Applied Mechanics
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Lecture No. # 14
Methods of Virtual Work and Potential Energy

In lecture fourteen, we will take up methods of virtual work and potential energy. This is a new topic. Quite different from the topic which we covered in last three lecture, namely, the properties of surfaces and moment and product of inertia.

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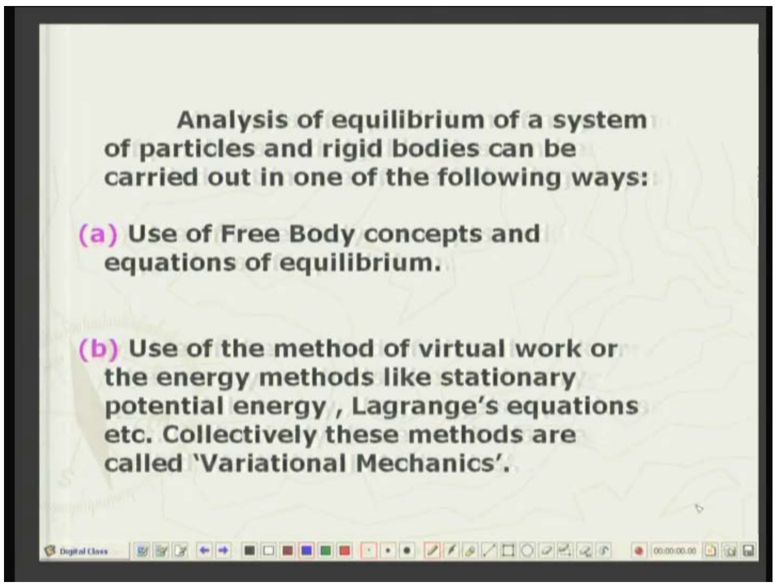

These two methods which I have mentioned deal with the equilibrium of a system of particles and rigid bodies. This analysis of equilibrium can be done in one of the two ways, number one, use of free body concepts and equations of equilibrium. You may recall that in lectures three, four and five, we were discussing equivalent force system, free body diagram, equations of equilibrium and all those related topics. So that is one way of analyzing the equilibrium. The other way is the subject of today's lecture and the next two lectures will be devoted to this. This deals with the use of the method of virtual work or energy methods like stationary energies. Stationary means maximum or minimum energy. Collectively all these methods or labeled variational mechanics, because they depend upon variational calculus, we will not be discussing in this course but it is a very interesting subject. Lagrange's equations also, we will not touch because of their maximum use is in the problems involving motion of particles dynamics problems but again, they are very versatile and very useful in analyzing the motion of particles and rigid bodies. Suppose we are discussing a system consisting of particles and rigid bodies or several sub systems which are interconnected with each other and the usual method is to divide this total system into individual free bodies and then set up the equations of equilibrium for each of the free body and this will then assemble all the equations of equilibrium of the free bodies.

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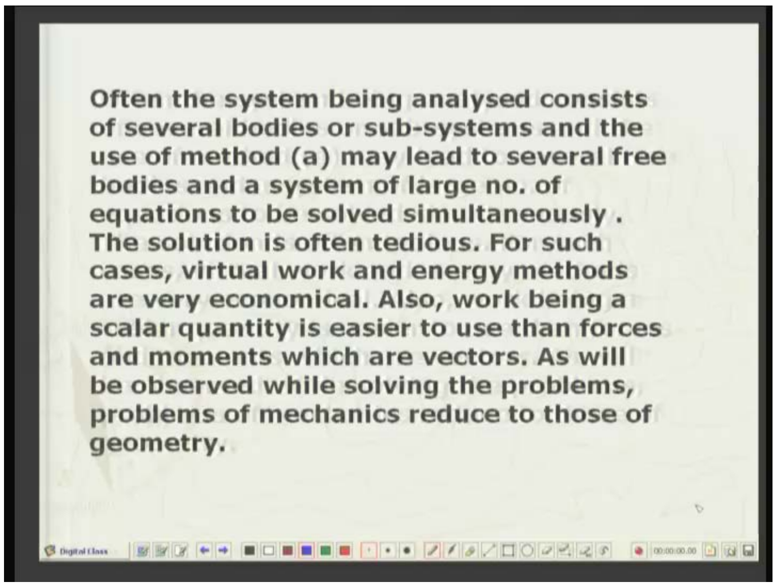

This will lead to a large system of simultaneous equations, algebraic equations or sometimes differential equation and then we solve them. So in general the procedure is quite lengthy and we need to use some numericals in many cases. This may lead to some numerical analysis.

Now, on the other hand, in the variational methods, we will concentrate on a very simple concept, that of work and energy. The advantage of work using work and energy instead of forces and moments, is that work and energy are scalar quantities whereas forces and moments etcetera, are vector quantities. Now, addition of a scalar quantities, multiplication, etcetera, are much easier than the corresponding mathematical operations for vector quantity. Also, we will see, when we take up examples involving work and energy that the equation of mechanics reduced to equation of geometry, where we will have to correlate small displacements in one variable with the other and so on and so forth. So, very often, it is seen that the variational methods or virtual work method lead to much simpler, economical solution to the same problem as compared to the free body concepts and a use of equation of equilibrium.

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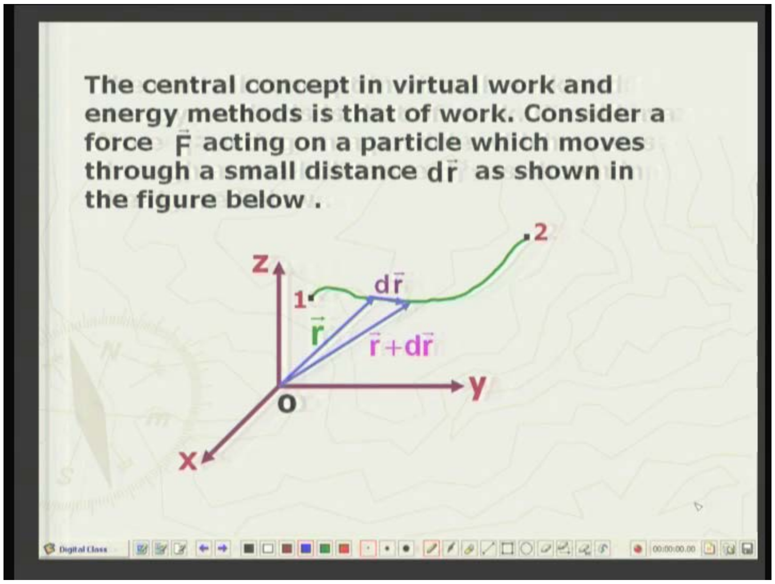

Well, let us begin with what is work in total. We know that work means that you apply some force and move the body. So suppose there is a block. You push it. You are doing some work but mathematically or analytically, we will define work like this. Suppose there is a force vector F which is trying to move a particle from position one to position two. Now, at an instant, the location of the particle is at the position vector r and when the motion has taken place through a small distance dr, the location of the particle is at the position vector r plus dr. So, the work done in this operation, the differential work dw is given as a definition F dotted with dr, that is, the force vector scalar product with the displacement vector dr.

