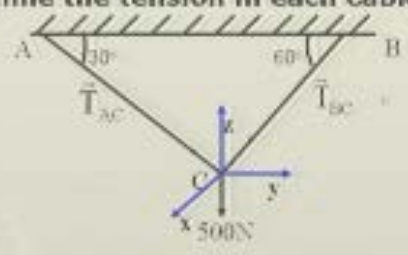


Next consider pulley D. Again a tension in the string T_1 which is equal and opposite to the T_1 which was acting on the rod AB. W_A is the weight suspended on this other free end of the string and these are the reactions at the pin joint between the pulley and the rod AC and these are the equal and opposite reactions i.e. D_x and D_y on the rod AC. In this way every contact point can be concluded, and it must have equal and opposite actions and reactions. This is how to draw the free bodies and the suitable reactions at the connections or contact points. Let us now fix our ideas about the conditions of equilibrium by taking some example first. Consider the example of concurrent forces and then for rigid bodies

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EXAMPLE

Two flexible cables jointly support a body weighing 500N as shown in the figure. Determine the tension in each cable.



The diagram shows a horizontal ceiling with points A and B. Two cables, AC and BC, are attached to the ceiling at A and B respectively. Cable AC makes an angle of 30 degrees with the ceiling, and cable BC makes an angle of 60 degrees. At point C, where the two cables meet, a weight of 500N is suspended. A coordinate system with x, y, and z axes is centered at point C. The z-axis is vertical, the y-axis is horizontal to the right, and the x-axis is diagonal downwards to the left.

The problem can be solved either by using vectors or graphically. At point C, there are three forces:

- i) 500 N
- ii) Tension \vec{T}_{BC}
- iii) Tension \vec{T}_{AC}

First example deals with two flexible cables AC and BC. They are contacted to a ceiling at point A and B making an angle of thirty and sixty degrees respectively to the ceiling. At the contact point between the two cables, that is point C there is a weight of five hundred or force of five hundred Newton's applied. These strings are flexible, so they can sustain only tension and we are asked to find out the tension in string AC as well as in string BC. All these forces i.e. T_{AC} , T_{BC} and 500N force are passing through point C. So it is a system of concurrent forces and let us now examine the equilibrium. We have to begin with the problem first. Draw the coordinate axis xyz, consistent with the right hand screw system.

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Using the equation of equilibrium for concurrent forces

$$\vec{F}_R = \sum \vec{F}_i = 0$$

$$\therefore -500\hat{k} + T_{BC}[\cos 60^\circ \hat{j} + \sin 60^\circ \hat{k}] + T_{AC}[-\cos 30^\circ \hat{j} + \sin 30^\circ \hat{k}] = 0$$

$$\therefore [T_{BC}(0.5) - T_{AC}(\sqrt{3}/2)]\hat{j} + [-500 + T_{BC}(\sqrt{3}/2) + T_{AC}/2]\hat{k} = 0$$

Solving component wise:

$$T_{BC}/2 - \sqrt{3}/2 T_{AC} = 0 \quad \therefore T_{BC} = T_{AC}\sqrt{3}$$

$$-500 + T_{BC}\sqrt{3}/2 + T_{AC}/2 = 0 \quad \therefore T_{BC}\sqrt{3} + T_{AC} = 1000$$

$$\therefore 4T_{AC} = 1000 \quad \text{or} \quad T_{AC} = 250\text{N}$$

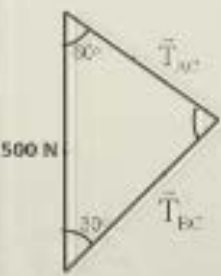
$$T_{BC} = 250\sqrt{3}\text{N}$$

Taking up the solution both graphically as well as analytically. Analytical solution. The condition of equilibrium for concurrent forces is that the resultant force which is sum of all the three forces is equal to zero. First of all we have the vertically downward force which is $-500\hat{k}$ unit vector, then the force in the string BC is resolved into the y and z components cosine 60 degree in the j direction, sine 60 degree in the k direction times the magnitude of the force T_{BC} . Similarly T_{AC} will be in the negative j direction component and vertical component in the k directions. These are the two string forces. Simplifying these components, i.e. we will write j component separately and k component separately, we get a single vector equation consisting of the j component and the k component.

