Applied Mechanics
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Lecture No. 2
Vector Analysis (Contd.)

Lecture 2 is a continuation of the topic of vector analysis. You may recall that we were discussing vector algebra and, in particular, the multiplication operation for vectors. We had discussed scalar product. We had discussed the product of a scalar quantity with the vector as well as the scalar product of two vector quantities, also called the dot product. In this lecture we will begin with the vector product of two vectors also called the cross product.

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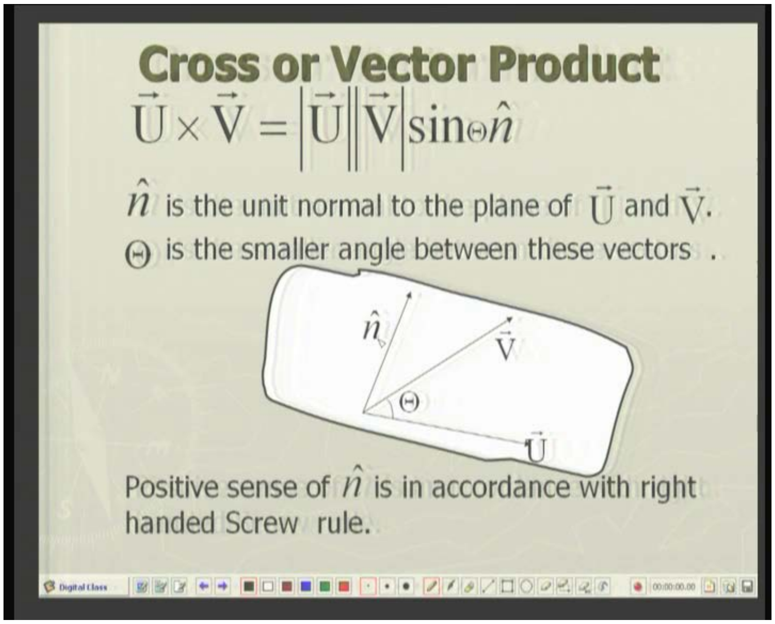

Let there be two vectors quantities, vector U and vector V. So we define the cross product of U and V namely U crossed with V as the product of the magnitude of U magnitude of V and sin of the angle between U and V. Again the smaller angle is considered. So sin theta. Now, since the this product gives me a vector quantity, the question naturally asked is, which is the direction of this quantity. The direction is obtained as follows. A unique plane can be passed through vector U and V as shown in this figure over here. This white is the plane containing both vector U and V and the normal vector to this plane, that is, vector n, is a unit vector which is simultaneously normal to vector U and vector V. Now the positive direction of n is obtained by the right hand screw system as we did for the coordinate axis, you may recall. So if I rotate the screw in the direction from U to V, that is the anticlockwise direction, then the right handed screw will move in the positive direction of n. So this is how you can obtain the positive sense of the unit vector n.

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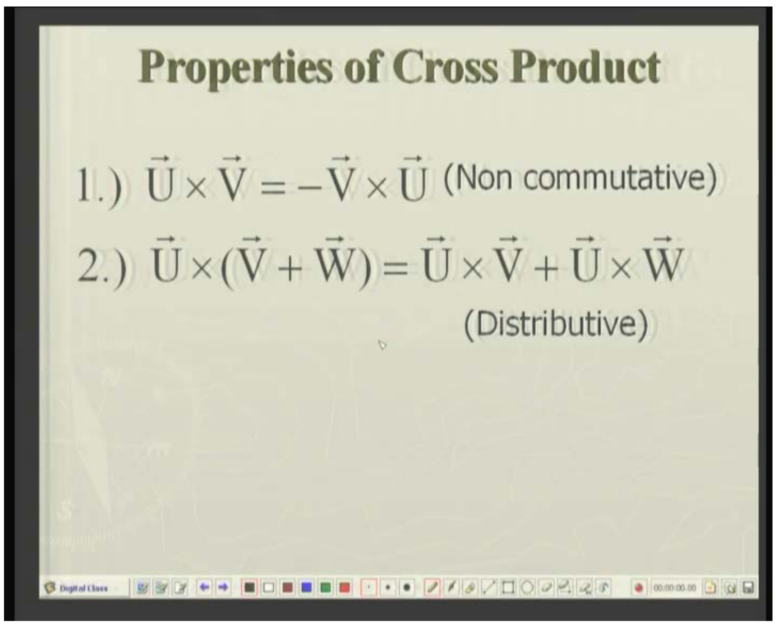

Now, there are certain properties associated with this vector product or cross product namely vector U crossed with vector V is no longer equal to vector V crossed with vector U, as we saw in the case of dot product. In this case, it is a negative quantity. So U crossed with V is equal to minus or V crossed with U, that is, it says non-commutative law. This is easy to understand because by the right hand screw system U crossed with V. So we rotate the screw from U to V. When you consider V crossed with U, you rotate the screw form V to U, so opposite direction. So hence it is a negative quantity. Well the distributive law is still valid. U crossed with V plus W, that is, the sum of V and W is equal to the sum of the product of U crossed with V and U crossed with W, that is, you first multiply individually and then add or you first add V and W and then multiply U with it. So this is the distributive law.

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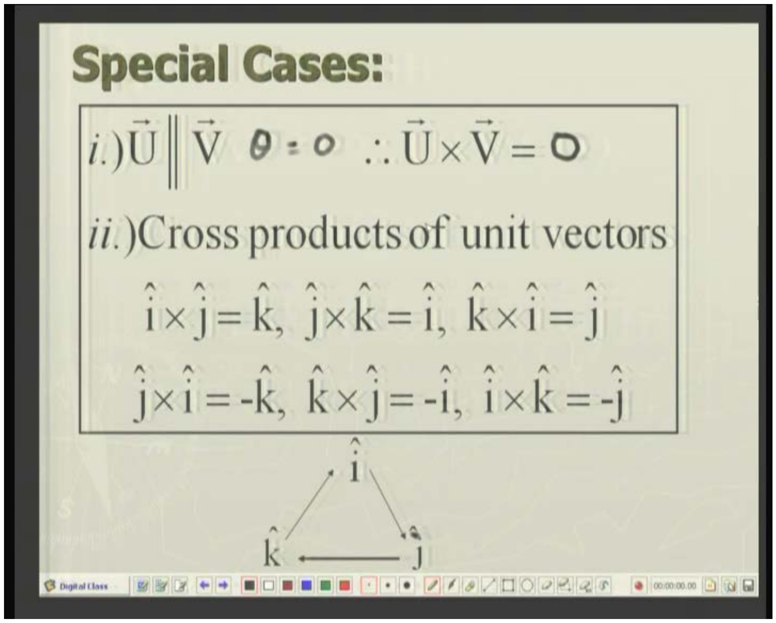

There are some interesting special cases of the cross product, namely, when U is parallel to V. Two vectors are parallel and their cross product is taken, what does it imply? When there are two parallel vectors the angle between the two, that is, angle theta will be equal to zero and sine of zero is also zero. So, obviously U crossed V is equal to zero. Now we take the cross products of individual unit vectors. Suppose I consider two unit vectors, i and j, that is, along x axis and y axis. You can easily see, let's say, this is the x axis, this is y axis and this is the z axis. So i crossed with j will give me the k vector, j crossed with k will give me the i vector, k crossed with i will give me the j vector. So i j k. So i crossed with j will give me k, j crossed with k will give me I, etcetera. If I reverse the operation, that is, j crossed with i, this will give me the negative quantity minus k vector and so on and so forth. So you can see in this operation, in a very simple way, i crossed with j is giving me k, j crossed with k gives me i, k crossed with i gives me j and so this is going in the clockwise direction. If I go in the opposite direction, that is, i crossed with k, which will give me minus j, j crossed with i will give me minus k, etcetera. So depending upon the order of vector multiplication, the unit vector is obtained.

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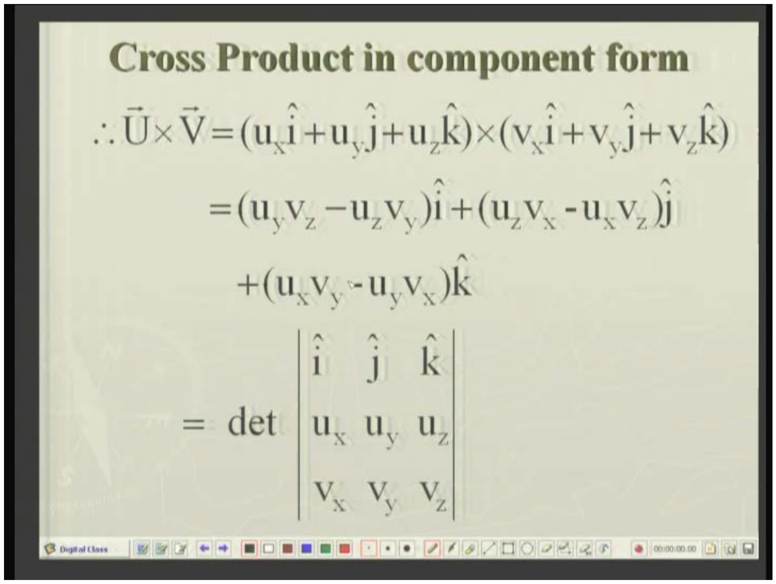

Let's see the cross product in the component form. The two vectors are U and V whose components are u_x, u_y, u_z and v_x, v_y, v_z respectively. So I multiply these two vectors now. Let me take the first term i a u_x. When I multiply with i unit vector with i unit vector we have seen the cross product is zero, i with j will give me k, i with k will give me minus j. So in this way, if next I take the second term, then third term and if you collect all this terms, you will get U crossed with V is equal to u_y v_z minus u_z v_y into unit vector i. Similarly u_z v_x minus u_x v_z multiplied by the unit vector j, etcetera. And this thing, this whole expression can be very easily checked to be equivalent to the determinant of this matrix and, which is this matrix, the first row is consisting of the unit vector i j k, second row is the components of the first vector and the third consists of the components of the second vector V. Okay. So if I carry out the determinant operation, that is, for finding out the determinant of this three by three matrix, the expression will be exactly as given here. Perhaps this expression is easier to remember than this. So you can easily check for yourself.

