Applied Mechanics
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Lecture No. 12
Properties of Surfaces (Contd.)

Today, we will take up lecture twelve. You may recall that in lecture eleven, we had introduced the second moments of area and products of area. Also, we learnt about the parallel axis theorem which enabled us to find out the second moments and products of area about axis which are parallel to a set of given axis passing through the centroid of the body. So under a translation of axis, we could determine the second moment of area and product of area when these quantities are given in some original axis. So we will continue with the topic on properties of surfaces.

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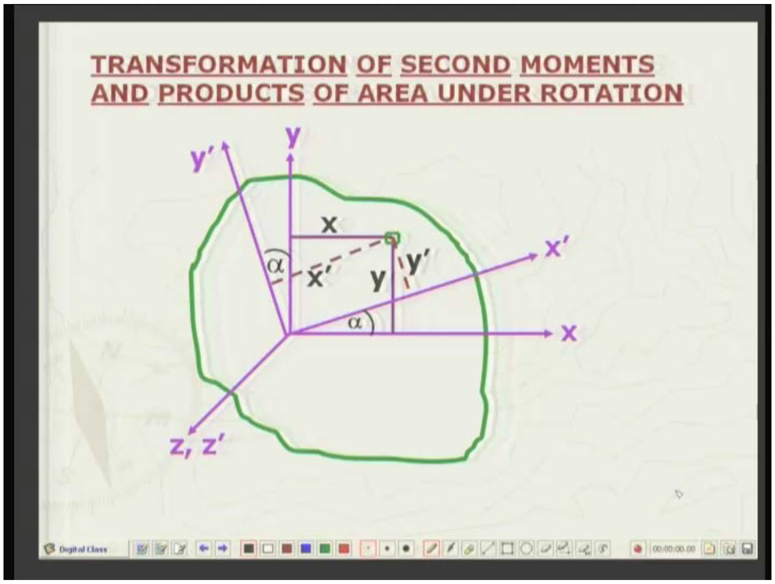

Today, we will look at the transformation of second moments and product of area when the axis are given rotation. For example, in this picture we can see that the axis xyz are given and a new set of axis is also given, which is x dash y dash and z dash. The new set is obtained from the old set of axis by rotation through an angle alpha as shown here about z axis. So that is why z and z dash axis are coinciding. So when such a rotation is given, what is the effect on the second moments of area and product of area? Can we correlate the new quantities with the old quantities?

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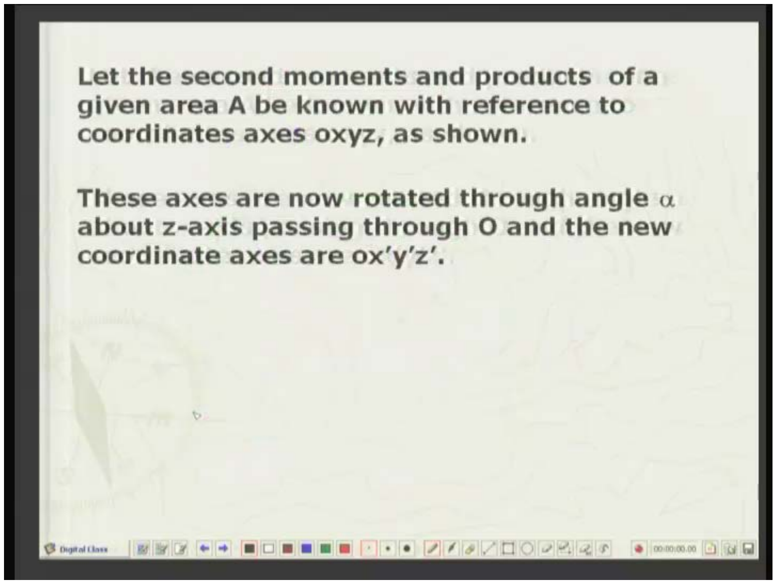

The answer to this question is yes, for, in order to obtain such a correlation, let us look at the new coordinates x dash and y dash.

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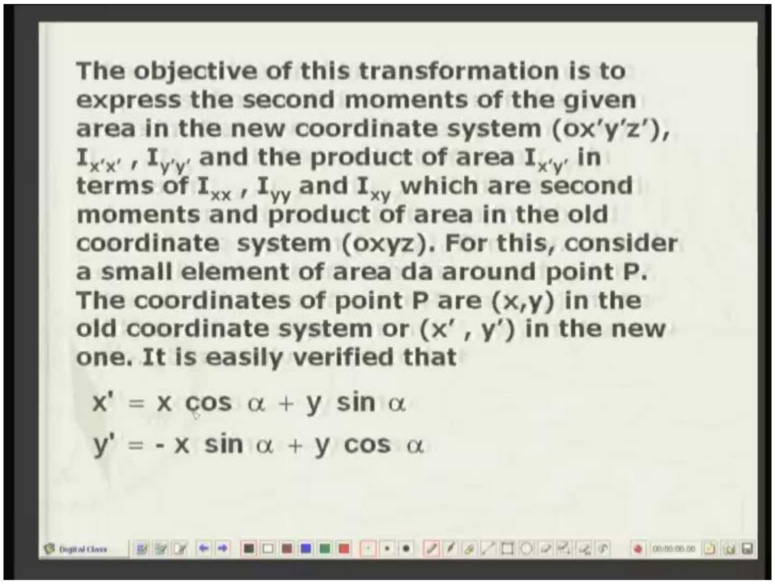

Going back to the figure, we will see that, let us say, suppose I consider an arbitrarily small element of the area, namely, the element is having dimension dx and dy, dx and dy. Now, this element is having coordinates x and y in this plane and in the new coordinates system, the coordinate of the same element are x dash y dash. We can correlate x and y with x dash y dash by simple trigonometry. For example, if I consider x dash, I can project it on old x axis and old y axis. Similarly, if I take y dash, I can again project it in x axis and y axis and we can see that the following relation will be obtained. That is, x dash is equal to the new x coordinate. That is equal to x which is the old x coordinate times cosine of the angle of rotation alpha plus y times sin alpha. Similarly, the new y coordinate, that is, y dash is equal to minus of x. That is, the old x coordinate sin alpha plus y cosine alpha.

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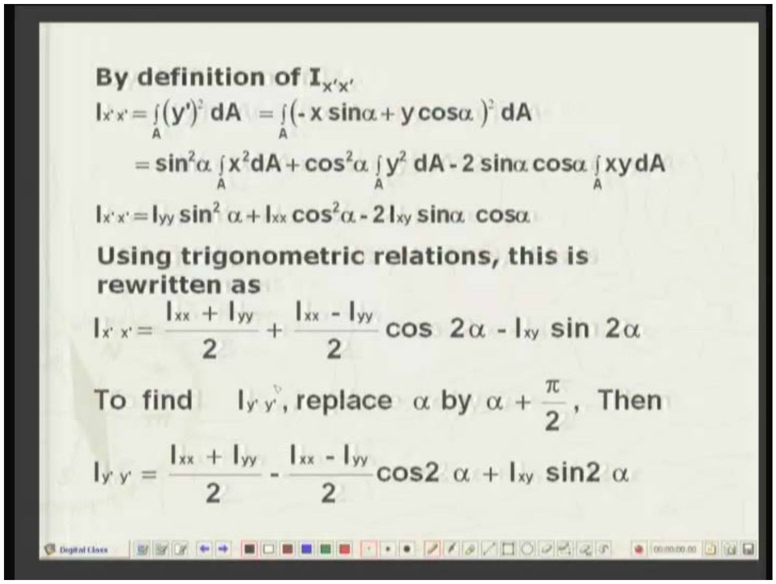