This vector equation is equivalent to two scalar equation i.e. j component is separately equal to zero and k component is separately equal to zero and a very simple solution is an algebraic equation for two unknowns. After substitution, etc. we get the final result that the tension magnitude of the tension in the string AC is 250N and that in the string BC $250\sqrt{3}\text{N}$. This is the analytical solution.

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Graphically



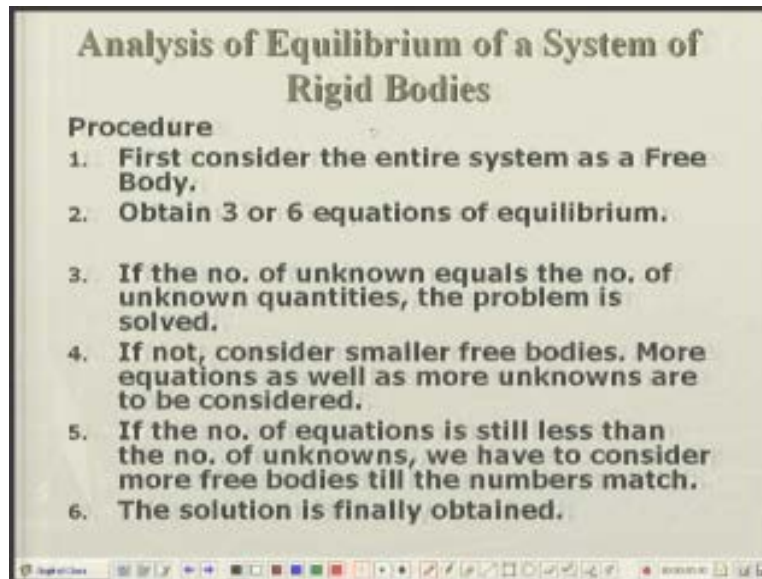
By Sine Law

$$\bar{T}_{AC} / \sin 30^\circ = 500 / \sin 90^\circ = \bar{T}_{BC} / \sin 60^\circ$$
$$\therefore T_{AC} = 500 \times 1/2 = 250 \text{ N}$$
$$T_{BC} = 500 \times \sqrt{3}/2 = 250 \times \sqrt{3} \text{ N}$$

Graphical Solution. It is the addition of all the vectors sum of all the vectors and by triangle law of vectors we can show that when the sum is equal to zero then all the three vectors will enclose a triangle. So it consists of the 500N force which is vertically downward. That is the force applied at point C and these are the two tensions T_{AC} which is at 60° and T_{BC} at 30° to the vertical or you can say that T_{AC} at 30° to the horizontal and T_{BC} is 60° to the horizontal.

Consider one centimeter to be equal to 100N. So you draw a five centimeter line at the two points of the straight line. You draw one line at 60° to the vertical and the other at 30° to the vertical. Wherever they intersect will be the apex of the triangle and then to find out the lengths of these two sides you use sine law of triangles. So T_{AC} divided by the sine of the angle opposite to the side that is 30° equal to the vertical 500N divided by the sine of this angle is incidentally 90° . So sine 90° and then similarly T_{BC} divided by the opposite angle sine 60° . Simplifying you will get the same result as the analytical solution provides.

