Applied Mechanics
Prof. R. K. Mittal
Department of Applied Mechanics
Indian Institute of Technology, Delhi
Lecture No. 16
Stability of Equilibrium

In today's lecture, which is sixteenth in this series on statics, we will be discussing stability of equilibrium. You may recall that yesterday we were considering in lecture fifteen, we were considering the equilibrium of a system of particles and rigid bodies which is subjected to only conservative forces.

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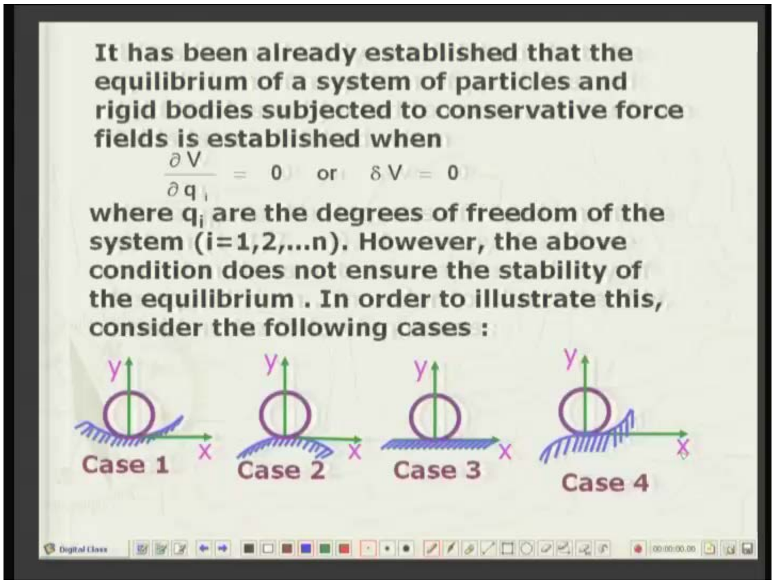

The characteristic of that system was that we could introduce the important concept of potential energy and the condition for equilibrium is that the first variation of the total potential energy which may consist of gravitational energy or elastic energy is zero. del v is equal to zero and the equivalent condition for a system of n degrees of freedom, let us say, q one q two up to qn is that the partial derivative of total potential energy with respect to each of these same degrees of freedom q one or q two or up to qn equal to zero. Now next important aspect to be examined is that once we have established equilibrium of the system, is this state of equilibrium stable or not? By stability or otherwise, we mean that if we disturb slightly the configuration of the system, will the disturbance grow or will it attenuate? That is, system comes backs to its original equilibrium configuration. So these questions are to be addressed and answered today. To motivate this discussion, I will consider a very simple example. Suppose we have a ball or a sphere lying on a curved surface. In one case, the surface is concave, the other is convex, third is level surface and fourth, changes from convex to concave at a point. So that is a kind of inflection surface. Now in all these four cases, you can see that the slope to the surface is zero at one point. That is this point where x is equal to zero, y is equal to zero. Similarly over here, over here and over here. Now suppose, let me examine this case one. I disturb it either to the right or to the left. Now the ball will tend to come back to x is equal to zero, y equal to zero state. It will come back to the configuration of equilibrium. In case two, if I disturb it slightly either to the right or to left, the disturbance will grow. It will go further and further away from this x is equal to zero, y is equal to zero point, that is, from equilibrium state. In the case three, whether I disturb it to the right to the left, neither the disturbance grows nor it attenuates. That is, the ball will or sphere will stay in the disturbed position and in four, if I give a disturbance to right, it will have a tendency to come back but it may overshoot that position and then it will go into a mode of growing disturbance.

So this also is to be treated as a kind of growth of disturbance case. Now let us say there is only gravitational force acting on each of these cases. So the height of the CG of the ball is a measure of the potential energy. Now in this state, the CG will be at this position and when I disturb it, CG will rise and then it will come back as the ball tries to come back. So it means that for equilibrium, the potential energy is proportional to the height of the ball. So when dy by dx is equal to zero, that is, the slope is zero over here, it means the potential energy is also dV by dx equal to zero. That is precisely the condition for equilibrium. So in all four cases from the principle of stationary value of potential energy, we see that these are the cases of equilibrium.

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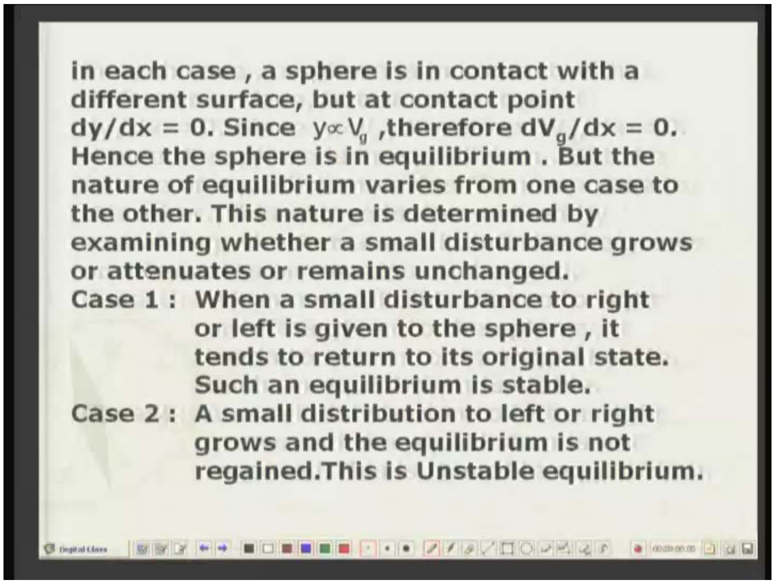

But we have the four cases as we have discussed and what is the distinguishing feature of all these four cases? In case one, the curvature is positive.

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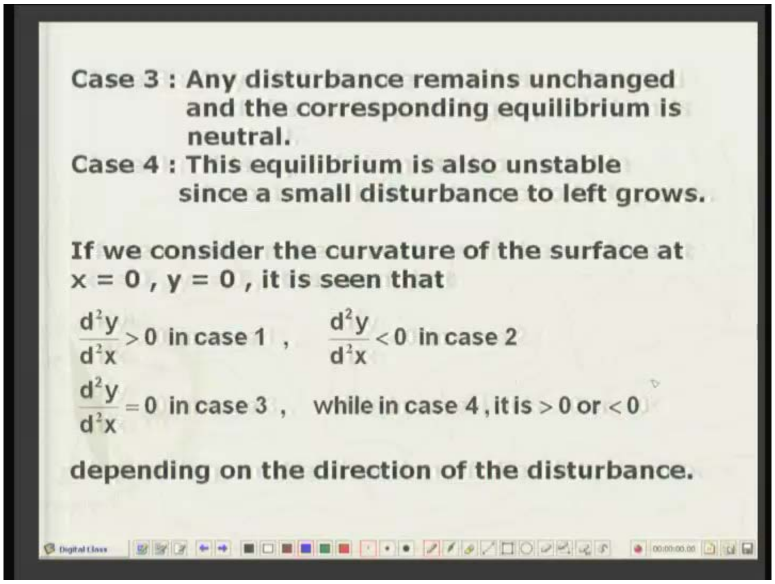

You can see it is a concave surface. So if I consider the curvature over here, that is the rate of increase of the slope, the positive curvature is nothing but rate of increase of the slope. In this case, rate of increase is negative because here slope is zero and then it becomes lesser and lesser. So, in case one, the curvature is positive. In case two curvature is negative. In case three curvature is zero. It is a flat surface and in case four, there is a non-definite curvature, that is, positive as well as negative and at the middle point, it is zero. So it is a case of change of curvature according to the place value of x. Since we have already said that the potential energy is proportional to the height of the ball, it means if we rephrase our conditions in terms of potential energy, then we will have the following conditions.

