Applied Mechanics
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Lecture No. 15
Methods of Virtual Work and Potential Energy (Contd.)

In our last lecture, that is lecture fourteen, we analyze the equilibrium of a system of particles and rigid bodies with the help of the principle of virtual work. Today we will start with another energy method or work method, namely the potential energy method. This is in a way offshoot from the principle of virtual work. Before we start with the principle of stationary potential energy, I would like to define some concepts which are very useful.

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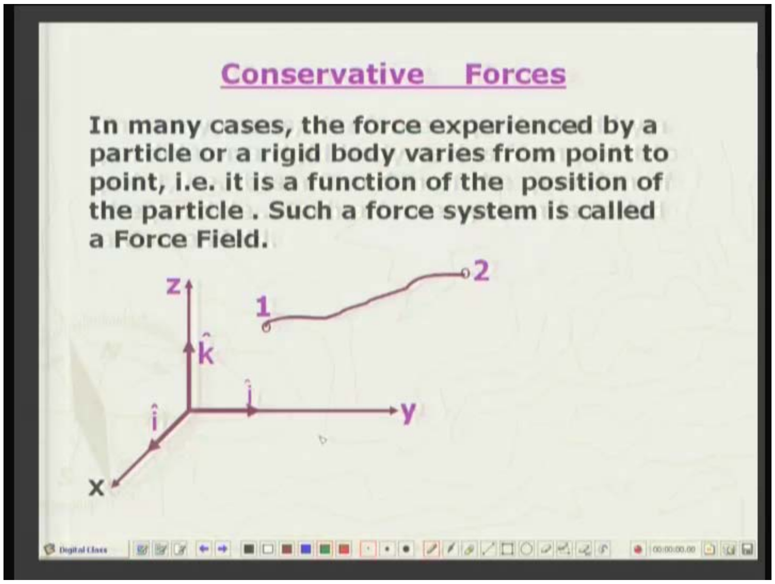

First of all we will talk about conservative forces but before I introduce what conservative forces are, let me talk about force field when the force is a function of the point in the given region of three dimensional Euclidean space. The force is called a force field. For example, if we have two magnets, we know that the force of attraction or repulsion depends upon the distance between the two. When the opposite poles are nearer to each other force is higher, when they are further apart, the force is less. Similarly for electrostatic charges or gravitating bodies. So in all these examples, we see that the magnitude and sometimes direction also changes from point to point. Similarly if we take the example of the ordinary linear spring, let us say this spring is connected to a given mass. Then depending upon the distance of the mass from the spring, that is, if the spring is stretched or compressed more or less, the force is also accordingly variable. So all these types of forces which depend upon the location of the point are called force fields. Now to define the conservative forces, let us say, a particle is moving in a force field and this field is described with the help of a Cartesian coordinate system xyz. i j k are the unit vectors along the coordinate axis. Now particle is starting from point one and it is traveling along any path, two point two.

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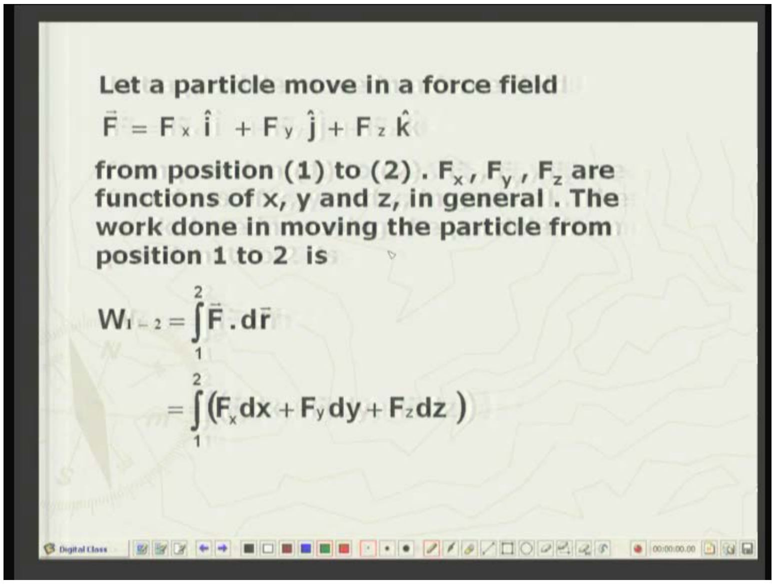

Let us say the force field is designated as vector F which has components F_x in the i direction, F_y in the j direction and F_z in the k direction. So vector F is equal to F_x i unit vector plus F_y j unit vector F_z in the k unit vector. We have already defined the work done as dot product of the force and the elementary distance vector dr. So F dotted with dr, that is the differential work over a small distance dr and if I integrate it from one to two, then I get the total work done in moving the particle from position one to position two. Taking the dot product we have F_x dx plus F_y dy plus F_z dz and this is integrated from point position one to position two.

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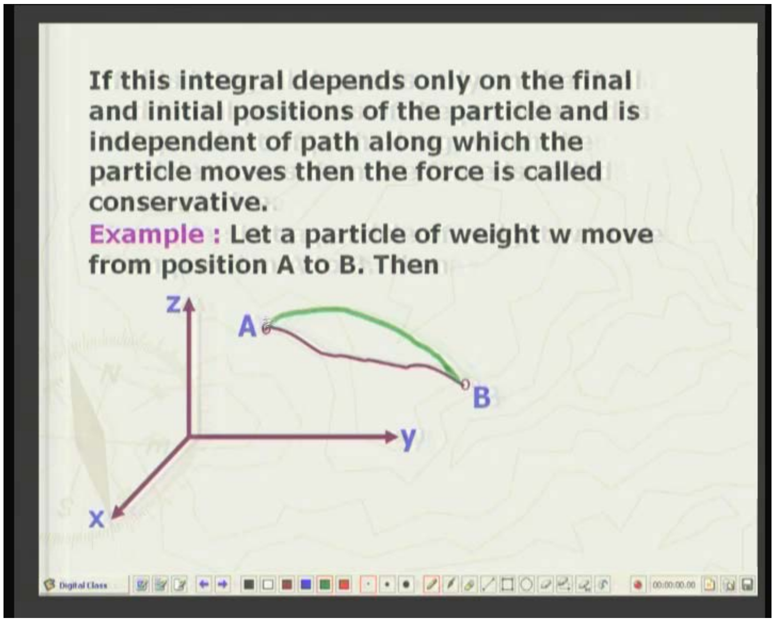

If this integral depends on only the initial and the final positions and it is independent of the actual path followed by the particle, then the force field is called a conservative force filed. For example, if these are two particle, points one and two, particle is moving from one to two. It can take up this path. It can take up this path. It can take up any arbitrary path like this. If we find that along any of these paths the work done is identical to any other path then the corresponding field of force is a conservative force field. Now let us take the given example of such a conservative force field. Let us consider a particle of weight W is to travel from the position A to B under the field of gravity, that is the only force acting is the gravitational pull.

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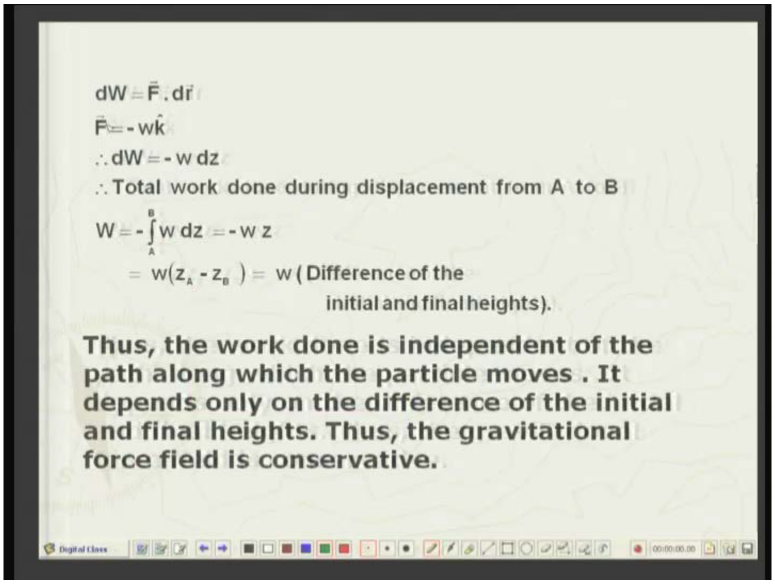

So, we can see that as defined the differential work done F dw is equal to f dotted with dr. Now the force gravitational force is downward in the z direction as according to this coordinate system z. The vertically upward is positive. So downward is minus direction. So we will have the force vector as minus w k unit vector. So taking the dot product, we will have minus W times dz. Now if I integrate from position A to position B, then we will have integration from A to B negative minus W times dz which we will give me minus w z from A to B and if I substitute the upper and lower limit and then take the negative sign inside. Then I will have W into the z coordinate of position A minus the z coordination of position B. Hence it is equal to W in to the difference of the initial and the final heights. So whether the particle has fallen from A to B, according to this path or this path or any other path all that matters is the initial height the final height. So essentially this total work done is independent of the actual path followed by the particle and it depends up on the initial minus the final height. So gravitational force field is a conservative force field. We can also look at this property in a slightly different manner.