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Analysis of Equilibrium of a System of Rigid Bodies

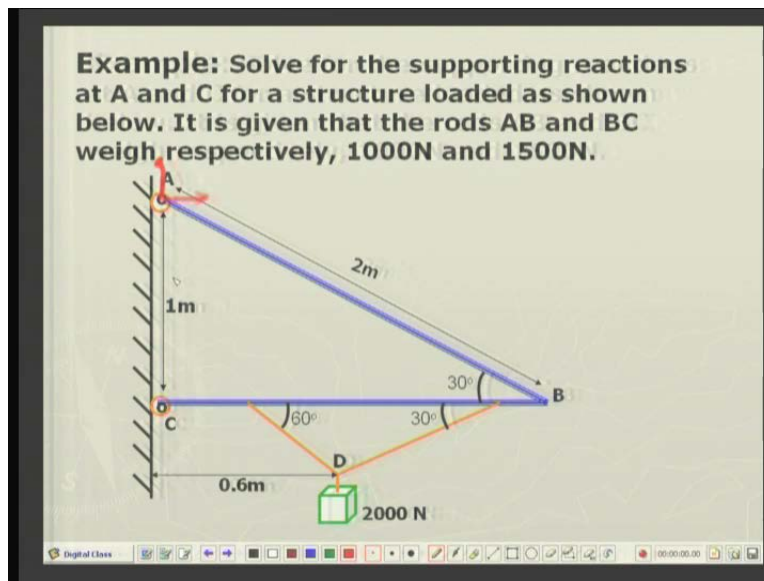
Procedure

1. **First consider the entire system as a Free Body.**
2. **Obtain 3 or 6 equations of equilibrium.**
3. **If the no. of unknown equals the no. of unknown quantities, the problem is solved.**
4. **If not, consider smaller free bodies. More equations as well as more unknowns are to be considered.**
5. **If the no. of equations is still less than the no. of unknowns, we have to consider more free bodies till the numbers match.**
6. **The solution is finally obtained.**

Analysis of equilibrium of a system of rigid bodies. Suppose we are given a system consisting of more than one rigid body which are in contact with each other or they are being supported by walls or floor or some other interconnection, etc. then first of all we will consider the entire system as separated from the walls or supports, as a single rigid body and then we will see how many reactions are to be found out. In the case of a plane system, i.e. when the all the bodies in the same plane forces are also in same plane, then naturally we will have three equations of equilibrium $\sum f_x$ is equal to zero, $\sum f_y$ is equal to zero and $\sum m_z$ i.e. the moment about the z axis equal to zero. But in the three dimensional case we will have six equations of equilibrium $\sum f_x$ is equal to zero, $\sum f_y$ is equal to zero, $\sum f_z$ is equal to zero, i.e. for forces and similarly $\sum m_x$ is equal to zero, taken about any arbitrary point $\sum m_y$ is equal to zero and $\sum m_z$. So if only three reactions are to be determined for the plane case then the three equations are equilibrium and the system can be solved. If there are only six reactions for the three dimensional case to be found out again we can solve, but in some cases the equations of equilibrium are not just sufficient to solve for all the reactions. If the number of unknown reactions equals the number of unknowns and also is equal to the number of equations of equilibrium, then we will solve the problem. If not then we will have to consider further free bodies i.e. we will break the entire system into separate free bodies

or subsystem and for each we will write down the appropriate equations of the equilibrium. We will come to a state where the number of total unknowns is equal to the total number of equations, then the problem is solved and we call the problem as statically determinate problem. If the number of unknowns is greater than the number of available equations of equilibrium then the problem is indeterminate.

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Consider the following example. Solve for the supporting reactions at A and C for a structure loaded as shown in the figure. It is given that the rods AB and BC weigh 1000N and 1500N respectively. There are two rods which are pin jointed at point B and at point A. The top rod is pin jointed to the wall or there is a pin connection and similarly at point C, there is a pin connection. There are two strings attached to the rod BC and these two strings themselves are simultaneously carrying the weight of 2000N at the point D. The distances are given 0.6m, 0.2m and this angle is 30°. The distance between A and C is 1m. We have to find out the supporting reactions i.e. reactions at A and C.

