Applied Mechanics

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Lecture No. 3

Analysis of Forces

The concept of forces is at the core of mechanics because the interaction between force and the bodies manifest itself through the motion of bodies and there are certain laws which go on this interaction.

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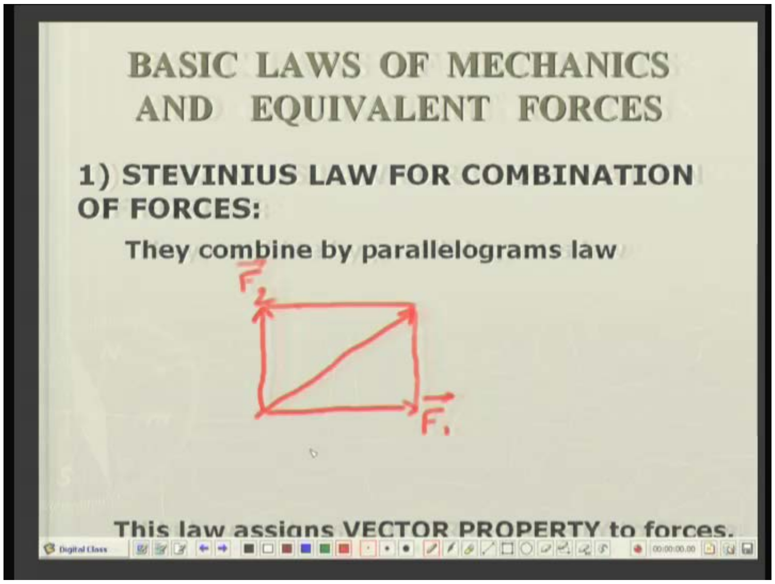

In the context of Newtonian mechanics, there are certain laws which have been annunciated over the a period of, let us say, about four hundred and fifty years and these laws, we will go through quickly now. First is the Stevinius law for combination of forces. Well, suppose there is a force F1, in direction as shown here, and there is a second force F2. Now, how do these two forces combine? We have seen that the combination is through a parallelogram law of forces or triangular law forces.

So I am just indicating the parallelogram law of forces, that is, the force F1 and F2, they form the two adjacent sides of a parallelogram and the diagonal of the parallelogram and gives us the resultant force. This law assigns the vector property to the forces. We move to the next law, the the laws of Newton's.

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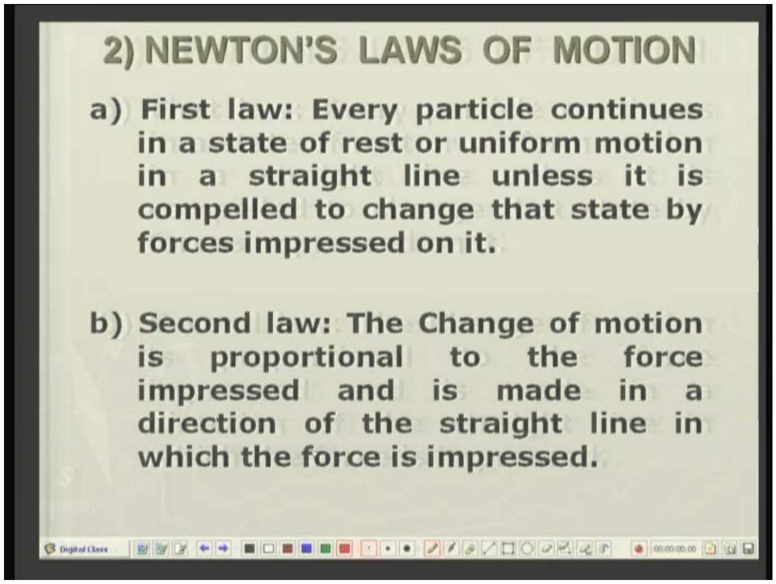

So first I will discuss the first law due to Newton: Every particle continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it. That is, if we don't apply any force on the particle or the bodies at rest, it will continue to be in rest, if the body or the particle is moving in a straight line with the uniform velocity, it will continue to do so, if we don't apply any forces but as soon as we apply the forces this state of rest or uniform motion will change. That is the first law due to Newton.

The second law of motion due to Newton is the change of motion is proportional to the force impressed and is made in the direction of the straight line in which the force is impressed. Well, what it says is that, as soon as we apply the force the earlier state of rest of motion will change and this change will be proportional to the magnitude as well as the direction of the force. If the magnitude of the force is higher, the change will be higher and, similarly, if the magnitude is lower the change will be small. Secondly the direction in which this change takes place is governed by the direction of the force. Now we look at these two laws of motion carefully these concepts of motion and rest. They need to be looked into in a great detail.

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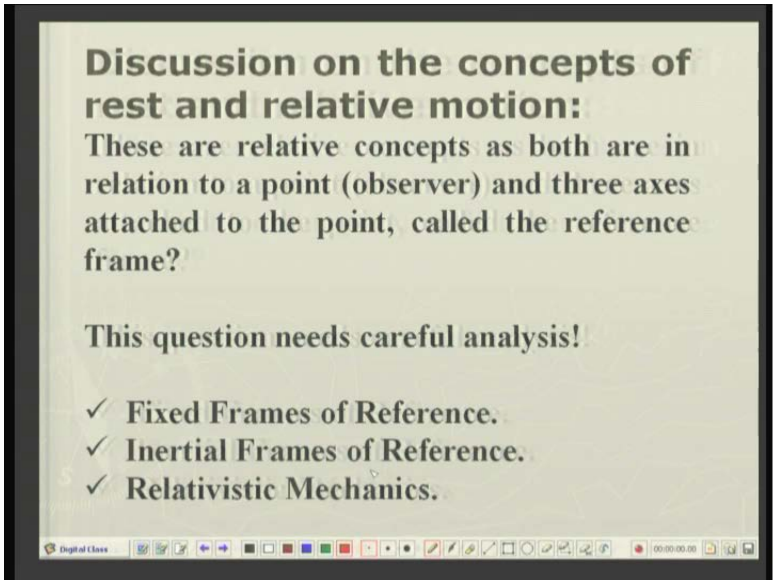

Well there are two or three concepts or notions associated with this. First is, how to define the state of rest or relative motion? Well it is a common experience, that, if you are traveling in a train, then if you look from the window of the train, the trees, etcetera or other things appear to be in motion opposite to the motion of the train. If the train is moving forward, the trees and other things on the ground will be appearing to be moving in the opposite direction. Well, to an observer or to a person who is standing on the ground, of course, the tree and other things are at rest. So it means the concept of rest or uniform motion is related to the person or it is a relative concept, relative to the observer. Well, for most of the analysis or problems encountered in Newtonian mechanics, it is good enough to take earth as the body which is at absolute rest but if you look at some of the problems involving motion of missiles or other bodies, like sometimes cyclones, etcetera, then we have to invoke the knowledge that earth is not at rest it is rotating about its own axis and also it is orbiting about the sun. So for such problems, the fixed observer cannot be taken as one standing on earth. It has to be taken, may be, an observer on sun for motion of galaxies and other heavenly bodies. If that is not sufficient, you may have to fix an observer very far outside the galaxies. So depending upon the nature of the problem, the fixed reference or fixed observer is to be chosen very carefully. Since we will be mostly dealing with engineering problems or, that is, structures or motion of bodies on earth, we will be considering a frame of reference which is fixed, relative to the earth.