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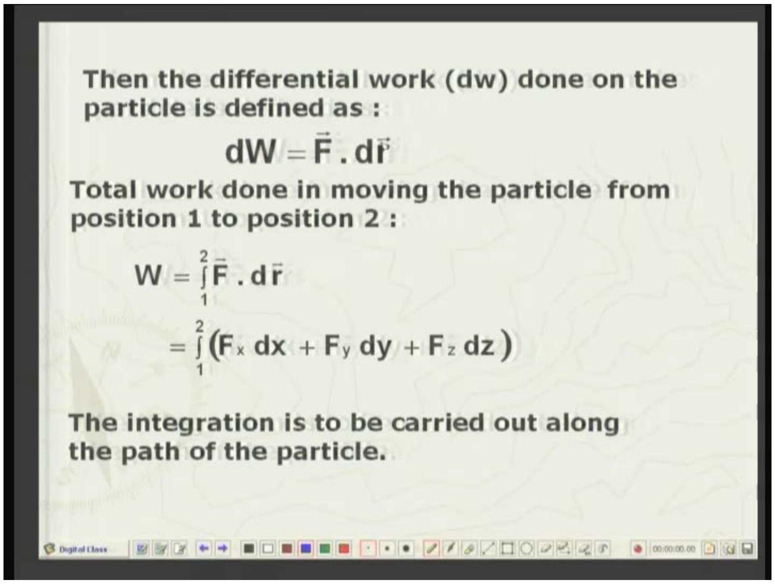

Since, you may recall from lecture one and two that the dot product is the product of the component of F along dr the vector of displacement. So if we integrate it to get the total work from position one to position two, here is position one, position two, so the force may be changing. The location is changing. Then we will do the integration from position one to position two of F dotted with dr and in the component form, this integral can be written as integration from position one to two Fx dx, which is the x component of F Fy dy plus Fz dz and mind you, this integration is to be always carried out along the path, that is, whenever you are moving from point to point, it is along the path. So in general, this integration can be path dependent.

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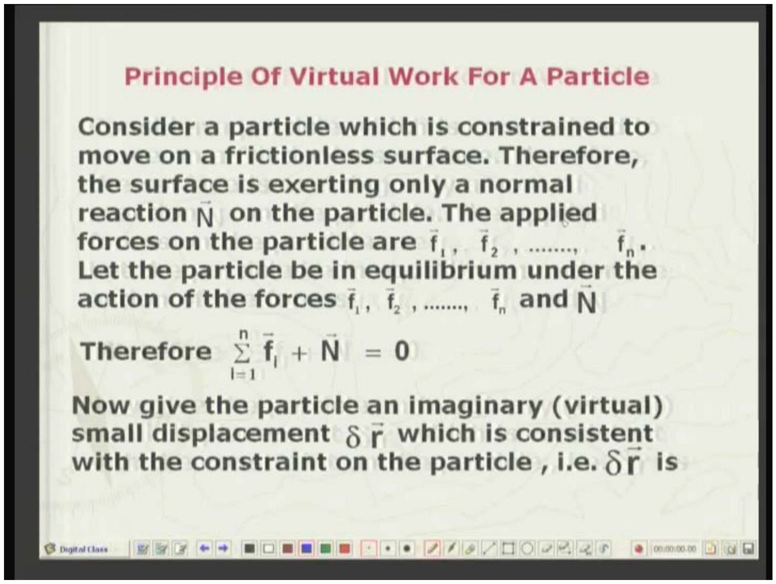

Now, after having understood the concept of work, let us now go to the principle of virtual work of a particle. First, we will focus our attention on the motion of a single particle. So consider a particle which is constrained to move on a given surface. That is the restriction we are placing for virtual work. So, it is constrained to move on a surface which is frictionless and smooth. So there is no friction force. The only interaction between the particle and surface will be through the normal reaction due to the weight or other component but there will be no tangential component as friction has been ruled out. So besides the normal reaction, the particle may be subjected to forces f one, f two, up to fn. So N number of forces given by vectors f one, f two, up to f n is also acting. So the total number of forces acting on the particle is n active forces, f one to fn and the next n plus oneth force is the normal reaction which is called the reactive force. Now if the particle is in equilibrium, then we know from equations of equilibrium of a single particle, that sum of all these forces is equal to zero. So we have sigma of fi I, going from one to N plus the reactive force N is equal to zero. Now, we give this particle a small displacement which we will call virtual displacement. I will mention why it is virtual, why not real and we will label it as delta r vector. Now the characteristic of this displacement is that this displacement is consistent with the constraints. The constraint on the particle is that particle is always on the surface. So that the displacement delta r vector is tangent to the surface at that point, at that current position and there is no normal component because if there is a normal component, it means the particle will lift off from the surface which will be violative of the constraint and why it is virtual and why not real.

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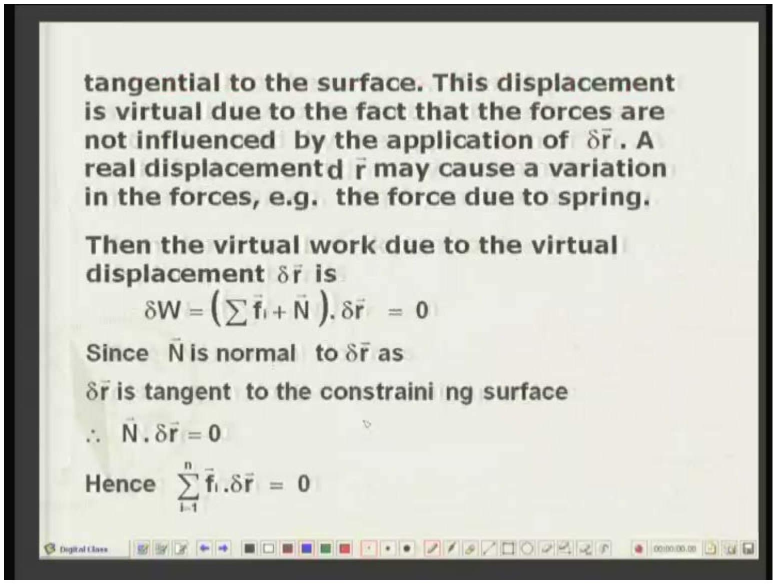

Real displacement may be labeled it as d r vector. See, when we give an actual displacement, some of the forces may change. For example, you take the case of a spring. Spring is compressed by an amount x. Now if I change the compression from x to x plus dx, then the compression of the spring changes. It means the force exerted by the spring on the particle will also change.

That is the real displacement but in virtual displacement, we give the displacement but we do not allow the forces whichever can change but we don't allow them to change. That is why this is the virtual displacement or imaginary displacement. Not the real one and to distinguish between the two types of displacement: Real displacement. They are both of them are of very small magnitude. So the real displacement will be designated as d r vector whereas virtual displacement we will designate as del r vector. So in such a virtual displacement, the work done by all the forces is simply the dot product and, you can easily see that the dot product of N with dr dell r is zero because N is normal to the surface. Dell r is tangential to the surface. Both are perpendicular to each other. The dot product of two perpendicular, normal vectors is zero.

So that is why this is equal to zero. So it means that the virtual work done is reduced to fi vector doted with the virtual displacement vector dell r. The summation of all these is equal to zero. This is called the principle of virtual work.