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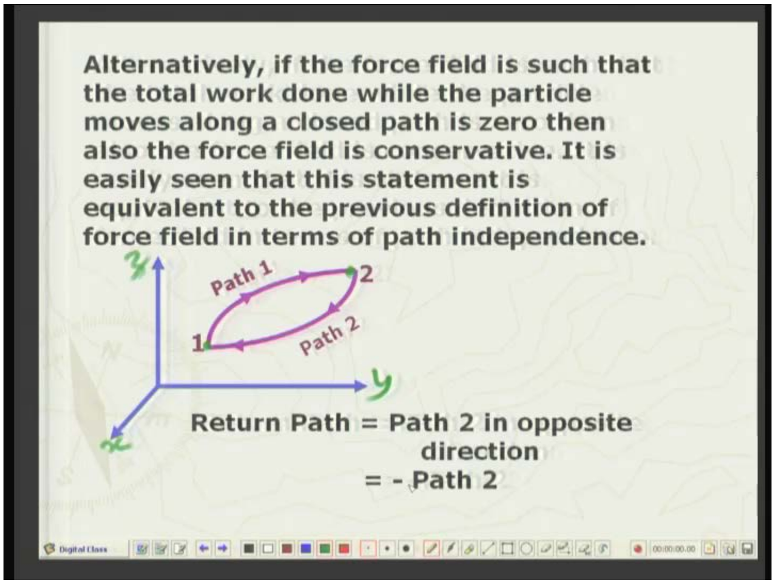

Suppose again I have a Cartesian coordinate system x y z and two positions, one and two, and to reach from one to two either I can follow path one or path two. Suppose I go from one to two by path one and then come back by path two, that is, I return to my initial starting point. So there the totally path is a closed loop. If I find that the total work done in a closed path is zero then also the force field is a conservative force field. It is very easy to see. Since we have already given the definition that in a conservative force field path does not matter. So it means if I go by path one, I will do certain amount of work and path two, I will do exactly the same amount of work but if I return from two to one by path two in the negative sense, that is, one to two, if it is in this direction two to one, work done will be negative of the previous one. So since the total work done in path one from one to two is same as work done along path two from the same one to two. So in the return, it is the negative and hence it is zero. Total work done is zero. So if you find that if a particle moves in a force field along a closed loop path and the total the work done is zero. Then also the force field is a conservative force field.

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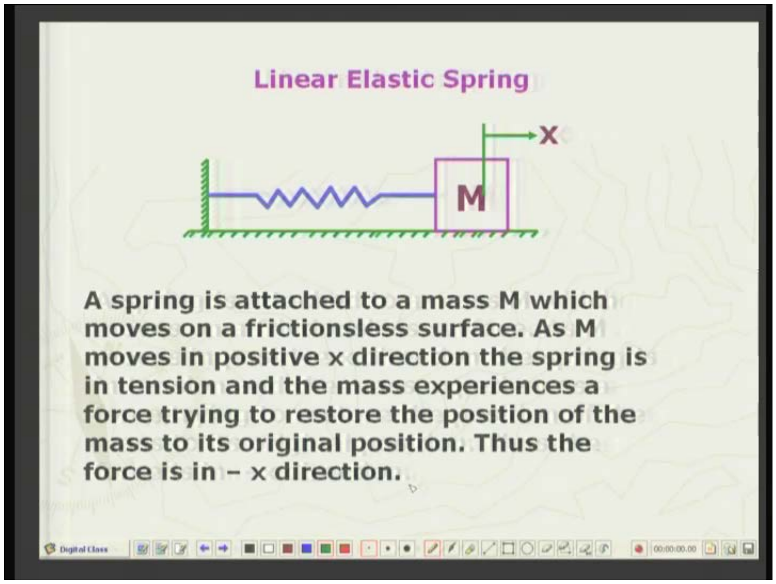

I have already given you an example of gravitational force field. Let us look at another interesting example, that of a linear elastic spring. Suppose here is a linear elastic spring with a spring constant K. That is the force required per unit extension or compression of the spring. So K is the spring constant and a mass M is attached to one end of the spring which is fixed. The other end is attached to the mass and mass can travel on a frictionless or smooth surface. Now if I pull the mass in the positive X direction, naturally this spring will be stretched and it will be in tension and it will try to pull the mass towards its original position. So there will be restoring force. Force will be in the negative X direction. Similarly if I move the mass in the opposite direction, the spring will be compressed and as a reaction, the mass will be pushed up pushed back. So again there will be a restoring force. So what we can say is that the force acting on the mass whether intention or whether this spring is in tension or compression, A is equal to minus K that is spring constant times the deflection X.

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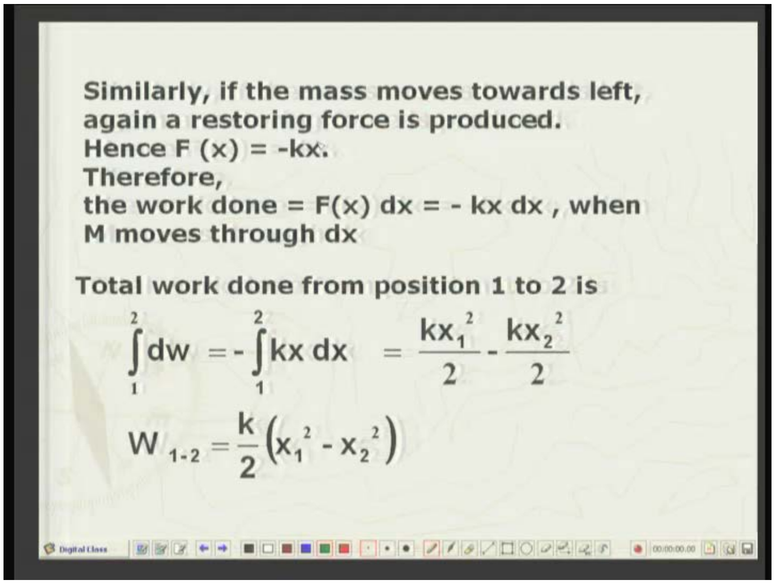

Now work done over a small distance is Fx into dx is equal to minus kx dx and if I integrate from one to two, this in elementary work, then I will have minus kx dx integration one to two. So again taking negative sign inside, you can have it. Work done from position one to two is k over two x one square minus x two square. Again, it is independent. It depends only on the final and initial position. So it means the force exerted by the spring is also a conservative force. Now let us try to put it in a slightly more interesting way. The same example which we have already seen, that is, a particle is to move from A to B.