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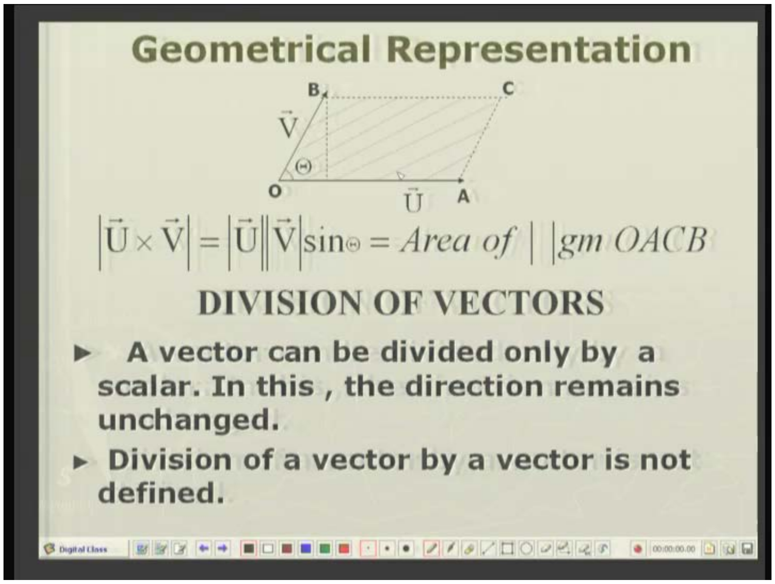

Let's now look at the geometrical representation of the operation of cross product. Here is vector U and this is vector V. So we have already seen, it is the magnitude of U and the magnitude of V times, the sin of the angle between the two vectors. So here is angle theta. You can see that this magnitude times the sin of this angle, so V times sine of this angle is this vertical altitude. So this gives me the area of the parallelogram O A C B which is equal to the base times the altitude. So the magnitude of U cross V is equal to the area of the parallelogram whose two adjacent sides are in the direction of U and V and the magnitudes are proportional to their magnitude. Okay. What is the direction of this? The direction of this cross product is simply the unit vector perpendicular to the plane of the parallelogram and again the sense is determined by the right hand screw system. Final algebraic operation for vector is the division of vectors. Well a vector can be divided by a scalar one. In this case the magnitude of the vectors gets divided by the scalar quantity but the direction of this quotient remains unchanged. So all you do is, you divide the magnitude and don't disturb the direction of the vector but the division of a vector by a vector quantity is not defined.

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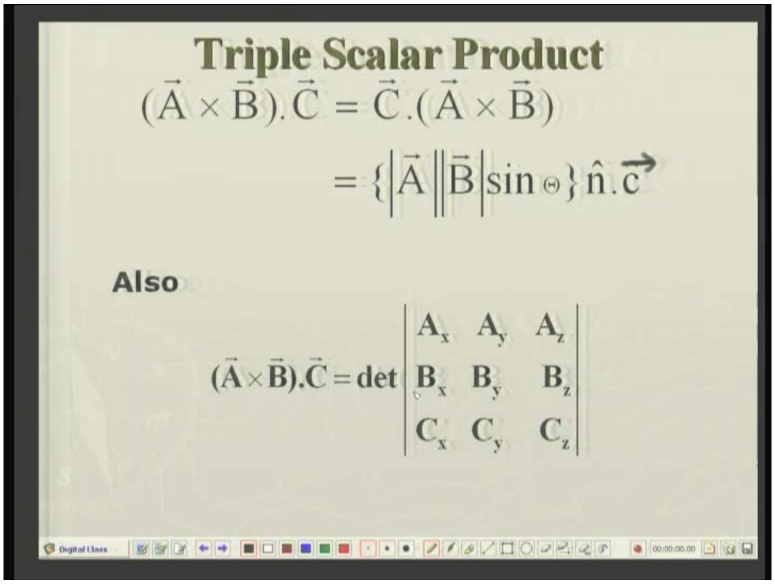

There is another type of a product which is sometimes encountered in mechanics, namely, the triple scalar product. Well it consists of first a vector product between two vectors A and B. For example, A crossed with B and this product is then dotted with vector C. So A crossed with B dotted with C is equivalent to C dotted with A cross B. This is due to the commutative law of dot product and magnitude wise, either this or these both are equal to magnitude of A times magnitude of B times sin theta. So this is a cross product of A and B and the direction of this product is the unit normal n dotted with the vector C. Okay. So this is the final expression for the triple scalar product and in the determinant form you can easily check that this will be equivalent to determinant of the matrix consisting of the three vectors A B C written Ax. So the first row is A_x A_y A_z, that is, the three components of vector A, second row is three components of vector B and the third row is the three components of vector C.

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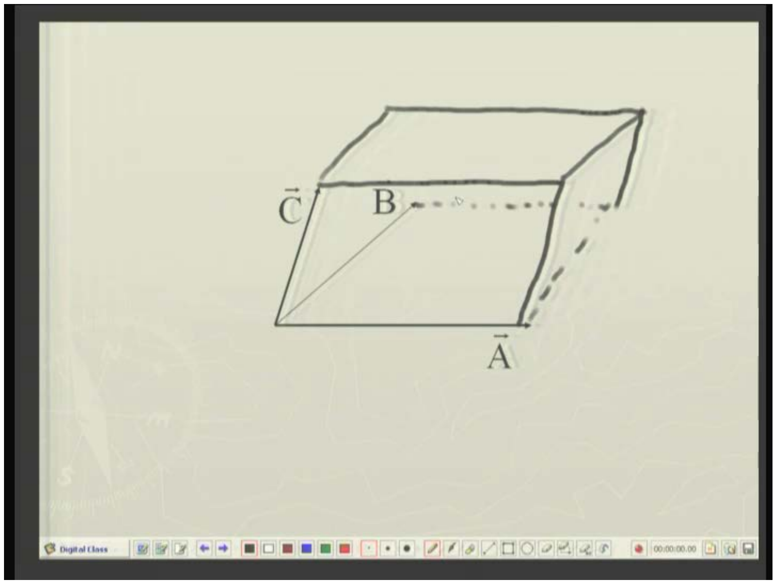

Well, as before, we will try to understand this triple scalar product graphically. Also here are three vectors A B and C. First, you will complete this parallelogram and then, you will complete this parallelepiped like this. So from C. So it makes a parallelepiped. The triple scalar product will be simply the volume of this parallelepiped. You compute the volume of this parallelepiped having three adjustment edges as A B C and you will find that the magnitude of this volume will be exactly equal to the magnitude of the triple scalar product.

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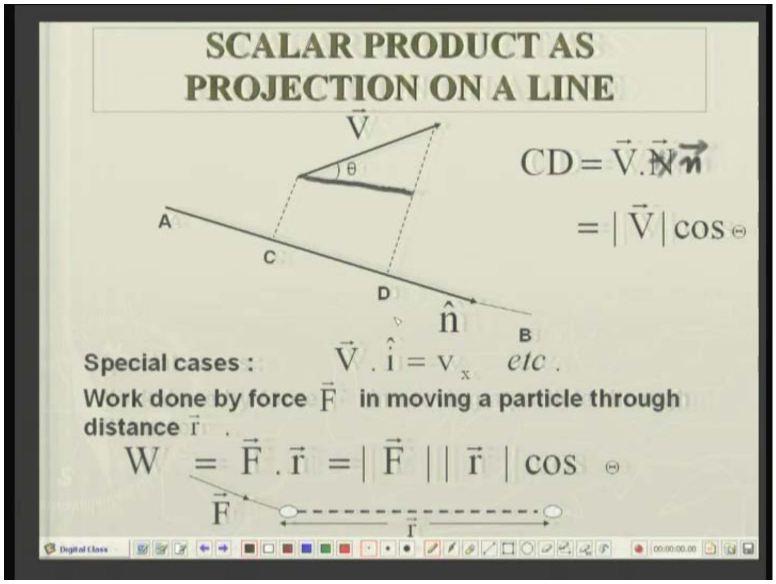

There is another geometrical interpretation for the scalar product, namely, it represents the projection of a vector on a line. For example, here is vector V and an arbitrarily given vector and an arbitrary direction of the unit direction in this is represented by unit vector n. Now if you draw the projection from the tip and the tail of this vector, let it be CD. So this length CD is equal to V dotted with unit vector n. n is the, let's say, it should be unit vector n. Okay. So this is equal to magnitude of V times the cosine of the angle between the vector V and the n unit vector. So this angle I have drawn. So this line is parallel to the unit vector n. Okay. So as special case, we can take the projection of vector V on the unit vector along x axis, that is, i vector, which will give me the component of V in the x direction V_x. Similarly if I take the projection of V on y axis, that will be V dotted with j. So this will give me V_y and similarly V_z. The application of this concept is in the determination of work done by wave force. You must have learnt in your earlier mechanics course, perhaps in physics or elsewhere, that is, the work done by force in moving a particle from position one two position two, that is, along a vector r is equal to magnitude of force time the distance time the angle between the force and the direction of movement, that is, magnitude F times, magnitude r times cosine of angel theta. So the angel theta is the smaller angel between the two lines. This is angel theta.