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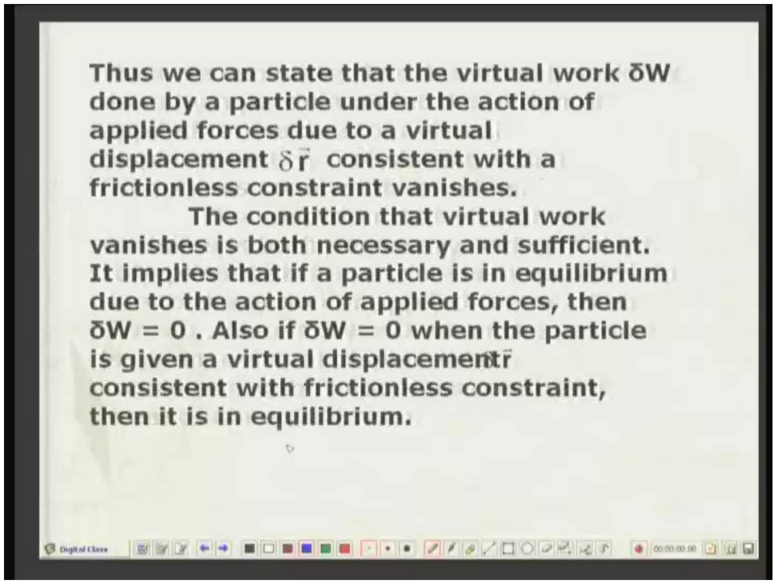

We can state that the principle of virtual work, del W, is a scalar quantity done by a particle under the action of applied forces due to virtual displacement del r vector consistent with the frictionless, smooth constraint , zero, vanishes. Well, this condition, although, it look so simple, is a very powerful condition and it is both a necessary and sufficient condition, that is, if this condition is followed, that is a system follows this condition, then the system is in equilibrium and vice versa. If the system is in equilibrium, then it must follow the a condition of virtual work. So total virtual work is equal to zero implies the particle is in equilibrium.

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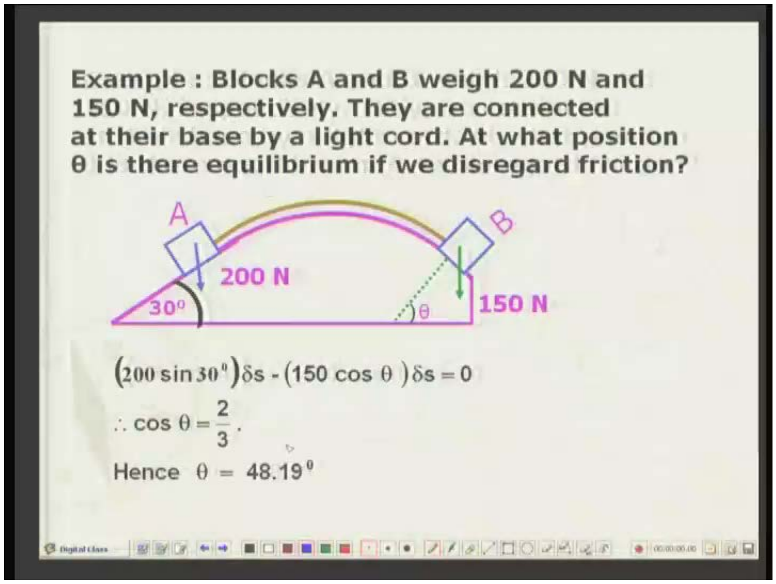

Well, we can illustrate this principle very easily with the help of a simple problem. Suppose there are two blocks, A and B. The weight of block A is two hundred Newton's, weight of block B is hundred fifty Newton's. At a given location, the position of B is making an angle theta to the horizontal and this inclined plane is at angle thirty degrees. The block A is on this state, incline plane. Both these blocks are connected by an inextensible cord or a metal band. So now the surfaces are assumed to be frictionless. Then the constraint, we can say, is frictionless constraint.

Well, we have to do it by the free body diagrams. You would first consider the free body A and there will be a tension in this direction and you will take the components in the normal direction and tangential directions. Similarly, you will have another set of equations for body B and so on, so forth. So the procedure is quite lengthy. Here, we will use the principle of virtual work. What I will do is that, I will give a small virtual displacement delta s to block A. So this displaced position is along parallel to this inclined surface, at a distance of delta s, since the joining band or the cord is inextensible. It means block B will also move through the same distance along this surface. So this is again delta s. So now, we will consider what is the work done by block A and block B and, since both the works are scalar quantity, we will just add or subtract. Here, you can see that there is a two hundred Newton force and the component in the direction of the displacement, that is tangent parallel to this thirty degree surface is two hundred sin thirty degrees, and this component into the displacement in the same direction delta S. That is the work done.

Whereas in this case, the component in the tangential direction to the surface will be in this direction and its value will be hundred fifty cosine theta because this angle will be pi by two. This angle will be pi by two minus theta. So sin of pi by two minus theta will give me cosine theta. So hundred fifty cosine theta. Now, this work is negative because the force is in this direction whereas displacement is in the opposite direction delta s. So that Is why we have put a negative sign here. So this total work done, virtual work done is set equal to zero for equilibrium and since delta s is arbitrary in magnitude, direction wise, it is always in agreement with the constraints. So direction, I cannot change but magnitude, since delta s is arbitrary. So this equation is satisfied very easily, if cosine theta is equal to two by three, that is, this is equal to this. So you can simplify sin thirty is half. So cosine theta will come out to be two by three and hence theta is equal to forty-eight point one nine degrees. So this angle theta comes out to be this. So you can see that the result is obtained effectively in one or two lines whereas in the free body method, it would have taken quite an effort.

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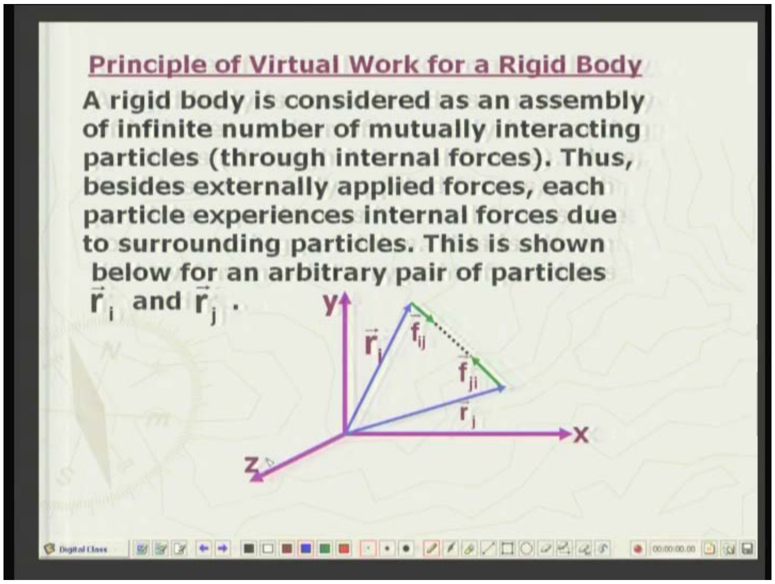

Now, we go to the principle of virtual work for a rigid body. Earlier, we were focusing on a particle. Now it is a rigid body. We will treat the rigid body as an assembly of a very large or infinite number of particles. Such that these particles are interacting with each other and this interaction is manifest through internal forces. Now how the interaction takes place. You consider, let us say, an arbitrary rigid body and you consider a pair of particles. Let me call this as i^{th} particle, this as j^{th} particle. So here is i^{th} particle, j^{th} particle. Their positions are in a coordinate system, r_i vector, r_j vector and the line joining the two positions is the r_{ij} vector, if we may call it. Now, by the law of gravitation, due to interaction of the particle, the particle i^{th} is being attracted towards particle j, by the force f_{ij} towards j and similarly, particle j is being attracted towards a particle i and since the action and the forces are taking place along this line joining, that is, r_{ij} vector. So you can see the interactive forces along the two interactive forces f_{ij} vector and f_{ji} vector. They are equal and opposite due to Newton's third law action and reaction being equal, being opposite.

