Applied Mechanics
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Lecture No. # 13
Moments and Products of Inertia (Contd.)

Today's lecture is lecture thirteen which is a continuation of lecture twelve, where in, we discussed or rather introduced the concepts of moment of inertia and product of inertia. These two concepts are very important for mechanics particularly in dynamics problems and during that lecture itself we set up a correspondence between moments of inertia and the second moment of area and similarly, for products of inertia, there was a correspondence with the product of area.

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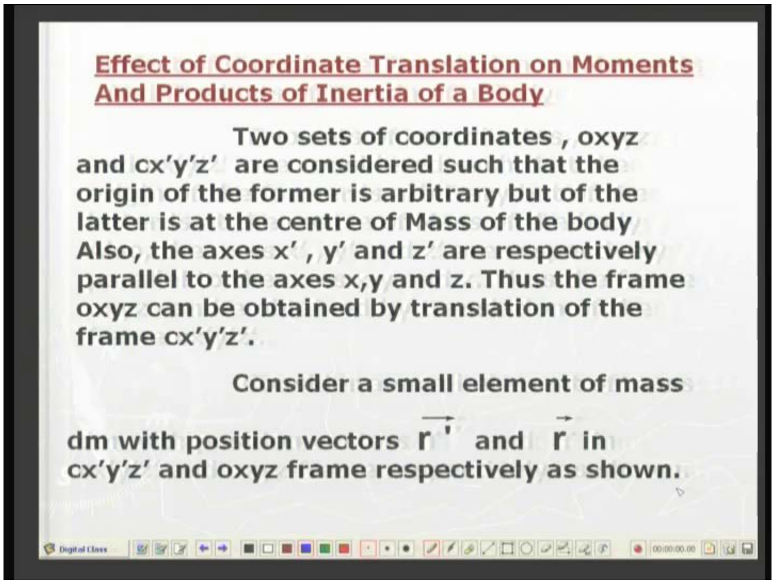

Let us start with the today's lecture now. Today, we will study the effect of coordinate translation on the moments and products of inertia of a body. Translation means the coordinates are moved without rotation.

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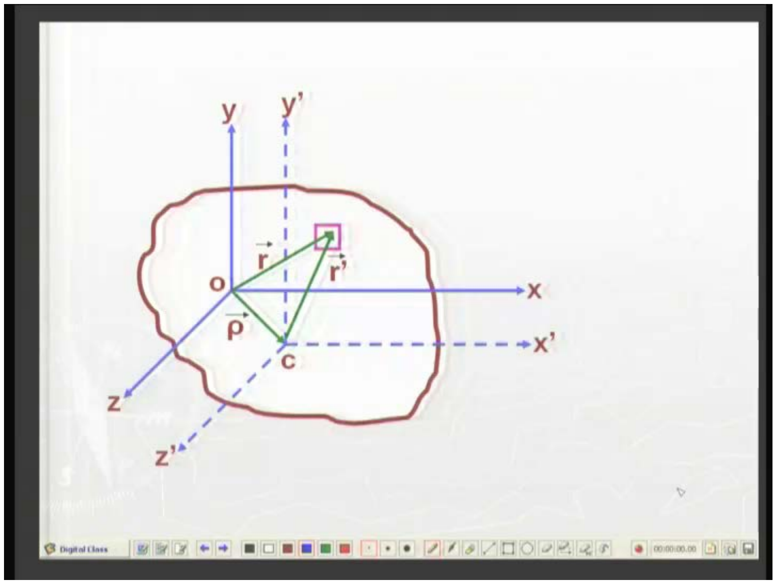

For example, if I consider an arbitrary body shown here and there are two coordinates systems. Cartesian coordinate system shown one is this dashed one. Center is c which is the center of mass of the body. So this coordinate system with origin at the centre of mass is x dash y dash z dash and there is an arbitrary coordinate system with origin at o and the coordinates axis as x axis, y axis and z axis. Now, the special feature about these two coordinates systems is that the x axis is parallel to x dash axis, y axis is parallel to y dash axis and z axis is parallel to z dash axis, that is, the axis have been shifted in a parallel manner. There is no rotation involved in shifting the axis. So when you have, let us say, the moments of inertia given in the c x dash y dash z dash at coordinate system and you want to find out the moment of inertia in the second coordinate system, that is, origin at o and xyz coordinate system. Then, what is the correspondence between the Ixx and Ixx dash and other quantities? How to set up a relationship so that I can get the moments of inertia in the new coordinate system in terms of the old coordinate system? Same thing for the products of inertia. So let us see how we can do this. Let us say that again referring back to the same figure, I will consider a small element of the given body. This element has dimensions, dx, dy, dz. It is a very infinitesimal element and the centre of this element has a position vector with respect to the old coordinate system, that is, the coordinate system at center c. So from center c to the centre of the element, the position vector is r dash. Similarly, the same element is located at a position vector r from the origin of the new coordinate system oxyz. So the distance between the two origins, o and c, is given by the position vector rho.

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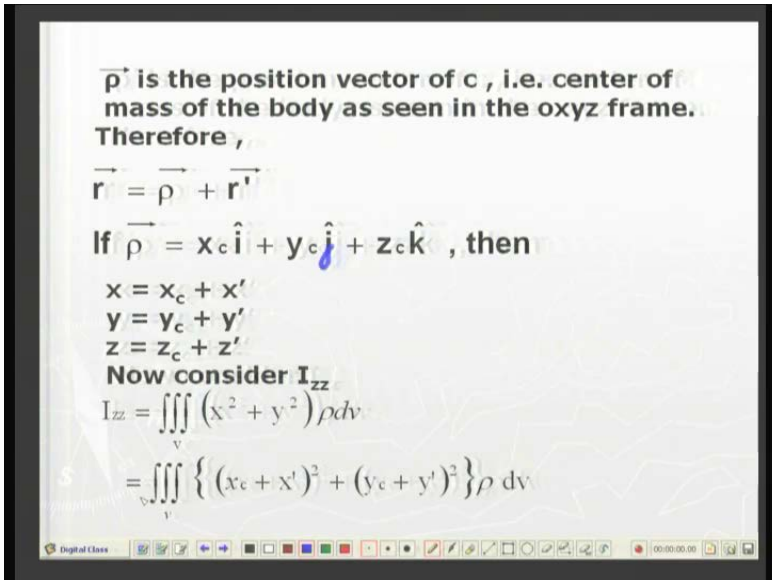

So it is very easily checked that the position vector r is equal to rho vector plus r dash vector. Now, this rho vector in the coordinate system at o, that is, the new coordinate system with the unit vectors as i j i j and k, the position vector of the centre of mass is Xci plus Ycj plus Zck, so that the coordinates of the center of mass in the new coordinate system are xc yc zc. So, substituting for rho from here, we can easily check that the x coordinate in the new coordinate system is equal to the x coordinate of the centre of mass plus the coordinate in the old coordinate system, that is x is equal to xc plus x dash y is equal to yc plus y dash z is equal to zc plus z dash. Now consider one of the moments of inertia, that is, Izz. By definition, this is in the new coordinate system. Izz is equal to the volume integral of x square plus y square into the mass density rho taken the integral over the entire volume. Now, substituting for x square plus and y square, we will have from this equation xc plus x dash whole square plus yc plus y dash whole square rho dv integrated over the entire volume.

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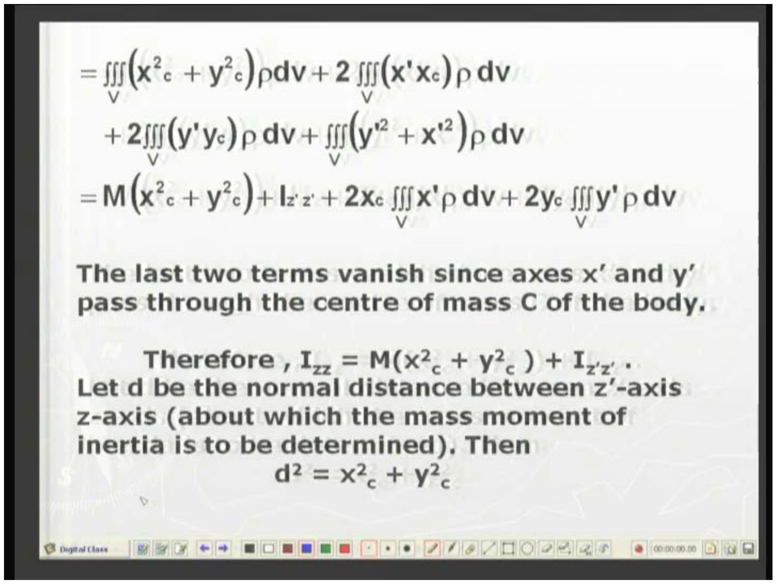

Now, we expand the squared brackets. So, by rearranging the terms, we can easily verify that the previous integral will be now equal to the sum of these integrals. One is the volume integral of xc square plus yc square into rho dv plus twice. These are the coupling terms x dash into xc rho dv in volume integral twice y dash into yc rho dv volume integral and then volume integral of y dash square plus x dash square rho dv. Now xc and yc, the coordinates of the centre of mass are constant. They are just fixed numbers. So I can take it outside and inside we will be left with rho dv volume integral which is nothing but the total mass of the body. So this first term of this expansion becomes mass into xc square plus yc square and the last term which is by definition, the moment of inertia around the z axis z dash axis in the old coordinate system, that is, cx dash y dash z dash the dotted coordinate system. So first and the fourth terms are taken care of and these two coupling terms. Now, again xc and yc are taken out as constants. So twice xc into volume integral of x dash rho dv and plus twice yc volume integral of y dash rho dv. Now, by definition, this is the x coordinate of the center of mass in the old coordinate system, that is, cx dash y dash z dash and this is the y coordinate of the center of mass in the old coordinate system.