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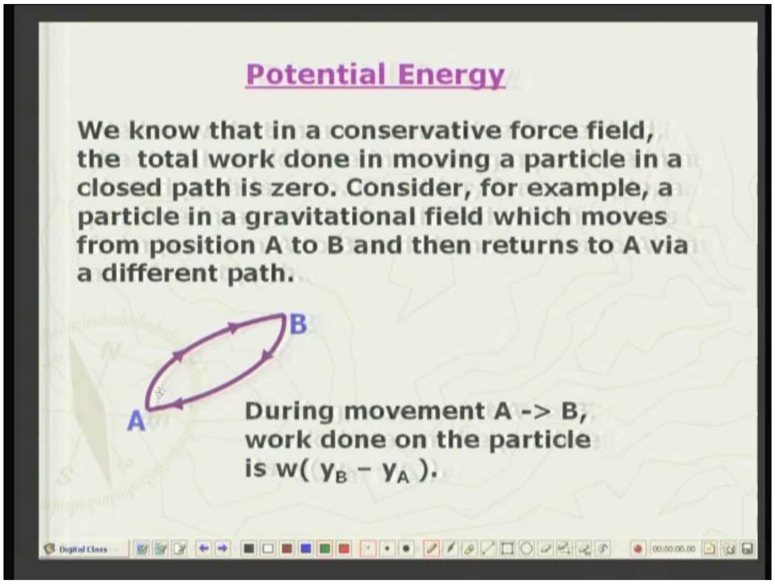

Now in moving it from A to B, certain amount of work has been done and when it returns by some other path or may be by the same path, it becomes a closed loop and means the total work is zero. Now that means that from B to A the particle has done work on the system. First of all, the system does the work and then in return, the work is lost. It means the particle has lost something to do the work and we can put it in a slightly different language, that is, a moving particle A to B, a capacity or capability was imparted to the particle. So that it can do the work while going from B to A. So that capacity to do or capability to do work of the particle is called the potential energy of the particle. Well, example, you can take a gravitational field from A to B. It has gained something by work being done on the particle, namely, it has gained height and due to that when it comes back to the original height or it falls back to the original height, it means it has lost something and that gain or loss is the potential energy. So that when it comes back it can impart work to some other system.

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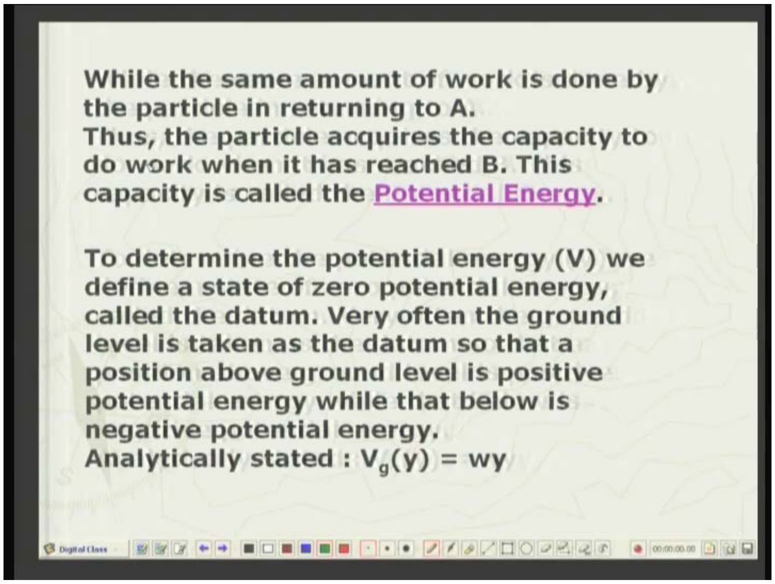

So we can say that the particle acquires the capacity to do work when it has reached B and this capacity is the potential energy. Well, in order to determine the potential energy of the given particle or a rigid body, we start with a state which we will call the datum state, the natural state or some identifiable state which we will call the zero potential energy state and this is called the datum. For example, these are most like an elementary example of gravitational field. So suppose on the ground level, I am taking it as the datum level, if the particle is laying, then it has zero potential energy. So when it is raised up, it will have some positive potential energy. When it goes below the ground, that is, underneath, then it has negative potential energy. So the basic concept is the datum level, that is the zero level of energy, and depending upon positive work done or negative work done, the potential energy will be accordingly higher or lower, that is positive or negative.

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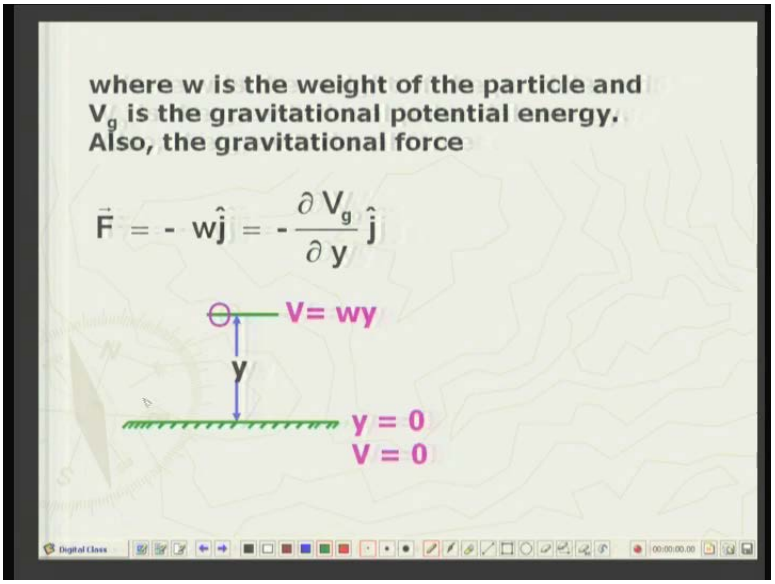

So I have illustrated this. Suppose there is a sphere or a ball of weight W, then at the ground level, that is y coordinate, the height coordinate is zero. So the potential energy is zero but when it has gone up, the potential energy will be w times y. If it goes down then it will be negative.

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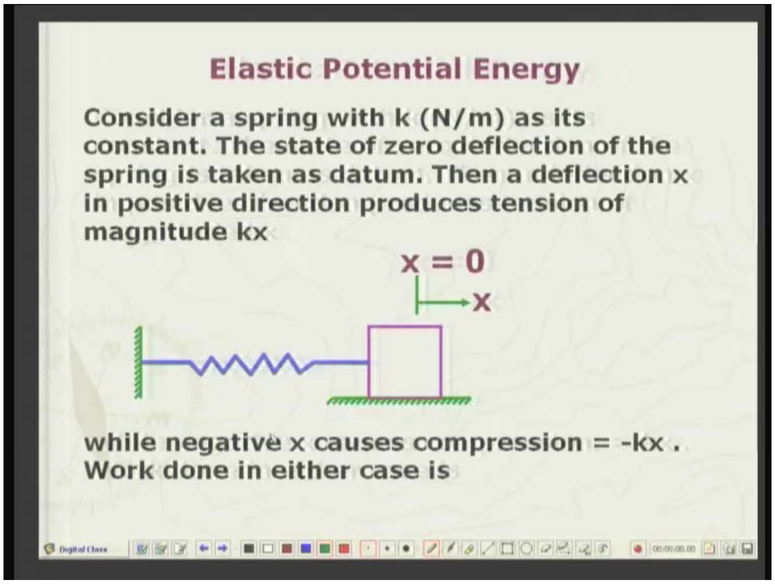

That was the gravitational potential energy. Now, let us see the elastic potential energy. Since this spring is an elastic element, the energy stored in the spring will be the elastic potential energy. Again, we will choose the datum for this system as the natural length of the spring, that is, unstretched or uncompressed length of the spring. So let us say when the mass is at position x and is equal to zero, the spring was in its natural state, no deflection, the energy corresponding to this is the zero energy. When it goes in the positive direction, let us say, it has reached over here, then we have already seen the energy is equal to half k times the displacement from the zero level, that is, x so half kx square and even if it goes in the negative direction, that is, spring is compressed, it will be half k minus x square, which is the square of minus and positive. So whether the spring goes moves or mass moves to the left or to the right, the energy stored or the potential energy in the spring will be always positive. That is the interesting aspect.