Now by definition of the new second moment of area about y axis, that is Ix dash. Ix dash is equal to, by definition, area integral of y dash square. When this integral is taken over the entire given area and substituting for y dash from the previous equation, we will have it minus x sin alpha plus y cosine alpha whole square area integral. This integrant is expanded and since alpha is constant. Therefore sin alpha and cosine alpha are also constant. So they can be taken outside the integral. So sin square alpha times area integral of x square dA plus cosine square alpha area integral of into area integral of y square dA minus two sin alpha cosine alpha into area integral of the product term, that is, xy Da. Well, these integrals yield the well-known quantities, that is, x square dA area integral will give me Iyy. That is in the old coordinate system. So Iyy sin square alpha. That is the first term. Similarly, y square area integral will give me Ixx. Therefore Ix plus Ixx cosine square alpha minus two times. The last term gives me the product of area in the old coordinate system two I minus two Ixy sin alpha cosine alpha. So we have basically set up a relationship between Ixx Ix dash x dash and all the three quantities in the old reference system, that is, Ixx Iyy and Ixy. Now this relationship can be recast in a slightly different manner by using trigonometric relations. So because sin square alpha and cosine square alpha can be expressed in terms of um cosine two alpha through trigonometric identities and after simplification, it is very easy to come to this final result, that is, x dash is equal to Ixx plus Iyy divided by two plus Ixx minus Iyy divided by two into cosine two alpha minus Ixy. Well, this two times sin alpha cosine alpha will be easily given in terms of sin two alpha. So, I have the second moment of area about y axis, that is in the new coordinate system, that is, x dash is equal to the corresponding terms in the old coordinate system and cosine alpha cosine two alpha and sin two alpha terms. I could do the same analysis, same procedure for I y dash y dash. That is the second moment of area about x dash axis. That is in the new coordinate system. Either I go through the same expressions, take the squares and simplify or there is an alternate quicker method, that is, if I replace alpha by alpha plus ninety degrees.

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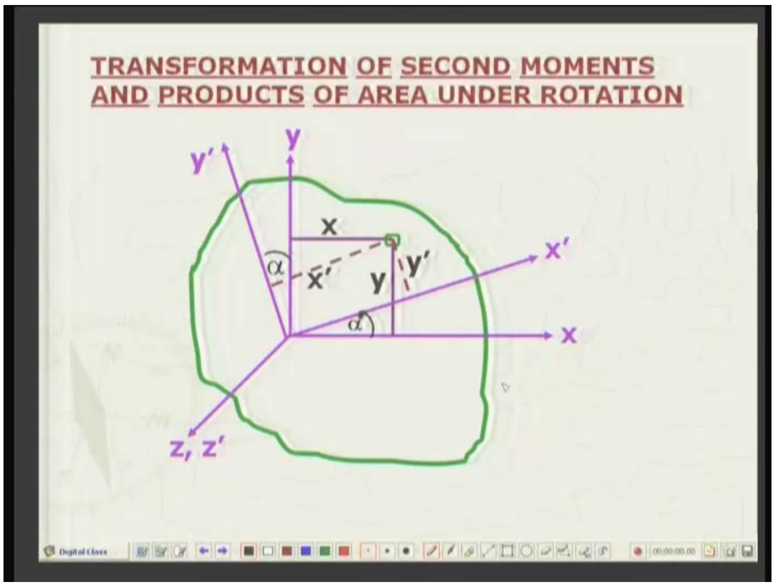

You can see from the figure that this y dash will make an angle of alpha plus ninety degrees with x axis old x axis. So if I substitute alpha by alpha plus pi by two, then the relationship is very quickly obtained. Iy dash y dash is equal to Ixx plus Iyy by two minus because of cosine terms. That will lead to cosine two alpha plus pi. So it will give me minus Ixx minus Iyy by two into cosine two alpha plus sin two alpha Ixy sin two alpha. Well, let us do the relationship for the product of area, that is, Ixy terms. So Ix dash y dash, that is, product of area in the new coordinate system is to be related with the corresponding quantities in the old coordinate system.

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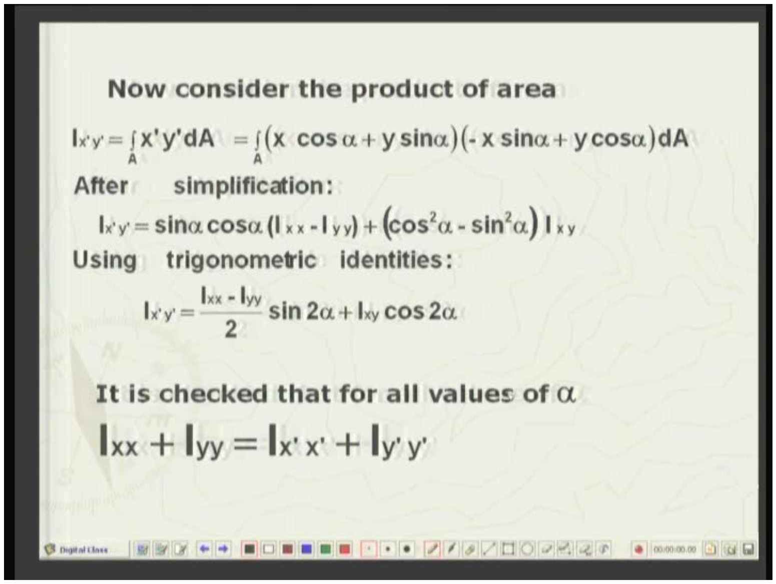

So by definition, this is equal to area integral of the product term Ix dash y dash and again, substituting for x dash and y dash from the relationship which we have obtained earlier.So you get this term for x dash, the second term for y dash and then you take the area integral over the entire area given area and this product term is now simplified and again going through the area integrals, we can come to this result. That is, Ix dash y dash. The product term in the new coordinate system is equal to sin alpha times cosine alpha into Ixx minus Iyy. These are the two second moments in the old coordinate system plus cosine square alpha minus sin square alpha into the product of area in the old coordinate system, that is, Ixy. Again, the square terms can be replaced by through trigonometric identities by the sin two alpha and cosine two alpha terms. So we will have Ix dash y dash is equal to Ixx minus Iy by two sin two alpha plus Ixy cosine two alpha. So what we have achieved is that the moment of area and product of moments of both the moments of area and product of area in the new coordinate system can be correlated with the three quantities in the old coordinate system, which we have already calculated. For example, with the help of these relations, we can calculate very quickly those quantities in the new coordinate system with these coordinate systems, related through rotation about an axis normal to the area, that is, z axis.