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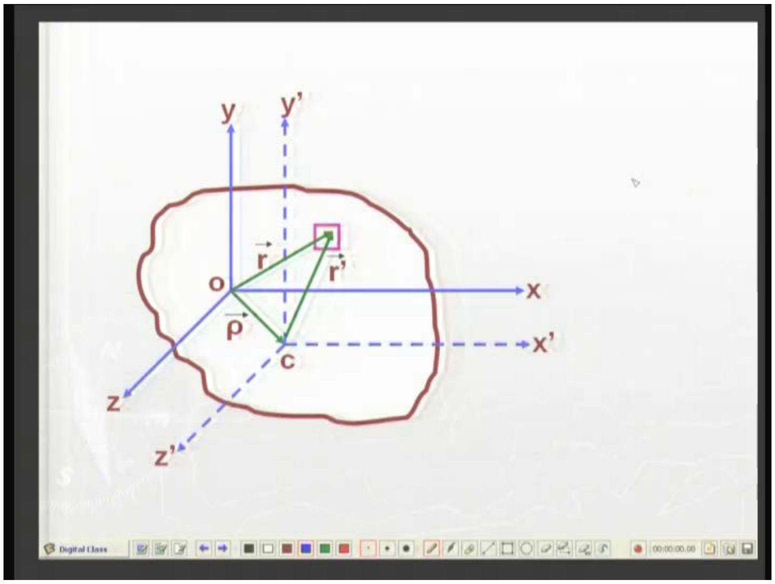

The old coordinate system, as you can easily see from the figure, is located at the centre of mass. So in the x dash y dash z dash coordinate system, the coordinates of the center of mass are zero, zero, zero. So, it means that this term and this term are zeros. These volume integral and they reduce to zero and hence the contribution is zero and I am left with Izz is equal to Mxc square plus yc square plus Izz dash z dash. So, I have Izz is equal to this expression. Now to understand the first term, this is the mass of the body and xc square plus yc square. This is the distance between the square of the normal distance between z axis and z dash axis. Again, refer back to the figure. So this is the z axis and this is the z dash axis. So the normal distance between the two squared. D square, the normal distance between z and z dash axis is equal to xc square plus yc square this by Pythagoras theorem.

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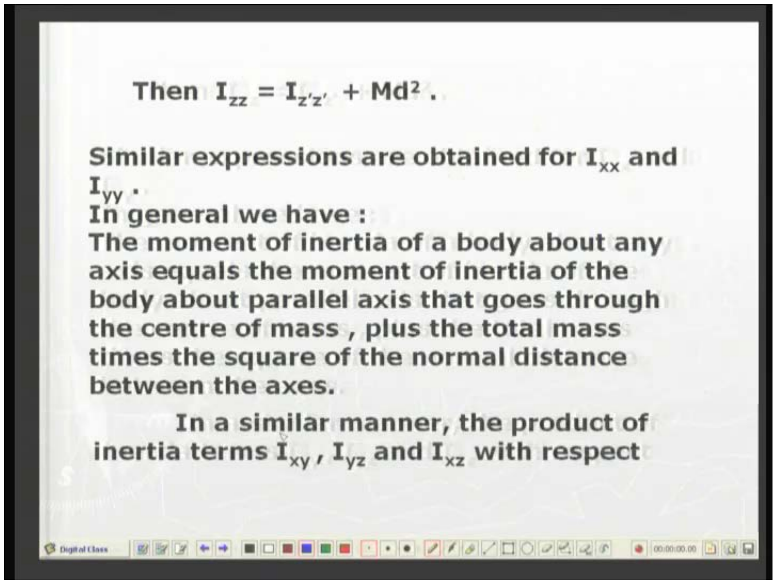

So it means, we have a very compact expression, that is, the z moment of inertia Izz is equal to in the new coordinate system. That is located at the origin and is at an arbitrary point o and equal to the moment of inertia about z dash axis in the old coordinates system with origin at the center of mass of the body plus the total mass into the normal distance between the z and z dash axis squared. Similar expressions can be obtained for Ixx and Iyy. For example, Ixx will be equal to I x dash plus mass times the normal distance square between the x axis and the x dash axis. So this is how we get the correlation and then, we can now turn our attention to the product of inertia, that is, Ixy Iyz Ixz.

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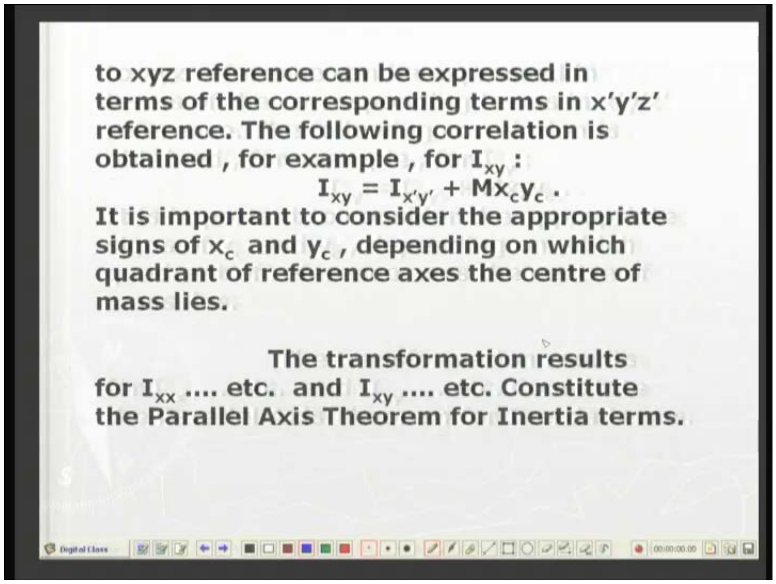

The procedure is exactly identical. That is you write down the expression in the new coordinate system and then expand the terms. That is, for example, in Ixy expression, you will have x and y terms which you can be written as xc plus x dash. Similarly, y will be written as yc plus y dash. Again, take the product. There will be four terms and so on and so forth, exactly proceeding in the same lines you will have, for example, Ixy is equal to ix dash y dash plus the total mass of the body into the product of x coordinate of the center of mass times the y coordinate of the center of mass as seen by the oxyz coordinate system. So you have to be careful in product of inertia terms, that is, you have to check whether the center of mass lies in the first quadrant or the second quadrant third or four quadrant. So properly, the positive or negative sign of the coordinates of the center of mass, that is, xc and yc and zc should be taken care of because in the moment, inertial terms are squared. The distance is squared. So negative or positive does not make any difference because when it is squared, it is positive but in the product of inertia terms, you will have xc and yc. So if xc is negative, for example, then this term will be negative. If both xc and ah yc are negative, of course then, it will be positive. So one has to be careful in properly accounting for the coordinates of the center of mass. This whole exercise is called the parallel axis theorem.

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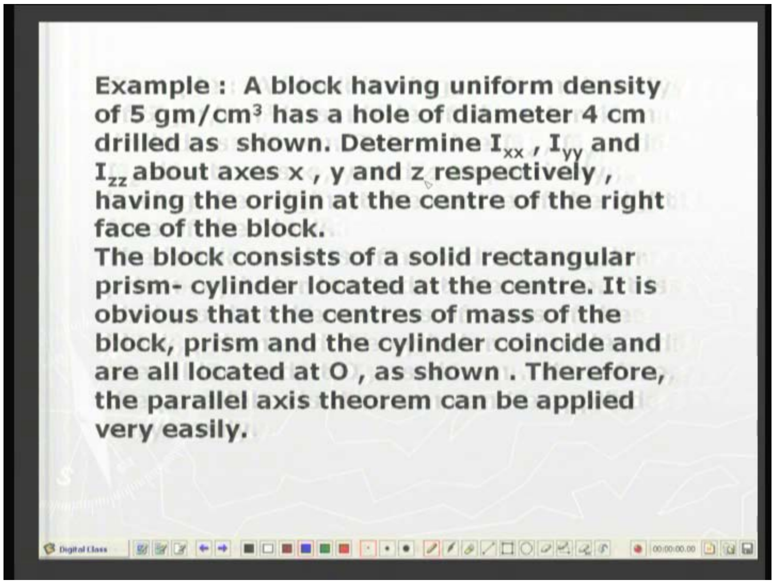

Now, let us try to apply this parallel axis theorem with the help of a simple problem. Let me show the diagram and then I will come back to the problem.