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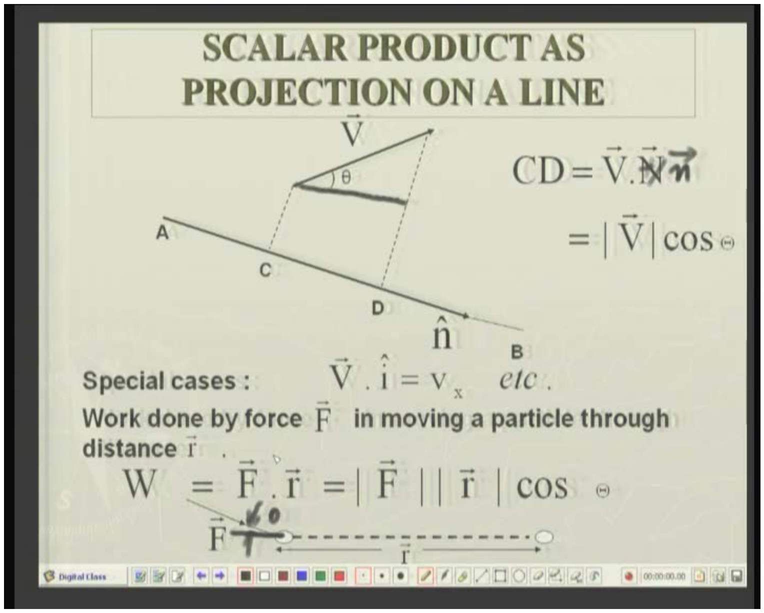

Okay. So this scalar product as a projection of line is interpreted as a physical quantity, namely, the work done.

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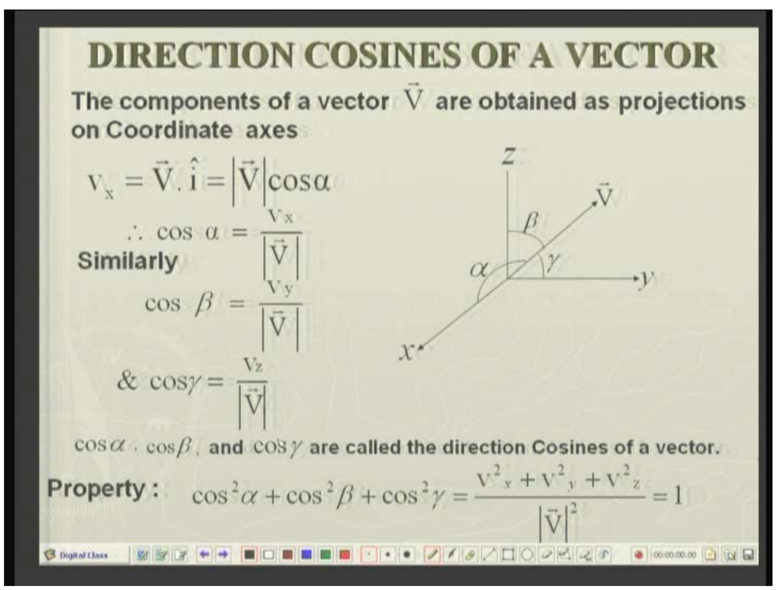

Now another useful concept associated with vectors is the direction cosine of cosines of a vector quantity. The components of vector V are obtained as projections on coordinate axis. For example, I have just shown you that V_x is equal to the projection of V on the unit vector i, that is, V times cosine alpha. So alpha is the angle which vector V makes with the x axis. What you should do is that, you pass a unique plane with vector V and x. Okay. You can always draw a single plane passing through two straight lines and you measure angle alpha in that plane.

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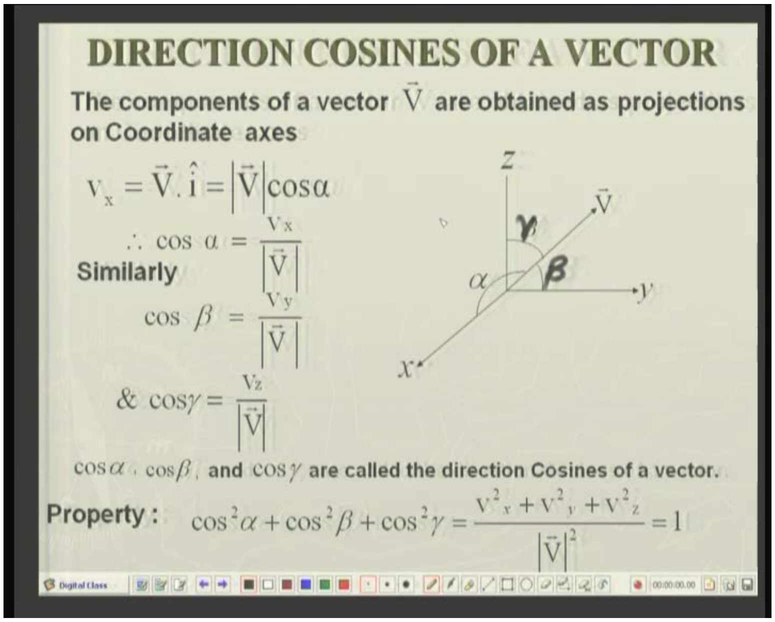

Similarly when you come to angle cosine beta, well then this is the angle between vector V and the y axis and this is the angle between vector V and the z axis. So each time you consider the unique plane containing the vector and the corresponding axis. So angle alpha is the angle between x axis and B, angle beta is the angle between y axis and V and angle gamma is the angle between V and z axis. Okay.

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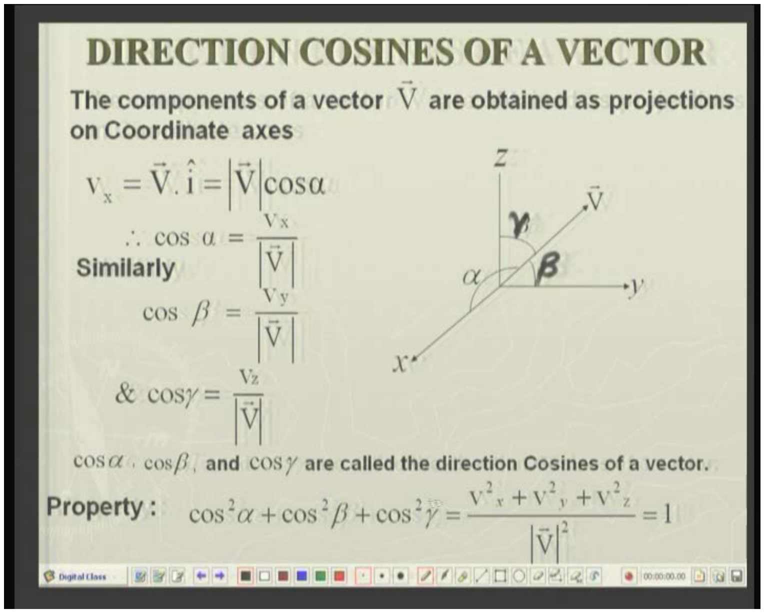

So we can show that the sum of the squares of the cosines, cosines square alpha plus cosine square beta plus cosines square gamma, is V_x square plus V_y square plus V_z square as given over here, divided by the magnitude of vector V squared and this is obviously equal to one because the numerator and the denominator are identical terms.