So point A is a pin joint and point B is also a pin joint. Hence two reactions at point A and two reactions at point B, meaning you have four reactions. So normally, we will require four equations to solve for four unknown reactions but in the plane of the body all

the forces and the structure is in single plane and only three equations of equilibrium are available. It means we have to do much more than that.

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First the entire structure (isolated from the supporting wall) is considered as a free body.
Equations of Equilibrium for a plane case:

(i) $\sum F_x = 0 \therefore A_x + C_x = 0$
(ii) $\sum F_y = 0 \therefore A_y + C_y = 1000 + 1500 + 2000 = 4500$
(iii) $\sum M_i = 0 \therefore -A_x - 2500(0.866) - 2000 \times 0.6 = 0$
Thus $A_x = -1200 - 2165 = -3365\text{N}$
 $\therefore C_x = -A_x = 3365\text{N}$

First of all we will write down the equations of equilibrium for the entire structure separated from the wall and taken as a single free body. $\sum F_x$ is equal to zero. So let me say that the horizontal reaction A_x then A_y , similarly C_x and C_y . So you can say that A_x plus C_x is equal to zero (sum of vertical force is zero). Also A_y plus C_y is equal to the sum of applied vertical forces, 1000N plus 1500N. These are the weights of the two rods and this is the applied force in the vertical direction at point D. So the total vertical forces are 2000N, 1500N and 1000N. The total vertical force is 4500N and then you take moments about any arbitrary point of all the forces. Consider the moments about point C. The A_y component is passing through C and hence no moment will be produced by it. So there will be moment due to A_x and the three vertical forces. These are the two terms which are producing moments and that solves for A_x is equal to -3365N

Now the significance of this minus is that we had taken A_x as pointing towards the right. Since the answer is negative it means in fact it is pointing towards left. Hence the up short is that you can choose any reaction in the appropriate direction but sense can be

arbitrary. If the answer comes out to be negative it means the actual force is in the opposite sense. So once we have this then using the first equation we can find out C_x which is equal to minus A_x which is 3365N. So this is positive it means the chosen direction is the correct direction.

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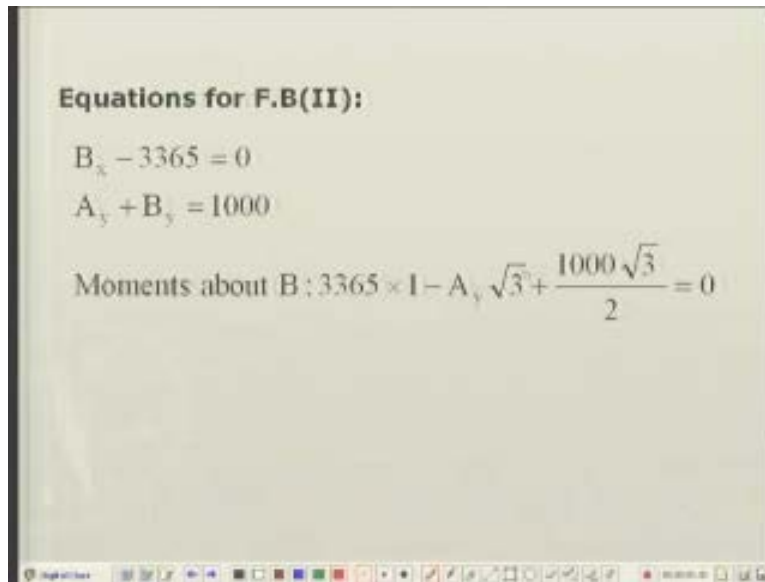
A_y and C_y cannot be obtained from above equations.
 To obtain A_y and C_y , we consider one more free body, i.e. Free body (II) shown.

Free Body (II)

Both free bodies (I) and (II) have 6 unknowns (A_x , A_y , B_x , B_y , and C_x , C_y) and six equations of equilibrium).
 Therefore it is a complete system of equations and can be solved.