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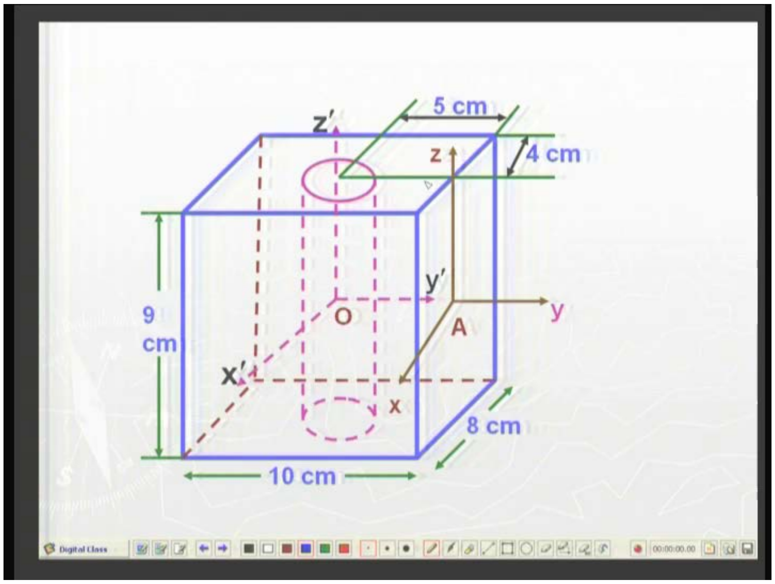

Here is a rectangular block. The outside dimensions are ten centimeter, eight centimeter and height is nine centimeter. In the center of this block, there is a cylindrical cavity from end to end where this magenta color dotted figure is the cavity. So the net body is a block minus the cavity. So we have been asked to find out the moment of inertia about a coordinate system whose origin is at the center of this right hand face whose dimensions are eight centimeter and height is nine centimeter. So A is the centre of this face and there is coordinate system xyz located at that point A and we have to find out Ixx iyy Izz and similarly the product of inertia terms. So go back to the problem. A block having a uniform density of five grams per centimeter cube has a whole of diameter four centimeter drilled as shown. So we have already seen. Determine Ixx Iyy and zz about x y z respectively having the origin at the center of the right face of the block. So I have already mentioned that the net body is the total cubic body minus the cylindrical cavity. So we can find out this moment of inertia and then by parallel axis theorem, shift it to the new coordinate system whose origin is at A the center of the right hand face. Well, for rectangular bodies, cylindrical bodies, spherical bodies, etcetera, the expression for the moments of inertia are available in many books, hand books and mechanics books, etcetera.s So we will not derive those expressions from the beginning because that will take lot of time. You can refer to those hand books or tables and let us see how we can do this. Now, let us have a coordinate system at the center of mass of the body. Now, where is the center of mass of this body, the outside rectangular block? The center of mass is at the midpoint of the three dimensions, that is, its location will be at four centimeter along this dimension, five centimeter along this and similarly, half of nine, that is, four point five is the height and the cylinder is also located at the same point.

So the center of mass of the cylinder and the rectangular block are coinciding and hence of the net block, that is, solid rectangular block minus the cavity will be also at the same location. So O is the center of mass of the net body whose dimensions along, let us say, length is five centimeter, along width is four centimeter, along height is four point five centimeter. So at that point, we have coordinate system x dash y dash and z dash parallel to the three edges of the block.

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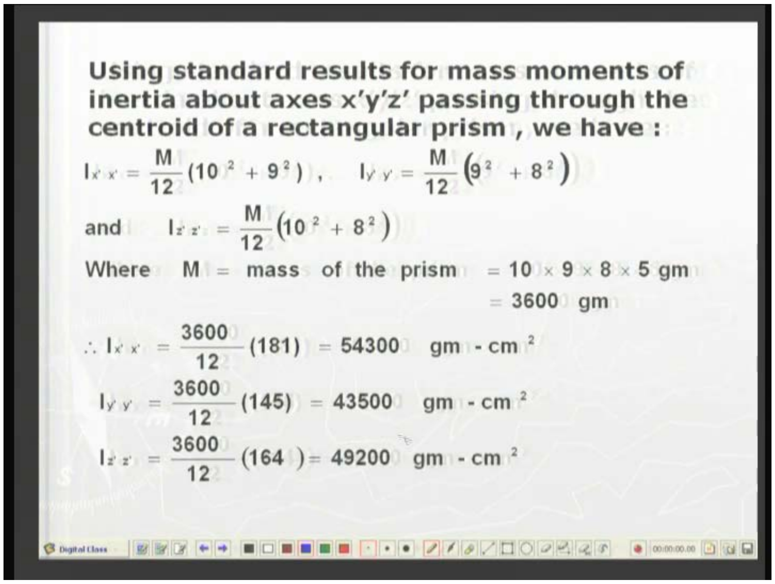

Now, for Ix dash x dash, what you do is mass of the block divided by twelve. This is for the solid rectangular block into the two dimensions perpendicular to the x dash x axis that is ten square plus nine centimeter square. Similarly, Iyy dash total mass divided by twelve into the two dimension perpendicular to y dash axis, that is, nine square and eight square and similarly, I z dash z dash. So the three moments in a inertia in the x dash y dash z dash are obtained. Now total mass of the solid prism, you can see, is ten into nine into eight. This is the volume. Now, this is five grams per centimeter cube, the mass density. So it comes out with thirty-six hundred and when I substitute in these three expressions and simplify this brackets, I will get Ix dash x dash as fifty-four thousands three hundred gram centimeter square. These are the units of the moment of inertia Iy dash y dash is forty-three thousands five hundred gram centimeter square and Iz dash z dash is forty-nine thousands two hundred grams centimeter square. So I have got for the solid block these three values and if I subtract from them the corresponding values for cylindrical cavity, I will have the moment of inertial terms for the net block.

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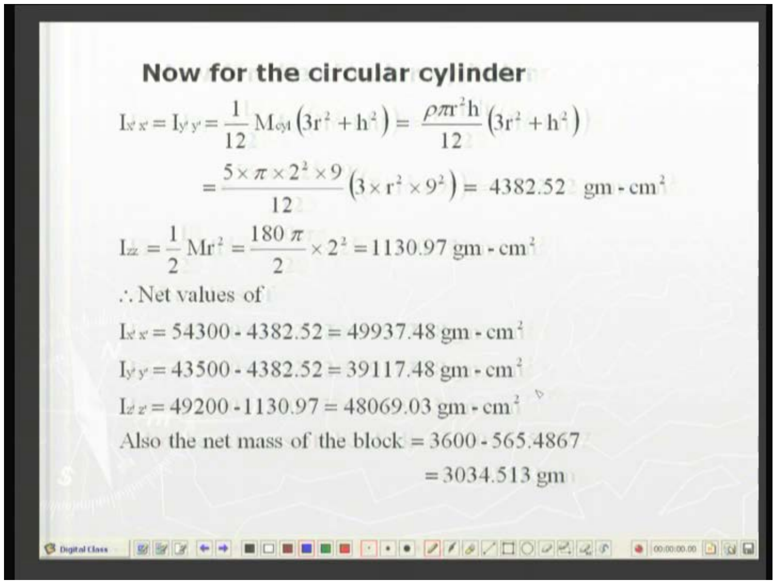

So for these cylinder again. From the hand book or from the tables, we will have mass of this cylinder, that is, if the cavity had been there. So one by twelve of that into three r square plus h square. Now since x axis and y axis are along the two perpendicular diameter of the cylinder.

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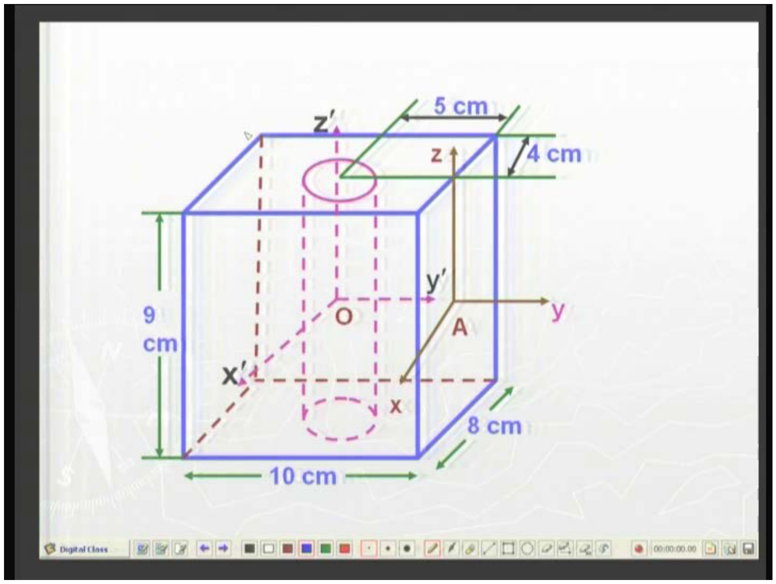

So it means Ix dash x dash is equal to Iy dash y dash for the cylindrical body. So when I substitute over here, everything is known. Radius is known. Four centimeter is the radius and the height is known, namely the height is nine centimeter height of the block. So after simplification, I will get forty-four three eight two point five two gram centimeter square and for the moment of the inertia about the z axis, I will have one by two Mr square from the tables. So M is already obtained from here. Pi r square is the area into height into rho is M. So you calculate it out. It will be eleven thirty point nine seven. So the net value for the net block, that is, the solid rectangular block minus the cylinder cavity. These are the three values Ix dash y x dash y dash y dash z dash z dash and the net mass of the block is three thousand thirty-four point five one three grams.