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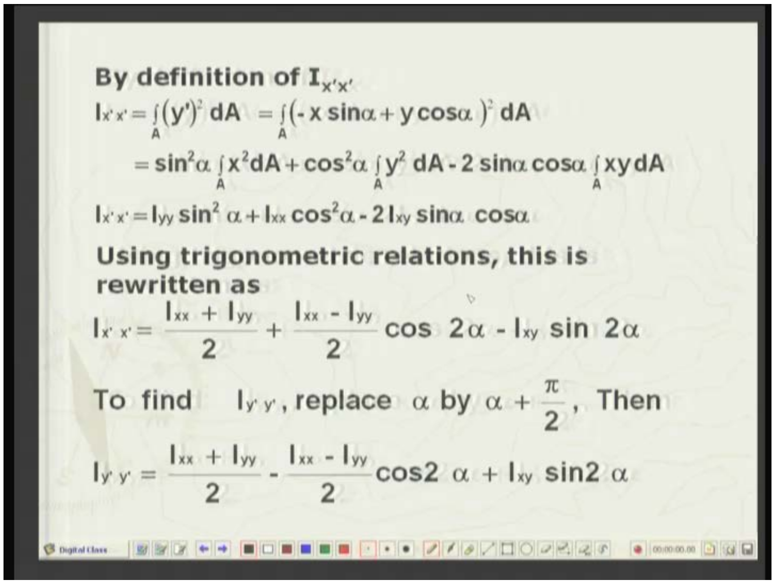

Now, looking at the relationship for I x dash x dash and I y dash y dash, if I add up these two equations which we have already obtained, then you can easily see that these two last terms will cancel each other and again, because there is a negative sign and positive sign here, these will also cancel each other. So we will be left with Ixx plus Iyy. So the sum of the second moments of area about x axis and y axis in the old coordinate system is equal to the sum of second moments of area about the new coordinates, that is, x dash and y dash. So this quantity is independent of alpha. So what does it mean? It means that for any alpha, the sum is constant. It is invariant. That is a very useful result and it is a very basic property.

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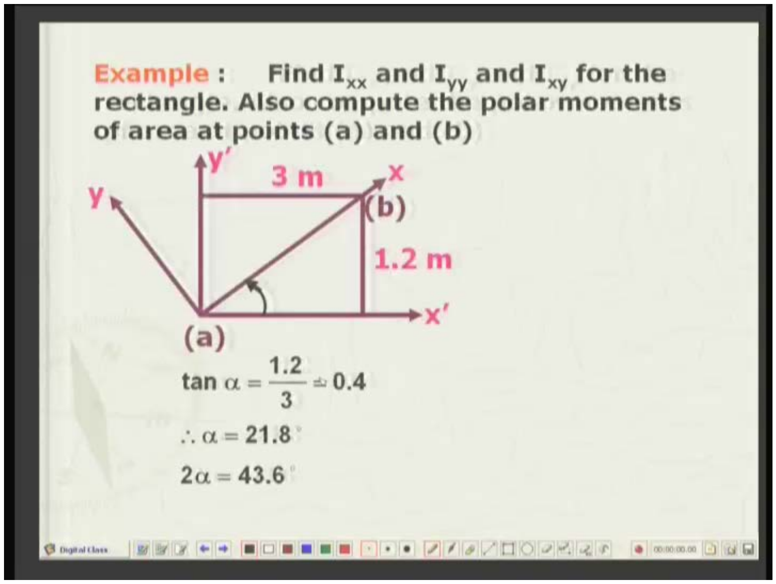

Now, I think we can all grasp these concepts better by taking up an example. The example is, find Ixx and Iyy and Ixy for the rectangle. This, a rectangular plate is given. We have been asked to find Ixx Iyy and Ixy. Also, compute the polar moments of area about axis passing through point a and b. If you recall, we had in our last lecture introduced the concept of polar moment of an given area. So first of all, what we do is, suppose I have these quantities. These axis x dash axis and y dash axis and the new axis is xy axis, which is obtained by rotation through, let us say, if I call this as an angle alpha, tangent alpha will be equal to one point two divided by three. This is one point two and this is three so point four. So alpha will come out to be twenty-one point eight degrees and since we need cosine two alpha and sin two alpha. In the relation, two alpha will be equal to forty-three point six degrees.

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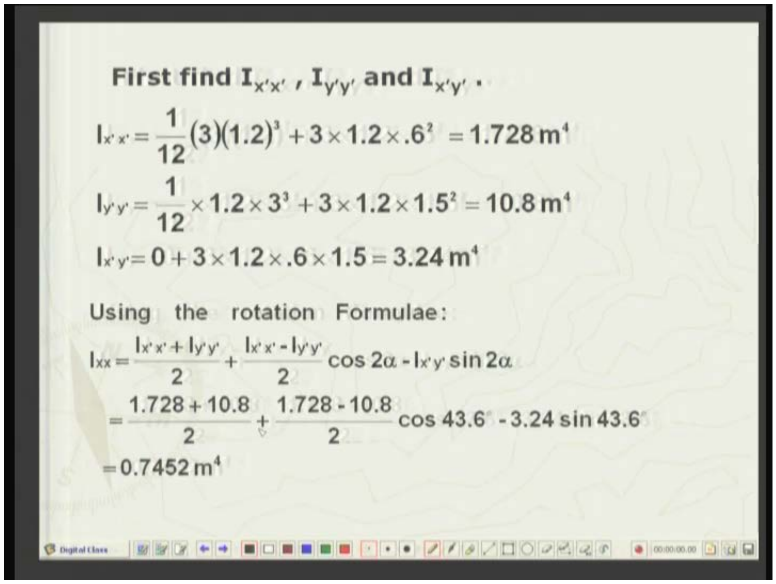

Now first, we will find out Ix dash x dash, Iy dash y dash and Ix dash y dash, that is, about this base and about this side. Well, if I have an axis passing through the centroid of the area, that is, through the centre of this diagonal, this, you can say, the half the width and half the base. So this is the centroid of this rectangle. We will first find out the second moments of area about these doted axis and then by parallel axis theorem, we can shift these to x dash axis and y dash axis.

So the procedure is once again like this. First of all, this rectangle is given. So forget about xy coordinates for the time being. I am considering only x dash y dash coordinates. Now, to calculate the second moments and product of area about x dash and y dash coordinate since the second moment of area about the axis passing through the centroid. That can be obtained from handbooks are or it can be very easily calculated. So, namely, it is bdh cubed by bd cube by twelve, that is, b is the width, d is the depth cubed divided by twelve. So, if we consider that formula, we can calculate, let us say, first of all, this axis second moment of area is easily obtained about this axis. Then this has to be translated. This has to be transferred to x dash axis by parallel axis theorem. Similarly, I will calculate the second moment of area about the vertical axis passing through the centroid of the rectangle and then shifted through parallel axis theorem. So that is the procedure.

So first, I will calculate Ix dash x dash. Now, width times depth cubed by twelve. That is about the middle axis and then by parallel axis theorem, a parallel axis theorem means area times the distance between the two axis squared. So area of the rectangle is three into one point two times the distance. This distance is one point two by two, that is, point six, So into point six square.

So that will give me one point seven two eight meter four. Similarly for I y dash y dash, that is, the second moment of area about the side. So again, we will have bd cubed by twelve. Now, b width one point two times d depth is three cubed by twelve plus, by parallel axis theorem, area times the distance square. Now the distance will be three divided by two one point five. So area times one point five square. So this is exactly what we have done. I y dash y dash is one by twelve into one point two into three cube, plus area is three into one point two times the distance square one point five square. So this comes out to be ten point eight. Now, for the product of area. The axis is passing through the centroid of the rectangle, they are symmetry axis rectangle and is, after all, a symmetric figure about the central axis. So we have already seen, that, if the axis are passing through the centroid, then the product of area terms are zero. So I xy about these axis will be zero. So if I want to shift them to x dash and y dash, then again parallel axis theorem for the product of area which means that the area times the distance between the x axis and into the distance between the y axis e e into d into a.