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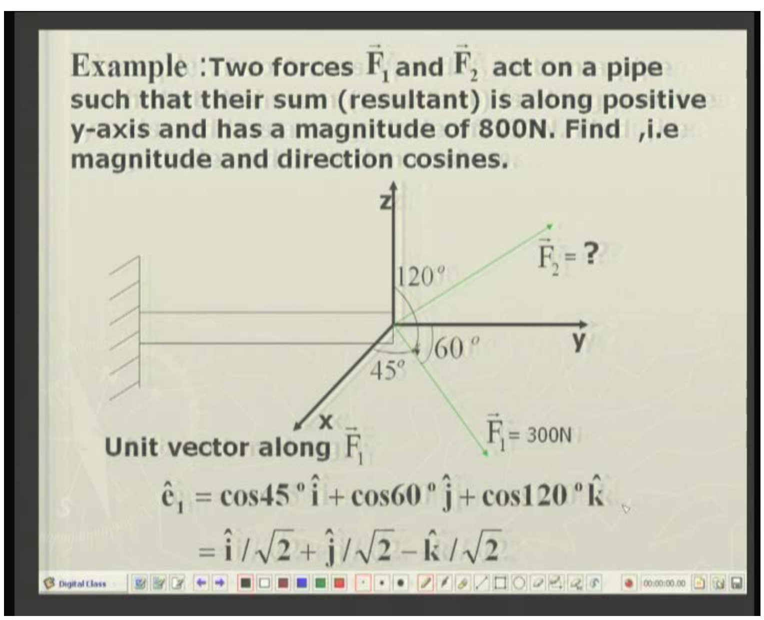

Let's take up an example to fix our ideas about the direction cosines. Suppose there are two forces F_1 and F_2 which act on the end of a pipe as shown over here. This is a pipe which is fixed at one end and at the other end there are two forces F_1 and F_2. F_1 is given in magnitude as well as in its orientation. Magnitude is three hundred Newton and orientation is obtained by three angles, which the force vector F_1 makes with x axis, y axis and z axis, namely, the angle forty five degrees, sixty degrees and hundred twenty degrees. Okay. We want to find out point F_2, that is, its magnitude and direction cosine, such that, the sum of the vectors F_1 and F_2 is along y axis. So the sum or resultant is along the positive y axis and has a magnitude of eight hundred Newton. So both the magnitude and direction are given. Well, to solve this problem, let's first find out the unit vector along the force F_1. Well, cosine of alpha is cosine forty five degrees. So it will be cosine forty five degrees unit vector i, cosine sixty degree unit vector j, cosine hundred twenty degree unit vector k. So well, you know the cosine of one forty five degrees is one by root two, so i unit vector root two divided by root two plus j unit vector divided by root two and cosine of forty five, hundred twenty degrees is equal to negative of root. This is one by two. So this will be minus. Please, sorry for the mistake. So it is one by two. So there is no under root sign over here.

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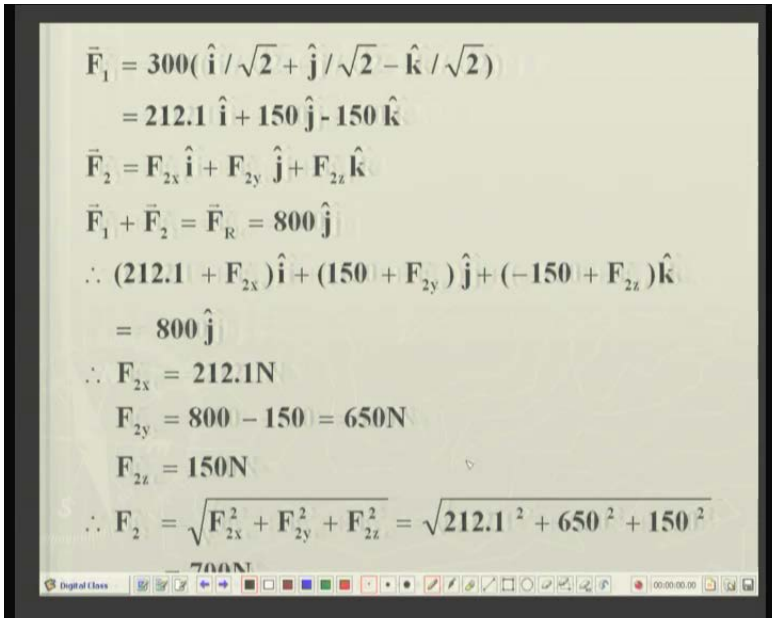

So you can easily see that this will be also equal to, there is mistake over here also, so you can find out F_1 is equal to two hundred twelve point one unit vector i plus hundred fifty, that is, three hundred by two unit vector j hundred fifty unit vector k. F_2, since we don't know its magnitude, we know only it in the component form, which will be F_{2x} i plus F_{2y} j plus F_{2z} k. So these are the three components. We know that the sum of these two vectors F_1 plus F_2 is equal to the resultant force and this is given as eight hundred in the positive y direction, that is, unit vector j. So we get the vector equation two hundred twelve point one plus F_{2x} unit vector i hundred fifty plus F_{2y} unit vector j minus hundred fifty plus F_{2z} unit vector k equal to eight hundred j. Alright. Now this vector equation is equivalent to three scalar equations, that is, compare the i components with i components which is zero on the side, j components with j components and similarly this will be equal to the k components with k components. So you can easily write this as, eight hundred j plus zero i unit vector plus zero k unit vector. Okay. So first we will compare the i components, that is, two hundred twelve point one plus F_{2x} is equal to zero. So this will give me F_{2x} equal to minus two hundred twelve point one Newton. Similarly, second will be ah eight hundred j. Eight hundred is equal to one fifty plus F_{2y}. So F_{2y} will be equal to eight hundred minus one fifty. So it is six fifty Newton and finally F_{2z} minus hundred fifty is equal to zero. So F_{2z} is equal to hundred fifty Newton.

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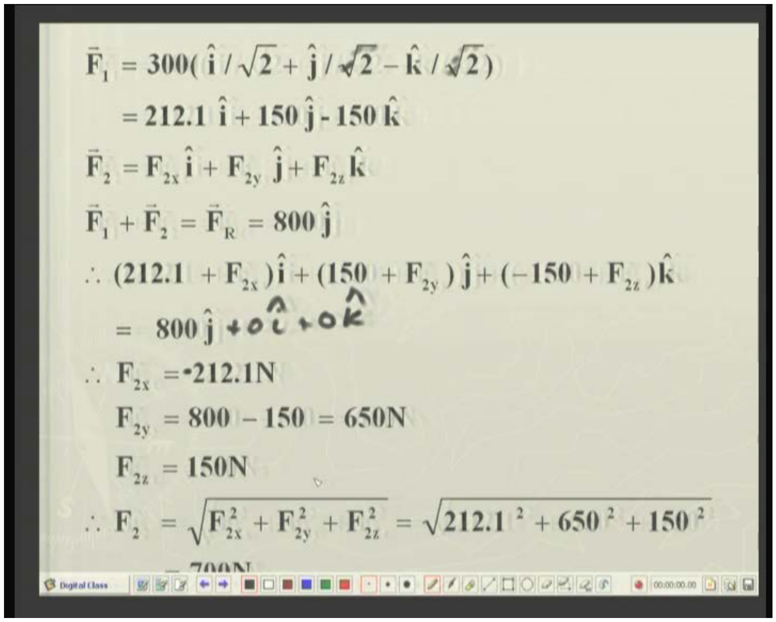

And if I find out the magnitude of force F_2 which is equal to the x component square plus y component square plus z component square whole under root, this will be two hundred twelve point one square plus six fifty square plus hundred fifty square. This will give me seven hundred Newton.

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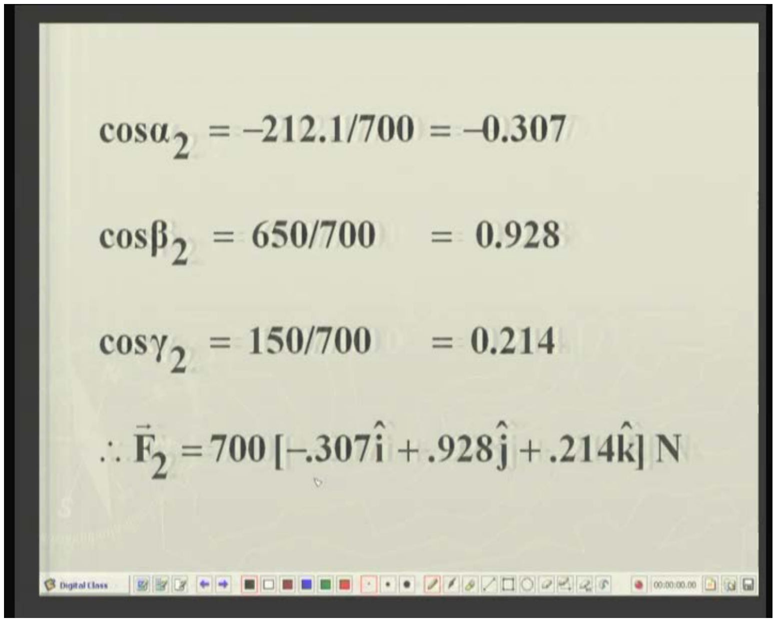

Now we find the individual directions cosines. Individual direction cosines are cosine alpha_2 which is equal to ah F_{2x} divided by the magnitude of F which is equal to minus two hundred twelve point one divided by seven hundred which is obliviously equal to minus point three zero seven and similarly for cosine beta, cosine gamma. So we will have F_2 as the magnitude times cosine alpha I, plus cosine beta j, plus cosine gamma k. So seven hundred into minus point three o seven i point two nine eight j point two one four k. So this is the final expression for the force F_2.