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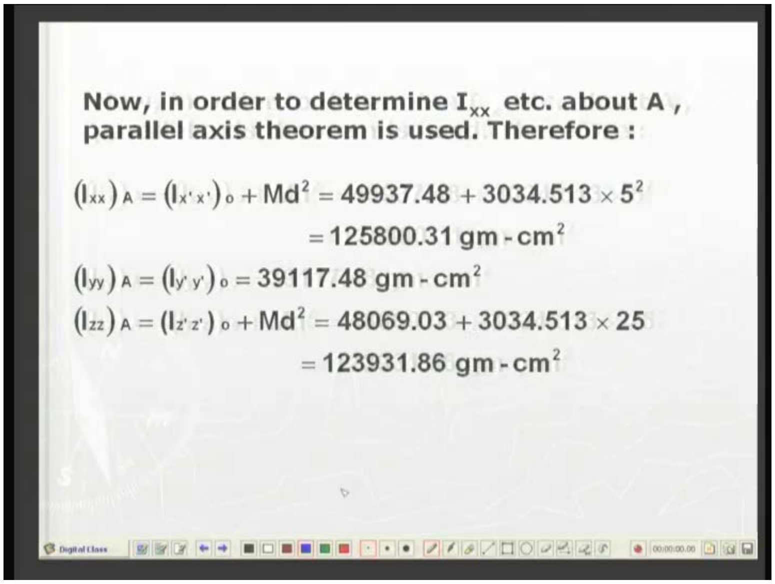

Now we are ready to apply the parallel axis theorem. Let us go back to the figure. Suppose I considered x dash axis and x axis. These are two parallel axis. The normal distance between the two axis can be obtained. This is half this dimension. So it means, Ixx will be equal to Ix dash x dash plus mass times d square distance between the two axis normal distance squared. So similarly for y dash y. Well, fortunately y dash and y axis lie on the same line. So there is no need to have a parallel axis theorem, that is, the normal distance between y dash and y is zero. So it means second terms contribute zero and then for the third z dash z dash, you can again find out the normal distance. So let us see. Sorry. Ixx about A, that is on the right hand face is equal to about the center plus Md square. So this five square will come out to be this value Iyy since they lie along the same axis. So this distance is a zero. So we don't have any change and similarly z axis. Again, these five centimeter. So d square is twenty-five. So this is how step by step you get the values.

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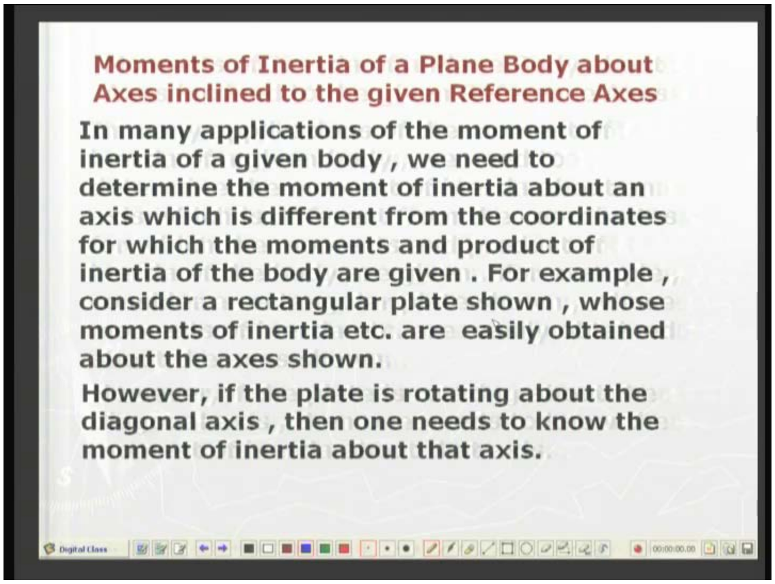

Now let us go to another aspect of moment of inertia. In the previous case, the two axis xyz and x dash y dash z dash were obtained by translation. There was no rotation involved, that is, x axis was translated parallel to itself to the new position. x dash axis y axis was translated without rotation to y dash axis and so on so forth. Now in many problems we are required to rotate the axis, no translation, but about the same origin, the orientation of the coordinate axis has to be changed.

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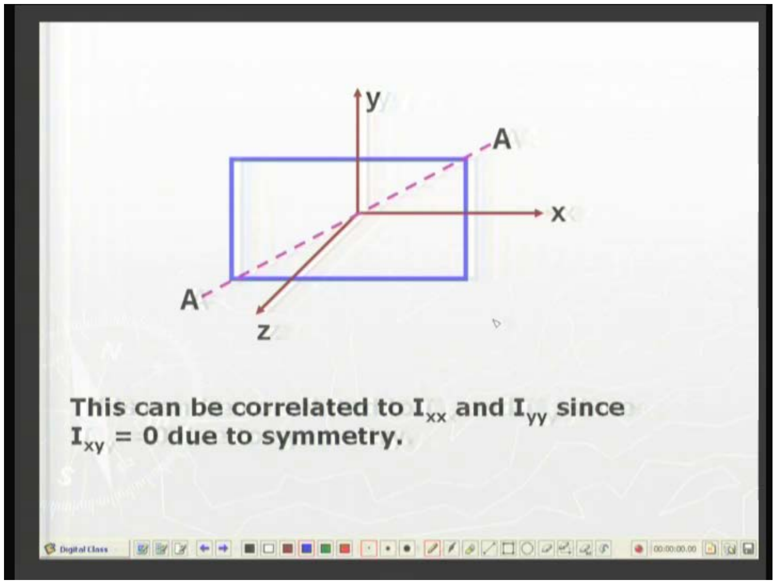

I will give you a very simple example. As you will learn in dynamics, the moment of inertia plays an important role in the rotation of solid bodies and that is moment or torque causing the rotation of the body and that depends up on the moment of inertia about the axis about which the rotation is taking place. For example, consider a rectangular plate which is rotting about its diagonal axis A, that is, X is passing throw the opposite corners of the plate. Now it means I have to find out the moment of inertia about this axis. This may not be easily available in your hand books or in your tables. So what you have do is that you take the value of the moment of inertia about the axis parallel to the sides, that is X axis and Y axis and of course Z axis is perpendicularly to the plane of the plate and then I want to transform these values of moment of inertia about to a new coordinates system. At least one of axis of the new coordinate system is along A A. The other can be perpendicular to this. So it means that I rotate the coordinate system about z axis through an angle a theta. Then x axis will be the new coordinate x dash axis which is coinciding, siding with A A. So it means under rotation. We want to get an expression connecting the new moments of inertia with the old moments of the inertia. So let us do this.

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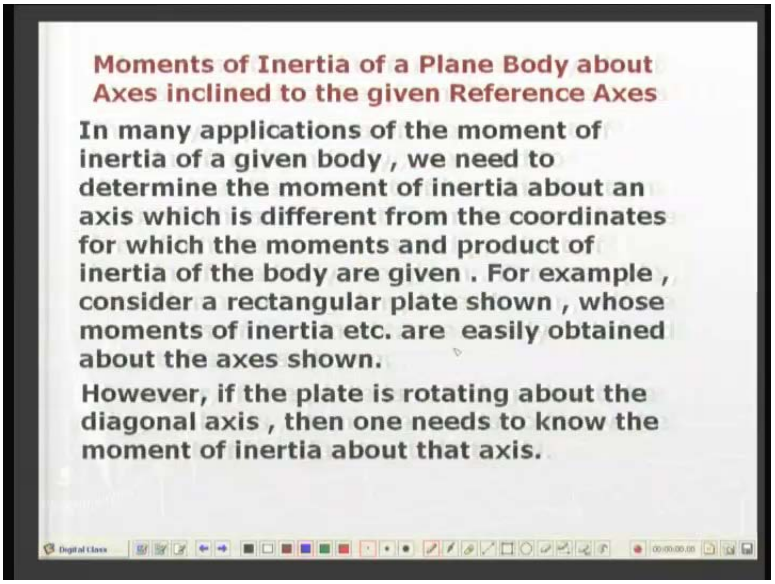

So as I said in many applications, this is needed and we will now start. You may recall at this point that we did the similar exercise in the case of second moment of areas and the product of areas, that is, if you are given a surface of a given area, then the second moments were converted about one given x set a. Coordinates were changed over to the second moments and products of area about the new coordinate system. So it was a coordinate transformation. So similarly we could have done it for the moments of inertia. The procedure is exactly on same line but here I have chosen a slightly different route. Here we will use the correspondence between the moments of inertia and the second moments of area in our previous lecture, that is lecture twelve, we had set up a correspondence between the two sets of quantity through rho t, that is, the mass density into thickness of a plate. In case the plate is of uniform thickness and homogeneous material, then the correspondence is very direct.