Now, suppose there are two frames of reference; one is fixed to the earth and the other is moving at a uniform velocity related to the first observer. Uniform velocity in a straight line. Then again, we cannot distinguish between the two observers because even if a body is observed in a frame which is under uniform motion, the forces will not be required for this motion because the forces are proportional to the acceleration. Change in motion, that is, a body moving at or an observer moving at a uniform velocity has zero acceleration. So it means that all those bodies, which are moving relative to each other at uniform velocity in a straight line, they will be treated at var and the set of all such observers is called the inertial set. Okay. And the third concept involved is the relativistic concept. Naturally when the speeds of bodies under analysis is very, very high as comparable to the speed of light, then as I have already made a statement classical or Newtonians mechanics is not adequate and we have to resort to relativistic mechanics due to Einstein. So for our course, for our analysis, it is sufficient to consider Newtonian mechanics and the perquisite for the Newtonian mechanics is that, we fix an observer to the ground, to our earth and then all set of observers or reference frames which are in relative motion with respect to each other through a uniform velocity along a straight line are at var. Such a set is the inertial set.

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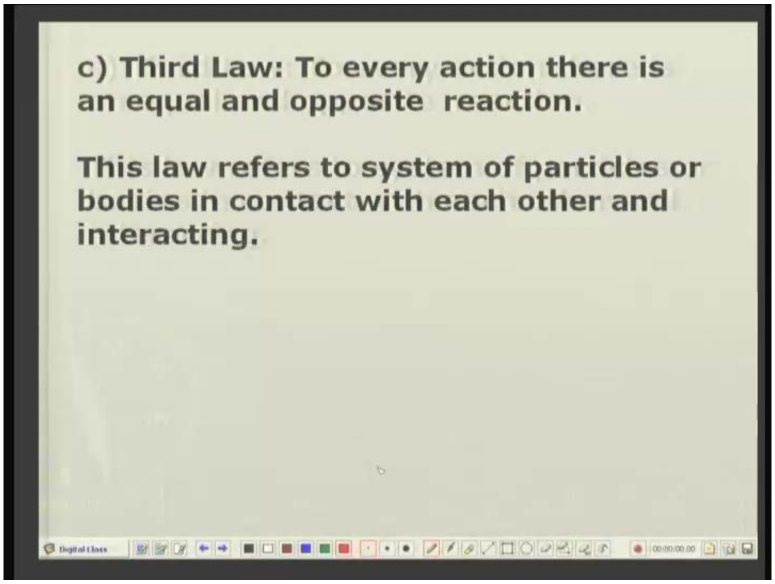

The third law of Newton states that to every action there is an equal and opposite reaction. Whereas, the first two laws, they were concerned with the motion of a particle or a rigid body. Here in the third law, we are discussing the interaction between the bodies themselves. So whenever there are two bodies or more bodies which are in contact with each other and they interact with each other, then the action of, let us say, body number one on two is equal and opposite to the action of body number two on one.

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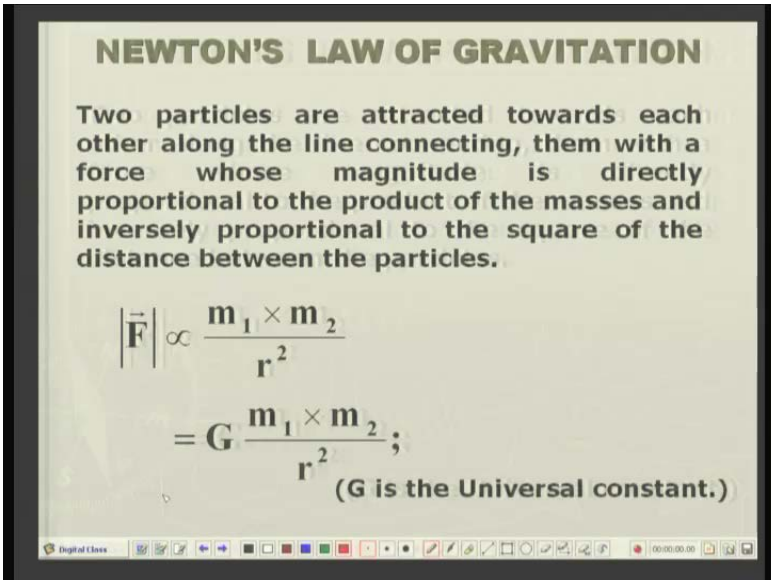

The fourth important law of classical mechanics or Newtonian mechanics is the law of gravitation. Whereas the third law was the interaction between the particles or bodies in contact, this is the interaction between the bodies which are separated from each other by some distance. So this is the law of interaction or at a distance. What does it say? Two particles are attracted towards each other along the line connecting them with the force, whose magnitude is directly proportional to the product of masses, of the bodies of course, and inversely proportional to the square of the distance between the particles. Let us look at this law more carefully. The two bodies have an interaction or a kind of attraction towards each other and this interaction or force of interaction is along a direction. So law states both the direction and magnitude is along a line connecting the two particles. So you connect the center points of these two particles and then join them. That defines the direction of interaction or direction of the attractive force and the magnitude is proportional to the product of the two of the masses. Please do not confuse mass with the volume or the weight. It is the amount of matter, as we have clarified in lecture one. So it is proportional to the product of the masses and inversely proportional to the square of the distance between these two particles. So when this law is stated analytically, it says that the magnitude of the force of interaction F is proportional to m1 into m2 divided by r square.

r is the distance between the two particles and if the proportionalities replaced by the equality sign, introducing a constant of proportionality G which is called the universal constant. Then the law states that the magnitude of the force is G times m1 into m2 over r square. Okay. So this is the Newton's gravitational law. Well this is illustrated graphically here.

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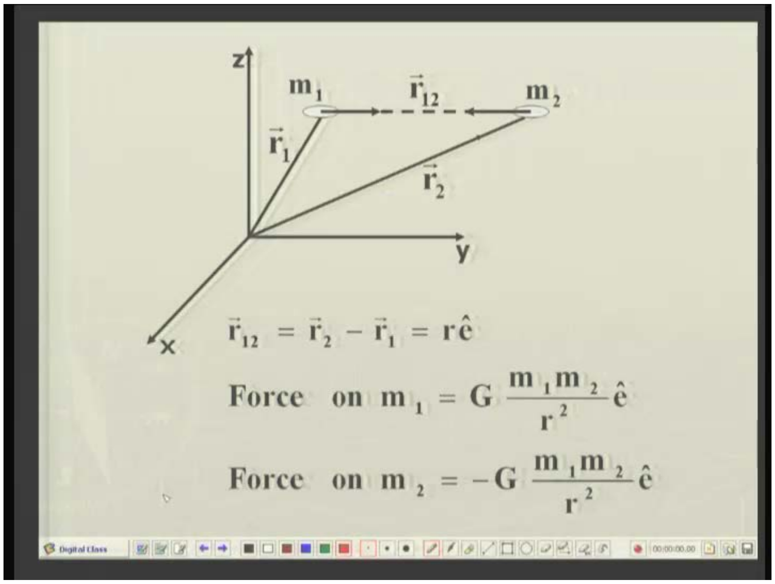

Suppose m1 and m2 are the two particles or bodies and the center of these particles are located in the Cartesian coordinates, so that the position vector of the center of m1 is r1 and the position vector of the center of m2 is r2. They are illustrated here. So the line connecting the mass m1 to m2, the straight line, is illustrated as the vector r12. Then r12 vector is equal to position vector two minus position of vector of one and this can be written as the distance, the scalar distance, r into the unit vector along this line of connection. Okay. e. So unit vector is e. Force on particle m1 due to m2 is given as, the attractive force in the direction of e. So G into m1 m2 over r square into the unit vector along the connecting line and conversely the force on m2 due to m1 is negative of the earlier force. So minus G m1 m2 over r square into e, so that, the vector is now in the opposite direction.