So look here, Ix dash y dash is equal to zero plus three into one point two. That is the area of the rectangle into half the side into half the base point six into one point five. So that comes out to be three point two four meter four. So it means, I have obtained Ix dash x dash, Iy dash y dash and Ix dash y dash about the sides of the rectangle and then, I have to shift these quantities about the inclined axis. Sorry. These inclined axis is angle alpha, which is forty three point six degrees. So x axis and y axis. I have to use the rotation formula. So if I rotate it now through forty-three point six degrees. Then I will get Ixx by use of the formulas, cosine two alpha. We will get it as point seven four five two meter four.

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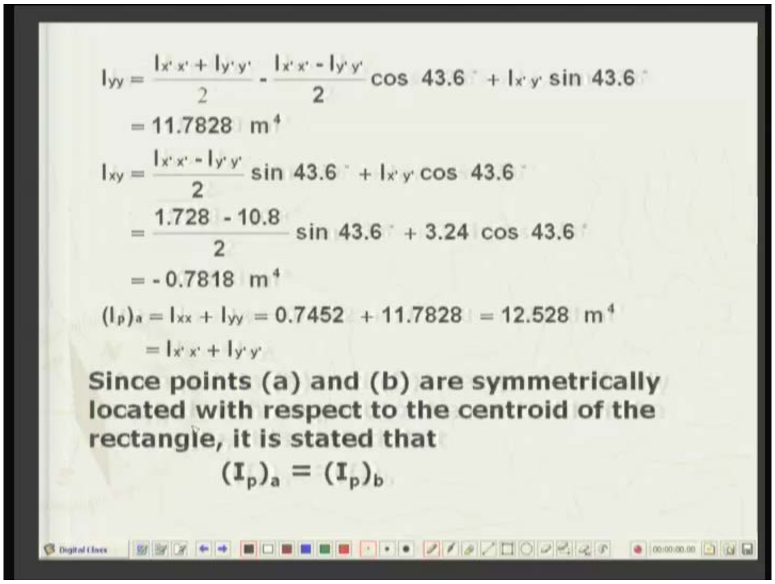

Then we go to Iyy, eleven point seven eight two eight meter four and then the product of area, that is, point seven eight with the negative sign point seven eight one eight meter four. You may recall that Ixx and Iyy are always positive. So indeed, we are getting positive results for Ixx and Iyy in any coordinate system but Ixy, the product of area term can be positive, negative or even zero. Here, we are getting it as negative. So it is all consistent with our general conclusions. So Ixy is obtained by the same transformation formula as minus point seven eight one eight meter four. Now, we want to find out the polar moment of area. The polar moment of area, recalling from last lecture, is the sum of the two second moments of areas, that is, Ixx plus Iyy. So I polar at this corner a and the second is corner b. So it will be equal to the sum of the two Ixx and Iyy.

So point four five two. That is Ixx and this is Iyy which comes out to be twelve point five two eight and this is the invariant quantity and for point b.

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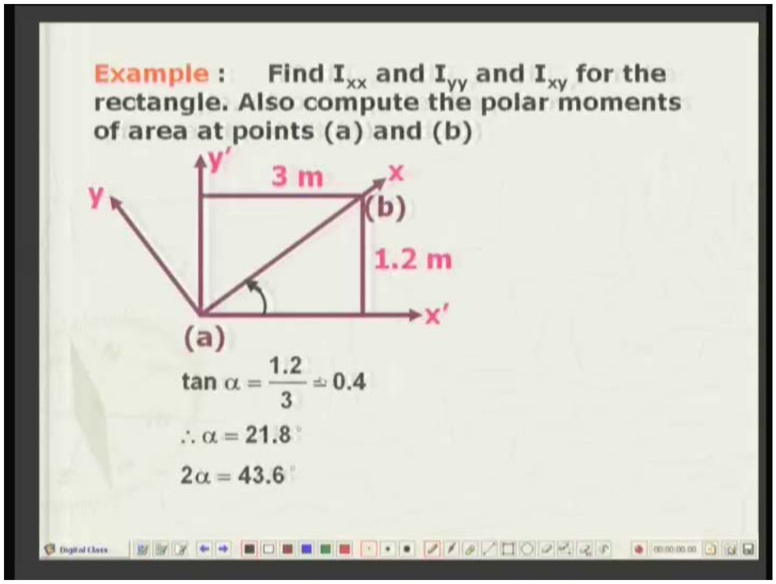

Since it lies symmetrically about the centroid. You can easily see that whatever happens to this, same is over here. So if our z axis is over here and z axis over here, then by parallel axis theorem, we can see that although on one side the distance is positive, on the other side, distance is negative but when you take the square, it will not make any difference. So it means that polar moment of area about point a is equal to polar moment of area about point b. So I polar about point b is equal to I polar about a. That is why we will have this result and the value will be equal to twelve point five two eight meter four. So that is all i have to say about the moments of area and products of area.

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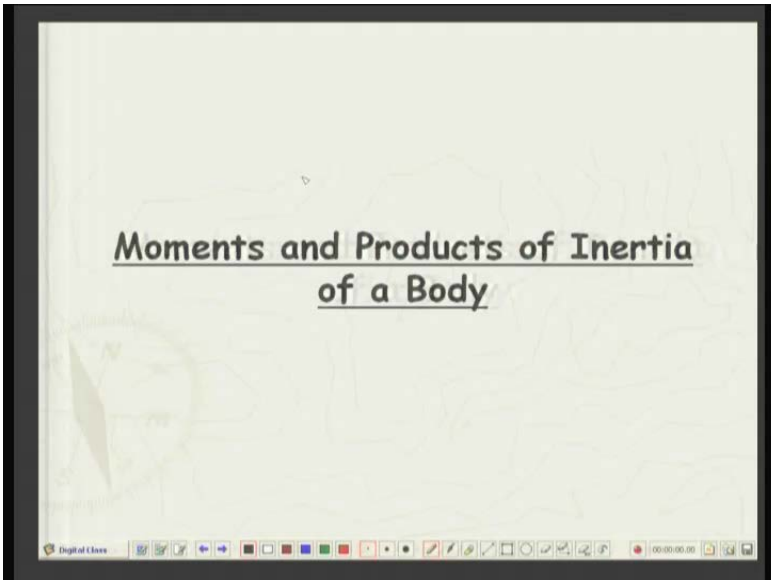

Now, we will continue our lecture but on a slightly different topic and that is the moments and products of inertia of a body. Now, we are no longer dealing with only areas, which is a two dimensional concept, we will be dealing with body, which is a three dimensional concept and we will also show that when the body is a plate like body, that is a thin body with uniform thickness, then there is a direct correlation between the second moment of area and the product of area with some quantities, which we will very soon be introducing. That is the moments of inertia and the products of inertia of a three dimensional body.