Now, in a rigid body, the distance between the two particles is always fixed because if the distance changes, then the body is deformable. So it means that the motion of i^{th} particle with the respect to j^{th} particle will be on radius. So it will be moving in this direction and similarly, the relative motion between j^{th} particle and i^{th} particle will be along the conjugate arc but the forces are along this path. Arc is perpendicular to this line. So the work done in any relative displacement between particle i^{th} and j^{th} particle will be zero because forces along the path joining the two particle and the displacement is normal to it. This can be said for any pair of particles. So what I want to conclude is that the work done in a rigid body due to the internal forces will cancel out, pair wise between any pair of particles. Hence the total work done will be zero. So, the internal forces do not contribute to any work. The only work which can be done by the rigid body or on to the rigid body will be due to the active external forces and constraints. Also do not do any work if these are frictionless constraints.

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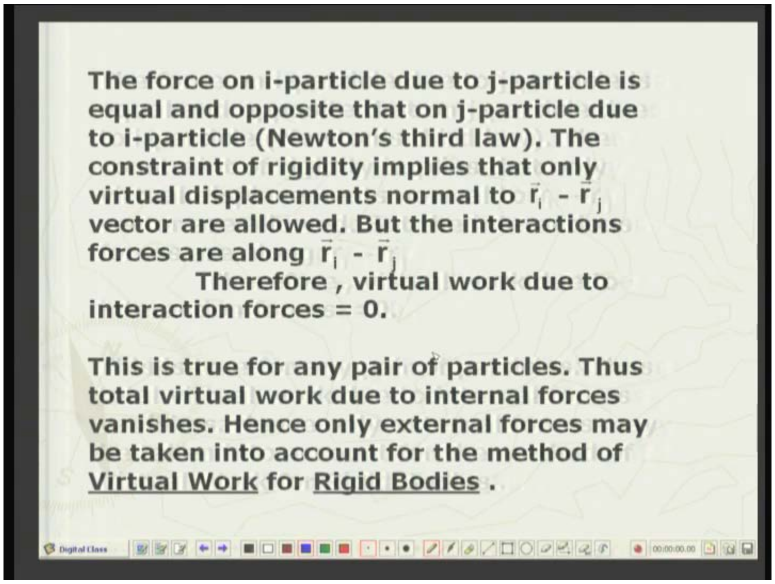

Hence, we say that the total virtual work due to the interaction forces is equal to zero. So again, for a rigid body, the equilibrium of the rigid body implies that the work done by the external active forces, that is, we again neglect the reactive forces. Due to constraint must to be equal to zero. Let me introduce another important concept in the same topic. That is the degrees of freedom.

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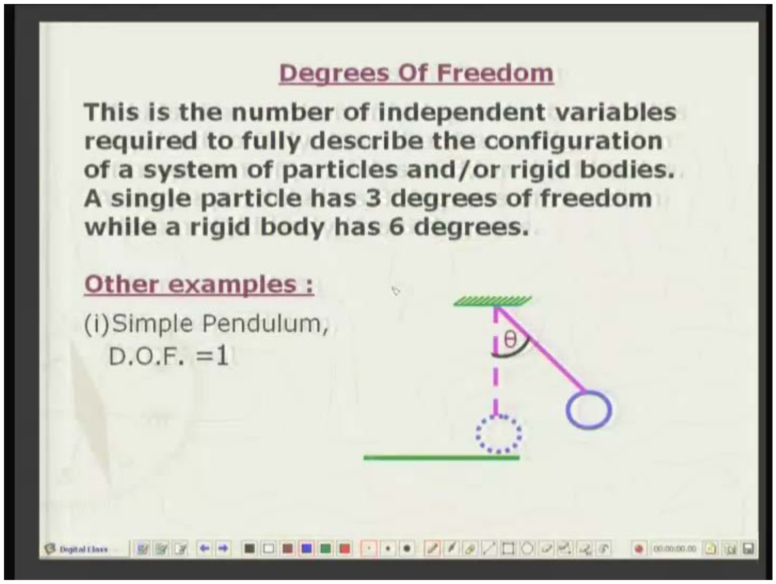

This is the number of independent variables required to completely describe the configuration of a system of particles or rigid bodies. You consider any system several particles. So one particle, one rigid body or several rigid bodies and determine how many independent variables are necessary to completely depict the configuration of the system and that will be the number of degrees or degrees of freedom of the system. Let me give you some simple examples. Consider a single particle moving in a three dimensional Euclidean space. So you require three Cartesian coordinates or cylindrical polar coordinates or any other coordinate system. Only three of them are needed to completely describe the location of the particle and these can be varied independently if you vary x. You do not have to vary y and z automatically or directly. So they can be varied independently. That is the crucial thing. If you consider a rigid body, for example, this pen, consider a particle on this body. Let us say, centre mass of the body. So the location of the center mass, that requires three Cartesian coordinate and then the orientation of the three body. As you will learn later on, you will require three angles called Euler angles to completely orient the body in any configuration you want and these three angles can be varied independently. So three translator coordinates x y z and three angular coordinates theta five and gamma. For example, six degrees of freedom for a rigid body. Now consider a pendulum here. How many degrees of freedom it has? Of course, we will restrict its motion in the plane. So, a pendulum can swing and this location can be given uniquely by the angle the string makes to the vertical. That angle theta is shown here. So just one specification, that is, specification of theta will be sufficient to locate it. For example, theta is equal to zero, theta is equal to five degrees, theta is equal to three degrees and so on, so forth. Minus five degrees, etcetera. So it has one degree of freedom. Let us go to slightly more complex system.

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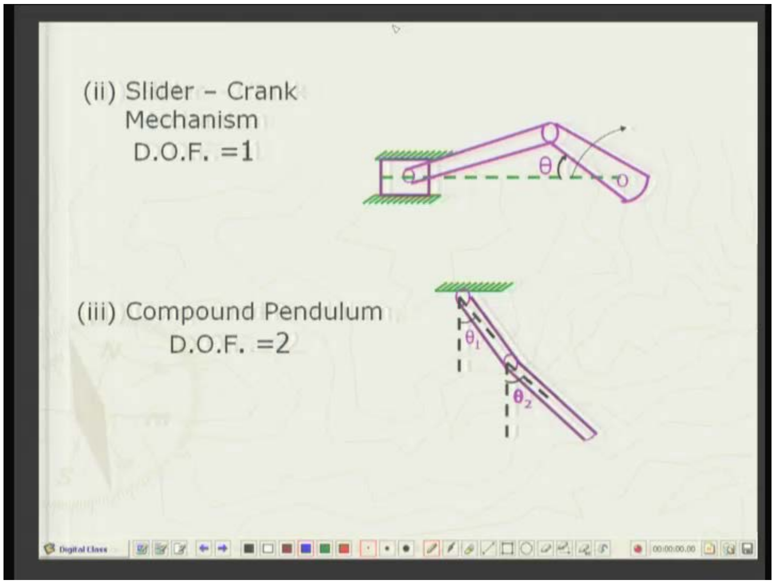

Here is a slider crank mechanism. There is a slider and there is a connecting rod and there is a crank. Incidentally, you are familiar with this mechanism in automobile engines. This is the piston, this is the connecting rod and this is the crank which turns the differential and then the motion is given to the automobile. So although it has three different bodies which are pinned with each other, the piston is always constraint to move within the cylinder, that is, the motion is always in the straight line along this and this crank is given a rotary motion. So this mechanism is also useful in so many machines. When you go to workshop, you can see this mechanism in use. Now, in spite of three digit bodies involved, you need only one angle specification, that is, theta, as shown here. To completely locate this configuration of the mechanism, for example, when theta is equal to zero, all the three will be in straight line. That is the only position available when theta is equal to ten degrees. Let us say, when theta is equal to twenty degree, thirty degrees, you can completely describe any location with the help of only one specification that of theta. So the system is again of single degree of freedom.