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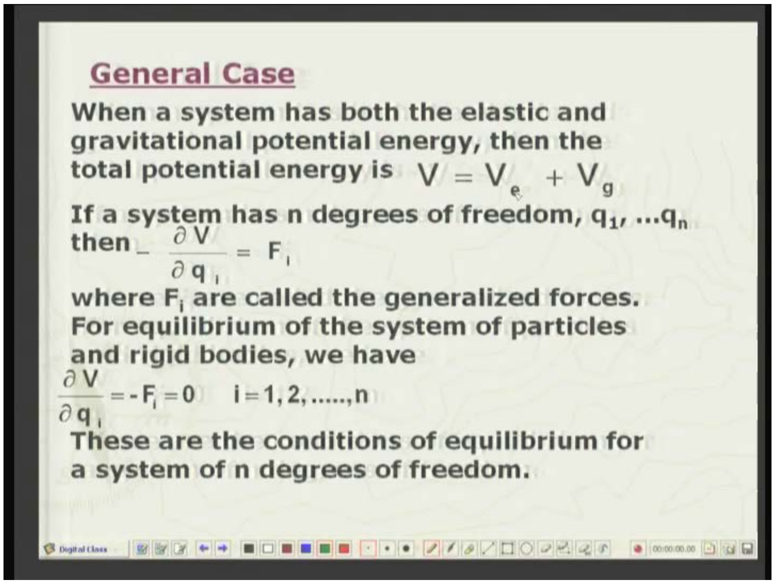

When your system has both elastic potential energy as well as the gravitational energy, since the energy is a scalar quantity, energy comes from work and work is scalar being a dot or scalar product. So energy is also scalar quantity and the scalar quantities can be added or subtracted like real numbers unlike vectors where we have to use the parallelogram or triangle law of a vector addition. Here a simple arithmetic addition or subtraction is needed. So if I add both potential and gravitational and elastic potential energies, then I will get the total energy potential energy V as Ve plus Vg in whether it is gravitational case or the elastic case.

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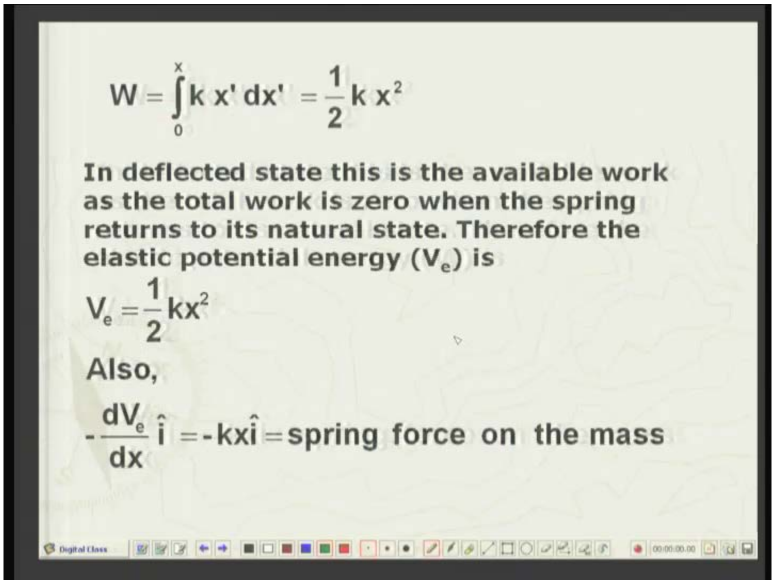

If you differentiate the potential energy, as we can easily see that here we have elastic potential energy, so half kx square and if I differentiate this with respect to x, I will get minus kxi unit vector i. So this is the spring force and similarly when I took the example of gravitational field. So again if you differentiate with respect to height, with a negative sign you will get the vector of gravitational force. So if you take the total potential and differentiate with respect to the corresponding coordinate with the negative sign, we will get the conservative force. We have earlier defined degrees of freedom in last lecture. So suppose for a large system which has many degrees of freedom, with the degrees of freedom are q one q two up to qn and you have expressed the potential energy in terms of q one q two up to qn. If you take the derivative of the potential energy with respect to any q qi, let us say a and put a negative sign, then it will give me the corresponding force Fi. This force may be the actual force or if the degree of freedom is an angle, then it will give you moment, etcetera, etcetera. So in general, we call these as generalized forces. So please remember that the derivative of the total potential energy with respect to any degree of freedom will give me the corresponding generalized force and for equilibrium, let us say, a given particle is subjected to several potential energies and if we want to examine the equilibrium of the particle, then each and every generalized force should be equal to zero.

It means dV by dq i that the derivative with respect to i th generalized coordinate is equal to zero. This is the statement of equilibrium of a system. Well, let us say, my generalized coordinates or degrees of freedom are x y z. Then the total force acting on the particle is equal to minus dV by dx unit vector i minus dV by dy unit vector j dV by dz unit vector k and this thing we can write in the notation of vectors as gradient of V. V is a scalar potential field and its gradient is a vector quantity. This is a well-known result from vector analysis. So minus del V minus, the gradient of the potential gives me the corresponding force and if I have a virtual displacement del r vector which is given as del x i plus del y j del y j plus del z k.

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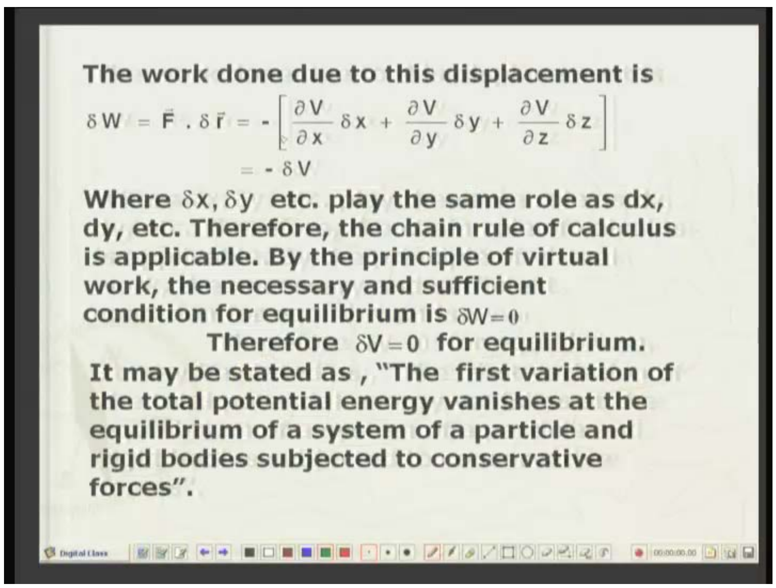

Then taking the dot product of the force with the virtual displacement, I will get the virtual work del W is equal to dot product of F with del r and if you carry out this product, then dV by dx into del x plus dV by dy plus del y plus dV by dz into d ah del z. That is exact nothing but minus of the del V, that is, the virtual where difference between the two positions of the potential energy.

Once again this virtual difference is due to the virtual displacement. This is not the actually displacement, that is the important thing is that in carrying out this virtual displacement, the forces don’t change. Otherwise it will be the actual displacement particularly for when a spring is involved. Any additional deflection will cause additional tension or compression and hence the force will change but when you give virtual displacement as compared to the real displacement the virtual displacement will not cause any change in the tension and compression. So that artificial situation you have to bear in mind. So this is sometime del V is called the first variation of potential energy. So the equivalent principle to the principle of virtual work is the that del V is equal to zero. So when a system is at equilibrium then the first venison of the total potential energy which will include gravitational potential energy. Spring energy is potential energy or any another potential due to a conservative force. So if I calculate the variation of that potential energy when a virtual displacement was applied and set that variation equal to zero, then the system is in equilibrium. This statement is both sufficient and necessary, that is, every system is in equilibrium. del V is equal to zero and if del V is equal to zero, then the system is in equilibrium provided our force field are, whatever force field acting they are conservative if there is reflection or any other dissipative mechanism then the field will no longer be conservative. It will be dissipative field. So then this principle of first variation of potential energy cannot be applied.