Now to solve for the other two unknowns A_y and C_y . We cannot solve from the previous three equations as the number of equations is less than the number of unknown. So let us take one extra free body i.e. the rod AB. In the rod AB, we have already determined that 3365 is the reaction in the horizontal direction and the force 1000N, the length is 2m and the unknown reactions are A_y , D_x and D_y . So total unknowns are three and the equations available in the plane for equilibrium are also three. It means we will be able to solve the problem.

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Equations for F.B(II):

$$B_x - 3365 = 0$$
$$A_y + B_y = 1000$$
$$\text{Moments about B: } 3365 \times 1 - A_y \sqrt{3} + \frac{1000 \sqrt{3}}{2} = 0$$

Equations of equilibrium for the free body number 2. Again sum of horizontal forces is zero, sum of vertical forces is equal to zero and now we will take moments about B. We have chosen B because two unknowns B_x and B_y pass through B and hence their moments is zero. So i will immediately get A_y .

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Solution :

$$C_x = -A_x = 3365\text{N}$$
$$C_y = 4500 - A_y = 269\text{N}$$

Since A_x is found to be negative, therefore its actual direction is towards left. All other reactions are along assumed directions.

Finally collecting all the results we have C_x equal to minus A_x which is equal to 3365N and C_y is equal to 4500 minus A_y i.e. 269N.

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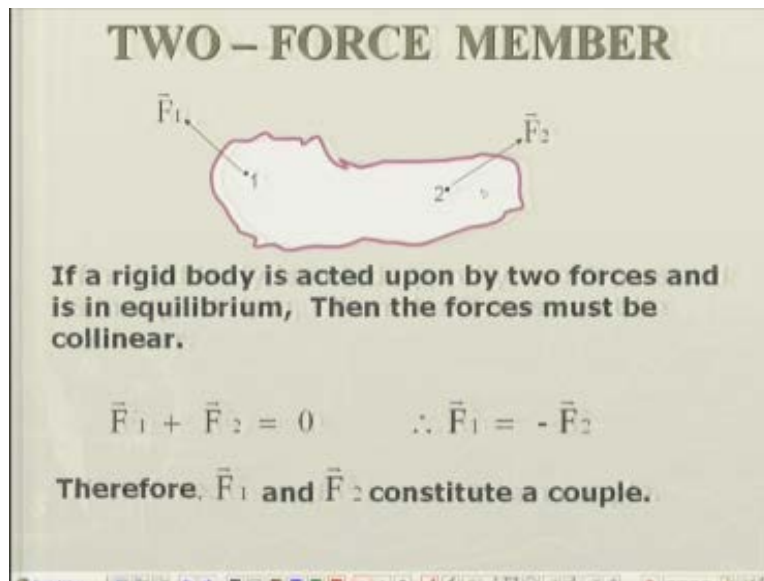
But for equilibrium, the couple moment $\vec{M}=0$
 $\therefore \vec{F}_1$ and \vec{F}_2 have a common line of action, hence collinear.