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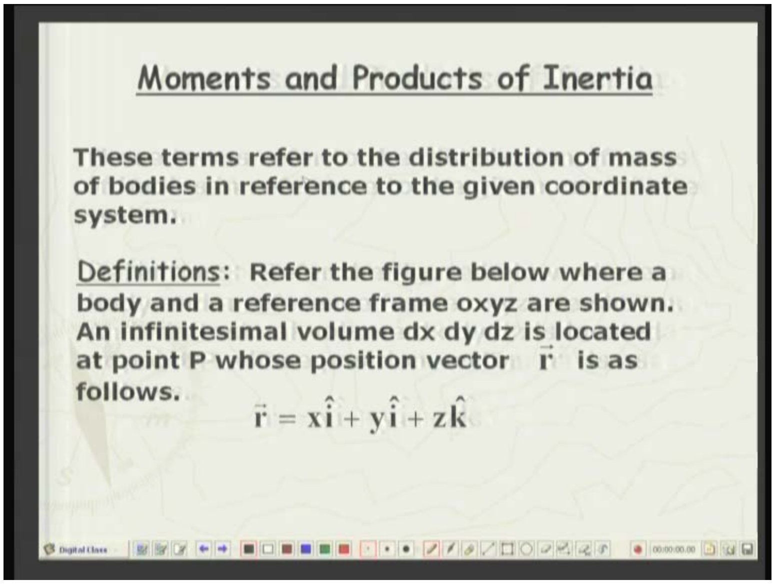

So let start with moments and products. These terms refer to the distribution of mass of bodies in reference to a given coordinate system, that is, if you are given a body and a coordinate system whether the mass is closer to the origin or farther away from the origin, how it is distributed around the origin, etcetera. So these concepts will be manifested through the moments and products of inertia. So let us define what these quantities are.

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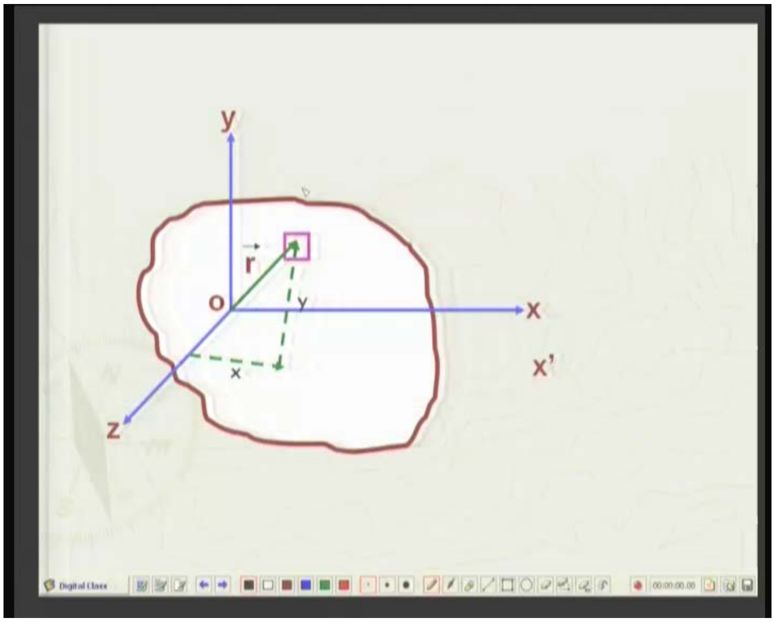

So if I refer to the figure here, let us say, this is a three dimensional body and an arbitrary coordinate system xyz and I consider a small cuboid of the body or element of the body whose mass is dm. So this element is located at a position vector r, which means that its coordinates are xyz. X coordinate is parallel to x axis, coordinate y is parallel to y axis and coordinate z is not shown, which I will be showing now. This is parallel to z axis. So if I now consider that element whose position vector is vector r equal to x times the unit vector I y plus y times unit vector j plus z times unit vector k.

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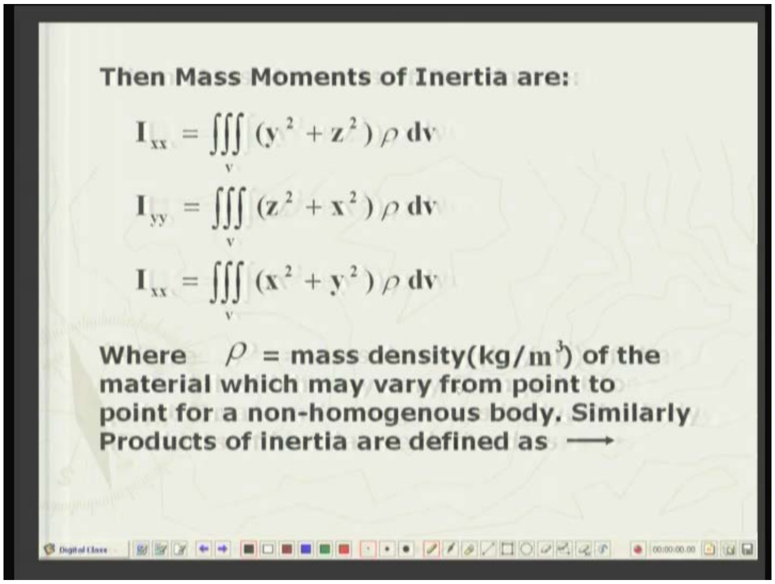

Then by definition, the mass moments of inertia are the volume integral over the quantity volume integral of the quantity y square plus z square the y coordinate square plus z coordinate square times the mass density. Rho is the mass density of the material in kilograms per meter cube. So rho times dv is nothing but dm, the elementary mass times the y square plus z square. This is simply the distance from the x axis by Pythagoras theorem. Similarly, Iyy is dm times the distance from the y axis square volume integrated over the entire volume of the body and similarly, I ah Izz is equal to volume integral of x square plus y square rho dv. So mind you, the quantity rho can be variable quantity. It need not be uniform. That is, rho can itself be a function of xyz. If the body is non-homogenous, let us say, body consists of mixture of several types of materials copper and steel or soil and water or something like that. So, from point to point, the density of the particles can be variable. Then rho is itself a function of xyz but on the other hand, if the body is homogenous, then rho is a constant quantity and it can be taken outside the integration sign.

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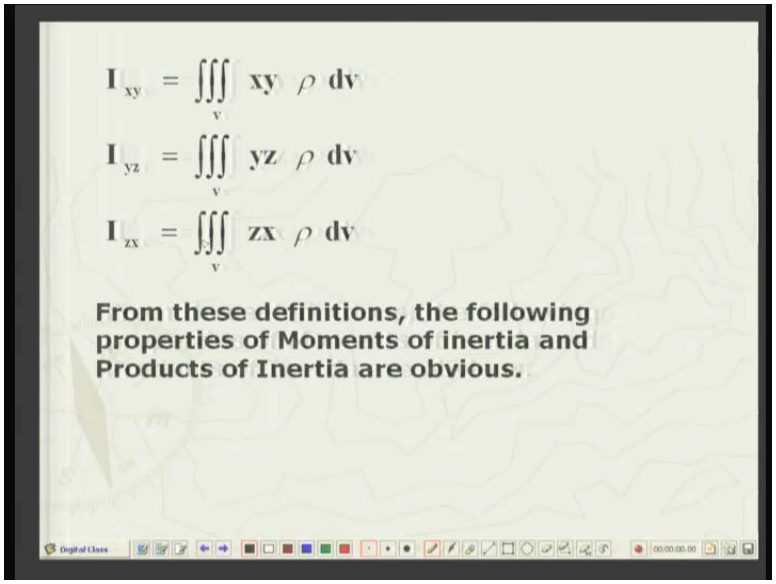

Second set of three quantities are the product of inertia quantities, Ixy, Iyz, Izx. Now definition here is much simpler. It is the volume integral of rho times xy, that will give you Ixy. Again, volume integral of rho time yz will give you Iyz rho time zx volume integrated, I will get Izx. From the definition, it is very easy to see that Ixy is equal to Iyx, Iyz is equal to Izx, zy, etcetera, zx, Izx is equal to Ixz. So the product of inertia terms are symmetric because the order of multiplication a into b is equal to b into a. That is a commutative process. So since that order does not matter, these quantities are also symmetric quantities.