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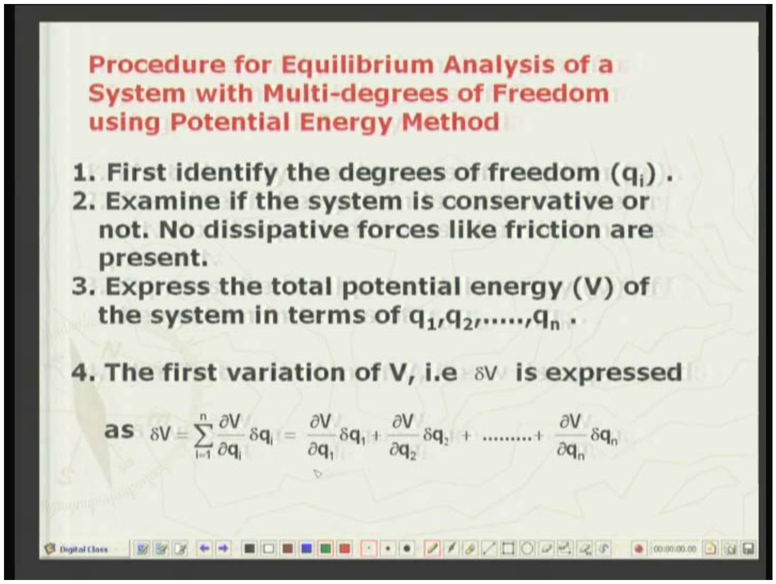

Now let us recapitulate and lay down the procedure for examining the equilibrium of a system with many degrees of freedom using potential energy method. So I will go over the step by step procedure. First you examine the system and identify what are the degrees of freedom, that is each coordinate or generalized coordinate should be able to vary independently, that is, without effecting the other coordinates. So first, identify the degrees of freedom. Let us say there is n degrees of freedom q one q two up to qn. Then the important thing is you examine whether the system which we are dealing with is conservative or it has some dissipative components, that is, a friction or viscous or dry friction or in hard system, some dissipative mechanism evolve heat.

So we should not have those dissipative components. So all it means our system is totally conservative system. Then the potential energy of the total potential energy of the system, that is, V is expressed in terms of q one q generalized coordinates q one q two up to qn. So V is a function of qi i going from one to n. Then you determine the first variation of V, that is, dV by dqi into del qi that del q one del two etcetera are the virtual displacements and take the corresponding partial derivatives multiply and add, you will get the total variation of the potential energy.

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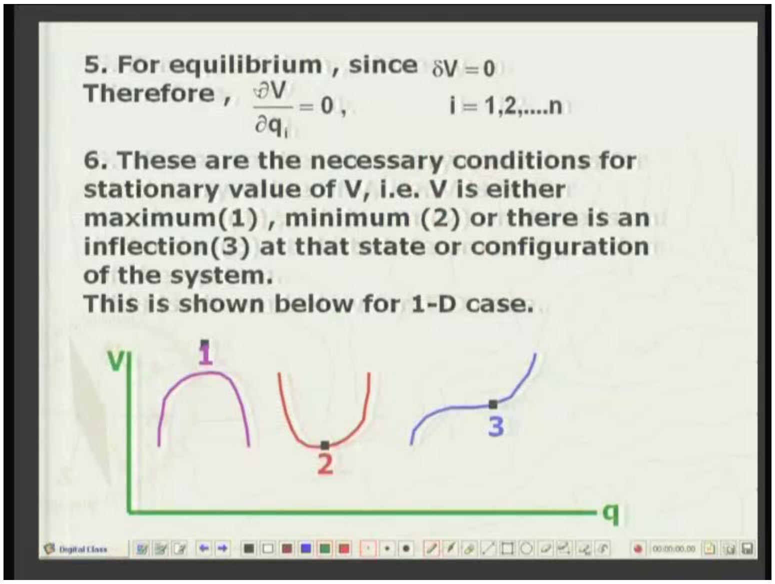

After having calculated this, you set it is equal to zero and equivalently it means each of the partial derivatives is equal to zero because dq del q one del q two, etcetera. They can be varied independent of each other. There is no connection. So it means in general, that statement is valid, del V is equal to zero is valid. If and only if all the partial derivatives are individually zero. Well, suppose this condition is found to be valid, then what does it geometrically or graphically signify? These are also from calculus. You know these are the conditions for the maxima or minima or stationary value of a function of several variables. So it means here, I have depicted it for V as a function of only one variable. When it is a curve, if it is a function of two variables, then it is a surface for and so on and so forth higher order surfaces. You can examine. So in a curve, one dimensional case, the function V, that is, the total potential energy, will be varying like this and at equilibrium, it can be maximum, it can be minimum or there can be an inflection point, that is, it changes from concave to convex. So that inflects. So all the three conditions will imply that the slope is zero. That is the tangent is parallel to ah q axis. Similarly over here. Similarly over here. So this is the graphical interpretation, that is, at equilibrium potential energy is either maximum, minimum or it goes through an inflection.

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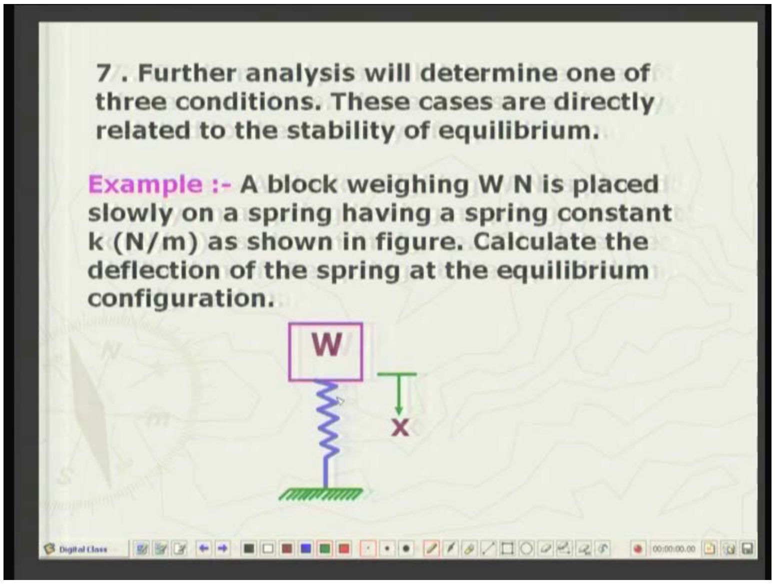

Well a further analysis which we will be taking up, perhaps in the next lecture, will show us that these three conditions will correspond to three qualities or three types of equilibrium: unstable, stable and neutral or something like that. Now let us take up a couple of examples to illustrate our principles. Suppose here is a block of weight W which is slowly made to rest on a spring. So this spring will be under compression. The spring constant K Newton's per meter. Deflection of the spring is shown here. Calculate the deflection on the spring at equilibrium configuration. So we have slowly placed it. Naturally, the mass will go down and what is the equilibrium position of the mass? As I said first, before starting, to compute the total potential energy. You have to fix your datum and since in this problem only one coordinate, that is the deflection or x coordinate is involved, it is a single degree of freedom system. That degree is x. I am taking downward as positive. X from here. So this is the unstretched condition. So downward is positive.

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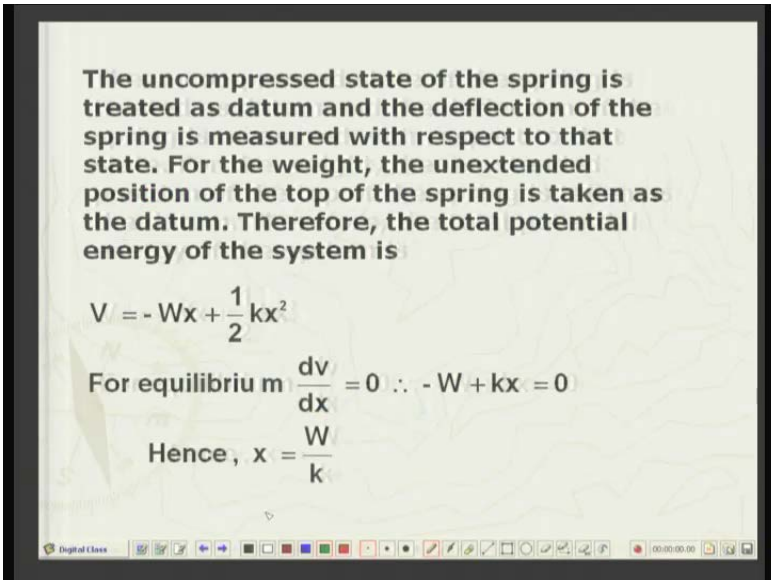

Now the natural state of the spring is taken as the datum for the spring and then when the mass is just touching the spring without its force being acting on the spring, that is, the datum for the mass, we will then compute the gravitational potential energy of the mass. W times x with a negative sign because it is proportional to the height. That is the difference above the datum.