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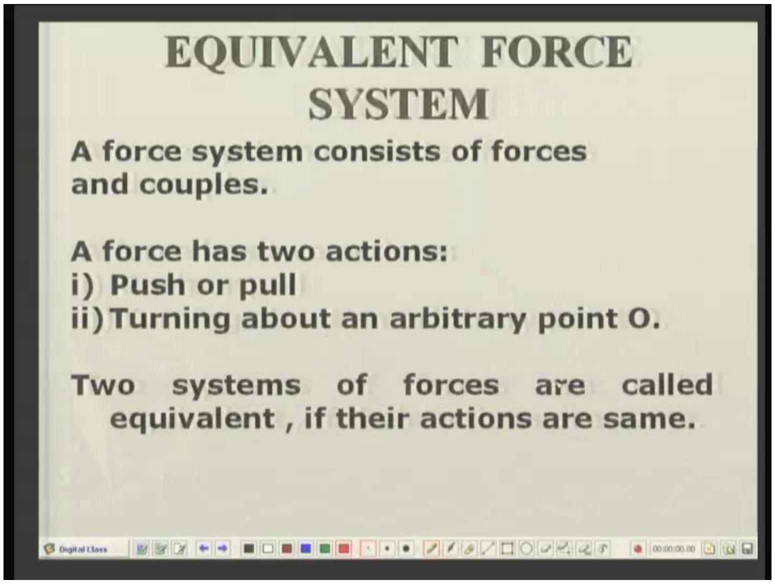

So after having discussed the basic laws of mechanics and understood the role of force, let us see how we can analyze various types of forces, may be, replace them with simpler forces or some other quantities of mechanics. So first we will consider equivalent force system. A force system consists of two types of quantities, one is called forces as such and the other is called moments or, for the time being we will have a pair of forces with opposite directions but separated from each other by some distance and they are called couple. But the couple itself manifests through a moment. So force has two types of actions, one is the push and the other is pull. So push. It pushes a body or a particle or it pulls a body. The second type of action of forces is the turning action, produced about a point arbitrary in space, designated as point O. So that is the second type of influence or action of the force. Now, two systems of forces, systems means forces consisting of individual forces or couples, two system of forces are equivalent if their actions are same, that is, they have the same push and pull effect as well as the turning effect. Now I will give you a few examples of equivalent system of forces.

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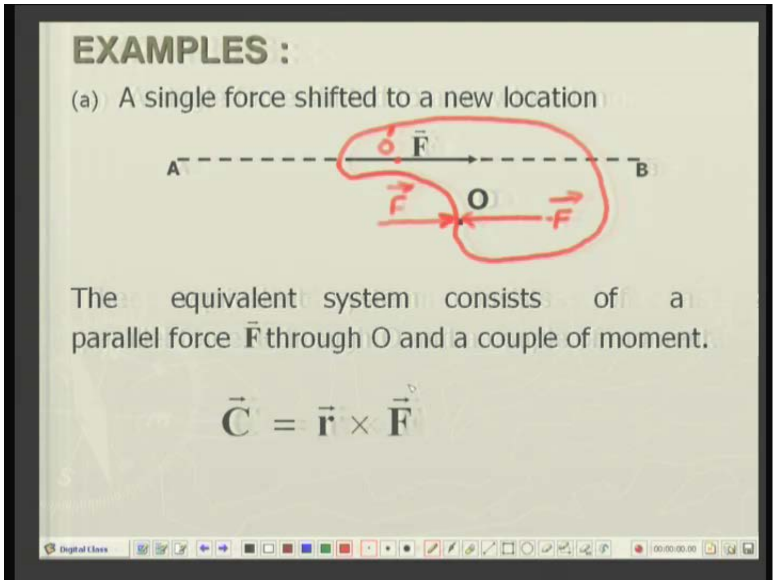

Suppose there is a single force F whose line of action is AB as shown here and I want to shift it to a new location. Earlier location is, may be, any point over here and now I want to shift it to a new point. What I will do is that, at point O, I will have two equivalent opposite forces. One is the force equal in magnitude and same in direction at point O. So I will call this as force F and an opposite force, same magnitude, same line of action but sense is opposite. So it is minus F. Okay. So one is F, the other is minus F. I will not change the given problem now. What I will do is that, I will take F force here and minus F force here and I will call it as a couple. So a combination of two forces, F and minus F, one passing through the given point, you may call it as O dash and the second force is passing through a new point O which is pre designated. So this constitutes a couple. So the earlier system of a single force is equivalent to a parallel force, same in magnitude and direction but shifted to a new position O and a couple of two forces, F and minus F and the moment of this couple. You remember, we have already discussed the moment of a couple which is equal to r cross F, where r is the position vector from point O to any point on the line of action. So suppose I take this as r, vector r. Okay. So the equivalent force system is a force F and a couple of moments C given by r cross F, where r is an arbitrary vector starting from O to a point on the line of action of the force F as given.

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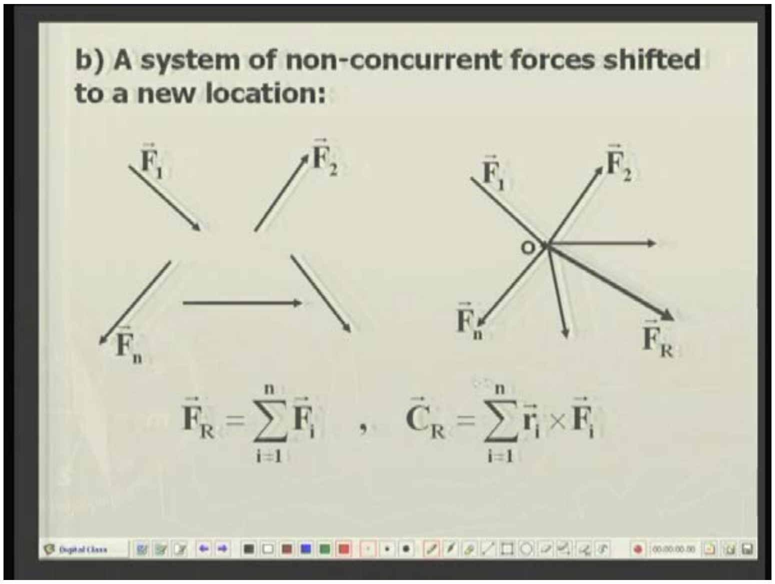

Second example of equivalent force system: Suppose we are given a, let us say, n number of forces, F1, F2 and so on, up to Fn. Okay. We want to shift all these forces, n number of forces, to a new location O. What I will do is that, using the previous example, I will shift the force F1 to a new position and in doing so, I will be creating a couple, with the moment of that couple as r1 crossed with F1. Okay. Similarly, F2 can be shifted to a new location and a couple is created, r2 vector crossed with F2 vector and so on and so forth, up to Fn, the nth force shifted to the new location and the additional couple will be rn vector crossed with Fn force. Okay.

So if I add up all these vectors forces with forces and moments with moments, then the sum of all these forces passing through the single point O is Fi summation, i going from one to n. This is the vector addition and the sum of all the moments of the couples. We have again discussed it earlier, that couples are free vectors. So, as long as, we know the point about which we have to consider the forces, then it is a material. When I add up all these moments of couples, this will be Ri crossed with Fi, i going from one to n. So the given system of forces is equivalent to a single resultant force FR and a plus is a single moment CR, which is the summation of all the individual moments. So what we have learnt is that, if we are given any system of forces we can replace them by a single resultant force plus a single moment which represents the moments of the couples involved in shifting the forces to a given point.