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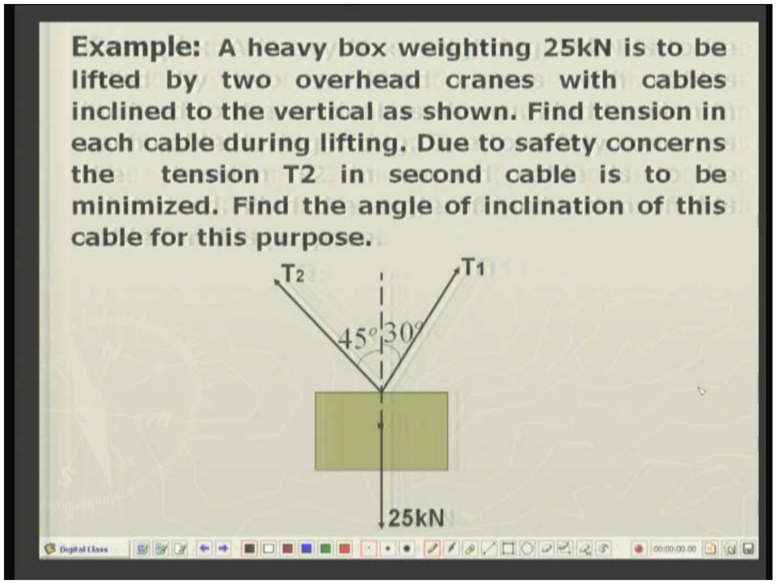

We will take up another example. A heavy box weighting twenty five kilo Newton is to be lifted by two overhead cranes with cables inclined. So in a factory situation you have a heavy box which is being picked up by two overhead cranes. So the cable for one is inclined at thirty degrees to the vertical and the second cable is inclined at forty five degrees to the vertical. So find tension in each cable during lifting operation. And the second part of the problem is that due to safety concerns the tension T2 in the second cable is to be minimized, otherwise it may break and there can be an accident. So find the angle of inclination of this cable for this purpose. So here is a box, here is a load acting downward, the weight, that is, twenty five thousand Newton's or twenty five kilo Newton's, and these are the two forces. So, for just lifting the box, just moving it up, the sum of these two vectors should be equal to exactly twenty five kilo Newton in this vertical direction.

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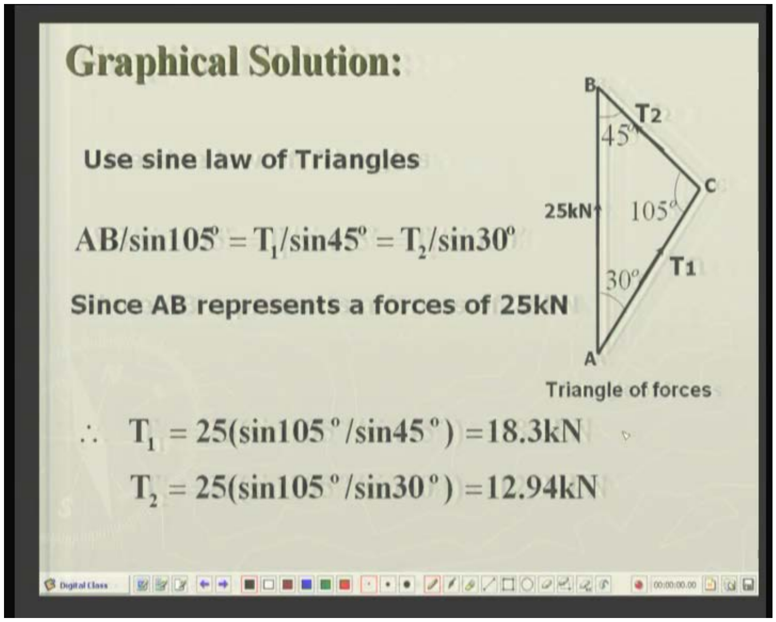

Well we can easily look at this problem in a graphical manner and we will use the triangle law of addition of two vectors. There is a vector T1 along the direction of the first cable, that is, at thirty degrees to the vertical and the second tension is at forty five degree to the vertical and both of them have to be equal to twenty five kilo Newton. So if we draw a triangle, first of all, we draw the vertical line of representing twenty five kilo Newton, may be, let's say, one centimetre is equal to five kilo Newton. So this is a five centimetre line vertical and along T1, from A we draw a line parallel to T one and from B a line parallel to T two wherever they intersect, that is the C point C, the apex of this triangle and you can easily see this angle at C will be equal to hundred and five degrees. And now you use the well-known, sin law of triangles. So A B divided by, this is A B, the opposite angle, that is, hundred and five degree. So A B divided by sine hundred five degrees is equal to T1 divided by the opposite sin of the opposite angle, sine forty five degree, T2 equal to T2 divided by the sin of opposite angle, that is, sine thirty degree. So we can easily see that from the first equation, because AB is twenty five kilo Newton that is given to us, T1 is equal to twenty five into sin of hundred five degree divided by sin of a forty five degree, which is eighteen point three one kilo Newton. T2 is equal to twenty five into sine of hundred five degrees divided by sin of thirty degree, that is, twelve point nine four kilo Newton.

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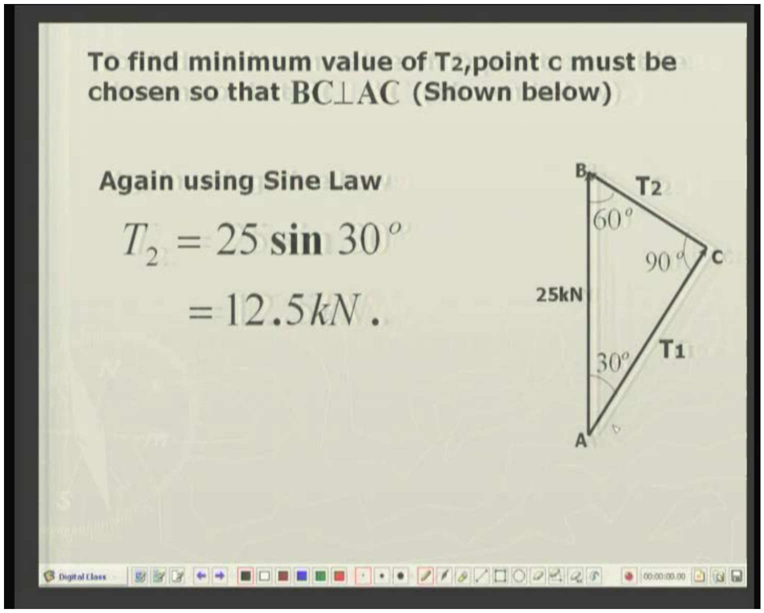

So both this cable tensions are easily obtained by a simple sine law of triangles and using the triangle law of addition of vectors. To find the minimum value of T2, it is a very easily understood graphically. Again, we have drawn the twenty five kilo Newton line, okay, and the direction of T1 is known to me. Of course, its magnitude is not known. Now, from point B, I will draw a line to represent T2, which will be the shortest distance between B and C. Okay.

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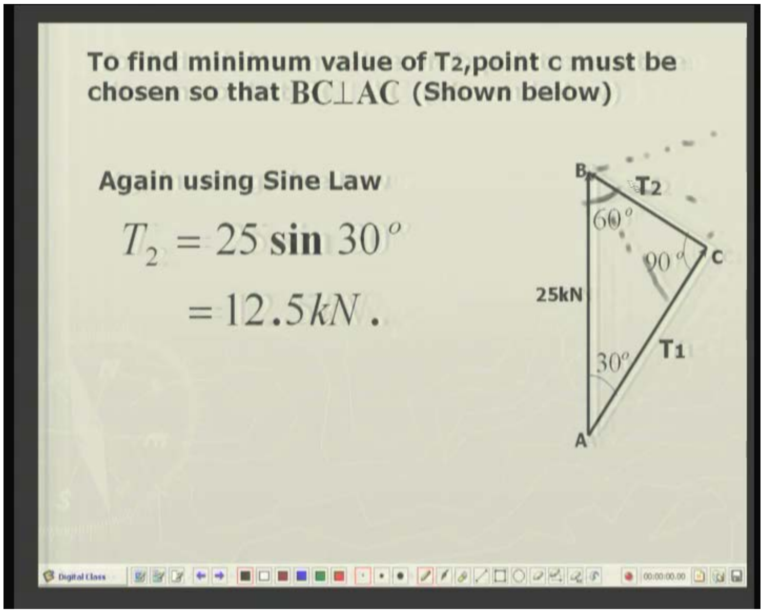

Let’s say, from B, this can be a one line. If I extend the line of AC, this can be another line, this can be another line, this can so. There can be several lines but which gives me the shortest distance? Naturally the line which is normal to line AC. So I will draw a perpendicular to line AC. So wherever it meets this direction of T1, that will define my points C. So once I have got thirty degrees and ninety degrees over here, this angle will be automatically equal to sixty degrees. Again you use sine law of triangles, you can find out that T2 will be equal to twelve point five kilo Newton and its direction will be sixty degree to the vertical. So this minimum tension in cable two is obtained when the inclination is sixty degrees.