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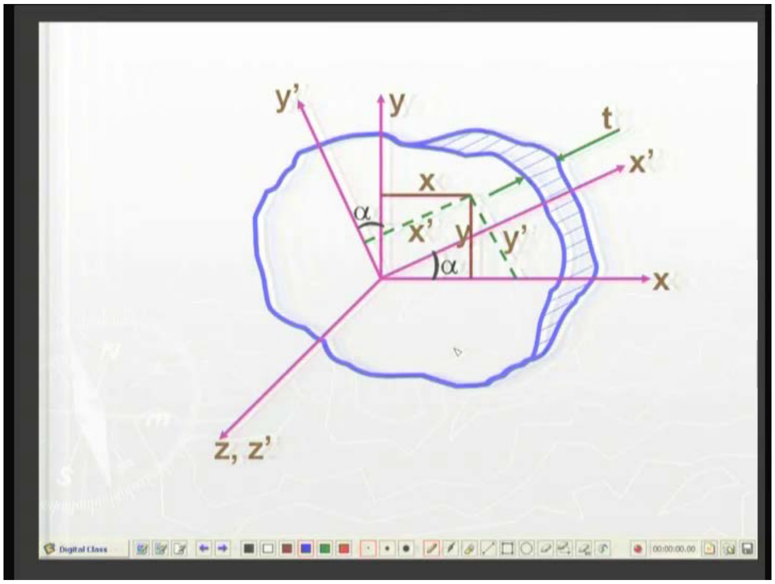

So here, let us say, a thin plate like body and for this, the coordinate system is x y z, the new rotated coordinate system. The rotation is through an angle alpha about z axis. So the new coordinate system will become x dash y dash z dash. Now if I consider any small element of the body here, then its x y z coordinates are x and y in the new coordinate system. It is x dash y dash. Sorry, this will be only up to this point. So this is not needed. This is only up to this point about up to the x dash x. So we can now project these x dash and y dash on to the x axis and y axis and set up a trigonometric relation and substitute but that procedure we used for the second moments of area. Now here we are using the correspondence.

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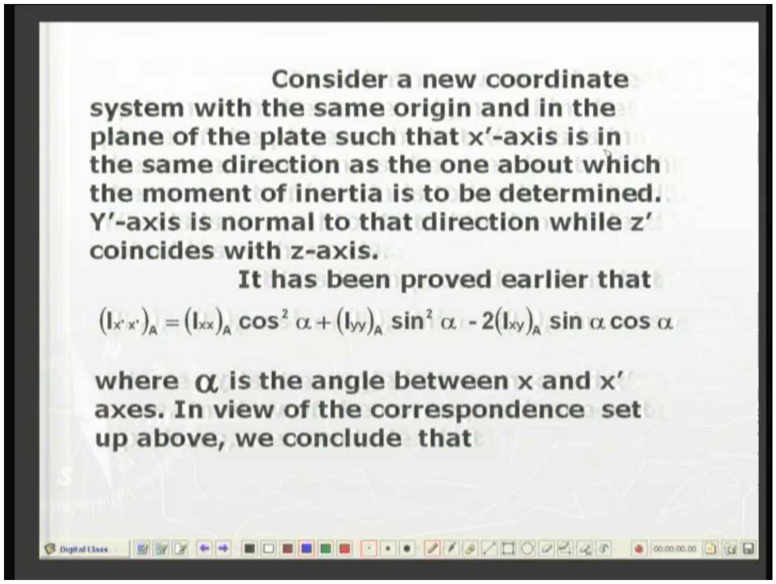

Well, it was proved earlier in, let us say, lecture twelve, that the second moment of area, Ix dash x dash a is equal to, that is in the old coordinate system is related to that in the new coordinate system and where a alpha is the angle between x and x dash axis.

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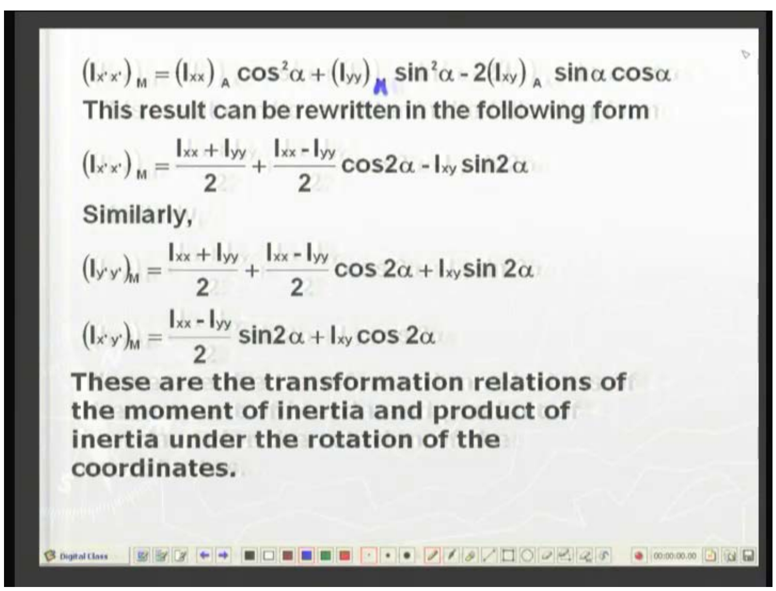

So in view of the correspondence setup there, we can conclude that similar result is obtainable for the moments of inertia, that is, Ix dash x dash moment of inertia. Mass moment of inertia is equal to the corresponding term, for the area Ixx a cosine square alpha Iyy. This is a and sin alpha minus sin square alpha minus two Ixi ah Ixy a sin alpha cosine alpha and if you use trigonometric identities, then we can convert this cosine square alpha sin square alpha etcetera into corresponding expression for cosine to alpha and then for the mass moments of inertia in the new coordinate system, Ix dash x dash is related to the corresponding terms of the mass moment of inertia in the old coordinate system Ixx plus Iyy by two. Here is the difference Ixx minus Iyy by two cosine two alpha minus the product of inertia Ixy sin to alpha. Similarly for the other two moments of inertia, that is, Iy dash y dash M and these are the expressions and again, using the correspondence we can have for the product of inertia terms.

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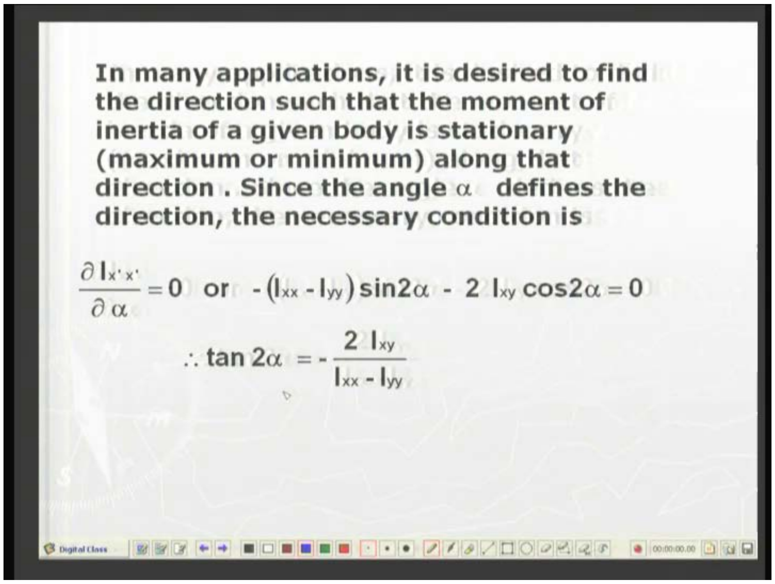

So it means we can go back from one coordinate system to another coordinate system whether there is translation or rotation by appropriate formulas. Now let us ask our self a question. In what coordinate system or in what direction the moment of inertia can be maximum or minimum, that is, what is that direction which will give me the maximum mass moment of inertia along that direction or minimum? So in other words, we want to find out those directions which will give us stationary values. You may recall from differential calculus, to find out the stationary value of a function, the derivatives of with respect to the variable have to be equal to zero. Similarly over here, to find out the directions along which the moments of inertia will be stationary, we will differentiate the expression with respect to alpha and set it equal to zero. So the expression for Ix dash x dash, when differentiated with respect to alpha, that is, angle of rotation and set it equal to zero. That condition will yield to us the maximum or the minimum values of moment of inertia. That is, we will get the alpha corresponding to maximum or minimum. So if I take the expression and differentiate with respective alpha, either here or here, whichever way you want to proceed, it is a material. So set it equal to zero and then simplify. Well this expression, when set equal to zero, is given over here and when I take it on the opposite side and divide, we will get tangent two alpha is equal to minus two times the product of inertia Ixy divided by Ixx minus Iyy. This is what the two dimensional case plate like body. For a three dimensional case, the procedure is different, quite complex and we have not considered because it leads to the Eigen value problem high grow and so on so forth. But for two dimensional bodies, this procedure is quite simple. So once we have this expression, suppose Ixy Ixx and Iyy are given to us, it means right hand side can be obtained and then I take the arc tan and I will get two alpha. Hence alpha.