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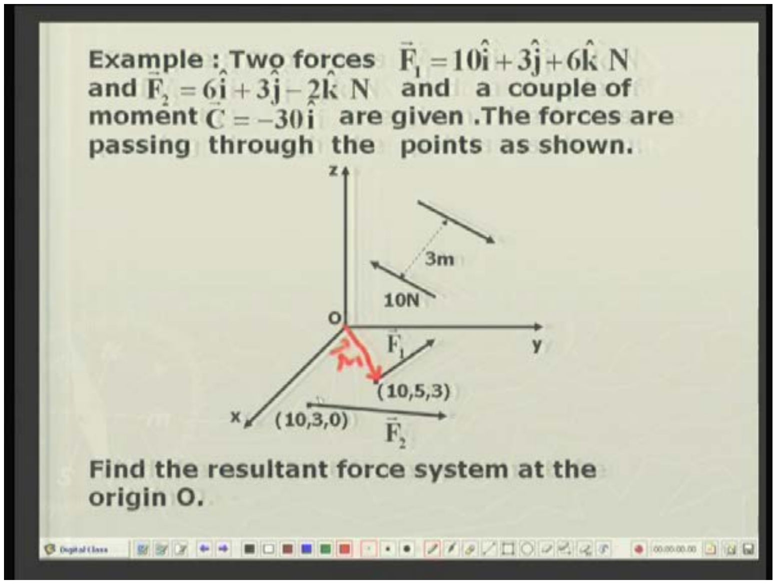

Let us fix up our ideas with the help of an example, which states that there are two forces, force F1 which is ten i unit vector plus three j unit vector plus six k unit vector plus F2 six i three j minus two k Newton's and a additional couple moment is given which consists of two equivalent opposite forces of ten Newton's as shown here, separated by a distance of three meters. So you can easily see, that the moment will be equal to three into ten thirty I, by right hand screw system, in the negative x direction. Okay. The forces are shown. Find the resultant force system located at the origin, that is, we want to shift all the forces to the origin and in doing so, what is the resultant source and what is the resultant moment? That is our objective. Well, the resultant force, since there are only two forces, is the force of ten Newton's are equivalent opposite. So they will cancel, their sum will be zero.

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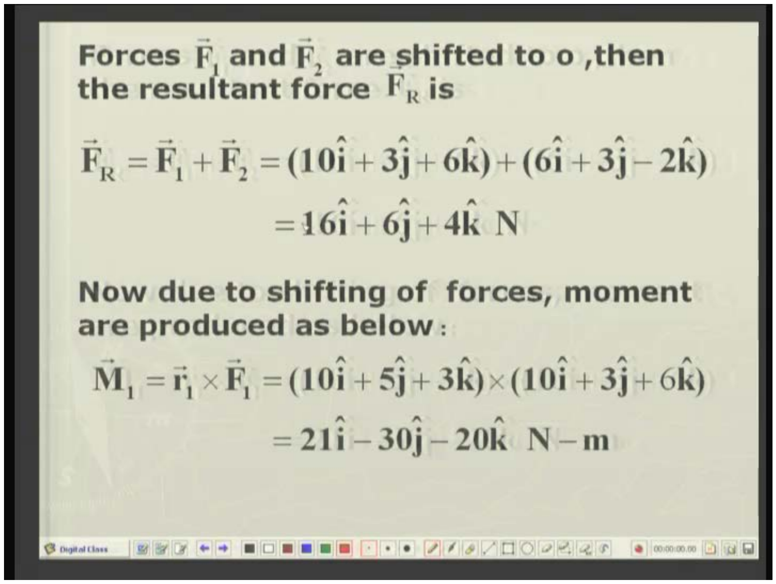

So the other two forces are F1 and F2, which are added vectorially to give us ten i plus three j plus six k plus six i plus three j minus two k. So the resultant force is sixteen i plus six j plus four k Newton's. That is simple. Now the moment due to the shifting of forces to the origin can be easily obtained. The moment of the force F1 is r one crossed with F1. If I go back to the figure, this is F1 and since any point on the line of action of the force is good enough, r one is the vector joining O with the r1. Similarly r2 can be obtained by joining O with ten three zero point. So the cross product is taken as r1 crossed with F1, similarly for the next moment, moment of the force F2, r2 crossed with F2. We have already seen how to calculate the cross product.

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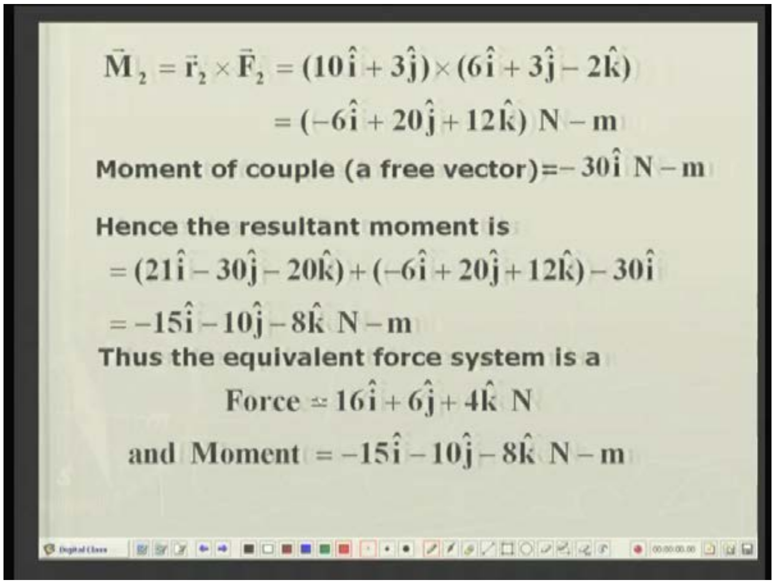

You can do it as the determinant of the matrix. So I will not go through the details of this calculation because you can do it. So the second moment is minus six i plus twenty j plus three twelve k Newton meters and the moment due to the couple, that is a free vector as we have illustrated earlier, is minus thirty i. So it can be supposed to be acting about any point in space. Hence the resultant moment of m1 m2 and the couple moment is addition of these three vectors and the resultant comes out to be minus fifteen i minus ten j minus eight k Newton meters. Okay. So the equivalent force system to the given force system is the resultant force of sixteen i plus six j plus four k Newton's and a moment of minus fifteen i ten j minus ten j minus eight k.

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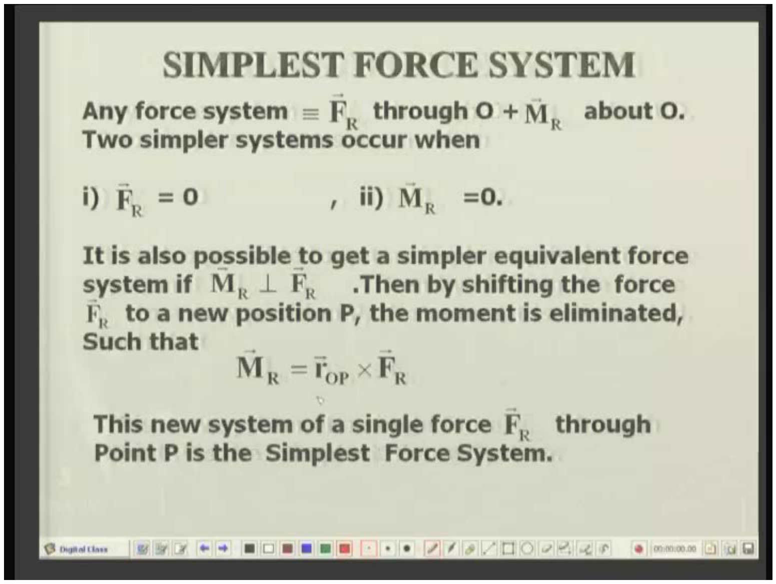

Now the question is, this equivalent force system consisting of the resultant force and the resultant moment, can we further simplify this? Is there is a way that these two things can be merged to get only one simple vector quantity? Well, the answer is, it is possible sometimes. What are those special cases? Well, if the resultant force itself comes out to be zero, then you are left with only the resultant moment and it cannot be further simplified. That is the only quantity. Okay. Or if the resultant moment is zero, then the equivalent system may consist of a single force FR but there is a situation when both FR and MR are not zero but still we can further simplify it, namely, the case when MR and FR are mutually perpendicular vector quantities, that is, their dot product is zero. In that case, we can further shift the force FR to a new position and the new position is to be selected in such a way, that the quantity MR is equal to the vector distance between the old position and the new position crossed with FR. Okay.