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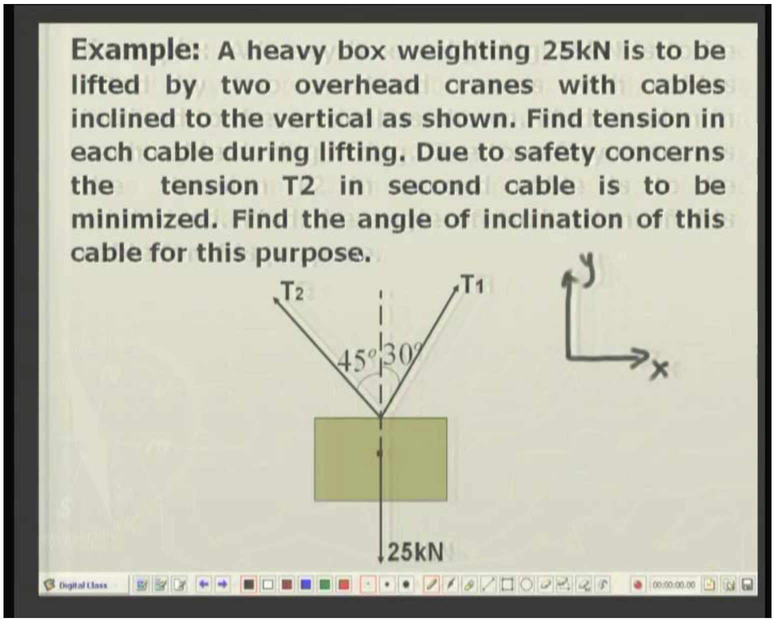

If we do the same problem analytically, well if I go back to this, you can easily see tension T1. Let's say, this is my x axis and this is my y axis. In all problems of this kind, first you should fix your axis. So this is my unit vector I, this is unit vector j. So if I resolve the components of T1, naturally it will be T1 cosine of sixty degrees in unit vector i plus T1 cosine of thirty degree unit vector j. But for T2 the i component is in the negative x direction, j component is still vertically up. So it will be T2 cosine of forty five minus i unit vector plus T2 cosine of forty five degree plus j unit vector.

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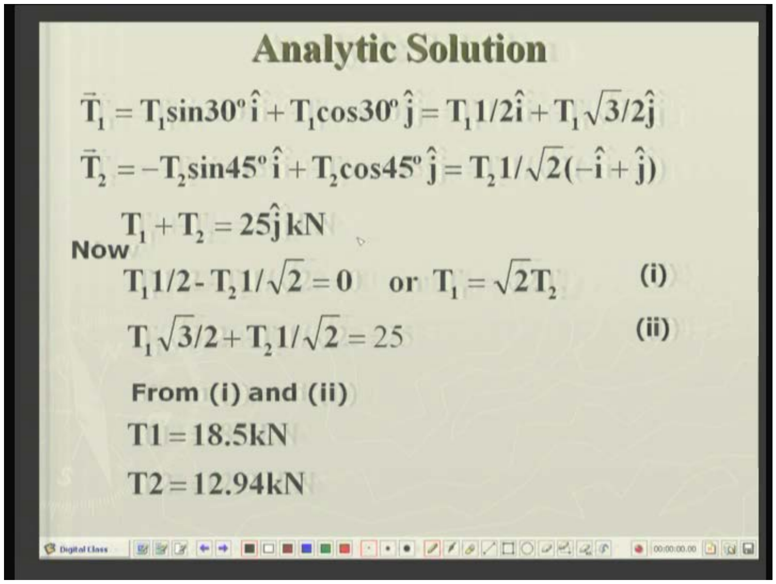

So accordingly we will have vector T1 is equal to the magnitude of T1, that is, simply, T1 divided by two unit vector I, that is, sin of thirty degrees and this will be T1 into sine of sixty degrees, that is, root three by two j. For T2, it will be, as I said, in the negative x direction. So minus T two sine forty five degrees minus T two cosine forty five degree. So these are the two component representations of the tensions T1 and T2. So sum of these two vectors T1 vector plus T2 vector is twenty five j kilo Newton. Now again, this vector equation is equivalent to two scalar equations and we can see that it will be T1 over two minus T2 over root two is equal to zero. So T1 is equal to root two times T2. From the second equation, you will find that T1 into root three by two plus T2 divided by root two is equal to twenty five. So solving these two equations for two unknown, simple, linear, algebraic equation, we will find T1 is equal to eighteen point five kilo Newton, T2 is equal to twelve point nine four kilo Newton. So same results, we had obtained graphically.

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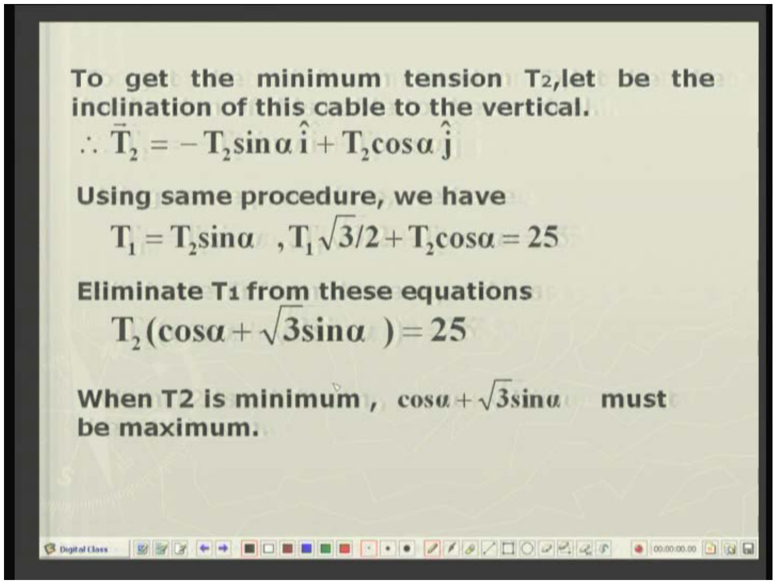

That is obvious but to find the minimum tension T2, there the procedure is slightly more involved. Suppose, when the cable T2 is inclined at angle alpha to the vertical, then the minimum is obtained. Then the vector T2 can be represented as, again, minus T two sin alpha because it is in negative x direction plus T2 cosine alpha in the positive j direction and we will again have a one vector equation equivalent to two scalar equations. So the equation will be T1 equal to T2 sine alpha and T1 root three by two plus T2 cosine alpha is equal to twenty five and from these two let us eliminate T1 which will give me this simple equation. Now to find out the minimum value of T2, naturally this term should be maximum because it has to divide twenty five. So when the denominator is maximum, T2 will be minimum. So the problem reduces to maximizing this expression, cosine alpha plus root three sine alpha.

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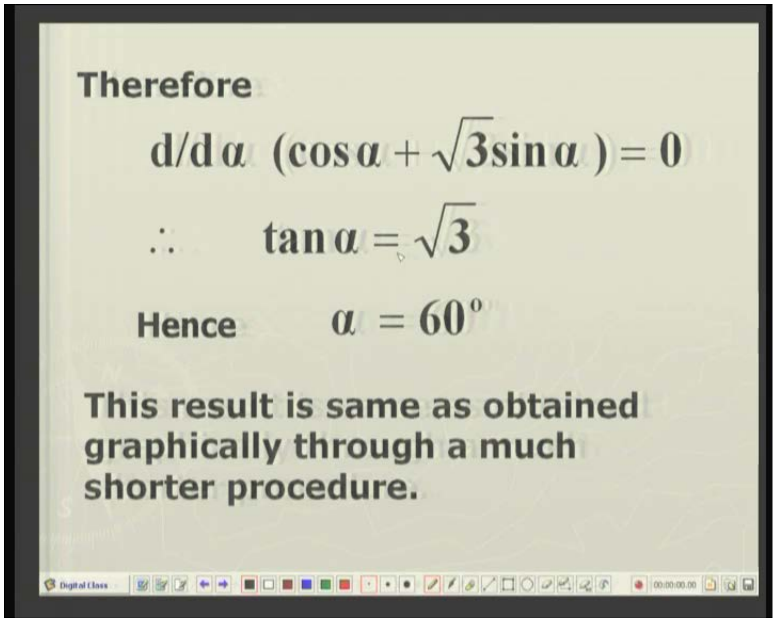

And this can be done by simple calculus, that is, the derivative of this expression should be equal to zero and when I take the derivative of this expression, I get tan alpha, which is equal to root three and hence alpha is equal to sixty degree. So again we come to the same conclusion, that when the second cable is inclined at sixty degrees to the vertical, the tension in that cable is minimized. Okay.

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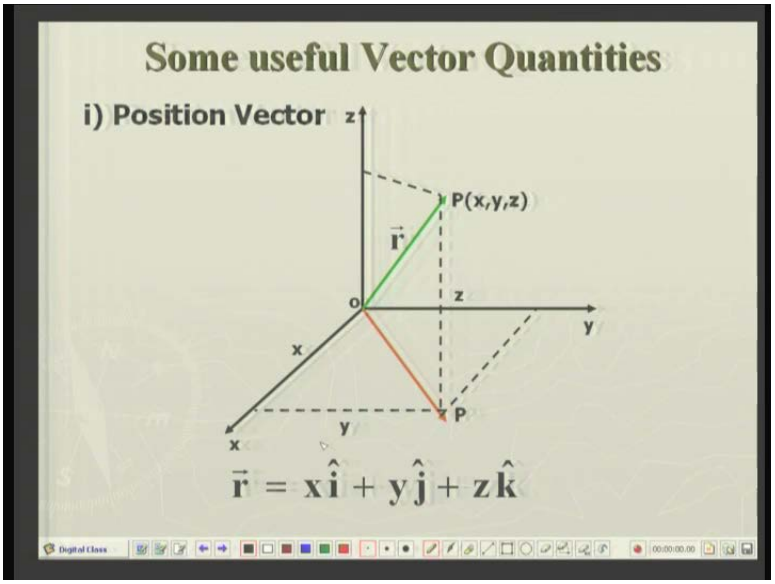

Now we come to some useful vector quantities, which will be encountered time and again during this course. These quantities are, first of all, the position vector. Suppose, there is a vector P and there is point P in three dimensional Euclidean space whose coordinates are xyz. Then the position of this point P with respect to the origin of the coordinate system is defined like this: You join this O with point P, then the vector, both in magnitude and direction, is called the position vector. So suppose I take the projections of vector r on x axis, y axis and z axis and we have already seen how take the projections graphically, that is, you project this vector r on z axis and one component on the xy plain. So this will be the projection along z axis and the red line will be projection in the xy plain and this projection is further projected on x axis and then on y axis. So this will give me quantity x, quantity y, this is x projection, y projection and this a your z projection and r is written as xi yj zk. So x times unit vector along x axis, that is, iy times unit vector jz times unit vector k.