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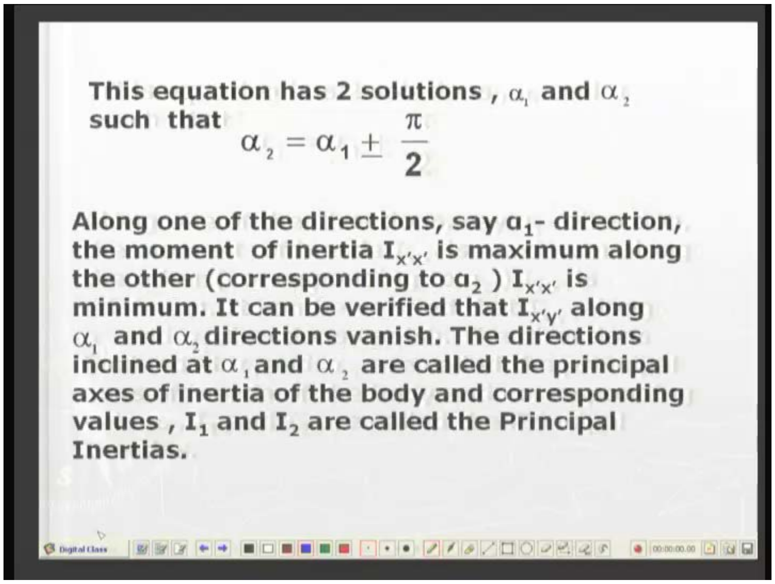

So this is exactly what is being done now. This will lead to two values. For example, if the expression is, the right hand side is positive, tangent of angle is positive in first and third quadrant. If it is negative, then it is negative in the second and the fourth quadrant of the plane. So I will get two values satisfying that equation for tangent to alpha and these two values or these two routs of the equation will be separated by pi by two ninety degrees two alpha will be separated by hundred eighty degrees first and the third quadrant or second and the fourth quadrant are apart of each other by hundred eighty degrees. So it means alpha two will be equal to alpha one plus minus ninety degrees or pi by two and once you have obtained alpha one and alpha two, you substitute back into this expression or this expression. You will get the corresponding values which will be one of them. Two values will be obtained for Ix dash x dash. One of them will be the highest possible value. The other will be lowest possible value.

So these are the Eigen values of the moment of inertia and these values of inertia are called the principle inertia and the corresponding directions or axis along which these happen to be maximum or minimum are called the principle axis or principle directions of inertia. So for a two dimensional body, that is a plate like body with uniform thickness, we can always obtain two directions. Call the principle axis of inertia about one direction, the value is maximum. The other direction value is minimum and these are so called the extreme or stationary values of inertia and these play a very important role in dynamics. Now suppose a body has Ixy zero. Then what happens? The numerator is zero. It means alpha is zero or ninety degrees, that is, the two axis x axis and y axis are themselves the principle directions. So whenever you have the product of inertia of a plate zero, if you find that a given plate has zero product of inertia, then the coordinate axis themselves are the principle axis of inertia. You do not have to do any further calculation and along one axis the inertia will be maximum and along the other axis it will be minimum.

For example, plate is circular. Then you take any two perpendicular diameters. They can serve as x axis and y axis because along that the product of inertia, Ixy will be zero. So it means any two diameters will be principle axis of inertia and fortunately maximum and minimum value will be identical. So because Ixx in this case will be is equal to Iyy, about any other diameter also, it will be same.

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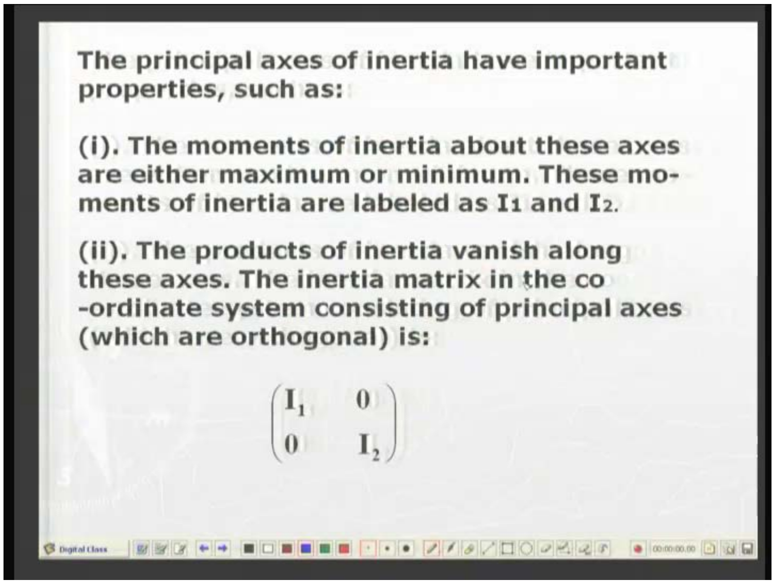

Well, I have already stated once you have the principle axis, then the product of inertia terms will be zero. So inertia tenser will be equal to inertia matrix is i one and i two. The diagonal terms are the principle inertia and the off diagonal is zero.

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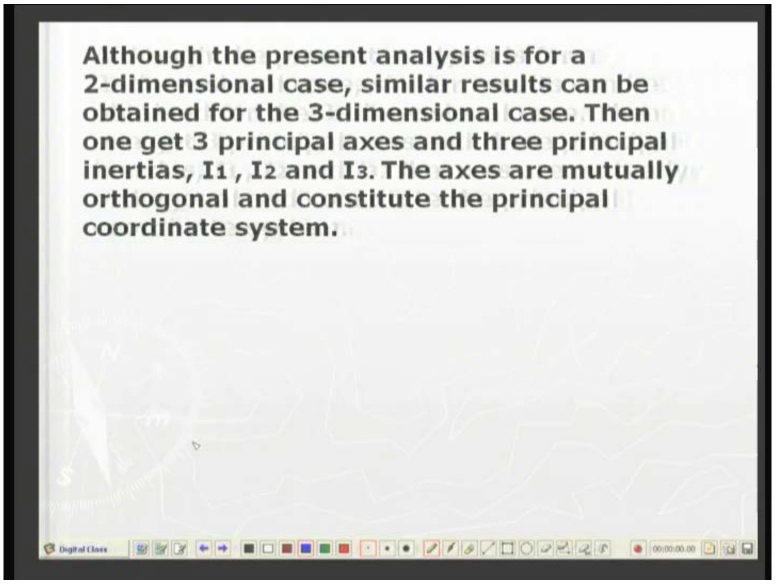

For three dimensional case, you can generalize it. Of course, the direct proof is, as I said more complicated in case of a three dimensional body, in general, there will be three principle axis of inertia and one of the inertia values along one of the axis will be highest. About the other, it will be lowest minimum and the intermediate about the third axis, it will be intermediate between the two and these three directions are always orthogonal. These are standard results from the Eigen value problem but we are not discussing them in detail. Let us take up one or two examples to illustrate what we have learnt today.

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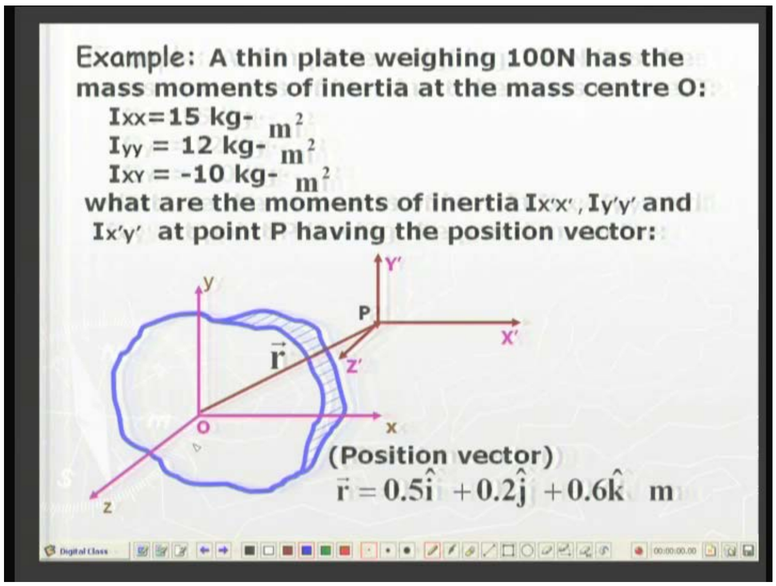

For I am restricting them to thin plate problems, a thin plate weighing hundred Newton has mass moments of inertia. They are given about the mass center O is the mass center, Ixx is fifteen kilogram meter square. Sorry. Its shift should not be shifted. It is kilogram meter square. Iyy is twelve kilogram meter square. Ixy is minus ten. See, the mass moment of inertia is always positive but product of inertia can be positive negative or zero. So here it is minus ten kilogram meters square. What are the moments of inertia about the new coordinate system x dash y dash z dash whose origin is at P whose position vector as seen from the xyz coordinate system? A is two j point six k meters. So the three coordinates of P are known to us.