Let us take up a case of two degrees of freedom, compound pendulum. There are two bars which are pinned together at over here and the top bar is also pinned to the support. Now, in this case, if it had been only one bar, may be theta one is sufficient or good enough to completely specify but now there are two bars. The top bar is specified by theta one and since there is a pin hear, theta two can be, that is, the inclination of the lower bar with the vertical, can be independently prescribed. It need not be same or half of this or one third of this. Let us say, theta one is ten degrees, theta two can be twelve point five degrees or fifteen degrees or even twenty degrees. So independently, we can prescribe. This is a system of two degrees of freedom.

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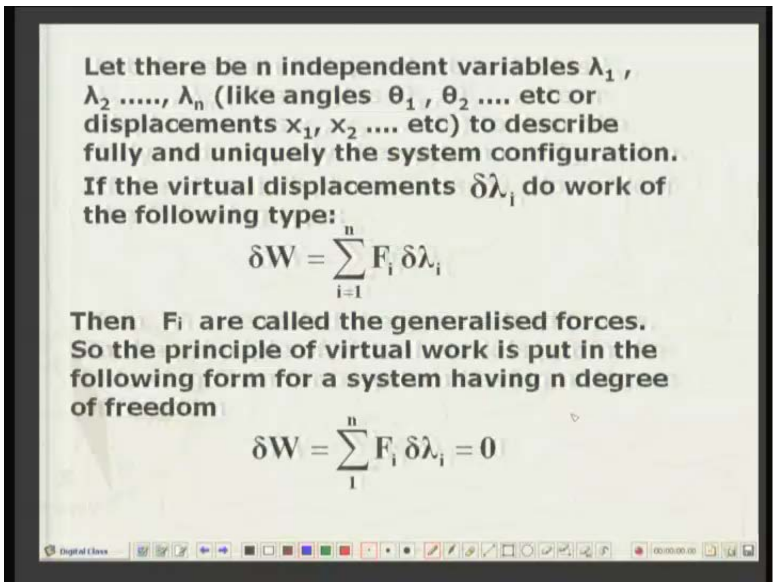

Now generally, these degrees of freedoms, we can designate as lambda one, lambda two, lambda three, up to lambda n. For n degree of freedom system and these lambdas can be angles or coordinates, theta one, theta two, up to theta i and then x one, x two, up to k. So I plus k is equals to n. So n degrees of freedom. Then, suppose I give virtual displacement to each of the degrees of freedom. Lambda one changes by lambda one to del lambda, one lambda two changes to lambda two plus del lambda two, etcetera, etcetera. N^{th} degree lambda changes to lambda n plus delta n. So if I change this system configuration by virtual displacements and virtual work done. In doing so, it can put in the form Fi times del of lambda i summation from one to n. Then Fi are called the generalized forces. For example, if the angles are involved, then the Fi will be the moment because moment times the angular rotation gives me the work whereas displacement time force gives me work. So that is why we have called them generalized forces. They are not always force. They can be moment, they can be torques, they can be anything else. So the total virtual work can be generalized as equal to the summation of generalized force times the virtual displacement of the corresponding degree of freedom and once we have computed the total virtual work, if this virtual work is equal to zero, then the system of n degrees of freedom is in equilibrium and vice versa. That is, if the system is in equilibrium, then the virtual work done is equal to zero. So that is the principal of virtual work for a system of n degrees of freedom which may be particle rigid bodies anything. So this is the most generalized form of the principal of virtual work.

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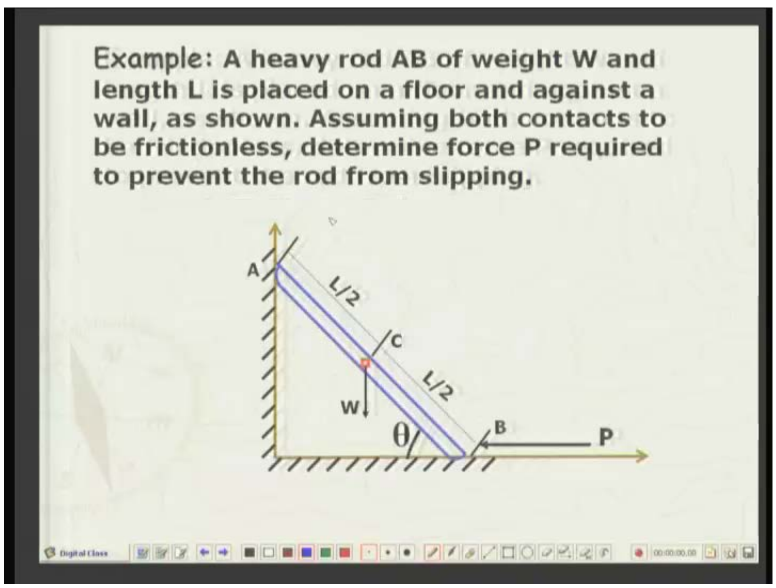

Let me take up one or two examples to illustrate this. First, I will take up this an example of single degree of freedom. There is a heavy rod which is resting against a wall and a floor.

Both these contacts, between the rod and the wall as well as between the rod and the floor are frictionless contacts. So the constraints are frictionless. The rod, due to its self-rate, may try to slide slip but this is preventing from doing so by a restraining force of P. You might have seen workmen use ladders for their work and second workmen stands on the floor to hold the ladder from slipping. So similarly, here we are holding the ladder or rod in position by a restraining force P. So heavy rod AB of weight W and length L is placed on the floor. As I have mentioned, seeming both the contacts to be frictionless. Determine the force P required to prevent the rod from slipping. Let us see, where are the forces? There are the two forces, P and W. So, if I give a slide displacement in theta, that is, theta changes to theta plus delta theta. Then what will happen? Both these points A and B will change the location. Now, force at B is horizontal. So the work will be done by this force in moving the point in the horizontal direction. Whereas the other force is the weight W passing through the center of gravity, which is, if the rod is uniform rod, then it will be half the length. So at the half length L by two, on each side, there is a vertical force W gravitational force. So the work will be done by the vertical displacement. So I have to find out the virtual horizontal displacement at B and the virtual vertical displacements at W. This is done in the next slide.