So but our X axis down ward, for, that is why we have taken it negative here and here it is minus x square which is simple x square. So half k x square is the elastic potential. The system of mass and the spring, the total, the system has the total potential of minus Wx plus half kx square. Then for equilibrium dV by dx is equal to zero and that will give me minus W plus kx is equal to zero and hence x is equal to W over k. A very simple problem but it illustrates all the points. That is the total potential energy of the system comprising of the weight and the spring is the contribution of the weight and contribution of the spring and then you simply take the slope or derivative and set it equal to zero. That will be giving you the equilibrium compression of the spring W by k. Simple.

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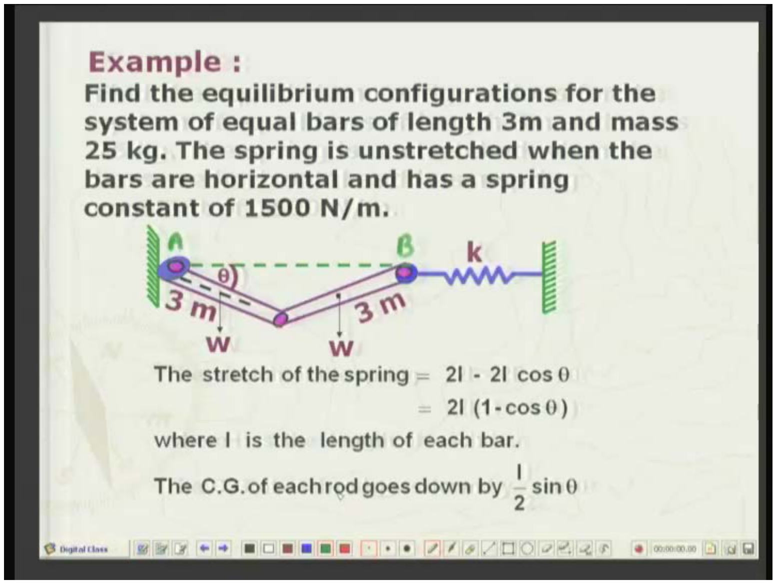

Let me go to a slightly more complex problem. This problem consists of two rods each of length three meters and mass twenty-five kg and there is a spring attached to one of the rods. The other rod is attached to a fixed support and the spring is also, on the other hand, attached to fixed support. So this distance is fixed between these two supports. Now the spring is un stretched when the bars are horizontal. So this datum state is defined, that is, when both the bars are horizontal, theta is equal to zero. So this will also be horizontal and at that time whatever the length, that is the total distance minus six meters, that will be the un stretched length of the spring and when the system is released due to weight, the bars will go down and simultaneously they will stretch the spring and spring will be storing elastic potential energy. These two bars will have the gravitational potential. So let us see first of all the stretch of the spring. What is the change in length of the spring? Originally let us say, it is this change in length will decrease in the length of over a because the total length is same. So if I can determine the decrease in the length from this point to this point, point A to point B, originally this was two l. So now it is two times cosine of cosine theta of l l time cosine theta. So two l minus two l cosine theta two l taken common out one minus cosine theta. So this is the change in length of the spring because the total length was to remain constant. So next, we see how the CG of the rod has gone down. What is the shift of CG? Originally bars are in straight line. So CG was somewhere here. Now it has gone down and centre of gravity will tell us how much the change in the potential energy is. So well the bars are assumed to be uniform. So the CG will be at half the length l by two times sin of theta so l by two sin theta. So we have elastic as well as gravitational potential. Easy to calculate.

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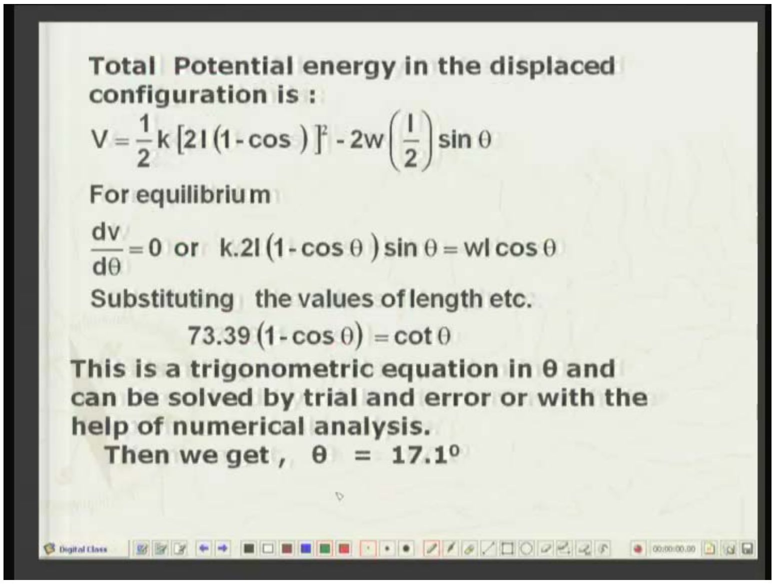

So the total potential energy of the system is for the first elastic component half k times change in length of the spring squared where two l into one minus cosine theta whole squared minus because it is going down. Each bar is going down W into l by two sign theta W into change in height and since two bars words. So minus two W l by two sign theta and for equilibrium, if I differentiate, there is only one degree of freedom. So dV by d theta total potential energy is capital V dV by d theta is equal to zero and if I take the derivative and simplify it, I will get seventy-three point three nine into one minus cosine theta is equal to cot tangent theta. This is a trigonometric equation which can be solved either by trial and error method or graphically or some numerical analysis techniques are available and if you apply any of these methods you will get theta is equal to seventeen point one degree. So it means just by calculating the total potential energy, and the interesting thing is, this potential energy is a scalar quantity, you can add, subtract, do anything and that has to be obtained as a function of the degree of freedom theta and differentiate it. It is equal to zero. As simple as that you will get the equilibrium configuration.

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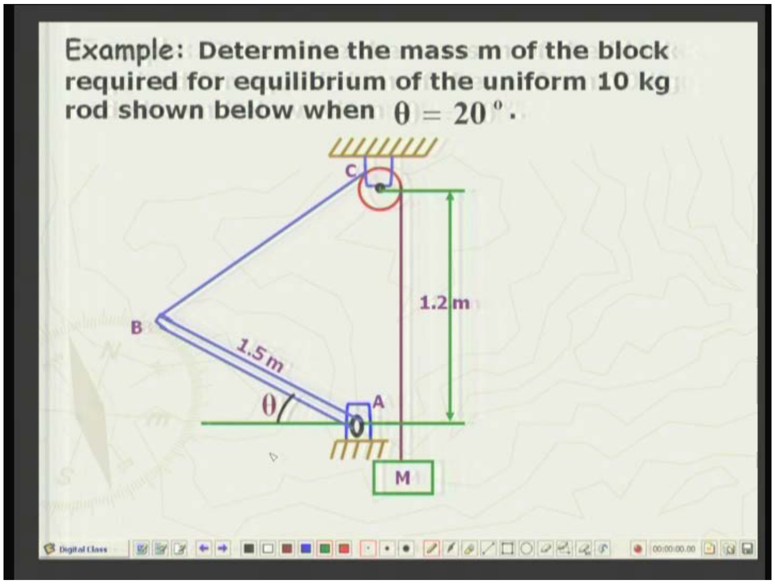

Let me take up a slightly more involved problem. Here. Determine the mass m of the block required for equilibrium of uniform ten kilogram rod. The rod shown when theta is equal to twenty degree. Now let me explain the problem. Here is a rod. One end of the rod is pined over here and the other end is attached to an expansible string which goes over a pulley and carries a mass M. So for a given value of theta, that is, twenty degree, what is the value of mass which is required to maintain the system in equilibrium? That is, system should be at rest. It does not go up or down. Now first of all, we will take the datum as theta is equal to zero degree configuration, that is, you can see when this bar A B is horizontal B is somewhere over here and A is a fixed point. So it is horizontal and then the string will be going like this and the mass will be at some position over here. So that is my datum configuration and any other configuration. There is given the angle theta. That is the only variable involved because the theta will uniquely determine the configuration of AB and since the total length is fixed, it will uniquely determine the position of M. So one degree of freedom system. So, what I have to do is that I have to calculate the change in the length BC. Of course, I am neglecting the diameter, etcetera of the pulley. So if I can find the change in the length of BC and since the string is inextensible, total length remaining same, I will know the change in height of mass M right. So let us do that. Only one type of potential energy is involved, namely, the gravitational potential energy.