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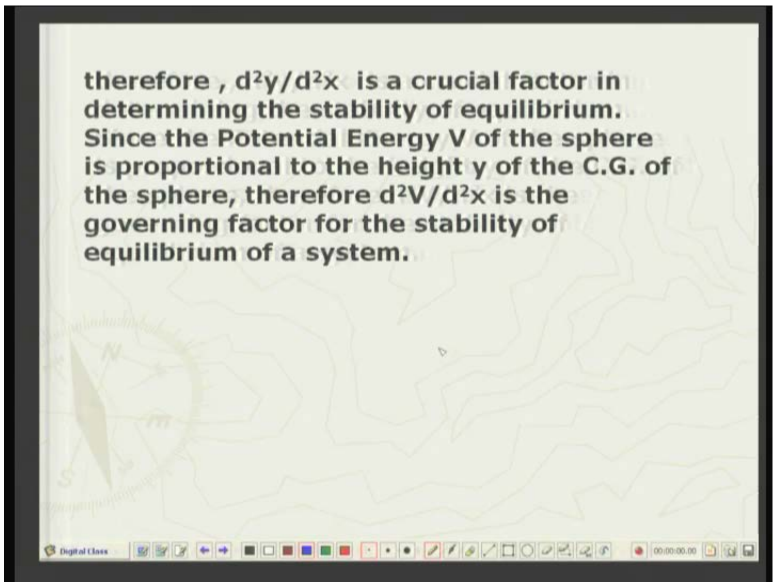

That is, if the second derivative of the potential energy with respect to the variable is positive, then the system will be stable. If it is negative, the system will be unstable. So we will examine from that point of view the stability of equilibrium.

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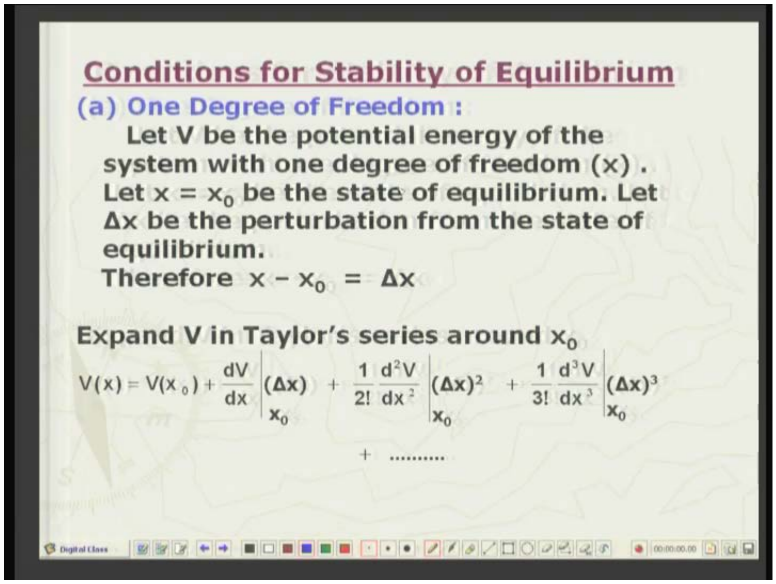

Conditions for stability of equilibrium: First of all, we will take up one degree of freedom case. That is just like the case we have examined a short while ago. So let V be the potential energy of the system with one degree of freedom and x is equal to x zero be the state of equilibrium. So it means when x is at x zero, the system is in equilibrium and now we give a small perturbation or disturbance delta x from the state of equilibrium. So we will define the disturb state by x and the equilibrium state by x zero. So x minus x zero is the disturbance or perturbation delta x. The potential energy of the system is the function of the variable. That is the function of the degree of freedom which we are calling x. So Vx, that is the potential energy in the state x that can be expanded in an one dimensional Taylor series around point x zero. That is the equilibrium state. So by Taylor's expansion, we have Vx is equal to V at x zero plus dV by dx evaluated at x zero times the disturbance delta x plus one by two factorial. Second derivative of V with respect to x evaluated at x zero times the disturbance square plus one third factorial. Third derivative evaluated at x zero times the disturbance cubed plus higher order terms. It is an infinite series. Now look here. This is the potential energy at the equilibrium configuration and at equilibrium configuration, we have already established the first derivative that is the slope is equal to zero. So the first term at equilibrium vanishes and I take Vx zero on the opposite side. So then we will see what happens.

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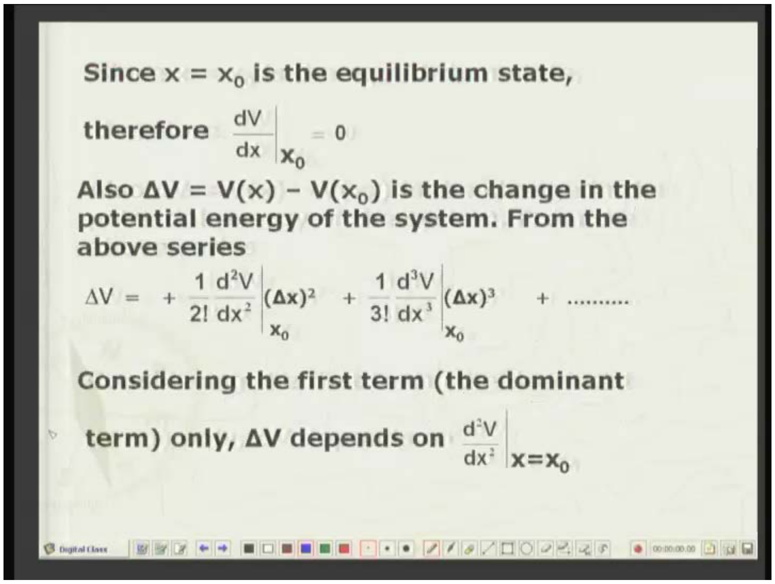

If we call the change in potential energy del V equal to Vx minus Vx zero, that is the deference of the potential energy between the perturb state and the equilibrium state. So that is del V, then del V is equal to one over two factorial. Second derivative of potential energy with respect to the degree of freedom evaluated at equilibrium configuration times disturbance square and then the third order term, fourth order term and so on and so forth. Now since delta x is very, very small, delta x square will be even much smaller but delta x cube delta x four will become much smaller.