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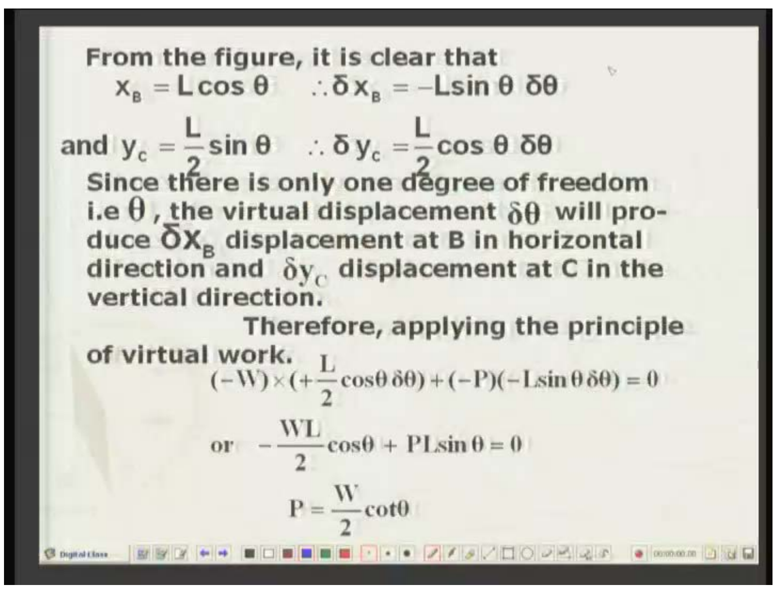

You can easily see that the coordinate X_B is horizontal coordinate of point B, L cosine theta. Therefore, I can find out delta X_B is equal. See, finding out the virtual displacement, you can use the same laws as of calculus. So cosine theta will give me sin theta D theta but instead of D theta, I will take the virtual displacement as del theta. So with the negative sign, del X_B is equal to minus L sin theta. Del theta Y_C, the vertical coordinate of the center of gravity of the rod, is L by two into sin theta. Again, use the differential method. So del Y_C is equal to L by two sin theta will give me cosine theta del theta. So both these virtual displacements are known and then finding the work done is very easy. You can easily see that if, let us say, P is moving in this direction, that B is moving towards the horizon. Then the center of gravity will go up because the point A will rise and with this, point C will also rise. So here it is positive work. Here it is negative work. So minus W into the shift in the center of gravity. That is going up L by two cosine theta del theta and for the horizontal force, restraining force P minus P in the negative direction minus L sin theta in the negative direction. This is equal to zero. So just one equation and then simplification. You get the result. If you had to do this by free body diagram, you have to assume the reactions. Here, write down the equation of equilibrium, moment, equation, etcetera, etcetera. So it needs much more effort than the virtual work method.

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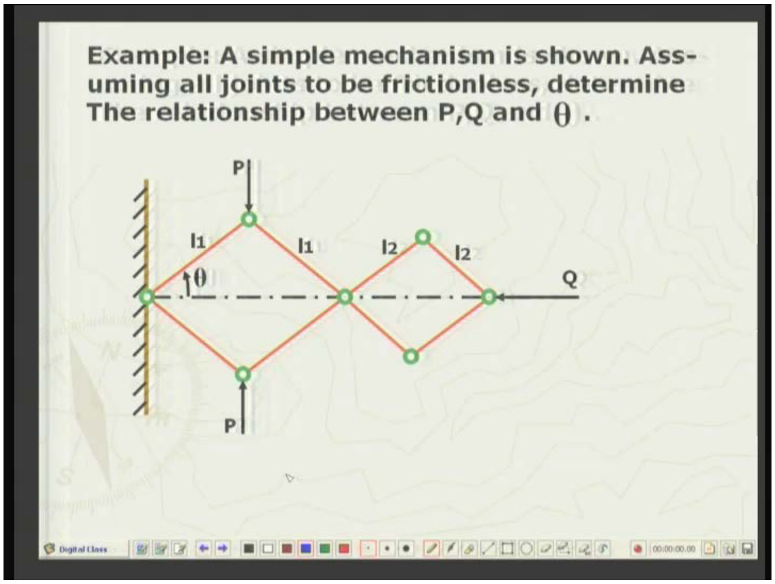

Let me take up another problem which is slightly more involved than the previous problem but again, it is single degree of freedom, a simple mechanism. So all these four rods are of length l one and these are smaller rods, length l two and these are pin jointed, all of them and this and this pin jointed to the wall and at this, we apply a force. There is a another force, P compressive force here. So this is the mechanism and these are the set of forces, Q and P, like this. So find the relationship between P and Q and angle theta. So you can easily see that when you change theta by a small amount, the whole configuration will change and this is the only degree of freedom needed to completely see the new configuration of the system mechanism. Again I may add the solving it by free body diagram and equations of equilibrium would need many free bodies and then equally large number of equations and they have to be solved simultaneously to get the final answer but here, we will do it very quickly by virtual work principle.

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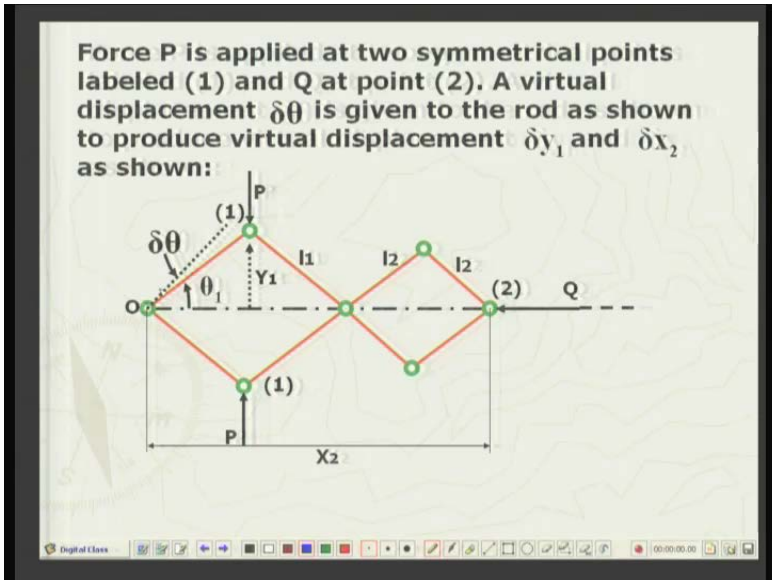

I give a small virtual displacement delta, theta one to angle theta one and you can see that this rod will be now in this location and as a result, this point will shift this point, will shift this point, will shift every point, will shift and that will be the new configuration. Forces are at this point and this point. Whatever work is being done by this force, same work will be done by this force because the mechanism is symmetrical about the horizontal X. So calculate this work. I will simply develop, to account for the work by the lower force and as the angle increases this point, all these points will shift horizontally and Q point of application of Q will also shift. so there will be work done by Q also. So total work done by all the three forces will be set equal to zero to determine the equilibrium configuration. Let us do that.