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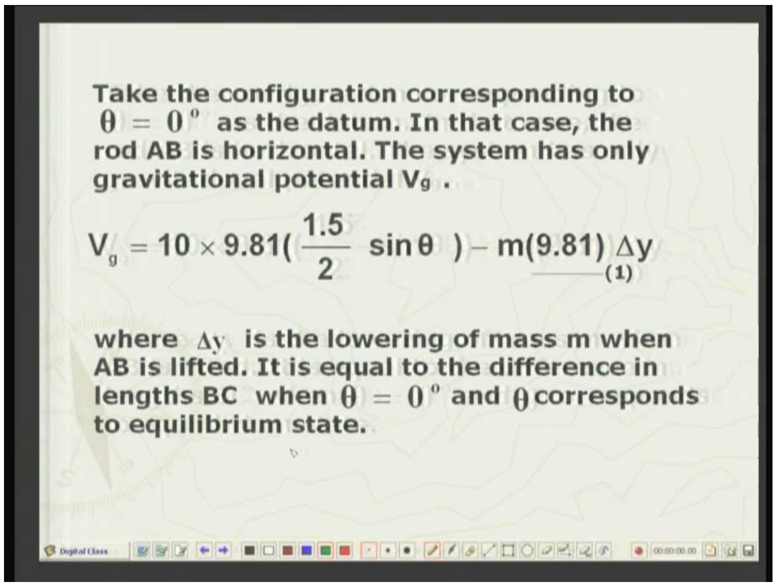

First of all, the rod is ten kilo gram in mass. So mass into G, that is, the weight. So ten into nine point eight one and then the center of gravity at the mid length of the rod length is l, is equal to one point five. So mid length is one point five to divided by two into sin theta. That is the height of the CG from the datum. So that is the potential energy of the rod. It is going up. So potential energy is positive and as the rod goes up, the mass is going down. So mass times nine point eight one, that is, the weight into the change in height del del y. So I have got the both the potential energy of the mass and the rod. To determine del y, all that is needed is trigonometry.

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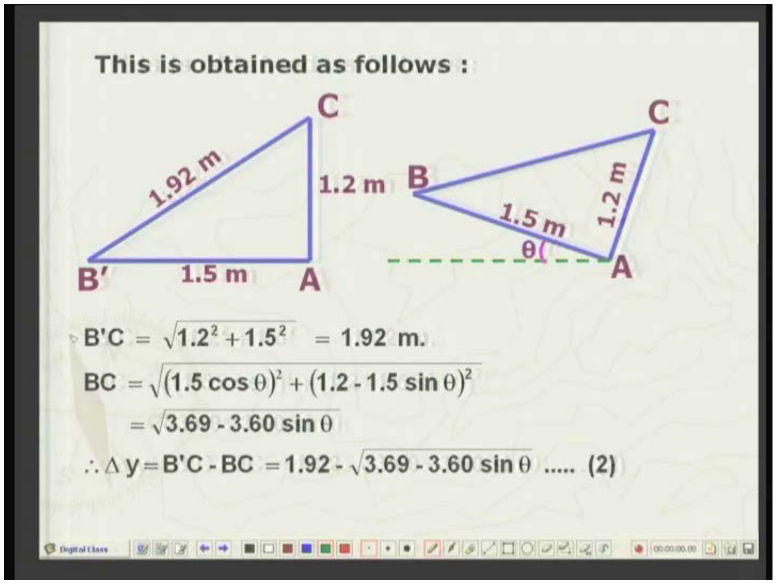

Datum in the datum condition A B dash C, that is this part of the figure, when it is horizontal, A B is here and C is over here and when it has gone through a rotation of theta, then this is the configuration. All you have to do is that you have to find out B dash C which from the Pythagoras we can easily see is one point five square plus one point two square whole under root which comes to be one point nine two. So this is one point nine two and when it has gone through rotation of angel theta, then it is to be obtained. Well, you can easily see one point five cosine theta, that is this and one point two minus one point. So why I have to find out this height? So one point five cosine theta square plus one point two minus that height. So this will come out to be three point six nine minus three point six sin square theta whole under root. So the difference between the two is B dash C minus B C is given by this quantity.

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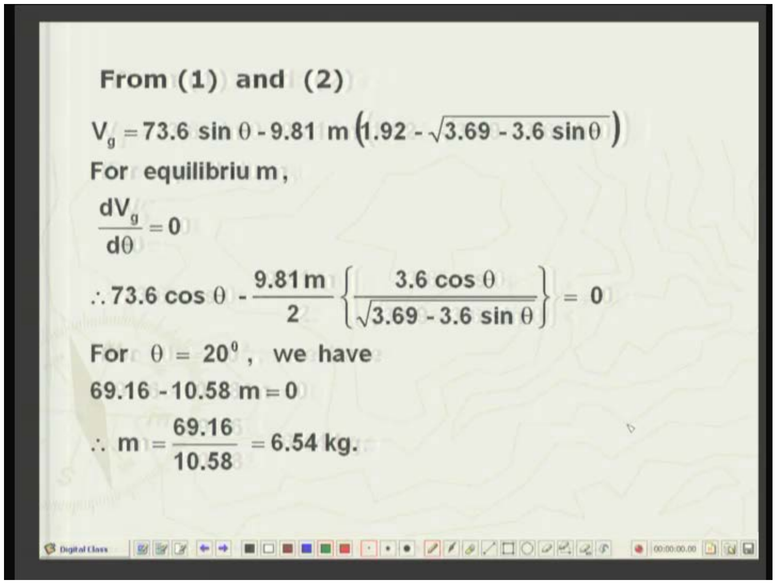

From one into two, we can find out the total potential energy of the system is seventy-three point six sin theta minus nine point eight one m into one point nine two into the three point six nine minus three point six sin theta and since there is only one degree of freedom dV by D theta is equal to zero. So I have this equation and since theta is equal to twenty degree is given to us, we will have sin theta substituted for sin twenty degrees and we will find that the mass required to obtain equilibrium is six point five four kilogram. So at theta inclination of the rod is equal to twenty degree to the horizontal. The necessary magnitude of the mass to maintain equilibrium is six point five four kilogram. Well, all the examples which I have taken are corresponding to one degree of freedom. So let us consider an example with two degrees of freedom.