So in this series, the largest term or the dominant term is the first term and since delta x square quantity is always positive whether disturbance is positive or negative. So the sign is a material. So it means del V the change in potential energy will be dominated by the this factor. So this is the dominant term. So we will look at the behavior of the second derivative of the potential energy at x is equal to x zero, just like in the example which I had given you, we were guided by the curvature d two y by dx square. That is also second derivative. So here for a general conservative system having a potential energy V will be again guided by deciding the stability of equilibrium by the second derivative of the potential energy at the state of equilibrium.

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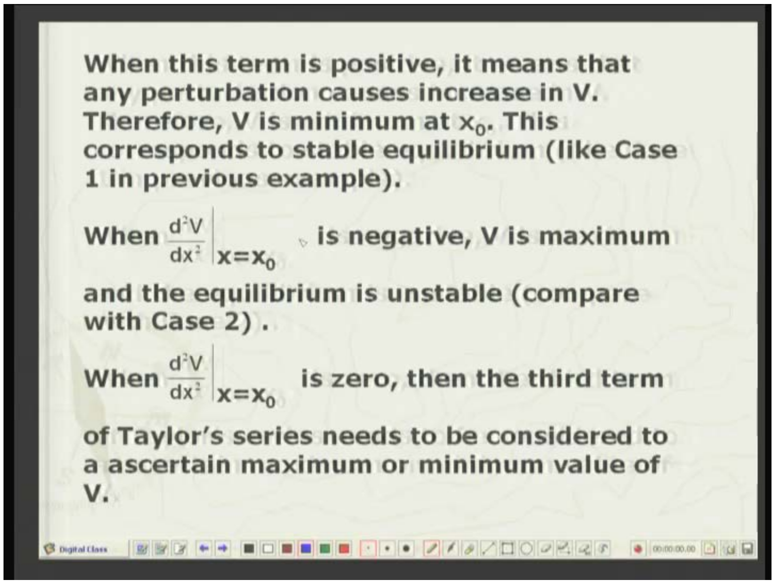

Well, when we have the first case, this term, that is the second derivative, is positive. It means that the perturbation causes an increase in V. Therefore the potential energy is minimum at x zero. So when the second derivative is positive, you come back to the figure here.

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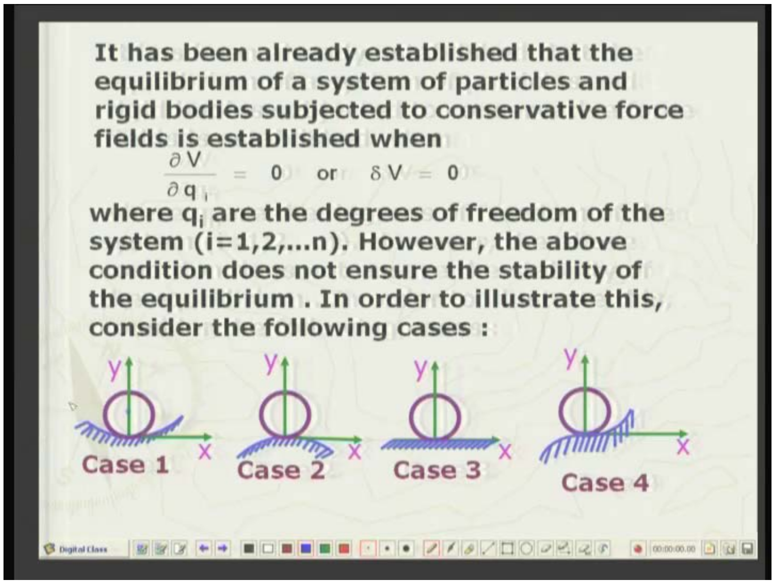

So when I give a disturbance on this side or on this side, the height increases in both ways. So it means that the equilibrium state is the lowest point. So similarly, in our case of potential energy of one degree of freedom, the first case corresponds to the minimum value of the potential energy because any disturbance either to left or to right will cause an increase in the potential energy. When this second derivative is negative, then it is parallel to the case two where whether you go to the right or left, the ball is going to a lower position. It means your potential energy is decreasing by analogy. So it means this is the case of maximum potential energy. So that any disturbed position will be having a lower potential energy and then we come to the third condition, that is the second derivative is zero. Then it is a level case. It means potential energy neither increases nor decreases at that point. So in that case either the potential energy is constant or at best, you go to the third order derivatives, that is, del cubed by V by dx cubed. So then we go to this term. Evaluate at the equilibrium state. So then you have to decide whether this term is positive or negative. If it is positive, you have a tendency towards stable equilibrium. If it is negative, again you will have an unstable equilibrium.

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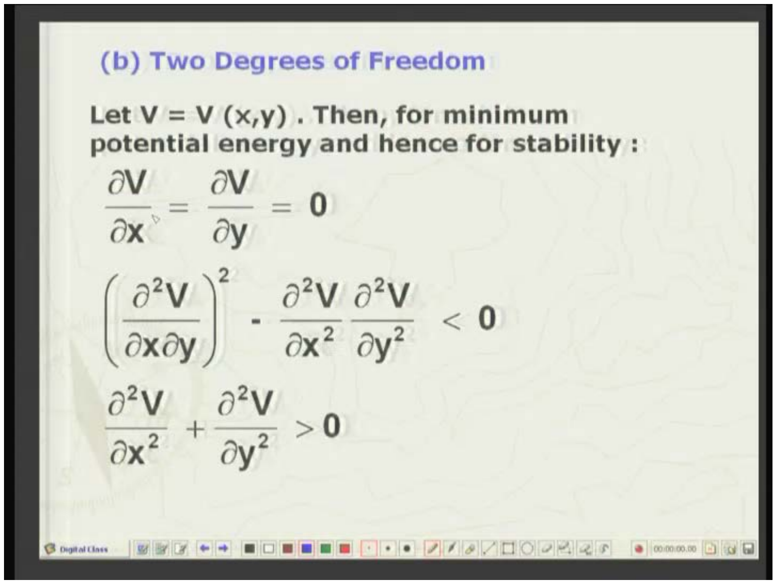

So, that was for one degree of freedom. The analysis was very simple that all you have to do is compute the second derivative of the potential energy as a function of that degree of freedom. Now when you go to two degrees of freedom, then the situation is more complex. Well for equilibrium, we know that let us say x and y are two degrees of freedom or q one and q two are two degrees of freedom. I am using x and y then for equilibrium since the first variation is equal to zero, it means that the partial derivates of the function V conserve potential energy function V with respect to x or with respect to y. Both are equal to zero. That is the condition of equilibrium and then to see whether our equilibrium is stable or unstable, we have to first consider there will be second order derivatives d two V by dx square d two V by dy square and the mix derivate d two V by dx dy. So first you consider this expression, that is the mixed derivate square minus the second derivate with respect to x square x multiplied by the second derivate with respect to y.

That quantity, when you evaluate this expression, should be negative. That is condition number one and then the sum of the second derivatives d two V by d x square d plus d two V by d y square should be positive. This is incidentally called the Laplacian. Laplacian of V is positive.