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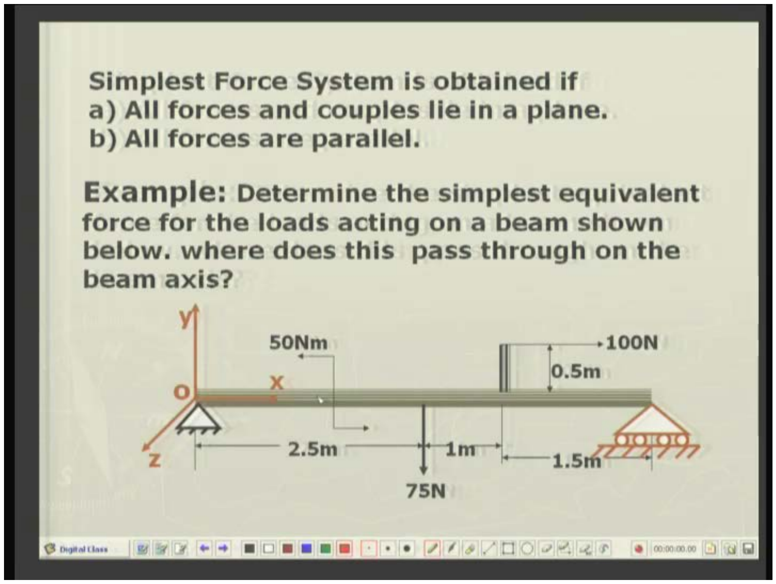

So to show that, we will make this statement, that the simplest force system, that is consisting of a single force is obtained if all forces and couples lie in a plane because when the couples lie in a plane, the moment of that couple is perpendicular to the plane and hence is normal to the given forces. So our condition is fulfilled. A single force can be obtained which is equivalent to the given system and second case is when all forces are parallel to each other. Then in shifting these forces to a single point, the moments will be created which are perpendicular to the direction of the forces. Again our condition is satisfied. Let us illustrate this with the help of examples here. First is the example of a beam which is subjected to a force of seventy-five Newton at a distance of two point five meters from the left hand end and second force is of a horizontal force of hundred Newton's which is acting on a bracket fixed to the beam. Let us say, this is the bracket which is welded to the beam at these joints and then on the tip of the bracket we have a horizontal pull of hundred Newton's and the distance between the tip of bracket and the center of the beam is point five meters and all other distances are given. Now let us see how we can replace this system consisting of a couple of moment fifty Newton meter, a force vertical force downwards seventy-five Newton's and a horizontal force towards right hundred Newton's.

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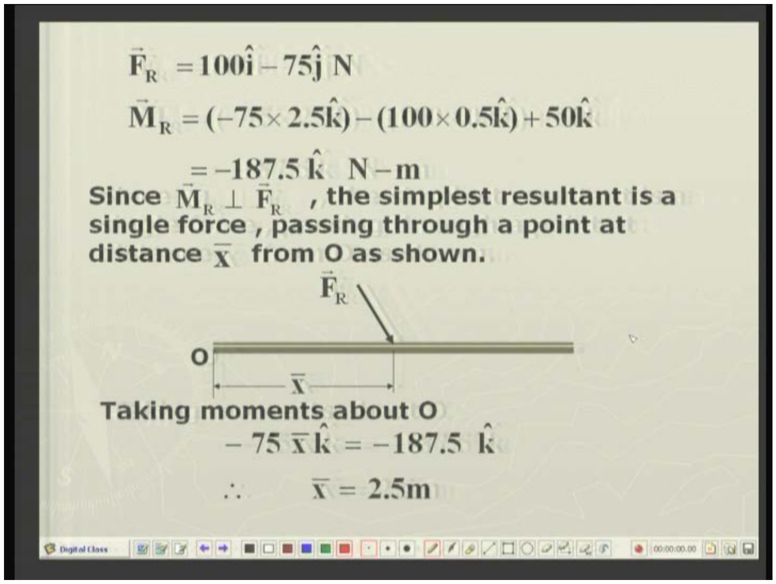

Well the resultant force, very easy to see, hundred Newton's, which is the horizontal force in the x direction and downward y direction seventy-five Newton force. So minus seventy five j and the moment of these forces, when they are shifted to the origin, going back to the figure, origin O and the x y z coordinates are also illustrated here. So we can easily see that the downward force minus seventy-five j, actually this is missed out. This is vector j unit vector. So crossed where two point five. Sorry for the mistakes. This is i unit vector okay and this is O. The moment of this force, when I shift this force to this point it will be creating a moment along the z axis in the inward directions. So you can see that when I shift this force, one force downward here and one force upward, the moment of the couple will be the clockwise moment, which will be in the minus z direction. So let us see, it will be four seventy-five into distance two point five in the minus and the other moment will be hundred into two point five. Again that will be in the clock wise direction. So minus k and this, the fifty k, is the moment of the couple already given. See once again let us look over the moments. See, when I shift seventy-five Newton's I will take two equal and opposite forces, one force in this direction, one force in this direction. So this vertically upward force and downward force, they constitute a couple whose moment will be a clockwise moment. So that will be in minus k direction. Similarly, this force of hundred Newton's to the right is shifted two point O along this line of action. So in doing so we will again generate a moment which is clockwise in the minus k direction.

So this is the first moment, this is the second moment and this is the given couple moment of fifty K in the plus in the anticlockwise direction. So the net moment is one eighty-seven point five in the minus K direction, that is, clockwise along the z axis and the forces are having components in the x and y direction. So their resultant force lies in the x y plane. So it is obvious that the moment vector MR and force vector FR are mutually perpendicular to each other. Hence we can further simplify this system of forces and replace this system by a single force FR passing through a new point, let us say, point p. Okay. So how can we get this? We have, as shown, the resultant moment minus one eighty-seven point five is equal to r o p crossed with the resultant force FR. So that will be equal to, let us say, the distance which is from the origin to the new point of action of the resultant force, x bar, so seventy-five into the distance, that is, x bar. So in shifting it, we will create again a clockwise moment. So it will be minus seventy-five x bar in the minus z direction that is minus k hat. So solving this one eighty-seven point five divided by seventy-five which is equal to two point five meter. So this x bar comes out to be two point five meter. So the original force system is replaced by a simpler force system passing through p whose distance is given over here. Let us take up an example of parallel forces.

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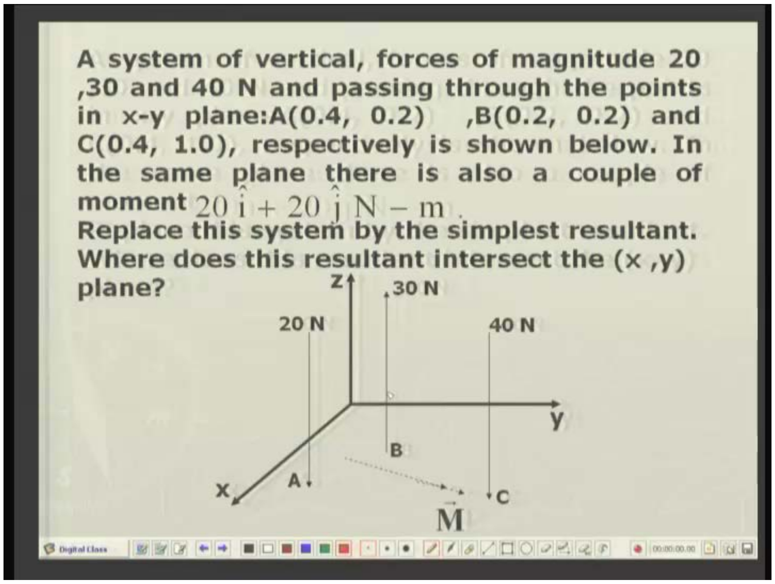

A system of vertical forces is given. There is a downward force passing at point A of twenty Newton's, a upward force parallel to Z axis of thirty Newton's passing at point B. Similarly, at C, forty Newton downward force. The locations of A, B, C are given and in addition there is a moment, indicated by two arrows, of magnitude twenty i plus twenty j Newton meters lying in the X Y plane. Okay. So replace the system by the simplest resultant. Where does this resultant intersects the x y plane? Let us see this.