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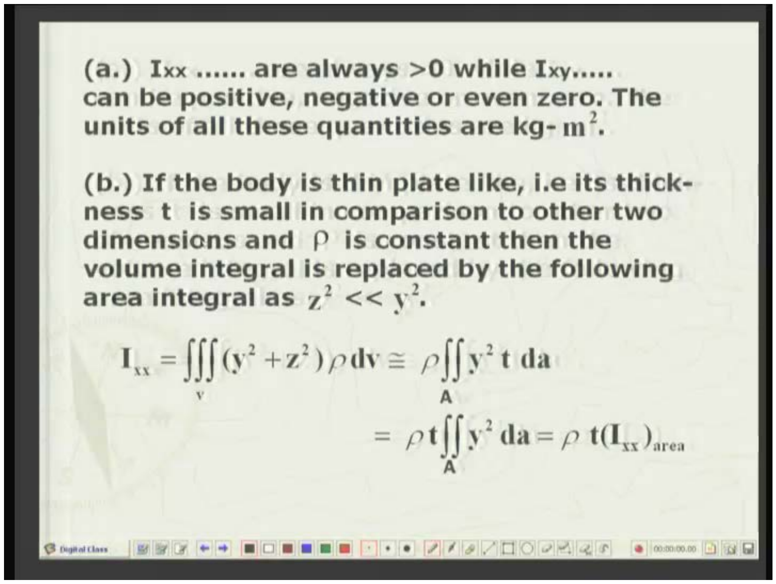

Now, let us look at some of the important properties of these moments of inertia and product of inertia terms. Well, first of all, the moments of inertia terms Ixx Iyy Izz involve sum of the squared quantities x square plus y square y square plus z square etcetera and square. When a quantity is squared, it is always positive and two positive quantities summed will always be positive. So it means that Ixx Iyy Izz will be always positive quantity. They will be zero only when the body reduces or degenerates to a point. In that case, the volume integral will be zero. Otherwise it will be non-zero quantity. So safely, you can say, that if the body is a finite body, it is not a zero volume body. Then these moments of inertia terms will be always positive quantity. Now, second point, if the body is thin plate like a body is in the form of a plate of uniform thickness and also, let us say, the body is having uniform density, it is a homogeneous body. So that t and rho are independent of xyz. So rho into t vector can be taken outside. So I can easily say, thickness is along z axis, then the volume integral will be converted into area integral by taking t and rho outside and also, since the body is thin the z coordinates of any point of the body will be much smaller than the y coordinate and z square will be even tinier or smaller than the y square quantity. So approximately, up to second orders of a infinitesimals, we can say that Ixx is equal to rho t taken out side into area integral which is nothing but Ixx area. This is Ixx mass moment of inertia. So to be specific, I should write m at the bottom. So Ixx m which means mass moment of inertia about x axis is equal to rho t times area second moment about x axis second moment of the area about x axis. Similarly, we can say about y axis and so on. The factor rho t is the mass density into thickness. So it will be the density of material but now expressed in terms of mass per unit area of the plate because thickness has been already taken into account.

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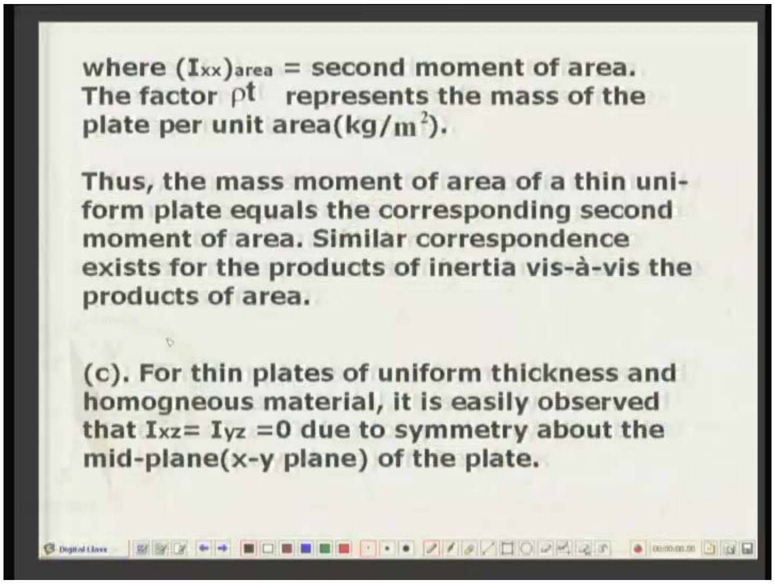

So it is rho. t is the kilogram per meter square, that is, mass per unit area. So we conclude that the mass moment of area of a thin uniform plate equals to the corresponding second moment of area. Similarly, we can say for the product of area. That is, the product of inertia for a thin homogenous plate has a correspondence with the product of area of the corresponding figure. Now if a thin plate has uniform thickness and consist of homogenous material, then we will take the x y plane as the mid plane of the plate. So that the top surface and the bottom surface of the plate are symmetric about the mid plane because the thickness is uniform and whatever lies above same lies below the middle plane. So then it is easily seen that Ixz and Iyz, because the area moment will be the mass moments of area, will be zero.

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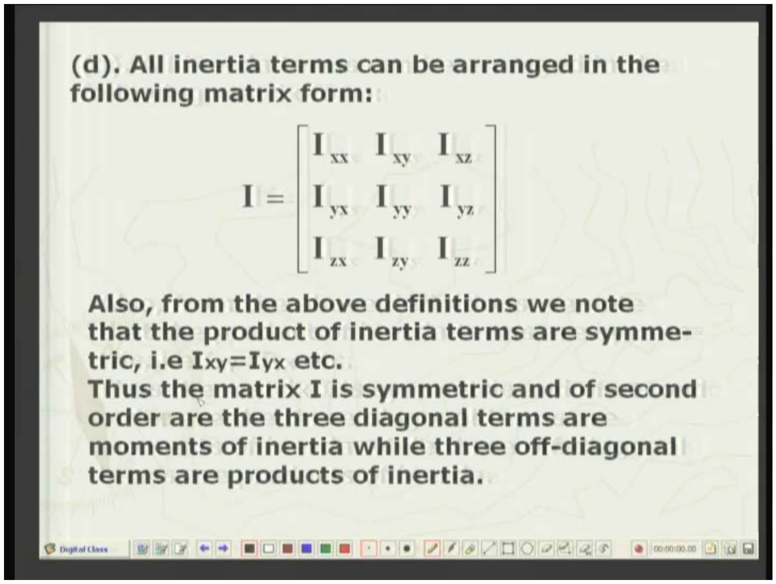

The mass products of all inertia terms can be arranged in the following matrix form. Ixx Iyy Izz. These are the three mass moments of inertia or simply moments of inertia and these are the diagonal term and the off diagonal terms. We have already seen that they are symmetric. Ixy is equal to Iyx. Izx is equal to Iz, xz, etcetera, etcetera and these are the product of inertia term. So although it is a three by three matrix, it should have nine terms but there are, effectively, six terms, three diagonal and three off diagonal because other off diagonals are symmetrically placed.