So as I said for stability, first we see insure that it is a state of equilibrium and then these two conditions have to be simultaneously met. For the case of instability, then again we have the condition that it is a state of equilibrium. That is understood and then this condition is same as in the previous case for the stable equilibrium case but the third Laplacian condition is opposite. In the previous case, we had this quantity as positive. Now it is to be negative or unstable equilibrium and again if it is equal to zero, that is, it is neither positive nor negative, then you have to go to the higher order derivatives and then the set of equation becomes more complex.

Similarly, if you have more than two degrees of freedom, q one q two q three, then the analysis of stability, when the relevant equations become much more complex because then you will have so many derivatives to worry about. So we will, for most of the cases, concentrate on one degree of freedom. I will take up one or two examples of two degrees of freedom also. So to sum up, if you have a system comprising of particles and rigid bodies which is subjected to only conservative forces, no dissipative forces like friction or some other viscous or dry friction or hydro or dynamic friction, lubrication, etcetera are involved, then the system can be examined in terms of the potential energy and the stability criteria in terms of the second derivatives of potential energy has been set up.

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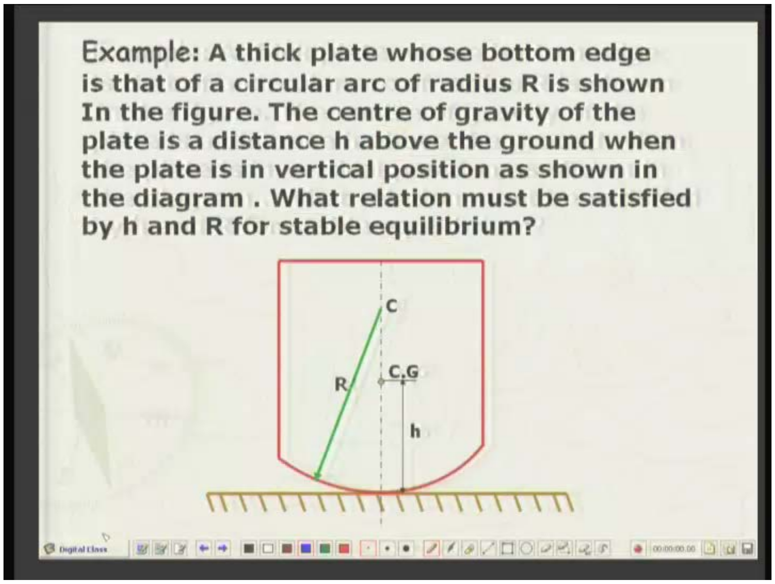

Let us go to some examples. First of all, I consider a very simple example. Here is a thick plate which is having a kind of nose which is circular in shape. So, a thick plate whose bottom edge is that of the circular arc. This is shown here. The centre of gravity of the plate is at a distance h from the lowest point. That is the point where it is resting on the level floor. When the plate is in a vertical position as shown in the diagram, what relation must be satisfied by h and R for stable equilibrium? So first of all, we will have to consider when is the equilibrium obtained and then examine whether equilibrium is stable or unstable. In all these problems, you start with the disturbed position and then we will see under what circumstances that position will be an equilibrium position. So this is what we are going to do.

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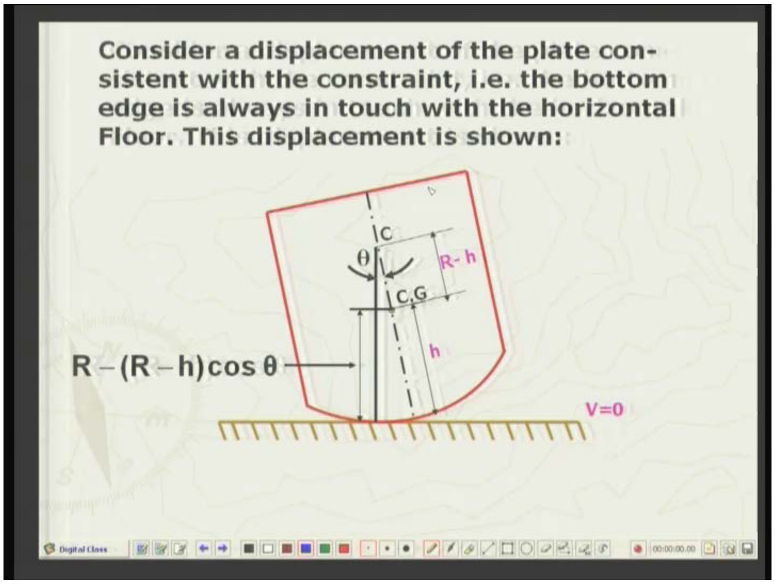

Here I have given a disturbance, that is, I have gradually tilted this plate and in tilting, the plate is being tilting without slip edge, that is always one point of the lower edge was in contact with the floor as shown here. So now in the tilted position theta is the angle which the centre line,this is a symmetrical body, of this plate makes with the vertical and theta is incidentally the degree of freedom because once you prescribe theta, you can see its inclination and that is sufficient to establish the configuration of this plate. So this is a system of one degree of freedom. Now there is no other force except gravitation force. So it means I have to calculate its potential energy in the disturbed configuration. This is the centre of gravity of the plate. So it means, first of all, you have to decide two things. How many degrees of freedom we have seen? There is only one degree of freedom and what is the datum, for, if it is a conservative system, what is the reference point for zero potential energy or datum? We will take the floor, that is, the lowest point of this body, as the datum and we call the potential energy equal to zero at that point. So this V is equal to zero state, that is, when it is just touching the floor. So CG in the vertical position, it was at a height h. Now we have to find out the height of CG. Obviously this is R the radius of the nose and you can see that this thing is total R. So R minus h. This is R minus h. This is also radius R.

So the depth of the CG from this center of curvature of the nose is R minus h and to find out the height of the CG, it means this R is the vertical line minus this height. This is R minus h cosine theta. So this is R minus h angle theta. So this will be R minus h cosine theta. So R minus R minus h cosine theta. That is the height of this CG.

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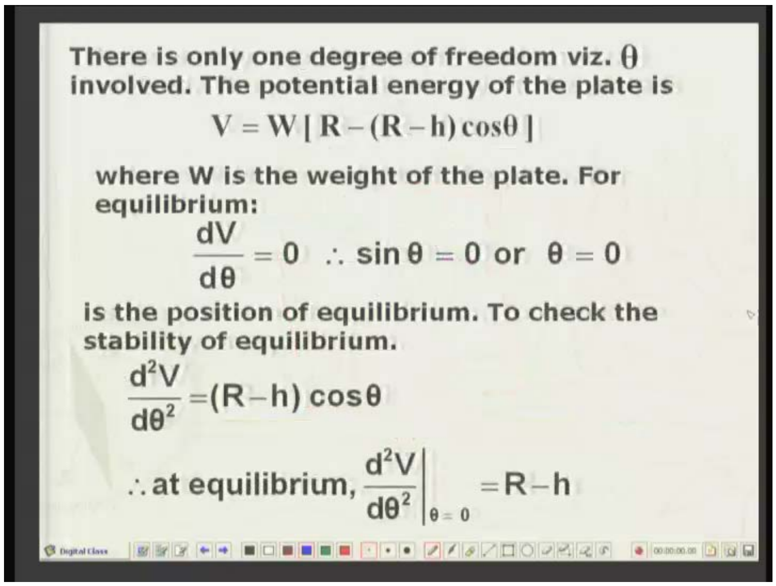

So from the datum, the potential energy is W, the weight of the plate. So W into R minus R minus h cosine theta. That is the potential energy in the disturbed configuration. Let us take the for equilibrium. The first variation is equal to zero or which is equivalent into the statement that dV by d theta because there is only one degree of freedom is equal to zero. So when I differentiate it, you will have first constant zero. So R minus h sin theta. So dV by d theta will be R minus h sin theta and this is set equal to zero. It is because R minus h is not zero. So it means sin theta is equal to zero. So one simple root of this is that theta is equal to zero. So it means the there is no inclination. The plate is standing vertical. This is the state. That is the plate is in the line of symmetries, vertical line. So that is the state of equilibrium. Now we want to establish whether this equilibrium is stable or unstable. For that we will now take up the second derivative.