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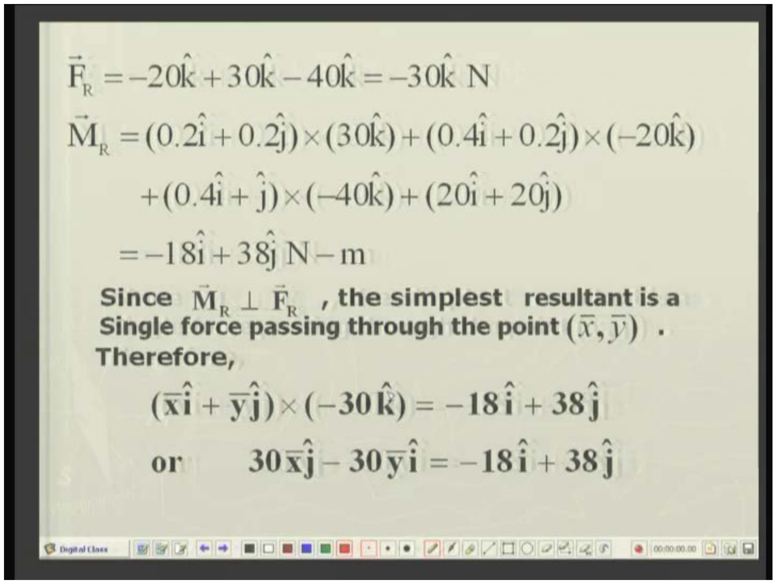

All the forces are parallel to z axis. So they are parallel to the K unit vector. So the first force, second force, third force, all were in the K direction. So the resultant is very easily obtained, minus thirty K Newton's, and in shifting these parallel forces to the origin, we have R cross F. You can again see, that the location of point A is point four i plus point two j. So the position vector of A is joining this point with this point. Similarly, position vector B is joining the origin to this point B and for C. So these are the given position vectors crossed with their respective forces and this last term represents the moment in the X Y plane, twenty i plus twenty j. So after carrying out this cross multiplication you add up all these vectors, you will get minus eighteen i plus thirty-eight j Newton meters force, in the K direction moment, in the x y plane because it has components in the x and y direction. So it means, the force is perpendicular to the resultant moment. Hence it is possible to replace it by a simpler force system namely MR. Let us say, that this simpler force system is passing through a point, x bar comma y bar. Okay. Then as stated, MR must be equal to r a o p crossed with FR. So x i plus y j crossed with the resultant force minus thirty k equal to the resultant moment MR, which is minus eighteen i plus thirty-eight j. So carrying out this cross multiplication and simplifying, we have this vector equation: thirty x bar j minus thirty y bar i equal to minus eighteen i plus thirty-eight j. This vector equation is equivalent to two scalar advance.

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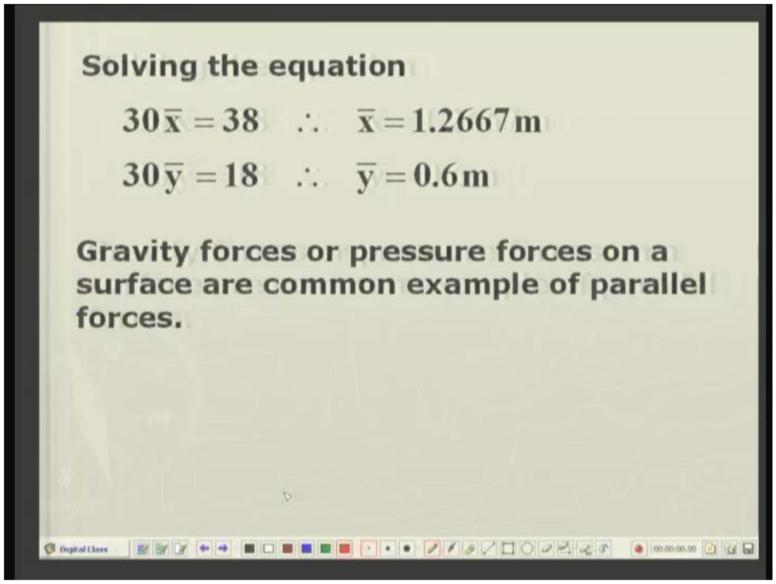

Solving these two equation, we get x bar is equal to one point two six six seven meters, y bar is equal to point six meters and hence we have replaced the given force system with the simpler one which consists of a single force parallel to the z axis but now passing through this new point x bar y bar.

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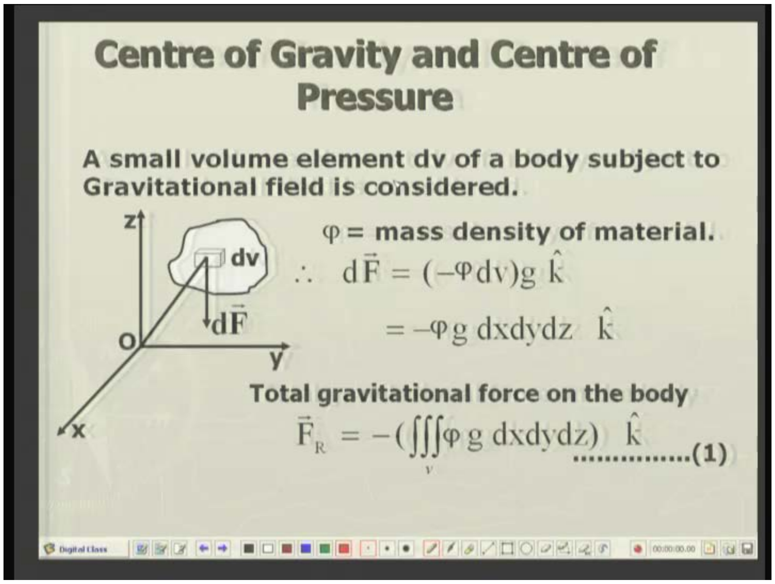

The application of this parallel force system is very useful namely in the calculation or in the determination of the centre of gravity as well as centre of pressure. Let us see. Suppose there is a body of arbitrary shape and size and we consider a small element of this body, dv phi is the mass density of the material. So each infinitesimal element is being pulled towards the center of the earth. This is the gravitational pull and gone by Newton's law of gravitational. So suppose we have this coordinate system x y z, z is the vertical direction. So the force of gravity will be in the minus z direction. So an infinitesimal element of the body of volume dv will have a force acting on it parallel to z axis in the negative direction as phi dv because mass density into the volume will give me the mass of this infinitesimal element times g the acceleration due to gravity and minus k is direction. So dv can be written as dx times dy times dz. So the elementary force on this infinitesimal element is minus phi g dx dy dz in the k direction. The total gravitational force on this body will be obtained by summing up all these parallel forces on infinitesimal elements and in the limit it will be replaced by volume integration. So the equation1 gives me the total resultant force on this body.