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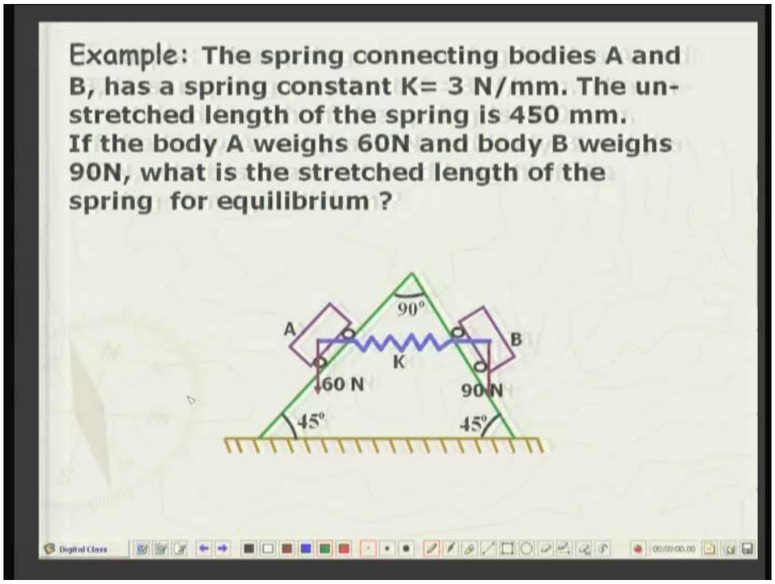

The example is like this. The spring connecting bodies A and B, these are two bodies which can slide down on these two inclined plains. No friction involved. Body A has a weight of sixty Newton, body B has a weight of ninety Newton's and they are connected with the help of a linear elastic spring with spring constant as three Newton's per millimeter. The un-stretched length of the spring is given as four hundred fifty millimeters and the weights are given already, sixty and ninety. What is the stretched length of the spring for equilibrium? Now the weight A and weight B can move independently. Of course, there is a spring attached into them but they do not have to be always horizontal. This can be at a higher level. This can be at a lower level and vice versa. So in that sense, they constitute a system of two degrees. I could have chosen the the displacement from this apex to this and second, as this to this but I have chosen slightly different generalized coordinates.

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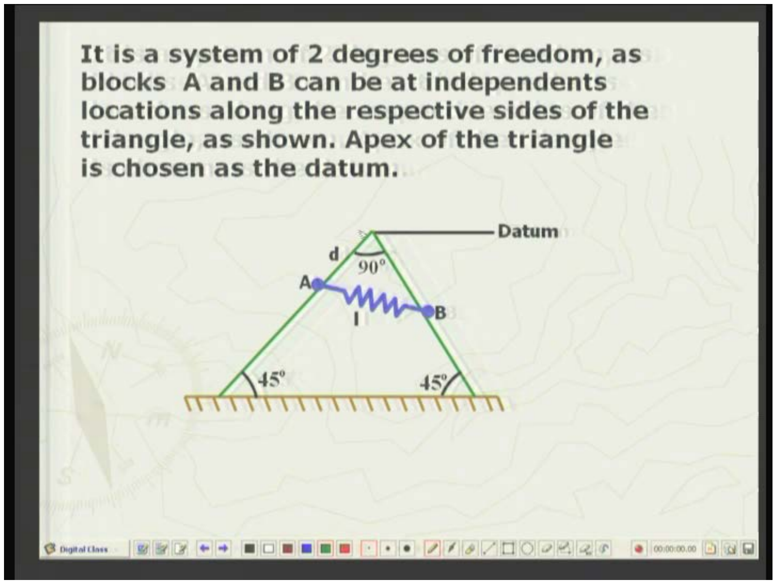

Let me show you. This apex I am taking as the datum for my system, that is, at this point the energy is zero and this is A and this is B and the distance of A from the apex is one degree of freedom. The other degree, you might have chosen this but for simplicity I have chosen the stretched length A B between the two because that is also uniquely determined as the second degree of freedom. So l which is the stretched length of the spring as the second degree of freedom. So d and l are our two degrees of freedom. Well, first of all, from the datum, I will find out how low the weight A is. So this is d. So I have to find out this height from the datum. What is the depth of A below the datum from point O? So that is D cosine theta and since it is below, it is negative. So weight into D cosine theta cosine forty-five degree with a negative sign.

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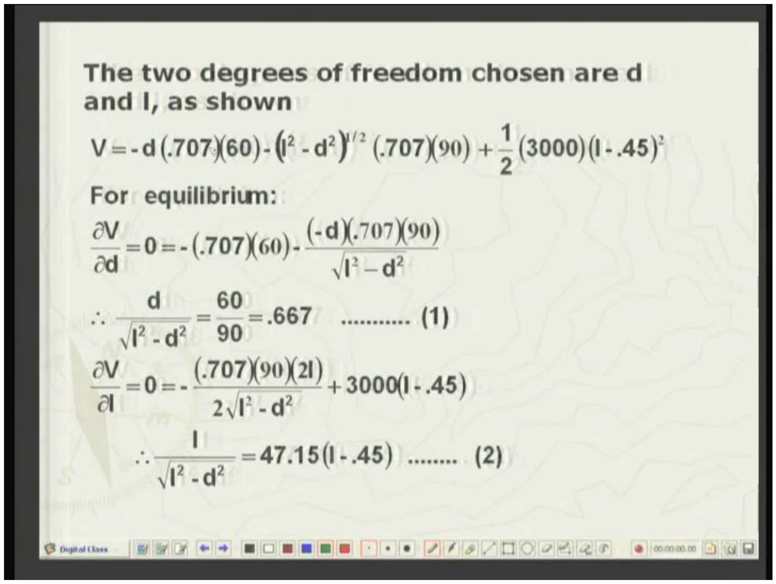

The second thing is the potential energy of the spring. So this is the potential energy of the spring, that is, three Newton's per millimeter means three thousand Newton's per meter. So I am using meter my unit for length. So that is why half k, that is, half into three thousands the final length l minus point four five, that is, the stretch whole square. So half kx square and weight B, what is this? So again l square minus because this is a ninety degree. So I can, by Pythagoras theorem, l square minus d square whole under root will give me this length and again, I will take to the cosine of this. So you get the contribution to the potential energy of weight B is l square minus d square under root into cosine of forty-five degree into ninety. So this is the total potential energy and once you have this as the potential energy, it has two degrees of freedom d and l. So for equilibrium dV by dd is equal to zero, dV by dl is equal to zero. I will get two equations. So well, differentiation is quite elementary. So one equation will be d over root of l square minus d square is equal to point six six seven and the other equation is l over l square minus d square is equal to this much. So from these two equations, I can determine the two unknowns. So for example, if I square square two and square one and subtract, it will give me l square over l square minus d square, this will give me d square over l square minus d square. When I subtract it, I will get l square minus d square over l square minus d square is equal to one and that will give me finally the value of length l which comes out to be four seventy-five millimeters.

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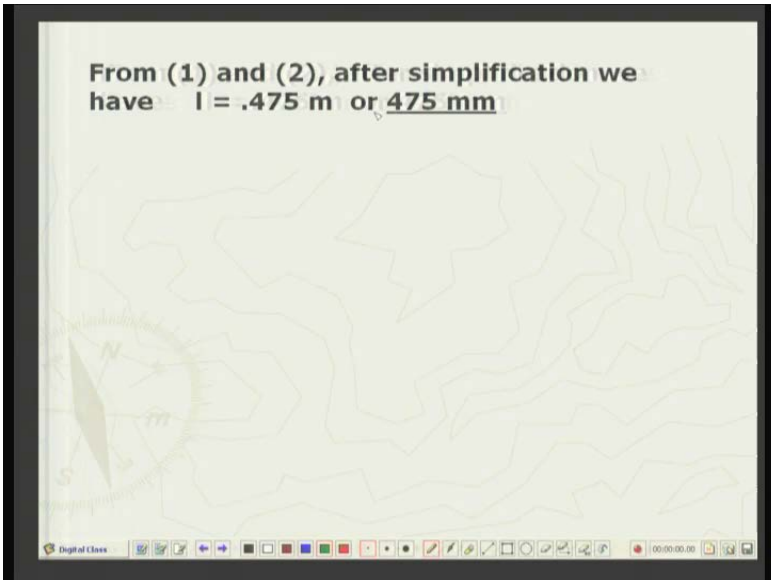

So there is stretch of twenty-five millimeter in the spring. So a system of two degrees of freedom can be very easily determined or established for equilibrium with the help of the stationary value or maximum, minimum or inflection value of the potential energy. In our next lecture, as I have already indicated, we will see how these three condition, maximum, minimum or inflection, will correspond to a quality of the equilibrium, whether the equilibrium is stable, unstable or it is neutral. So when the system of forces is conservative, no dissipative forces are involved. Then we can get the analysis of equilibrium in a very simple way, that is, you will take the first variation of the total potential energy, set it equal to zero and that will define the equilibrium. So that is all for today's lecture.