So again it is easy to calculate. It will be R minus h cosine theta and when theta is zero, that is, at the state of equilibrium, it means second derivative at theta is equal to zero is simply R minus h because cosine of zero is one. So for stable equilibrium, this quantity has to be positive. For unstable equilibrium, this quantity is negative. Positive means R minus h is greater than zero or R is greater than h. So if the radius of curvature is larger than the height of the CG from the lowest point of the plate, then you have a stable equilibrium. If on the other hand, the radius of curvature is lower than CG, that is R is less, that is here as shown the centre of curvature is above the CG, that is a stable configuration because then R is greater than h but if R is over here, that is the nose's very curved radius is smaller then the equilibrium is unstable and of course if R is equal to h, then we have to examine the higher order derivatives.

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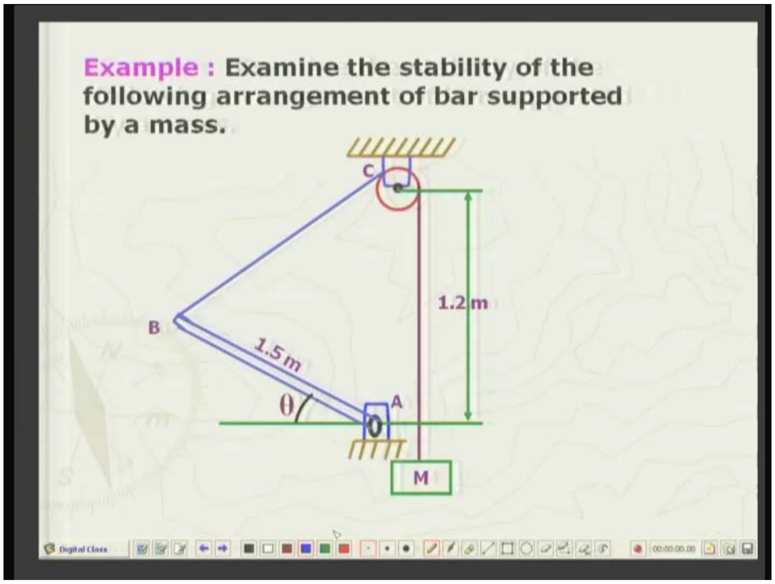

So this is exactly what we have concluded and now I will take up another example. Well this example was in fact discussed in the last lecture, lecture fifteen, where we were only concerned with the equilibrium configuration. We did not pay attention to the stability of equilibrium. You may recall that there is a heavy rod which is pinned at one end and the end is going through a string over a pulley and carries a mass M and then we wanted to find out whether when theta is equal to twenty degree, what is the value of M which will maintain the equilibrium of the state? When we were solving this problem, we set up the potential energy because there is only gravitational force involved. All the forces are conservative. So potential energy is easily defined in terms of the potential energy of the rod as well as that of the mass and if you recall we had set up this expression.

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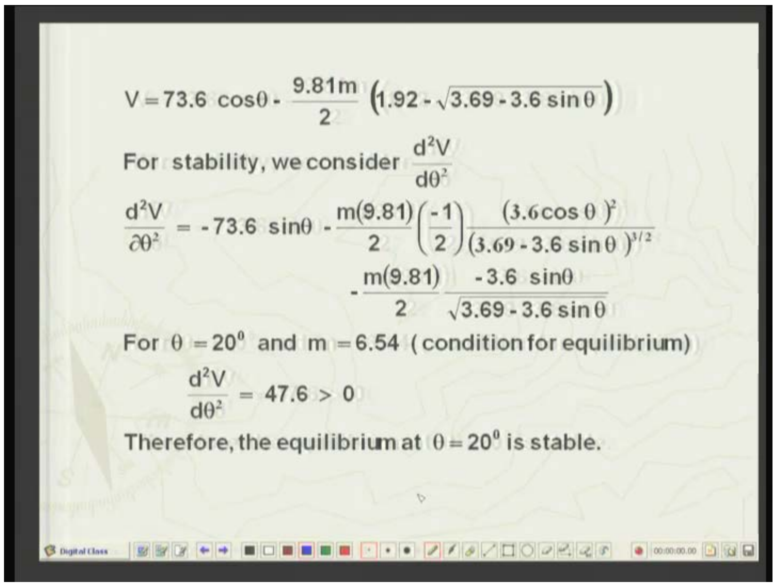

In our last lecture, for the potential energy, V is equal to seventy-three point six cosine theta minus nine point eight one into mass M divided by two into one point nine two minus square root of three point six nine minus three point six sin theta So when we took the first derivative equal to zero, then we came to the conclusion that if our value of M, that is, mass, is six point five four kilogram, then the body was in equilibrium, for theta is equal to twenty degrees. That was the result of tangents last time. Now for stability, we will take the second derivative. Again it is very simple to take the second derivative. Cosine theta will give me minus sin theta and then in this expression, you will find this is the derivative and after simplification, when theta is equal to twenty degrees, that is, you substitute theta is equal to twenty degrees and M is equal to six point five four in all this calculations. You will find that the second derivative comes out to be forty-seven point six which is greater than zero. It is a positive quantity. Hence it is greater than zero. So it means this configuration of theta is equal to twenty degrees and M is equal to six point five four is a stable configuration. So that if there is a small disturbance, that is, mass slightly vibrates up and down, the equilibrium will not be disturbed forever. The disturbed configuration will be such that it will come back to equilibrium. That is the interpretation of stable equilibrium.