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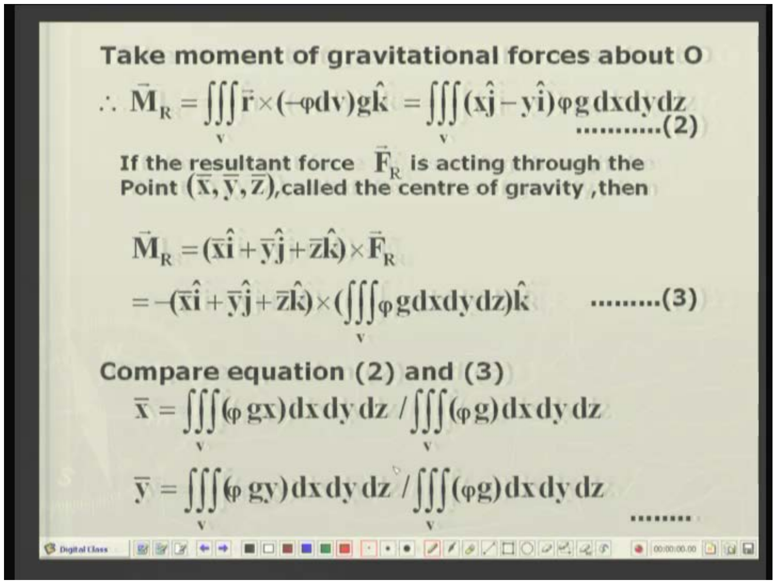

Take the moment of the gravitational forces on each and every infinitesimal element about point O which is the original of the coordinate system. So r crossed with the elementary force. When we integrate it, it gives me the resultant moment. Now we have MR and FR. This can be replaced by a single force. Now how can we do it? m the resultant moment should be equal to the resultant force passing through the point x bar y bar z bar. So it means, MR is equal to minus, I am taking out this negative sign from here, so, minus xi plus yj plus zk crossed with this resultant force and when I substitute MR over here and the right hand side again, we will have a vector equation equivalent to two scalar equations and from there, we will get x bar and y bar which is given by volume integral phi gx dx dy z dy dz divided by phi g dx dy dz. So similarly, y bar is given by this expression.

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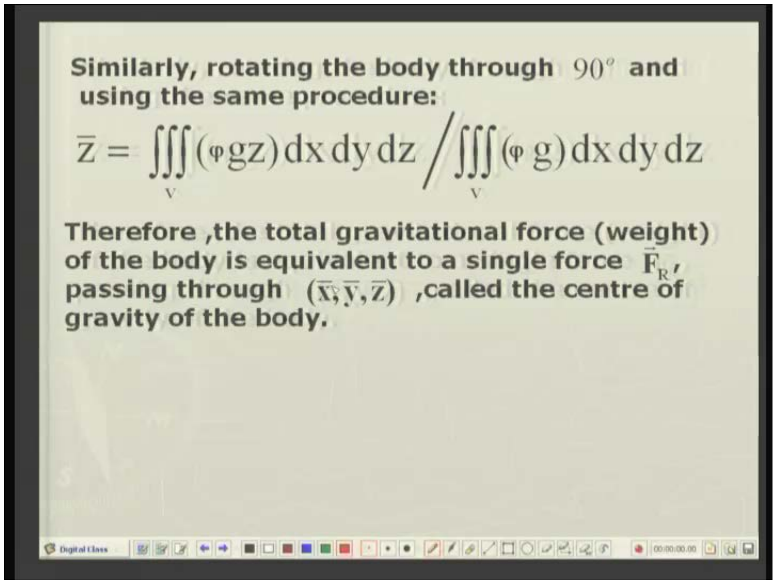

Now suppose I turn the body through ninety degree and do the same operation, then I can, in a similar manner, find z bar which is phi g z bar dx dy dz integrated over the entire volume divided by the magnitude of the resultant force. So we have obtained this x bar, y bar, z bar. The point through which the resultant gravitational force is passing, such a point is called the centre of gravity of the given body. On similar lines, we can consider the pressure acting on a plane surface. The concept of center of gravity was for the volume. Now volume is the three dimensional manifold. For the surface, which is a two dimensional manifold, we have, similarly, the concept of the centre of pressure.

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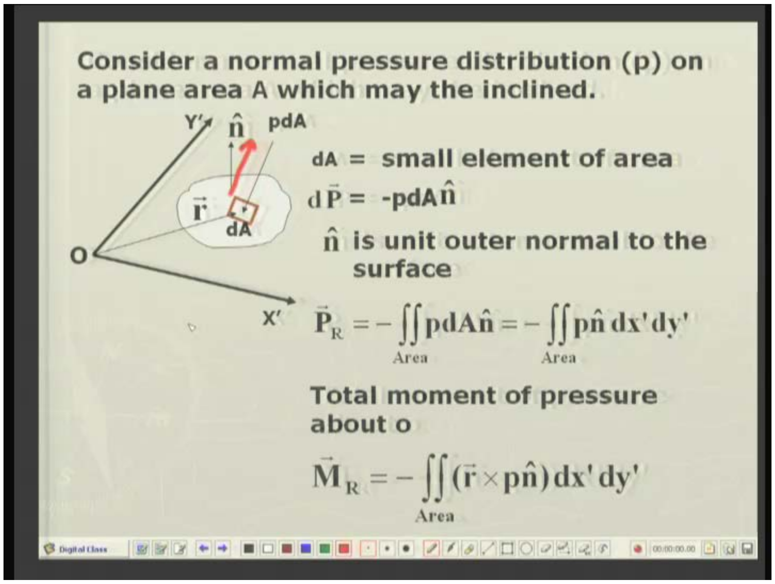

Suppose here, it is an inclined surface. It need not be horizontal. Let us say, it is inclined at a certain angle to the horizontal surface and in that plane of the surface, I have x dash, y dash as coordinate axis and in the same manner I consider a very small element of surface, da, which can be written as dx dash dy dash. Now the unit normal to the surface is the vector n and the pressure is acting on the surface. So, on this small element of surface, this pressure, which is force per unit area, p times da, that is, the area of this small element, is acting. Actually this unit vector should be in the negative. So it is in the negative of the unit vector, the pressure is acting. Pressure is compressive pressure. So it is in the opposite sign as to the outer normal. So dp is equal to p times da, that is, the magnitude times the unit vector in the opposite to the outer normal, that is, minus n unit vector. Resultant force of pressure is obtained by integrating over the entire area as given over here. So this is equal to minus p unit vector in the outer normal direction dx dash dy dash and we can shift these parallel system of forces. These forces are shifted to the origin by considering these moments as usual.

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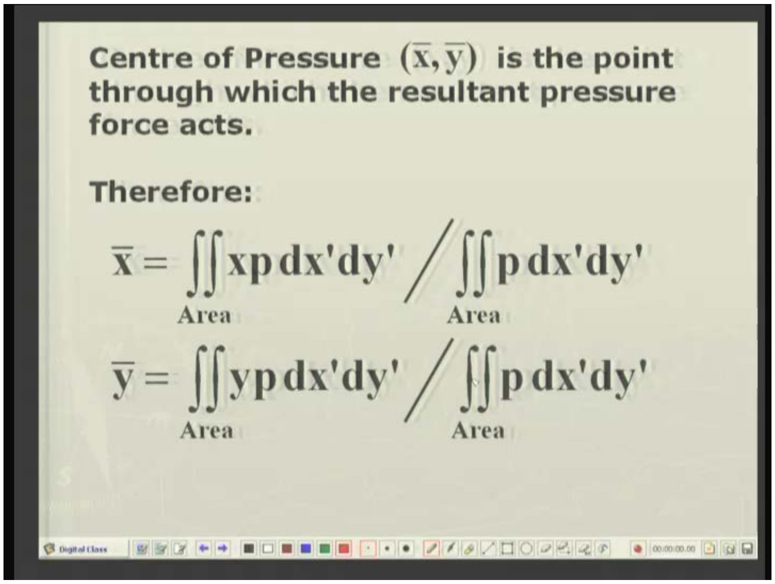

Let us define the center of pressure of x bar y bar as the point through which the resultant pressure acts. So carrying out the same analysis as for the centre of gravity, we see that x bar is equal to xp dx, that is, the area integral of the moments divided by the area integral of the pressure force, that is, the resultant magnitude of the resultant pressure force and similarly y bar is given by this expression.