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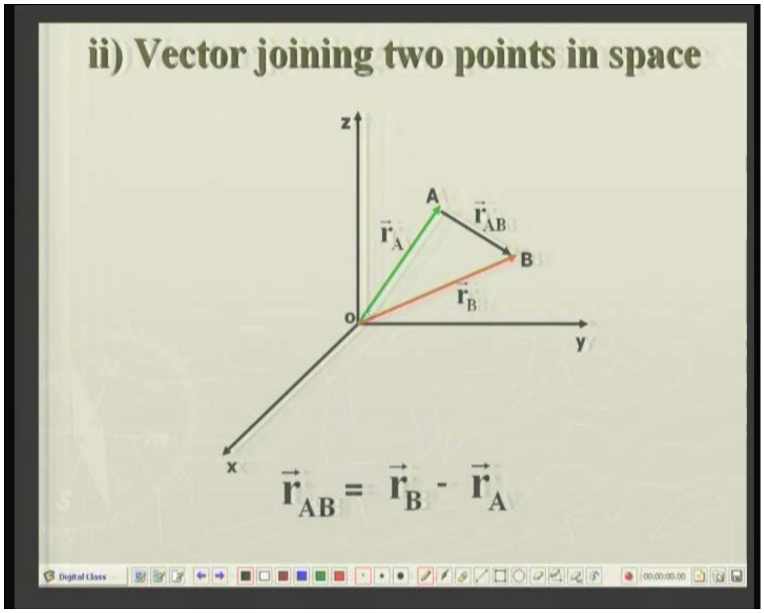

Suppose there are two points in this space, point A and B. So the unit vector of A is r_A vector unit vector B is the r_B vector. Okay. Then the vector joining A to B which I am writing as r_{AB} vector is equal to the position vector r_B minus the position vector r_A. Okay. So, in other words, the sum of r_A plus r_{AB} is equal to r_B vector. Okay. Green vector plus black vector gives me the the red vector and conversely the difference between these two, r_{AB}, is equal to r_B minus $r_{A.}$

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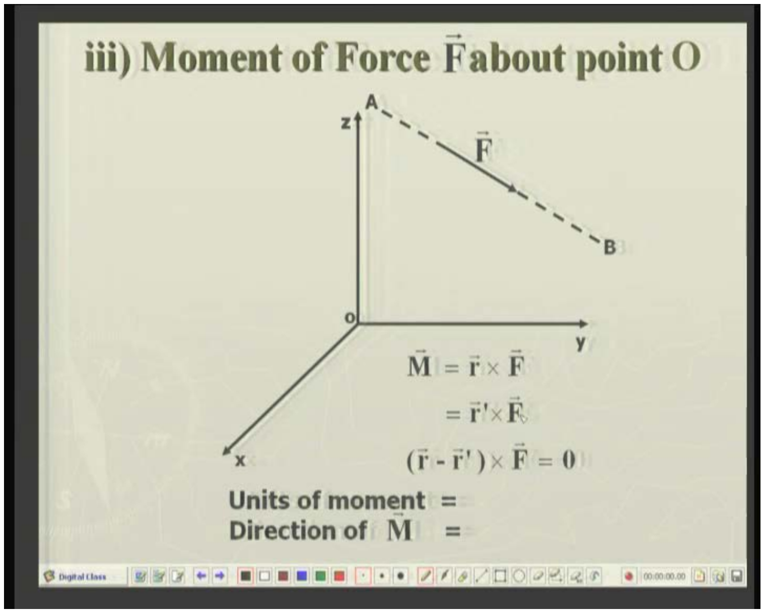

Let us now define a very important quantity associated with the force because this represents the turning action of the force. Force has two actions, one is push and pull and the other is the turning action. Now, suppose there is a force vector F, the line of action of this force is the line AB and the movement of this force about the given point O is defined like this: A movement M is equal to the cross product of the r and F. Okay.

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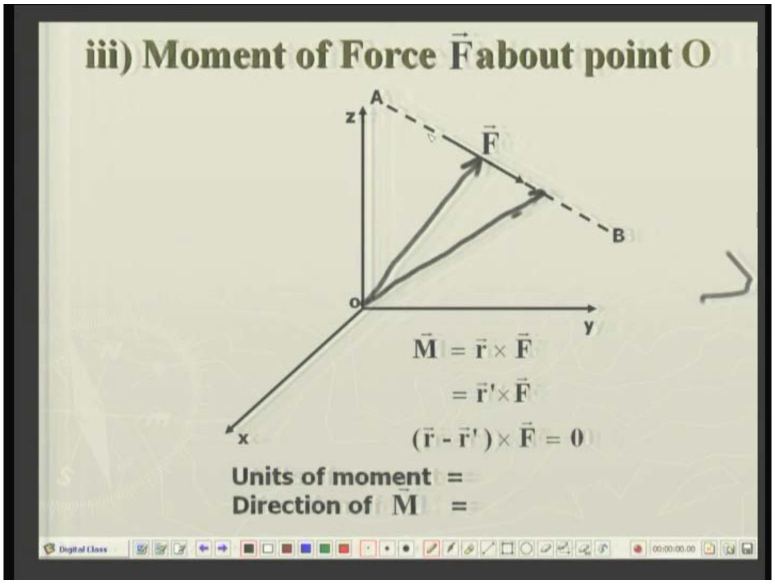

Now what is vector r? Vector r can be any point on the line of action of force F, whether I take this as a point or this as a point or this as a point. So as you can see, whether this as the point or this as the point. So these are the different three unit vectors. Sorry. Let me draw these again. Let us say, it can be this vector or this vector or any other vector. Okay. So whether I define this as vector r or vector r dash, it is r cross F or r dash crossed with F and you may be asking will it give me the same quantity, where, if I multiply with the r or r dash. Yes because we can easily see that r minus r dash vector will be a vector along the point joining these two tips. So it will be parallel to F. So r vector minus r dash vector will be parallel to vector F and the cross product of two parallel vectors, we have already seen, is equal to zero. Now this is the expression for your movement. What are its units? Well, force is in Newton's and the units of the position vector r are simply the length of the vector. So this is the Newton meters.

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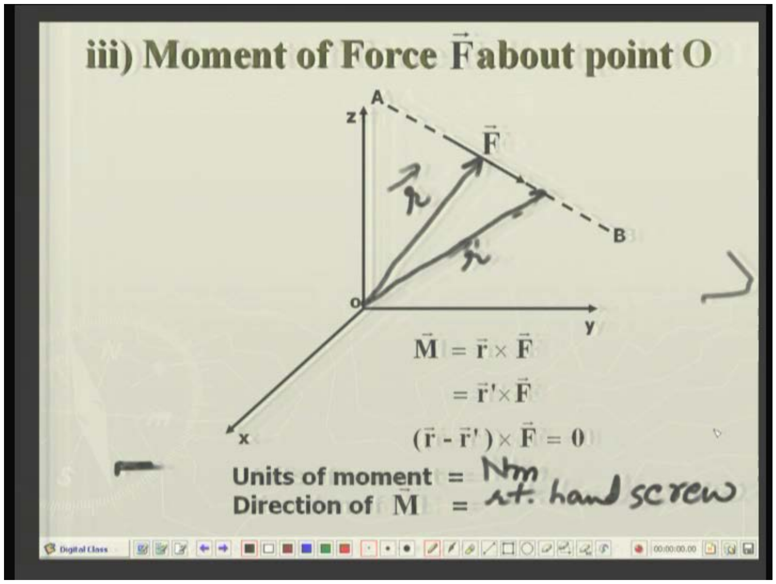

So the units of the movement are Newton times meter. It is a derived unit and what is the direction of M? Just as we fixed the direction of u cross v, that is, when we go from vector u to v and move the right handed screw. Accordingly, the direction of the movement screw gives me the direction. So r cross F means, if I take the unit vector r and unit vector F, you can easily see the movement of the screw will be into the plane of the screen. So you will determine the direction of the movement according to the right hand screw system. Okay. So we have seen both the direction, magnitude and the sense of the movement vector.