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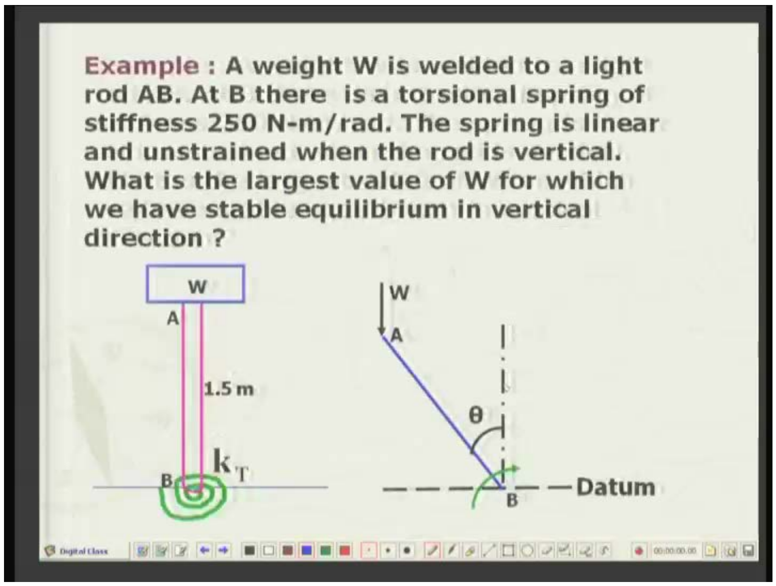

Let me take up an example where we will have two types of potential energies and this example is, let me read it out, A weight W is welded to a light rod. This is rod AB, a light rod, let us say made up of aluminum or some polymer and W is sitting on it connected to it. The length of the rod is one point five meter and at the lower end of the rod there is a torsional spring just like in your watches. So this torsional spring is trying to, if there is a tilting of the rod, bring it back to the vertical position just like your linear spring. If you disturbed it, it tends to bring it back to or restore it to the equilibrium. So similarly here, if you give an angular disturbance, then it comes back. It tends to come back to its original configuration and the stiffness of this torsional spring is naturally in terms of restoring moment Newton meters per radian per angular disturbance to two fifty Newton meter per radian. This spring is linear and unstrained, when the rod is vertical. So he has defined the datum for the spring, that is when it is vertical, the spring is untwisted. That is it is in a natural configuration and hence potential energy is zero.

What is the largest value of W for which we have stable equilibrium in vertical direction? So what is that W which when slightly increased will cause unstable equilibrium? That is the interpretation. Well, as I said, we will fix the datum. Two types of energies are involved. The potential energy of the weight because the rod is light. So we will neglect whatever potential energy of the rod is there and second potential energy is that of the spring elastic potential energy. So the zero state, that is the vertical state, is the zero state of potential energy of the spring and this is the datum for the W. Second thing we will do is that we will disturb the system through an angle theta. So I will take anti clockwise positive. So I have disturbed it to angle theta and weight will be now acting over here and we will find at what height the weight is now. So what is the potential energy of the weight?

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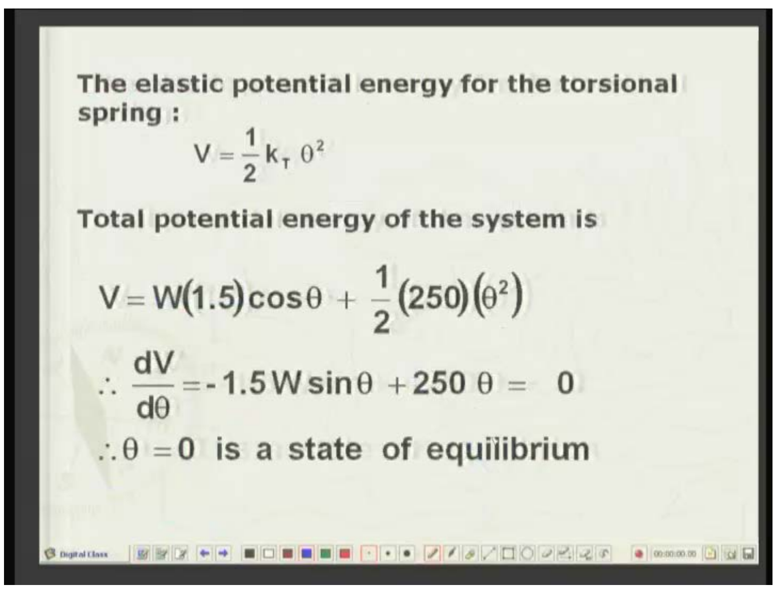

First of all, the elastic potential energy of the torsional spring is half kT theta square just like we have for linear spring half k kx x square. So similarly for the torsional spring, you can, exactly in the same manner, prove that the potential energy of the spring is half times the torsional stiffness, kT of the spring times angle of rotation square and the total potential energy of the system will be W into its height, that is the weight times. Sorry. The weight times the height. Originally it was over here. So it has gone like this. It means this length is one point five the length of the rod into cosine theta. That will be the vertical intercept. So that is the potential energy of the weight plus half two fifty. That is the torsional stiffness of the spring theta square. So this is the total potential energy and if I differentiate it and set it equal to zero, that will be the state of equilibrium. So that is a very simple step. So cosine theta will give me minus sin theta and this will give me two theta. So I get this expression and one root is obvious, that is, when theta is equal to zero, this term is zero, this term is equal to zero. So it means theta is equal to zero when the rod is exactly vertical, that is a state of equilibrium. Now we have to examine whether this equilibrium is stable or not. So what we do is take the second derivative. So sin theta will give me cosine theta and this will give me just two fifty.

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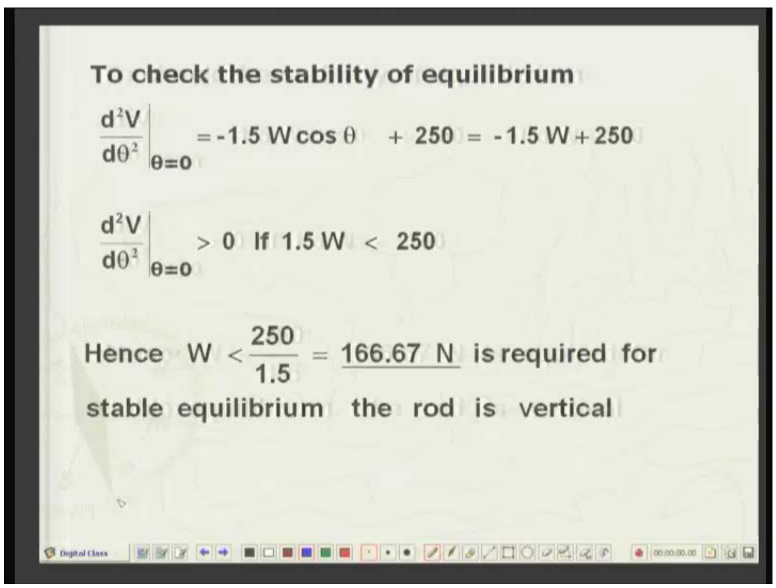

So you will have minus one point five W cosine theta plus two fifty and since theta is equal to zero for equilibrium, it means uh minus one point five W plus two fifty and since for stability of equilibrium, we want that this should be greater than zero positive. This is a negative term. So you can take it on the opposite side. So one point five W is less than two fifty or W should be less than two fifty over one point five which is one sixty-six point six seven Newton's. So as long as the weight on the rod is less than this, we will have stable equilibrium. As soon as it exceeds, we will have unstable equilibrium. So you imagine that here is a rod, I am slowly increasing the weight sitting on it and up to one sixty six point six seven Newton weight, the rod is straight vertical in a stable manner. As soon as this limit is exceeded, this equilibrium becomes unstable. Not that it will immediately collapse. What it means is that, if I give a small disturbance to the rod, this disturbance will grow and immediately the rod will fall down and hence the weight will fall down. W is equal to one point six. One sixty-six point six seven is the critical weight and similar problems arise in many other situations. One important thing is about structures. So in, let us say, columns, etcetera, they can withstand certain weight and if that weight is exceeded, the columns buckle. So, you will be learning in solid mechanics courses later on that buckling or the straight column becomes a curved column. That phenomenon of instability which happens for long thin columns subjected to compressive loadings. So this example is very useful in understanding instability phenomena.