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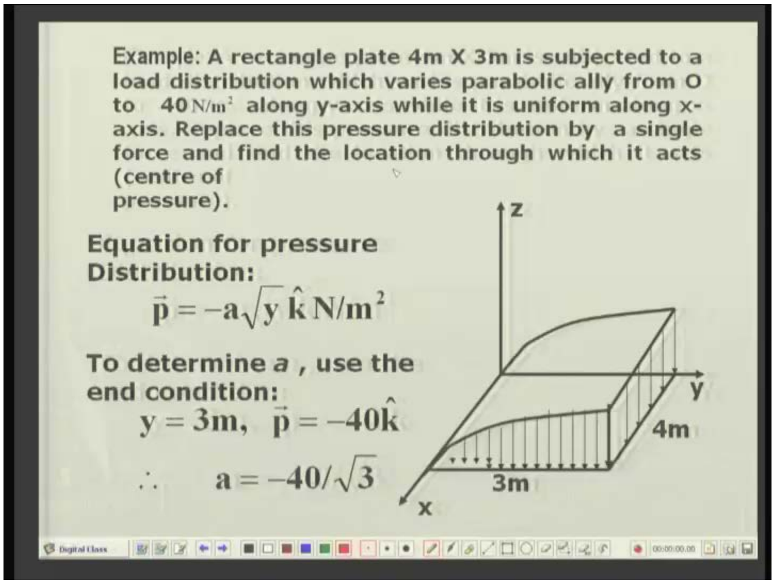

Let us consider one simple example, where there is a distribution of pressure, which pulls a plate of size three meter by four meter lying on the plane x y and z is normal to it. So there is a distribution of pressure, a parabolic distribution. So the rectangular plate of four meter by three meter is subjected to a low distribution which varies parabolically from zero to forty Newton per meter square. So it is a move. It increases in a parabolic manner, zero at this and forty at the other corner but it is uniform along the x axis. Along the y axis, it is increasing but it is uniform along the x axis. Replace the pressure distribution by a single force and find the location through which it acts and this location is called the centre of pressure. Okay.

Let us see. First of all, we will find out the pressure distribution in the proper vectorial manner because the force is acting downward and parallel to the z axis. So its unit vector is k and due to parabolic distribution, it is proportional to the square root of y and a is the constant of proportionality, which is to be determined from the boundary conditions. Well, we know that when y is equal to three meters the pressure is forty. So if I substitute y is equal to three meters in this equation and simplifying it, I can easily get a is equal to minus forty divided by root of three.

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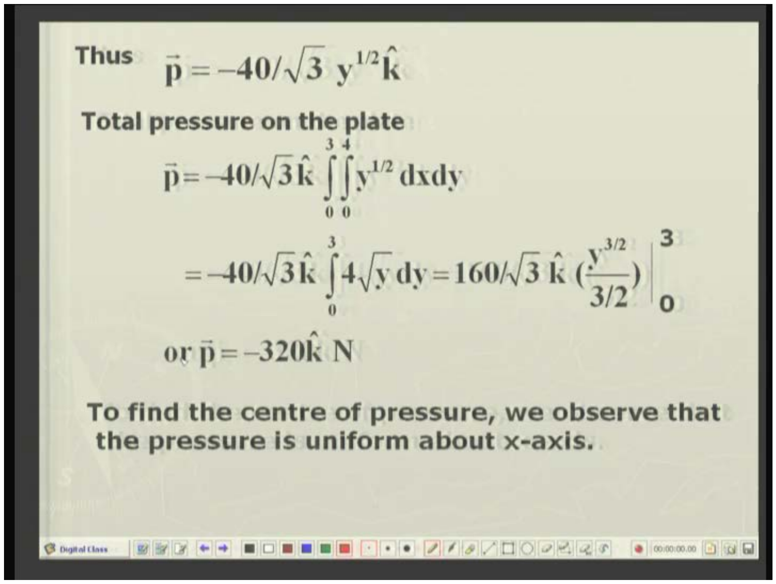

So the pressure in a vectorial manner is written as minus forty divided by root three into y under root in the k direction and if I integrate it over the entire plate, I get the total pressure force and this is obtained as minus three twenty Newton's in the k direction. To find the centre of pressure, we will take the moments and do the analysis.

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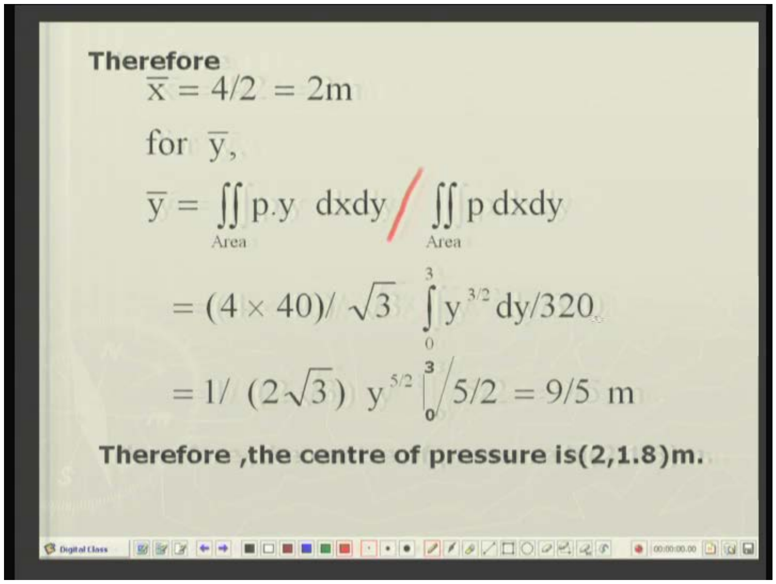

Well because the distribution of the force along x axis is uniform, rectangular distribution, the x coordinate of the centre of pressure will be the mid length of this rectangle, that is, x will be four by two. Always remember, for the rectangular distribution it is the center of the rectangular. So it is four by two, that is, two meter for y bar, that is, y coordinate of the center pressure. We take the moments about x axis. So py into dx y, that is, divided by this should be divided by the net force/ pressure force. So carrying through this is simple integration. The net force, you remember, is three twenty which was found earlier.

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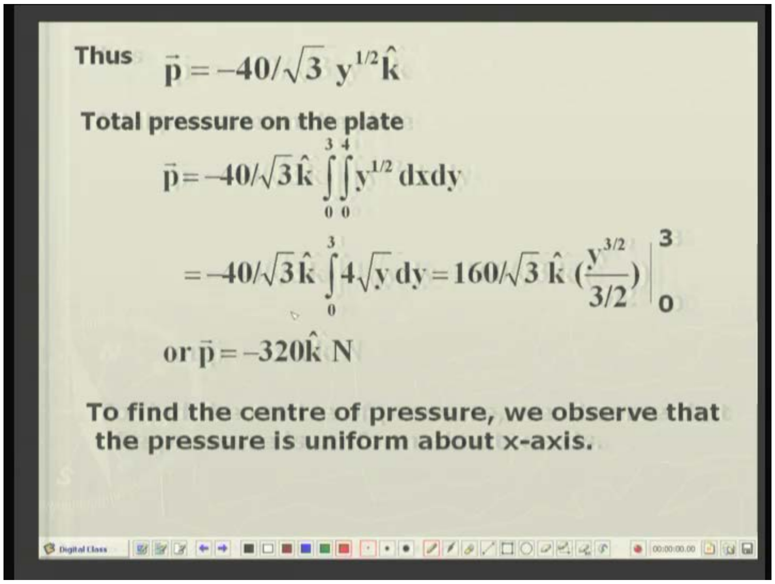

So in the denominator, we will have three twenty and the y coordinate will be obtained as nine by five meter. So the center of pressure is two meters and one point eight meters. Two meters on x axis, one point eight along the y axis. So I will close the today's lecture here. In the next lecture, we will be considering the equilibrium of forces.

Thanks very much.