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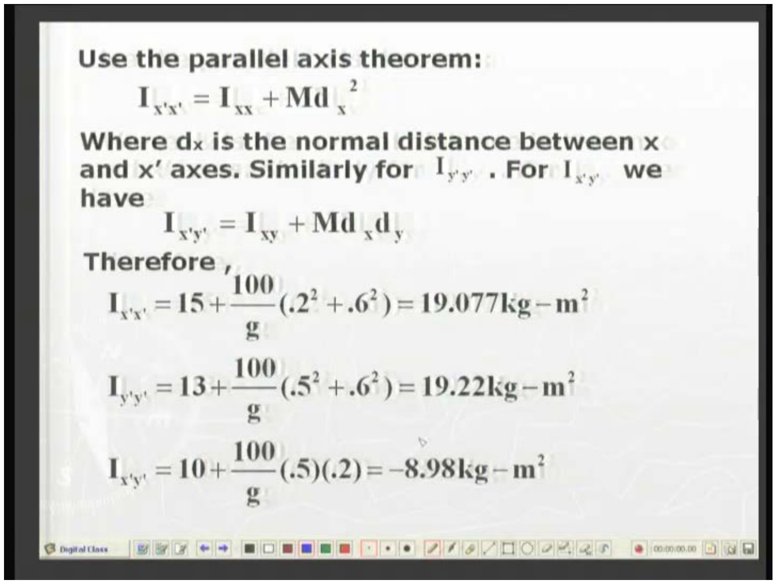

Well it is a direct use of the parallel axis theorem. So Ix dash x dash is equal to, in the new coordinate system, corresponding moment of inertia. In the old coordinate system, plus total mass into d x square the distance between the x coordinates old and new the normal distance between the two squared is the normal distance between these two. Similarly, for product of inertias, we have this formula. The rest is just substitution. Fifteen is the Ixx and then the mass is, you have, hundred Newton is the weight. So divided by G nine point eight one, you will get mass in kilograms. So then the normal distance between the two axis, will be by Pythagoras theorem, the z z coordinate square plus y y coordinate square of the point P. So this is exactly hundred by G point two square plus point six square. So this comes out to be nineteen point zero seven seven kilogram meter square. Iyy, exactly on the same line, the distance between the two normal distance between the two y axis is point five square plus point six square under root. So you square it up and for the product of inertia, in a new coordinate system, old coordinate plus mass times xc xp yp. X coordinate of p by times the y coordinate of p and this will come out to be minus eight point nine eight one nine eight kilogram meter square. So that was fairly elementary problem.

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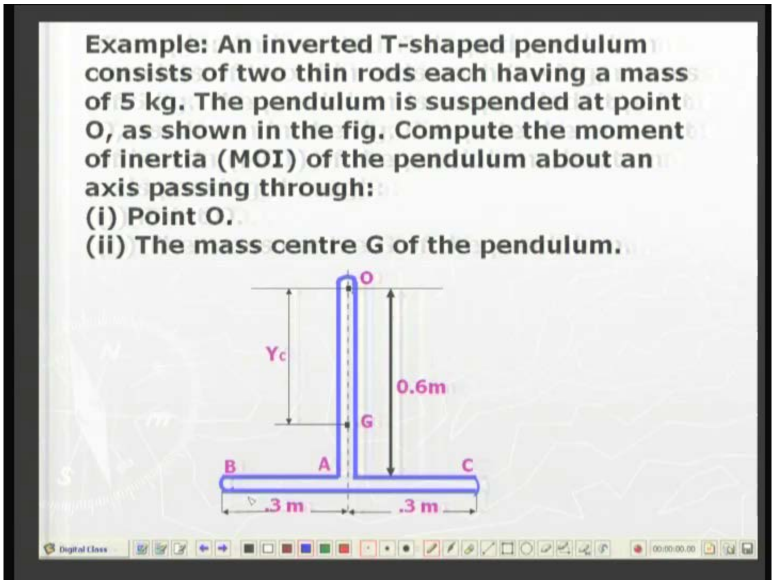

Let us go to a more interesting problem now. Here is an inverted t which is being used as a pendulum which consists two thin rods one vertical and the other rod is horizontal both the rods are having dimensions point six meter and point six meter thickness is much smaller than the other dimension. So, each vertical of these limbs of the t section are weighing five kilogram and they are having a mass of five kilogram and the pendulum is suspended from the top point O as shown in the figure. Compute the moment of inertia of the pendulum about an axis passing through the point O and point G, that is, axis perpendicular to the screen. So at this, t pendulum is swinging or oscillating about either O or about G. So we have to find out the corresponding moments of inertia.

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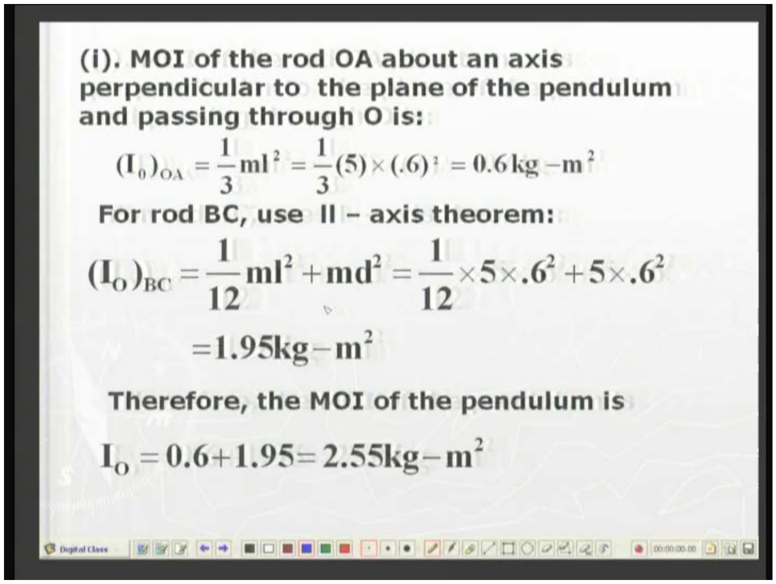

Well, using the information available in the hand books, we will neglect the rounding of the corners. We will consider them as rectangular bars. So moment of inertia of the body OA, that is the vertical part. So I have to calculate the moment of inertia about the extreme end an axis passing through the centre of extreme end. So that is given by one by three times mass into length square. One third of ml square and mass is five kilogram and length is point six, as I said.

So five into point six square divided by three, that is, coming out to be point six kilogram meter square. Now rod B. First up, we will find it about its own central axis and then we shift it. You understand, we have this axis which is a moment of inertia of a rectangular bar about its middle axis can be obtained very easily from the handbook and then it shifted from here to here. So we will have one by twelve ml square plus md square. Once again about this axis passing through the middle. It will be ml square by twelve and then, in a parallel manner, it is shifted over here. So the distance between these two is point six meters. So, we will have md square. So mass divided by twelve into point six square plus mass into point six square through one point nine five kilogram meter square. So the total moment of inertia about the point O. The top point is sum of these two expressions. This is coming out to be two point five five kilogram meter square. Now to find out about the center mass G. First off, we have to locate the center of mass and for that centroid of the cross, action has to be found out. Yc is equal to the first moment of the two masses, that is vertical part and the horizontal part divided by the total mass. So that you can easily see. Both the parts are five kilogram each. So five into point three. Well, this center of mass of this vertical part is at half the depth point three total depth is point six center mass is of the horizontal part is at total depth point six. So we have five into point three plus five into point six divided by the total mass ten. So the location of G is four point five meters from the top and then since we know the moment of inertia about an axis perpendicular to the screen passing throw O the top point and we have to find it of about the center of mass. So again use parallel axis theorem instead of doing it from initial formulas, you can use the parallel axis theorem at this stage and yc is known the distance between the axis along the y axis is known. So simple calculation will give me this IG as point five point five kilogram meter square. So this exercise has repeated use of parallel axis theorem.