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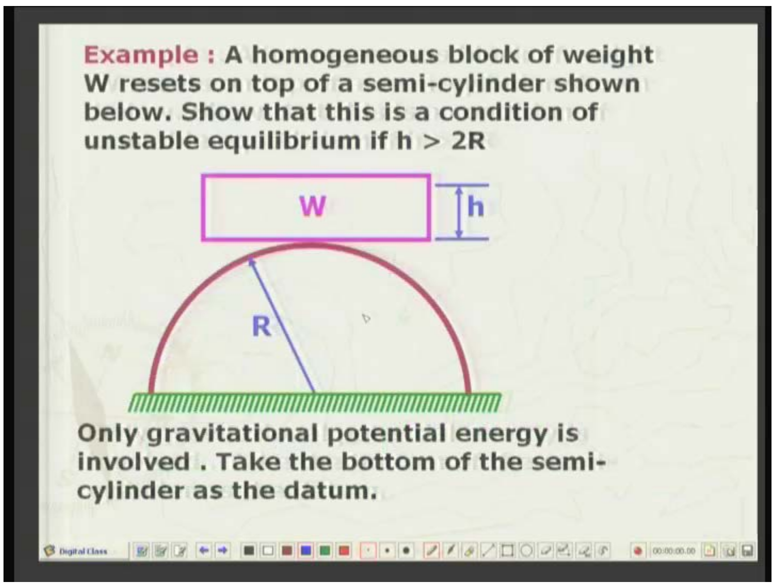

Let me take up another example which is a very interesting example. Here is a homogeneous block of weight W which rests on a dome which is in a form of a semi cylinder and shows that this condition is unstable. If h this height of the block is greater than two R, that is, the diameter of this dome or twice the radius of this dome. Well again, we have only one type of a force, that is, the gravitational force. So it is a conservative system. We will work with the potential energy and we will treat this ground level as our datum for potential energy, that is, the potential energy of the block will be measured with respect to or the height of the CG of this block will be measured with respect to this floor on which this dome is fixed.

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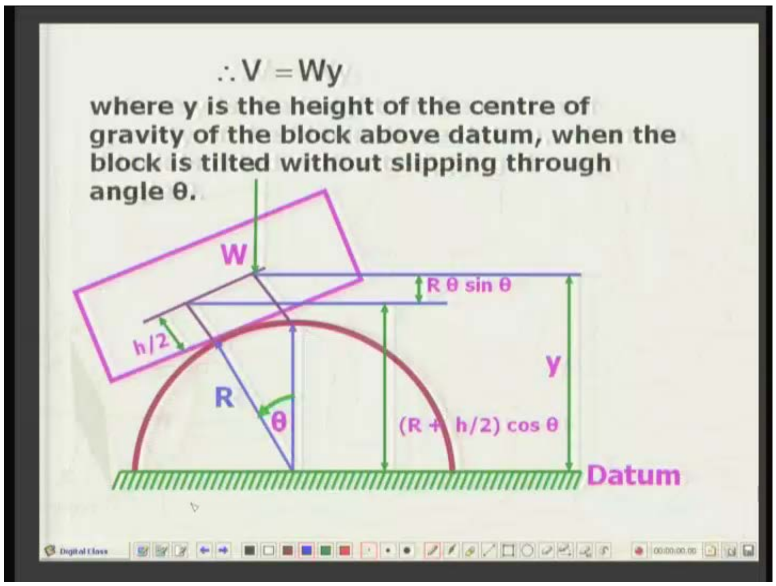

So as usual, we will disturb the system through an angle theta, that is, while tilting this block, we have to bear it in mind that at no stage there is slip taking place. That is, this tilting is without slippage. So that this length a is equal to R into theta. This radius R into theta is arc length. So this arc length is this length and by parallelogram, this is the length. So that is very important, that is, any tilting or disturbance should be without any slippage, that is, the contacting points are always at rest with respect to each other. Now rest of the problem is actually almost a problem of trigonometry because this is the CG of the block. I have to find out the total height y of the CG which consists of the R plus h by two is this total from this point to this point and then this length is parallel to this. So this is due to no slip condition R theta. R theta is the arc length. So this length is also R theta and hence this length is also R theta. So that this difference because this will be R theta sin theta, that the angle here is also theta. Why is it theta?

Always remember that the angle between two lines is equal to angle between the normal. So the angle between R and this vertical is theta. So horizontal line is normal to the vertical and this line is normal to this radial line. So it means this is also angle theta So this is R theta. Angle is theta. So vertical distance will be R theta sin theta and the vertical projection over here is R plus h by two. This is the total length R plus h by two R plus h by two into cosine theta. So the total height from the figure is R plus h by two cosine theta plus R theta sin theta. So once again, this evaluation is simply trigonometry and geometry of the problem.

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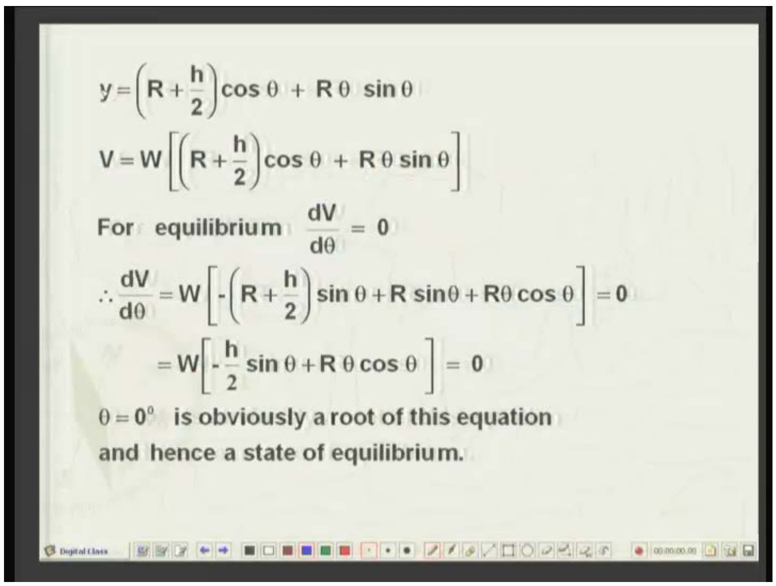

So after having understood it, the matter is very simple. The height of the CG y is R plus h by two cosine theta plus R theta sin theta. So the total potential energy of the system will be the weight of the block. It is supposed to be acting through the CG. So W is the weight of the block times y that is R plus h by two cosine theta plus R theta sin theta. So after having established the potential energy of this system, let us examine the equilibrium. That is, this is a system of one degree of freedom. Only dV by d theta is equal to zero. Well once you have dV by d theta equal to zero, rest is simple derivative of this expression. So you can easily see that cosine theta will give me minus sin theta and this will give me two terms first differentiation of theta and then differentiation of sin theta. So these are the two terms and this is to be set equal to zero. After little simplification because you can see minus R sin theta will cancel out R sin theta. Here, so minus h by two sin theta plus R theta cosine theta is equal to zero. Well, there is a theta here, there is a theta here and cosine zero is one. So obviously theta equal to zero is a root of this equation. It satisfies this equation. Hence it is the state of equilibrium. So what does that mean? Theta is equal to zero means that this configuration when you have block sitting on the highest point is vertical.