Compression **Tension**

For curved or segmented bodies

The concept of two force members

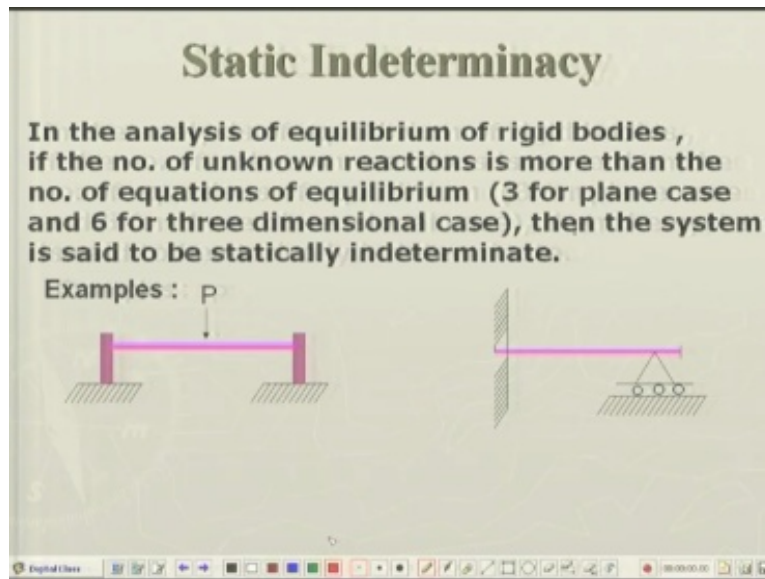
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Sometime there is a rigid body which we have told is subject to only two forces and the body is in equilibrium. As we will be seeing in next lecture when we deal with structural mechanics the problems those members or elements of a truss, they are good examples of two force members because two forces are acting on the ends of the members through pin joints. So the direction of the forces can be any arbitrary direction. So at point 1 and point 1 there is F_1 and F_2 . Now the number is in equilibrium and this body is in equilibrium. So by conditions of equilibrium the sum of the two forces F_1 and F_2 is equal to zero. It means that F_1 is equal to minus F_2 . In other words this body is subjected to two equal and opposite forces i.e. it is subjected to a couple. Now if the couple produces a moment let's say the moment is vector C , the condition of equilibrium for the rigid body is that C should also be equal to zero otherwise the body will start rotating and it will no longer be in equilibrium. Thus it means to fulfill that condition, the moment of the couple should be equal to zero i.e. the two forces should be collinear forces. So whenever a rigid body is subjected to two forces and the body is in equilibrium, at best it can be subjected to two collinear forces and there can't be other possibility. Accordingly if the two forces are both pointing into the body as shown in figure F_1 and F_2 i.e. they are going into the body then the body or the member is said to be under compression, whereas if the two forces are going outwards from the body as shown in the figure, then the member is said to be in under tension.

For example consider an angle shape for a curved body and let's say if two forces are passing through these two pins then the pins will be joined by a line and the forces will be acting along this line. Similarly if the member is a curved member then it will be again passing through these two pins which are along the same line of action. So these two simple conclusions are very useful in the analysis of trusses.

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Statical In-determinacy. When the number of unknowns is larger than the number of available equations then we cannot solve for all the forces and moments required to maintain body in equilibrium. Some other concepts like deformations or other things are required for solving the problem. In that case, the structure is called statically indeterminate structure. Many civil structures are statically indeterminate structure.

A simple case will be to consider a beam or a bar which is pin jointed at the two ends into the two supports and then the number of reactions will be two reactions at one end and two reactions at the other end. It may be suggested to a vertical force, a horizontal force, a moment, etc. Well this structure cannot be solved with the help of the equation of equilibrium alone and hence this structure is statically indeterminate structure.

Another possibility is a beam which is fixed into the wall that is grouted into the wall and on the other end it is also supported on a roller end roller support. Then we have said that there are three reactions at the fixed end: vertical reaction, horizontal reaction and a moment reaction. So the total number of reactions is four reactions and the number of available equations of equilibrium is three again. We have an indeterminate problem.

Well so in this chapter or in this lecture we have seen how to analyze the equilibrium of either a single particle or a single rigid body or a system of rigid bodies which are interconnected with each other and the crucial aspect of this analysis is that if you are given a system of rigid bodies which are in contact or interconnected with each other, then the method to choose appropriate free bodies because the choice of free bodies will determine how easily or how quickly you can solve the problem. Once you have chosen the free body it is equally important to replace the contact points through a system of reactions. Reactions may be force reactions or moment reactions and their directions, whether it is a pin joint, roller joint or a flexible string, correct connection, etc. and then for more complex problem we should first examine whether the problem is statically determinate or indeterminate. In this course we will be almost always concerned with statically determinate problem and important application of these concepts will come in the next lecture namely when we take up structural mechanics. Till then we will see how to use these concept.