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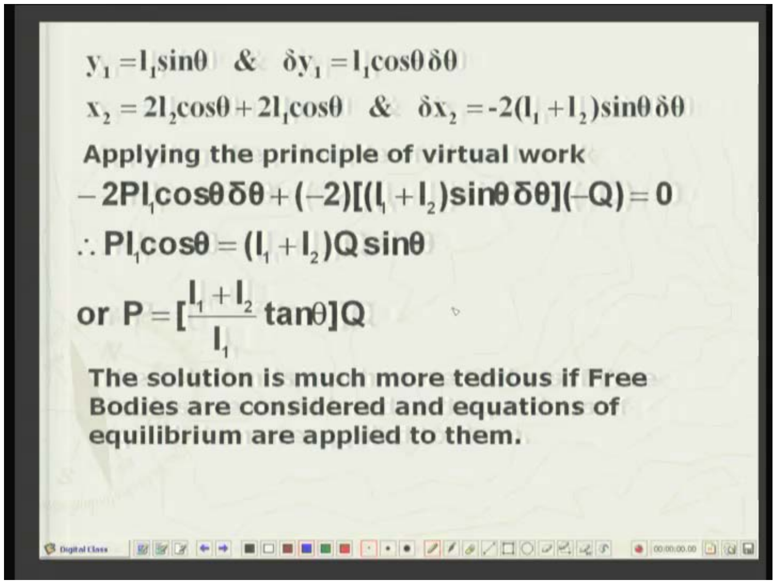

Well, y one is l one sin theta and x two, let us see from the figure what is y one. This is y one. So this is l one. So is l one sin theta or theta one and this is x two the horizontal coordinate of point two. So you can see that this will be twice l one cosine theta half here. So l one cosine theta one l one cosine theta one and similarly, l two and this angle being equal to this, this angle being so, this will also be theta one. So l two cosine theta one and similarly, over here. So you can easily see that x two will be equal to two l two cosine theta plus two l one cosine theta and you take the virtual displacements of y one and x two. Del y one is equal to l one cosine theta. Del theta and delta x two is equal because of cosine, there will be negative sign l one plus l two sin theta del theta. Now applying the principal of virtual work for this mechanism, let us say, again going back to the picture, angle is increasing in this direction. You can easily see that the points will be opening out. So force is downward. The opening out is upward. Similarly, force is upward, opening out is downward. So they are in opposite direction. So negative work will be done. Whereas Q is in this direction and displacement of point two is also in the same direction. So here, it is positive work. So that is very important to recognize or to ascertain the direction in which the work is being done. So you will have the work done by the P forces and the work done by the Q forces both headed up and set it equal to zero and you will have the final equation after simplification as, P is equal to l one plus l two divided by l one into tangent theta in to Q. So, we have relationship between theta Q and P. This is about what we were asked to do and you can see that our problem of mechanics has been reduced to a problem of trigonometry. That is the beauty of this system. That is, if you can describe your configuration of your system properly, with appropriately chosen degrees of freedom, then the problem becomes very easily tackled.

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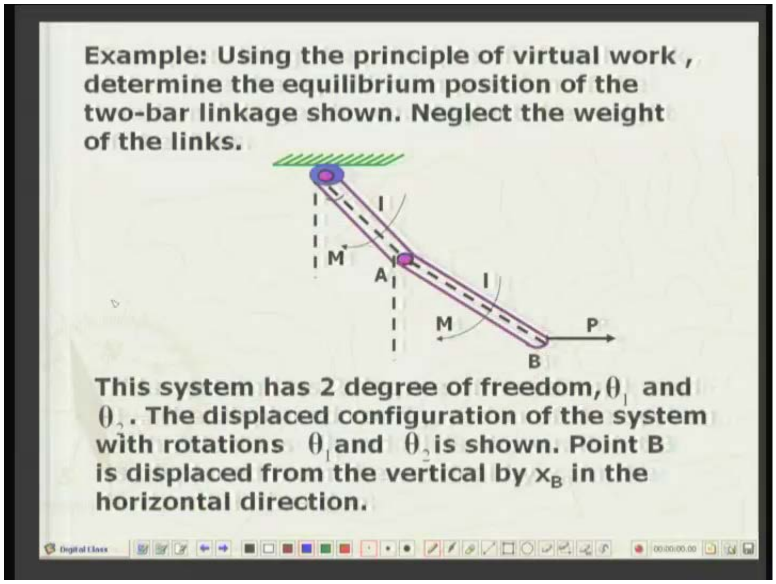

Let me take up an example of two degrees of freedom. Here is a compound pendulum. It consists of two rods. The top rod is pinned to the rigid surface and rod one and two are also pinned together. Length of each rod is l. There is a moment applied to each of the rods and a force P in the horizontal direction. So this is actually a two bar linkage mechanism. We are neglecting the self-weights of the links and determine the equilibrium position of the linkage system. This is, obliviously, a system of two degrees of freedom. The inclination to the vertical of the top link, inclination to the vertical of the lower link. We can call this theta one, theta two. The horizontal displacement from the vertical, we can find out from the point B which called X_{B}.

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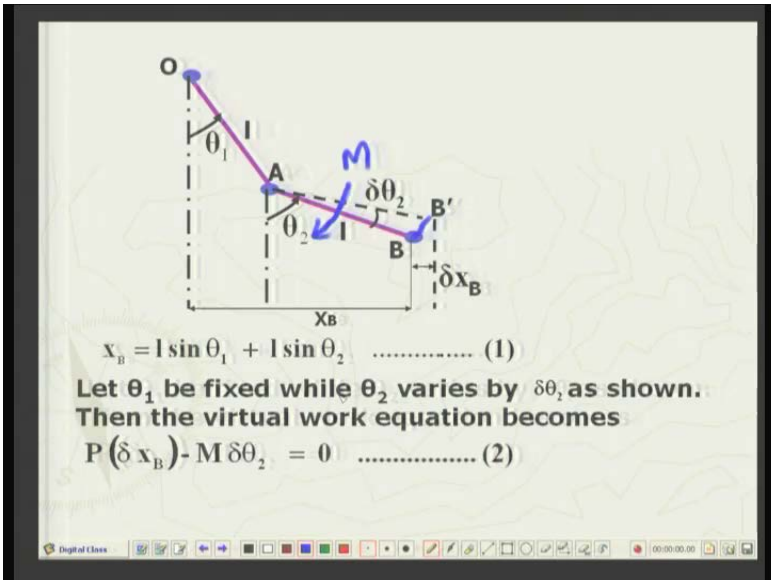

Now, let us say, this is theta one, this is two, as I have mentioned and X_B will be this horizontal distance plus this horizontal distance, which will be l into sin theta one plus l into sin theta two. Both the links are of same length l. Now, suppose theta one is fixed. I do not vary theta one. I give a virtual variation to theta two. So that theta two becomes theta two plus del theta two. Then what are the displacements involved? Well, first of all, due to this point B will shift to B dash and this shift will have a horizontal component delta X_B and there is angular virtual displacement delta theta two and there is moment, as I have shown you in the diagram, a clockwise moment applied. So, there will be work done in increasing the displacement from theta two to theta two plus delta theta two because there is an additional displacement and this is the moment M. So this moment will do work. Both are in opposite direction. So minus work is done here whereas there will be work done in shifting this delta X_B into P.

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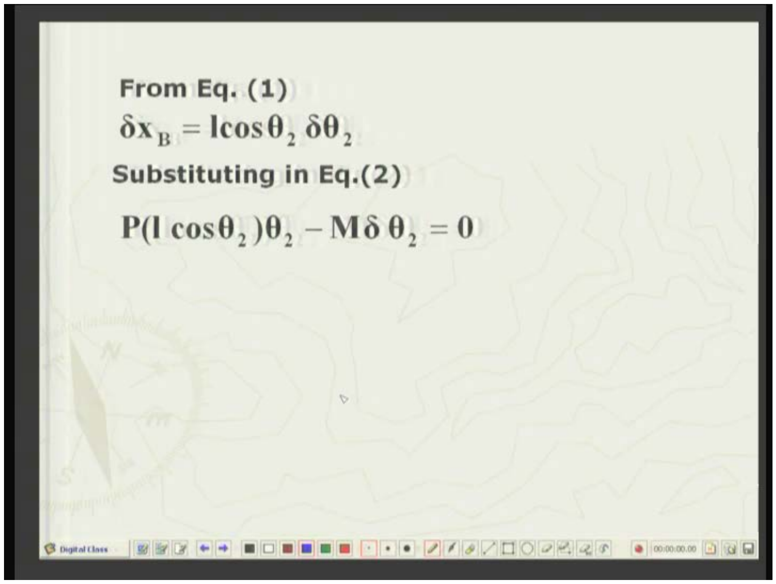

So if I further analyze this from equation one which I had shown earlier, if I take the virtual displacement of $X_{B,}$ then it will be delta X_B is equal two l cosine delta l cosine theta to into delta theta two.