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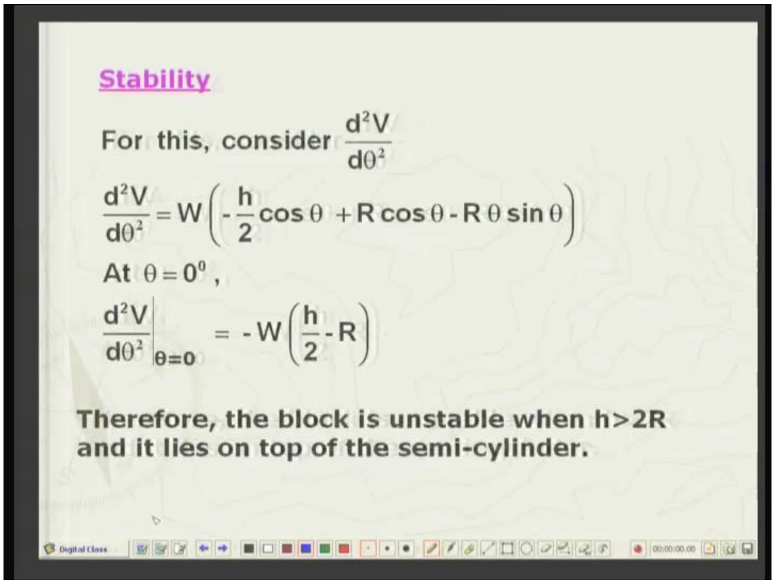

Now let us examine the stability of our system. Again take the second derivative. We will have second derivative from the previous expression. Once again differentiating it, set theta is equal to zero because that is the state of equilibrium. So we will have, after setting equal to zero, this will be one, this will be one, this will be zero. So minus W into h by two minus R right and if this quantity is positive or take the negative sign on the other side, if this factor is negative, then the system is stable and so on so forth. So the block is unstable, when h is greater than two R. So h is greater than two R means this will be positive and the derivative will be negative. So this is the condition for instability of the equilibrium and if it is h less than two R, then the system will be in equilibrium.

So I think with these examples, we have seen how to examine or how to analyze whether a given state of equilibrium is a stable state or an unstable state. If it is a stable state, it means that a small disturbance, I am not talking of large disturbance; the system will go into an entirely deferent mode. So I am talking about only a small disturbance. Whether it will grow or it will attenuate. We are talking of only the conservative system and if the second derivative is positive, then the disturbance will attenuate and system is stable and if it is negative, then the disturbance will grow and grow and it will leave the equilibrium configuration after a while and it is unstable. So these methods which we have discussed, the potential energy method and virtual work method, as I have emphasized in the beginning and I will like to emphasize once again, they are very robust, very powerful methods, not only for mechanical systems. Actually similar procedures have been applied in many other systems, electrical systems, thermal systems, etcetera and the basic concepts are exactly on same lines that if we define a potential function, it may or may not be mechanical potential, may sometime even be chemical potentials, electrical potentials. So this simple analysis is the starting point of stability in many contexts and that is why you must understand these concepts fully and they will be useful in many, many, many problems.

So this was the last chapter or last lecture in this statics part of the course on mechanics. If we recapitulate in next five minutes what we have learnt in these sixteen lectures, well, in the first one or two lectures we were discussing about the mathematical preliminaries. Mainly, we were discussing vector analysis because you might have realized that throughout we have used vectors and we have used them very profitably and economically and I hope you will continue to use vectors in your higher courses also and then we came to the description of what are the various simplifications or assumptions made in particle mechanics. I have not discussed rigid body mechanics as a separate entity. So what are various types of forces? Then we were also discussing about the basic laws of mechanics, Newton's laws. Secondly, Stevinius laws which established at force is a vector quantity, etcetera, and we had some discussion on equivalent force systems. That is a given set of forces and moments were replaced by the simplest possible force system. Force system means force and forces and moments and then we came to the concept of equilibrium of forces acting on a particle or on a rigid body and for a system of rigid bodies, it was essential to break up the given system into smaller systems or subsystem in terms of free bodies and all the contact points were replaced by the reactions which can be force reactions or moment reaction and then we applied the conditions of equilibrium.

Using those concepts, we went into structural mechanics where we applied the concepts of free body as well as the equations of equilibrium to trusses as well as beams. We found out that in case of trusses, the forces in trusses and the reactions from the supports. In case of beams, we derived the shear force distribution and bending moment distribution along the length of the beam and this led to shear force diagram, bending moment diagram, etcetera.

Finally there was another useful application of these concepts namely in case of friction or a system having friction. Friction was identified as a force which tries to prevent motion and if the motion is already taking place, it hinders motion. It appear to be a kind of unwanted force but then there were very technically useful applications of friction, namely in truss bearing, belt and pulley drives, etcetera, and we examined those cases in detail solving several examples of practical application. After discussing friction, we came to the properties of moment, properties of ah surfaces namely the first moment of an area, second moment of an area, the centroid of an area, etcetera, and then we came to a parallel concept. That is the moment of inertia and product of inertia.

Although we have not made much use of moment of inertia and product of inertia, they will be very much needed when you go to dynamics of rigid bodies in this second module that is the dynamics. So there you will need it and also in higher courses, solid mechanics, etcetera. The second moments of area and first moment of area will be needed in the theory of beams, a very important application of solid mechanics and finally we came to the principle of virtual work which provided us with the alternate method of analyzing the equilibrium but it was restricted to some constraints, that is, let us say, particle is moving on a surface which is a smooth surface. So all the dissipative of friction forces were left out of consideration. So if all the active force are non-dissipative, then the equilibrium can be examined by applying certain virtual displacements and computing the work done during those displacements and setting total virtual work equal to zero.

The advantage of that method was that, unlike forces and moments which are vector quantities, work is a scalar quantity. So the work contributed by several particles or several forces can be simply added or subtracted and secondly, the problem of mechanics can be almost reduced to that of trigonometry or geometry and hence the analysis becomes very simple. No internal reactions need to be taken into consideration. Only outside active forces, whether it is for a particle or for a rigid body, and when these forces further happened to be conservative forces, then it is very advantageous to use the concept of potential energy because conservative forces mean the work done by any path is same. The final advantage of the method of potential energy was that we could examine the stability of the system in a very natural and simple way. This is what we have discussed today. So with this, I close this module on statics and somebody will be taking up the next part on dynamics. Thank you very much.