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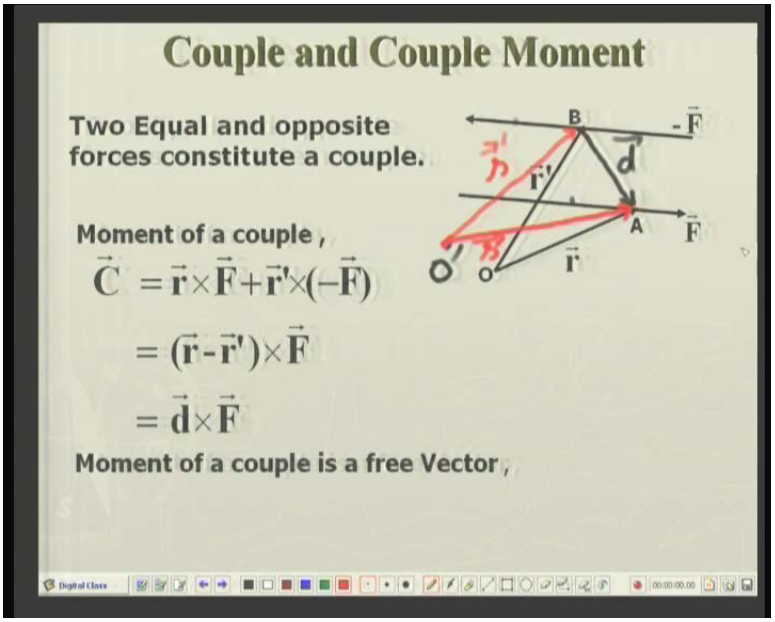

Suppose there are two parallel forces, force F and force minus F. Okay. That is, their directions are same their magnitude are same but the distance is opposite to each other. So one is force F and the other is force minus F. Suppose I choose an arbitrary point O in this space and I find out the movement of force F about O and then the force of the movement of force minus F about O. Add the two movements, so it will be, r crossed with F plus r dash crossed with minus F. So it is obviously r minus r dash vector crossed with F and this difference vector, you can easily see, is the vector joining A and B to A. So this is vector d. Okay. So the movement of a couple is defined as the vector joining these two points, arbitrarily taken, on the line of action of the F and minus F forces, crossed with the magnitude of that force. Now I make a very useful statement, that this movement obtained in such a manner is a free vector, that is, it is independent of the choice of O. First of all, we have seen the independent choice of A and B. I can chose B over here, A over here or B over here, A over here. All of them will give me the same magnitude, same value of the movement of the couple. And second independence of freedom is the choice of point. Suppose I choose this as O dash. Okay, now let us choose a different colour. Now from O dash again I choose point A and B. Same points because the choice is arbitrary. So why not choose the same points, B on force minus F vector and A on. So the corresponding vectors will be, let us say, I will call this as vector s dash and then i join with A. So this I will call vector S. Okay.

Now go through the same calculation. So it will be, S crossed with F plus s dash crossed with minus F vector. So again the difference vector between the S and S dash will be d. Okay. So will get the same value d crossed with F. So moment of couple is a very interesting quantity, that is, it is independent of the point about which the moments are taken, O and O dash, and for calculating this I can choose any two points on these two lines of action of the forces. So hence it is a free vector.

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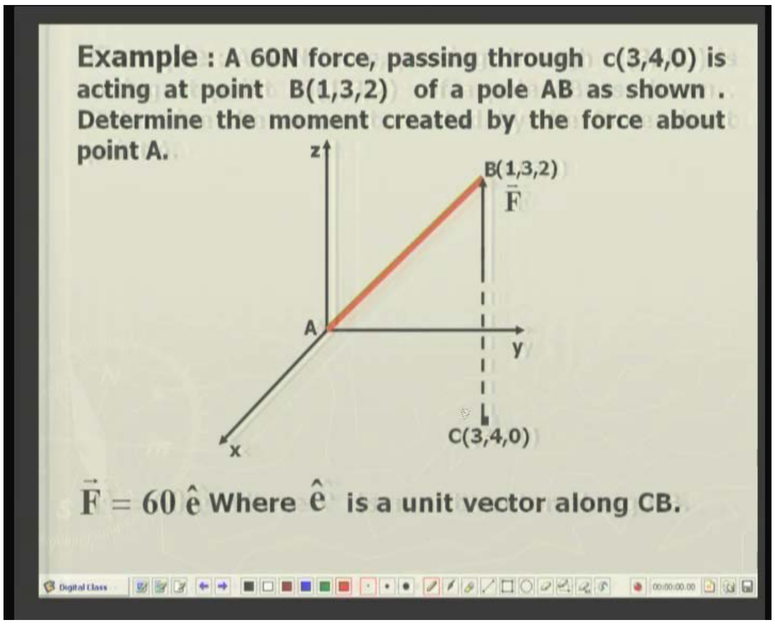

Let me consider one example, on how to take the moment of the force. For example, there is a force of sixty Newton, which is passing through the point c, whose Cartesian coordinates are three, four, zero and it is acting on the end of the rod AB. The one point of the rod is at the origin, the other is point B whose coordinates are one, three, two. So the line joining the point C to B is the line of action of force F and its magnitude is sixty Newtons. Determine the moment created by the force about point A. Okay. First of all, we will represent the force, magnitude is sixty time the unit vector along the line of action, that is, e the unit vector which is along line CB.

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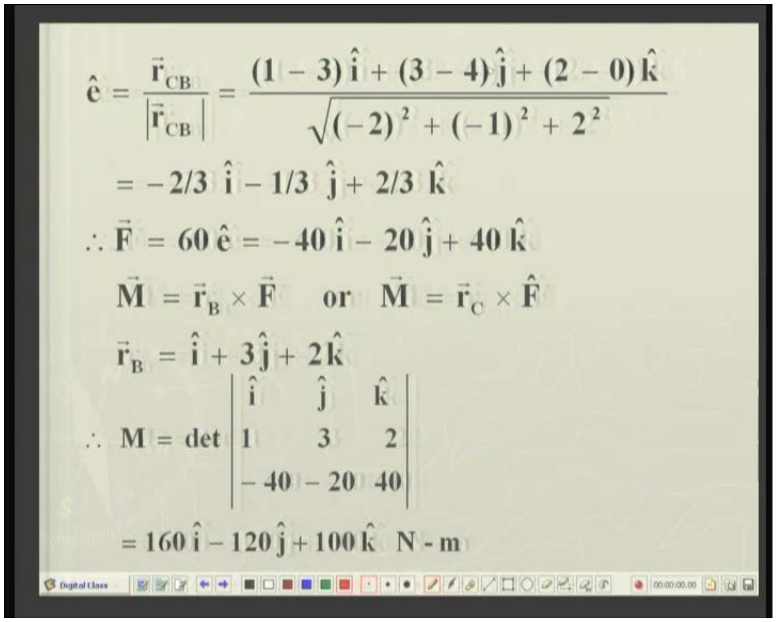

Well the line joining C to B is the vector Rcb, divided by its magnitude, which will give me, obviously, the unit vector and the vector rCB will be the difference between the corresponding coordinates, one minus three.

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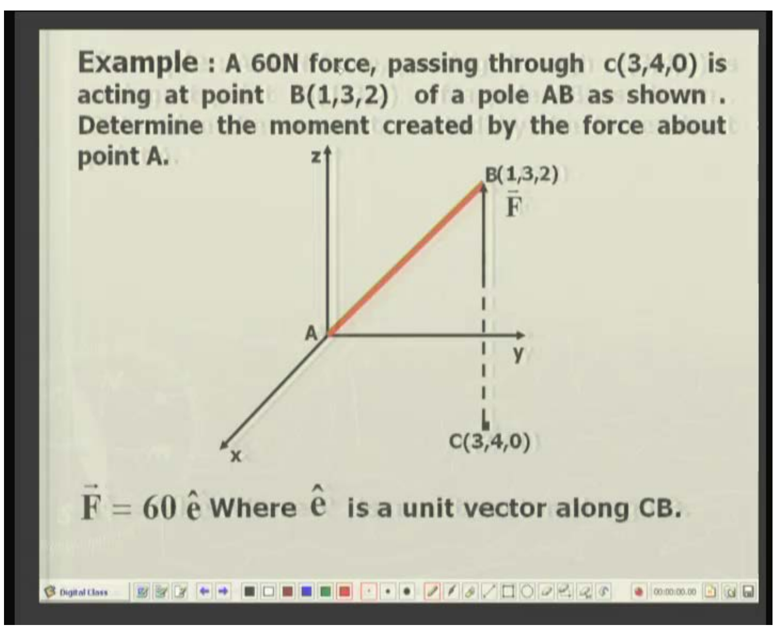

You can easily, see if I go to pervious slide, one minus three that will be X coordinates, three minus four Y coordinates, two minus zero that will be the Z coordinate. So accordingly I have the vector rCB given above and divided by its magnitude, that is, the under root of the sum of the squares. So minus two squares plus one minus square plus two squares. So this gives me the unit vector. You can check that its magnitude will come out to be one. So the force F will be equal to minus forty i unit vector minus twenty j unit vector plus forty k unit vector. Now the moment m is equal to the distance of the origin because we are taking the moment about the point A. Let me go back. We are finding out the moments about point A. So one point is this. The second point is, you can choose any point along the line of action, why not choose point B, because this is already known or we can choose point c. You will see that both the choice will give me the same final result. So I have chosen point B. And accordingly, rB, the position vector point B crossed with F and this will also be, as I said, rC crossed with F. So representing the cross product as the determinant of a three by three matrix, i k j, magnitude of rB and the components of the rB, one three two, components of F minus forty minus twenty plus forty and if I carry out this evaluation of the determinant, it will come out to be hundred sixty i minus hundred twenty j plus hundred k.

So we close our vector analysis. At this point we have seen that there are many operations involved in vector analysis and it can represent very useful quantities like force, movement, position and similarly, we can represent velocity, etcetera. In our next lecture, we will be concentrating on the application of vector analysis through forces namely the equilibrium concepts, free body concepts, etcetera. Thank you very much.