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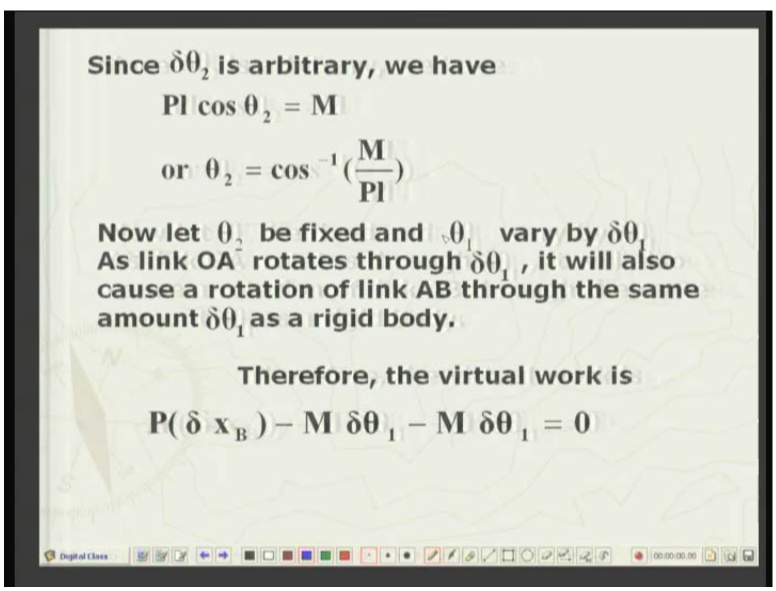

Once we substitute in to this equation and simplify, we will get theta two is equal to cosine inverse M over P l. So at equilibrium, the angle of the lower link to the vertical, that is, theta two is cos inverse of M over Pl. Now, to find theta one, we will fix theta two and theta one is varied because they can be varied independently. That is by virtue of the definition of degree of freedom. So theta two can be varied independently which we have done already. Now theta one can be varied independently. Theta two is fixed. Now, there is an interesting point here. When I vary theta one as a and do not vary theta two as a, theta one is varied. A will shift. So that will automatically shift the low rod also. So there will be one motion due to the rigid body rotation around the top point O due to theta one. So please keep this in mind.

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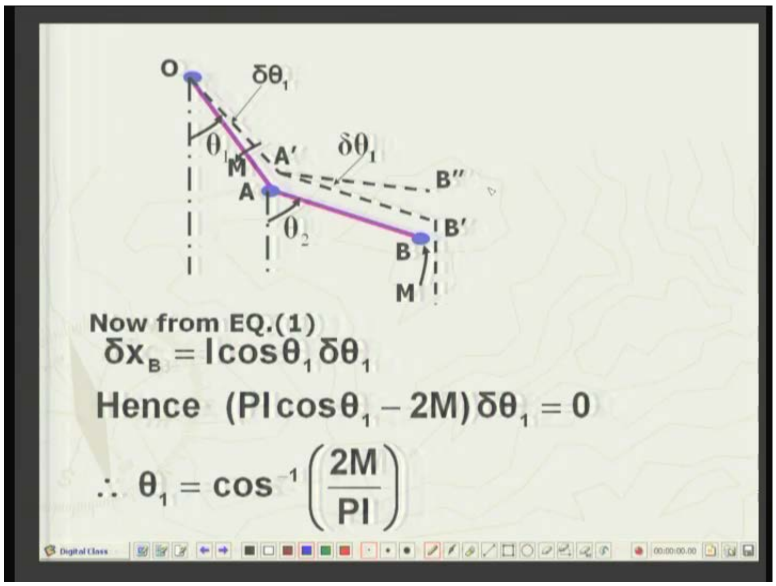

So we will now look here. First of all, as theta one is varied, this will be given. First of all, as a rigid body, it will move parallel to itself and then there will be an additional del theta one. So this theta del theta one will affect the top rod as well as low rod. So please remember this. So now from the equation one which we have written earlier del x two is l cosine theta one into del theta one. That is very simple. May be, you look at the equation once again. This is the equation one. So theta two was fixed. So there is no variation.

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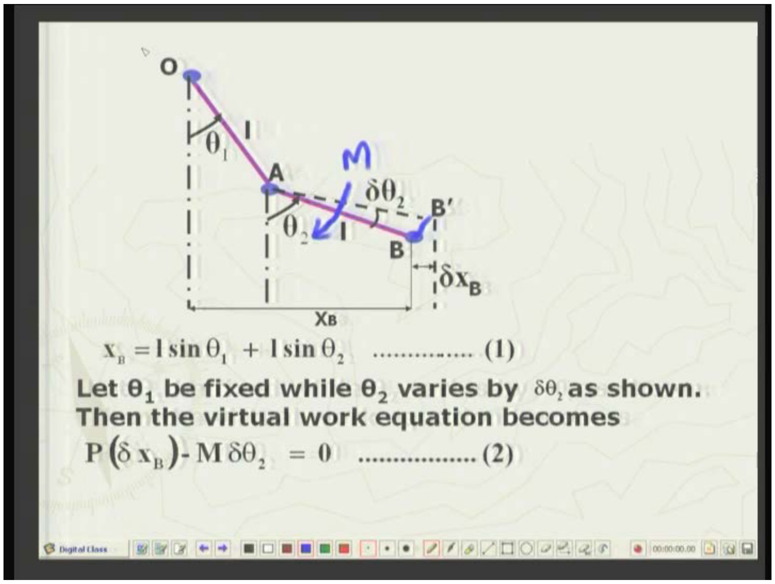

So there is variation only in theta one and then, we will substitute in the equation for the virtual work. Then Pl cosine theta one into del theta one, that is, the work done by P force and since both these rods are subjected to M one, a moment M here as well as moment M over here as shown here. So, the work will be done by both these moments. So that is why, minus two M del theta one is equal to zero. So theta one will be cosine inverse two M over Pl. So, we are seen, by using the principal of virtual work, the equations of equilibrium are dispensed with, that is, otherwise, we would have to consider the top link separately, lower link separately and then solve these equations simultaneously here. We have given a virtual displacement to the top separately and to the bottom link separately and each one gives us one variable at equilibrium. So this procedure of virtual work is really very economical and the big advantage is that we are working with scalar quantities. So adding or subtracting the work done is very simple whereas working with vector quantities like forces and moments, you have to use parallelogram law of forces or triangle law of forces and so on so forth. So, I close this principle of virtual work here. In our next lecture, we will start with another aspect of work, that is, for conservative forces.

I will define what the conservative forces are and how we can recast the principle of virtual work into the potential energy method, which is a very strong method. It is a very versatile method. It can be applied to many situations. Thank you very much.