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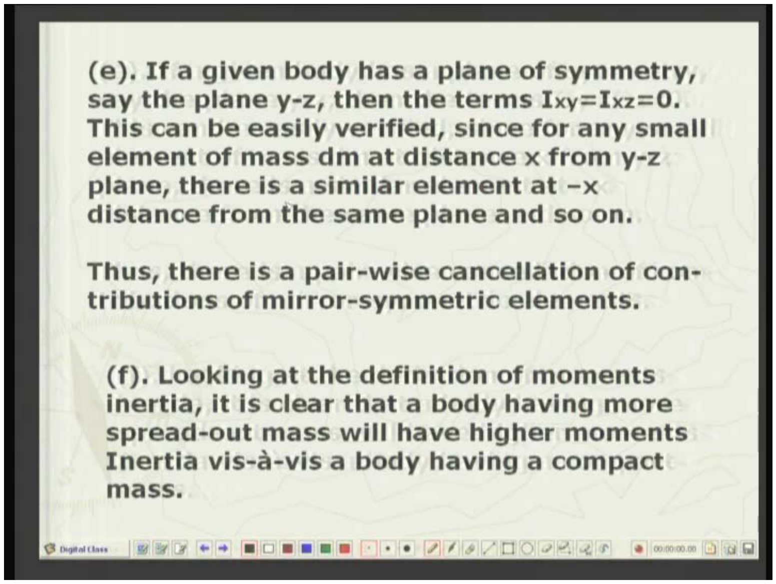

So, again, if the given body has a plane of symmetry, let us say, y z plane is a plane of a symmetry, then from the definition of the product of inertia terms, if I go back quickly to that definition, there is an element at a particular distance from that plane.

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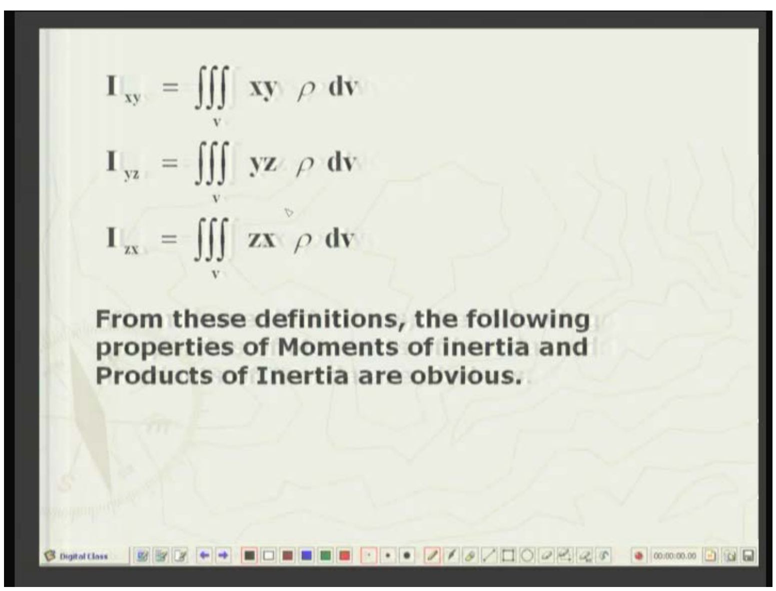

So because of symmetry, there will be a corresponding element on the mirror image location. Then the contribution of these two symmetric elements will cancel out because one is at a positive distance from the middle plane and the other is at a negative distance. So whenever there is an axis of symmetry, you can be sure that the two products of inertia terms will be zero because of that symmetry. That is, if yz is a plane of symmetry, then Ixy and Ixz will be zero. So this can be easily verified since for any small element, at a distance x from y, if there is a similar element at a distance minus x from the same plane. Looking at the definition of the moments, again, since Ixx Iyy, that is, mass moments of inertia terms, they have x square y square z square terms. So if the body is spread out, that is, during volume integrals, you have elements which are at a larger distance xyz from the given axis, then their contribution will be more. So if the body is spread out, then the moments of the inertia terms will be larger.

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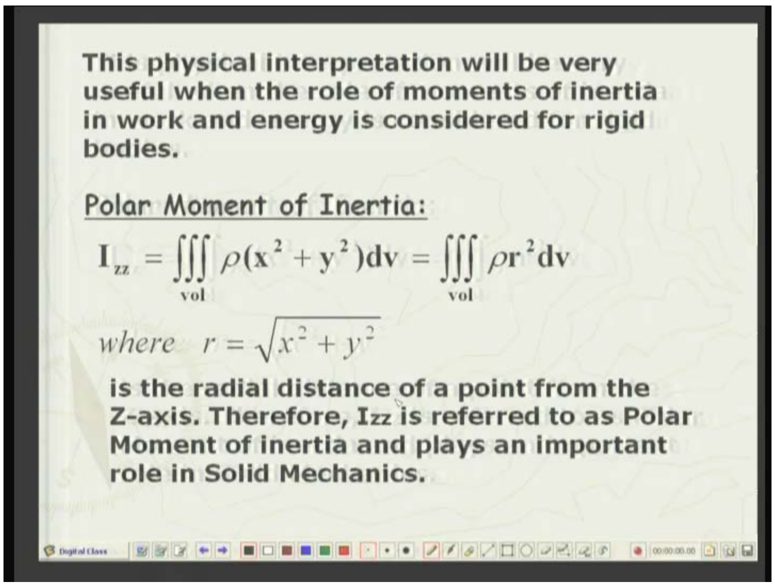

Now, we can also have polar moment mass moment of inertia. Well, I am not carrying the term mass again and it is understood, when we were talking about mass moments of inertia, that is, suppose I want to find out Izz, that will be I rho times x square plus y square volume integral and x square plus y square will be the radial distance of the elements from the z axis r is equal to x square plus y square. So what does this mean? You can label it as the polar moment of inertia about z axis. Similarly, Ixx can be written as y square plus. So it will be the polar moment of inertia about x axis. So in this manner, we can use the concept of polar moment of inertia.

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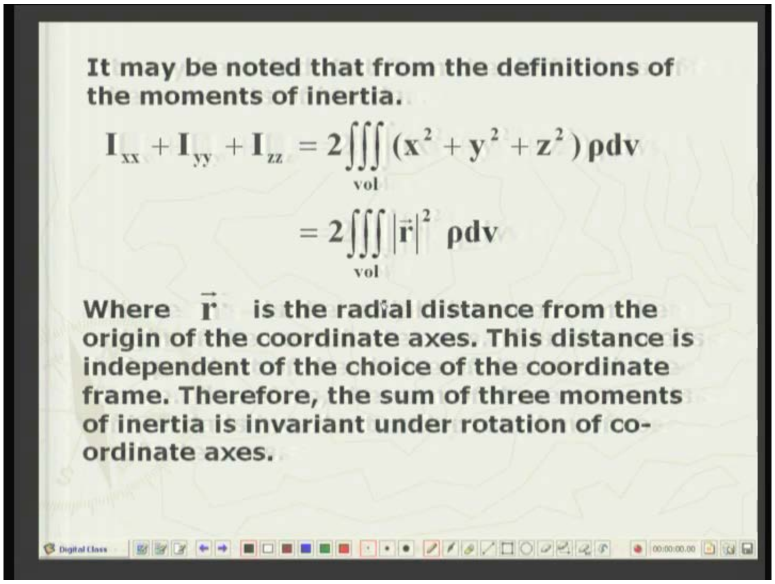

If I sum up of all the three mass moments of inertia, that is Ixx plus Iyy plus Izz, then using their corresponding expressions and adding them up, we will have twice the volume integral of rho times x square plus y square plus z square. What is this x square plus y square plus z square? This is the radial distance of the particle from the origin of the coordinate system. So it will be r vector absolute value square. So r vector is the radial distance. Now this quantity is a magnitude of a vector quantity for every particle of the body. It is independent, whether I take the xyz like this or this or this, if the origin is not changed. So for the same origin, this absolute value of r vector is independent of the choice of coordinate system. So it means that the sum of the three mass moments of inertia is invariant quantity. It is same in all the coordinate system. This property is very useful in solid mechanics.