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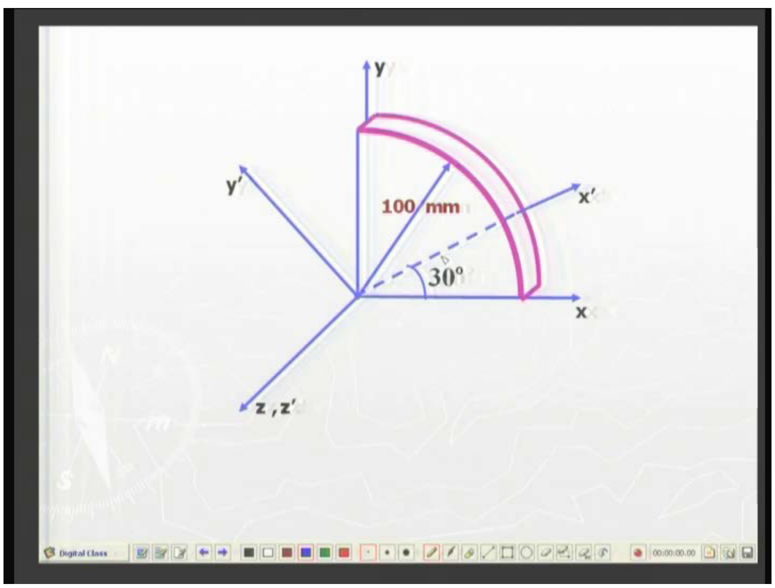

Let us have one more final example and then will close this chapter. Here is a plate a thin plate which is a quadrant of a circle. Various dimension are given and the one coordinate system x y z is passing through the mid plane of the plate and now this coordinate system is rotated through an angle thirty degrees about z axis. So the new coordinates are x dash y dash and z and z dash coinciding. So this is the middle plane over here. So you can see that this is how the location of the coordinate plane x y is or x dash y dash is.

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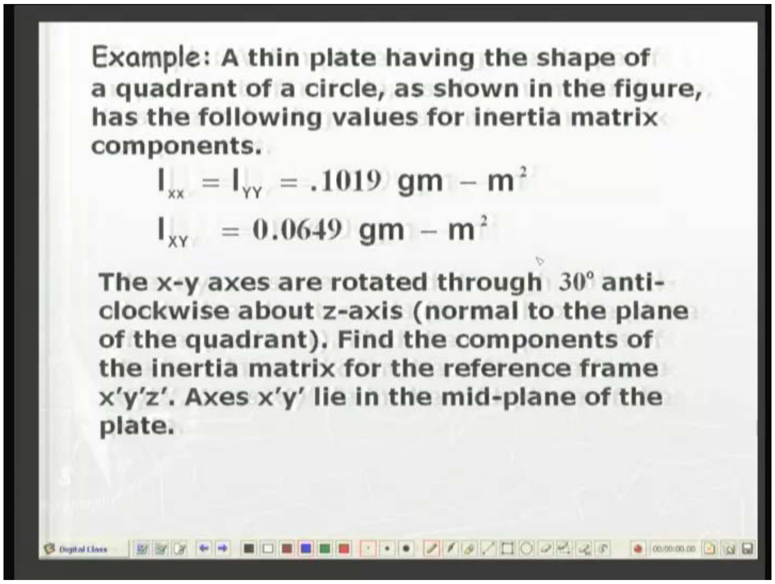

Sorry, first, let me read the problem. A thin plate having a shape of a quadrant of a circle is shown. This has the following inertia matrix. Components Ixx is given. Iyy is given. Both will be because of symmetry equal point one zero one nine grams meter square and Ixy is given as point zero six four nine gram meter square. The axis are rotated through thirty degree anticlockwise, as we have seen, normal to about an axis, normal to the quadrant. Find the components of the inertia matrix for the reference frame x dash y dash z dash ah x dash y dash lie in the mid plane of the plate.

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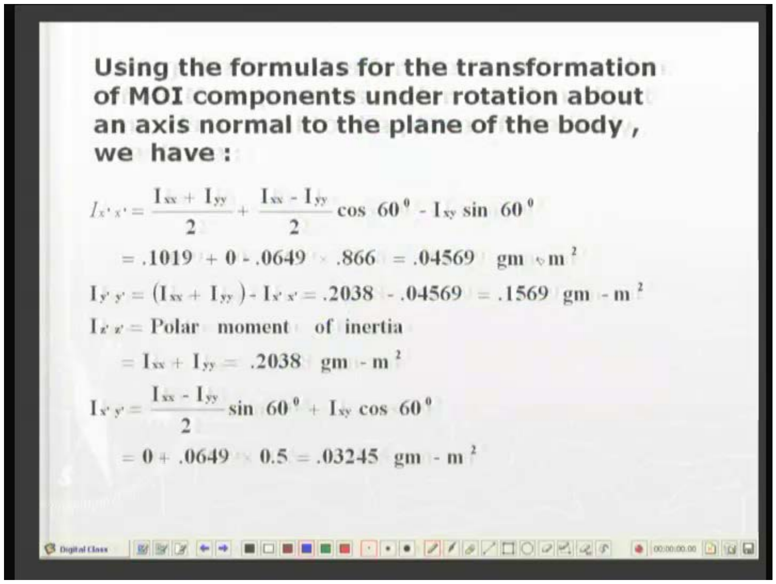

So I have already explained the problem. So let us go to the calculations. Well, we have to directly use up the formulas which we have derived for the rotation from the correspondence principle of the second moment of areas. So Ix dash x dash for the moment of inertia is Ixx plus Iyy. That is the moment of inertia values in the old coordinate system plus the difference between the twos divided by two cosine two alpha alpha is thirty degree so cosine of sixty degree minus the product of inertia times sin two alpha that is sin of sixty degree. Rest is because both of them are equal Ixx and Iyy are equal and this is divided by two. So first term will be point one zero one nine second term because of equality is zero and the third term will be minus Ixy is point zero six four nine into sin of sixty degree under root three by two point eight six six. So I have got it. Iy dash y dash. Again, using the formulas, we will have point one five six nine and then the puller moment of inertia, that is, Iz dash z dash, which is sum, of do you remember for thin plate we have already proved, Ixx and Iyy. So if I add these two, I will get the puller moment of inertia which is Iz dash z dash. So point two zero three eight grams meters square and finally the product of inertia in that plane. Ixx and Iyy are both being equal. So the difference between the two is zero and so the first term is zero and second term Ixy was already given to you point zero six four nine into cosine of sixty degree which is half that is point five. So this is what we have got. Finally, all the components of the matrix.

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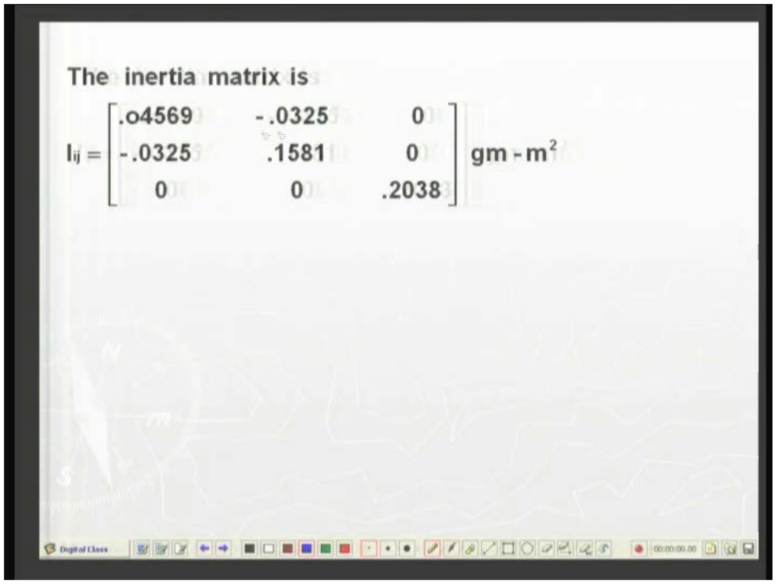

So if I put it in the matrix form Ixx Iyy Izz, these are the Ixy terms. Now Ixz and Iyz, because plate is of uniform thickness, the xy plane because it is lying in the mid thickness, you can say the plate is symmetric about the xy plane and hence Ixz and Iyz are zero. This is what we have earlier stated due to symmetry. So it means these two terms will be zero. So we have got the complete matrix of the inertia terms for this problem for this quadrant of a circle. So I think now we can close or sum up the whole chapter of moment of inertia, the product of inertia and the second moment of an area and product of area. We have seen how the shift of the axis parallel to the older axis brings in the changes in the moments of inertia terms, etcetera, product of inertia terms, second moment of area, etcetera, how the rotation effects and what are area of principle axis of inertia principle directions, etcetera, etcetera and then also for if the body is a thin plate we have a direct correspondence between the moment of inertia terms and the second moment of areas and similarly product of inertia terms and this product of areas. So these concepts will be very useful in the dynamics of bodies as well as later in later courses in solid mechanics when you are analyzing beams of different shapes and cross sections. Thank you very much for your attention. In our next lecture, we will be starting with virtual work and energy methods of analyzing equilibrium.