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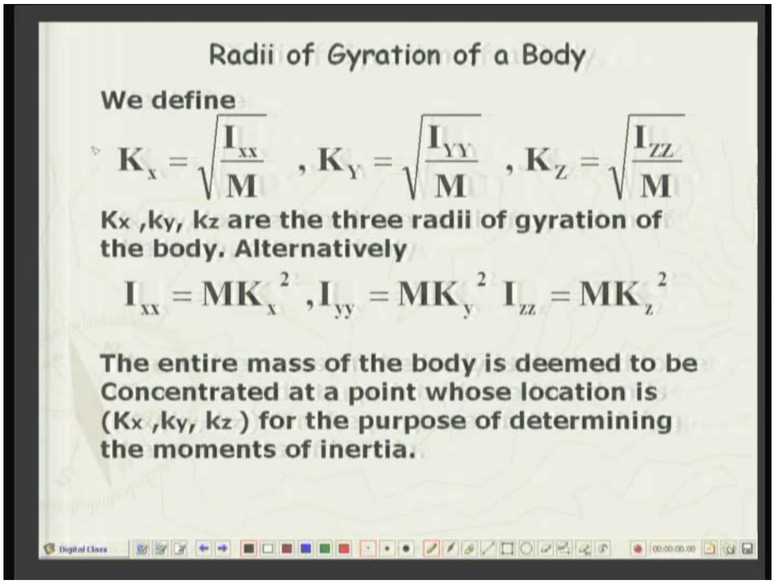

Now, let us introduce radii of gyration of inertia of a body just like we had radii of gyration of area of a given area. So we will have corresponding quantity for the three dimensional body. We will define Kx, that is x radius of gyration, as Ixx divided by the total mass of the body square route Ixx over m square route. Similarly, Ky Iyy over m square route etcetera etcetera or alternately, Ixx is equal to mass of the body times Kx square. Iyy is equal to mass times Ky square and Izz mass z square. So what is the physical interpretation of these radii of gyration? You can treat them as the three coordinates of a point where the entire mass of the body can be assumed to be concentrated. So, let us say, you shrink the given body continuously till it reduces to a total mass at a point. It keeps on shrinking and by conservation of mass, the total mass is not being reduced but the density is being made higher and higher. Then we can say that the entire mass is located at a point and what is that point? Its coordinates will be Kx, Ky, Kz. This is from the point of view of the moment of calculation of the moment of the inertia. For the calculation of the first moments of mass, we will have the center mass or centroid for this moment of inertia you have the radii of gyration.

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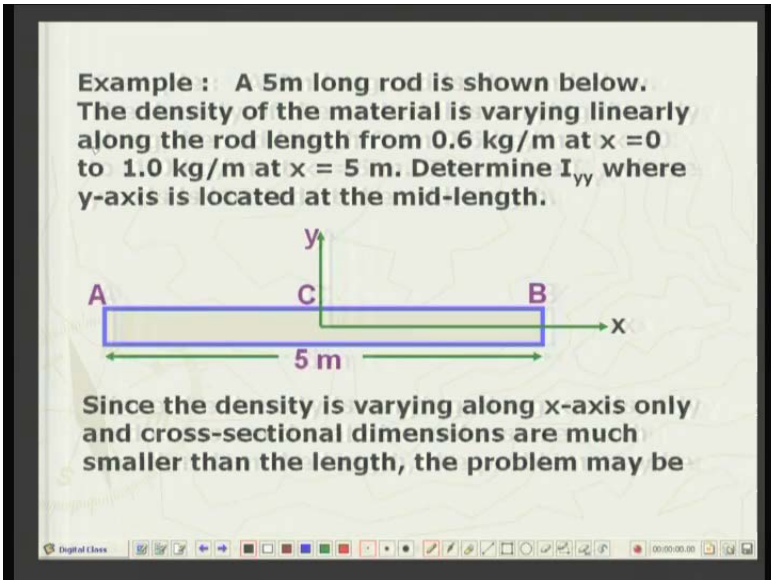

Let us try to take up some simple example to see how we can calculate the mass moment of inertia of various terms. Let me take up start with a simple one dimensional example. That is, I am treating the body to be one dimensional. There is only one variable, x variable. Although there is an area of cross section and mass, etcetera, we will say that this area of cross section is very small as compared to the length, which is, the lateral dimensions are small as compared to the length and effectively, the cross section is uniform. The density is uniform. So that rho can be at most function of x. So a five meter long rod is shown. The density of the material is varying linearly along the rod length. So it is no longer a uniform density. It is a variable density. It has a density point six kilogram per meter at x equal to zero, that is, it is one point, zero kilogram at x is equal to five. I think I may need a correction. Instead of, let us say, if this point A is at minus point two five meter, instead of x is equal to zero, I will have minus two point five meters and this will be plus two point five x is equal to plus two point five. So at A, the density is point six kilogram per meter and B, the density is one point zero kilo gram per meter at two point five. Sorry for the error.

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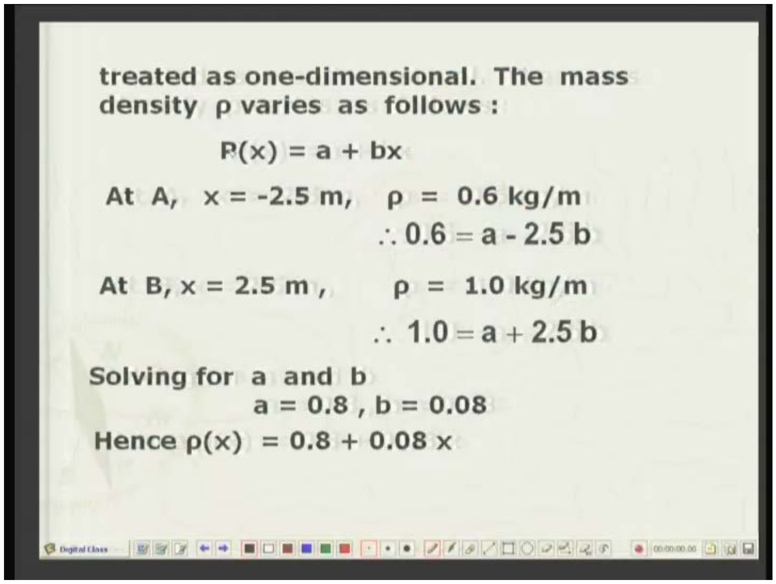

Let us see this density rho as a function of x because of its the linearly varying density. So the simple equation A plus Bx, rho as a function of x is A plus B x at point A which is corresponding to x is equal to minus two point five meters. So rho is point six. If I substitute x as minus two point five, A minus two point five B is equal to point six at the right hand end, that is, at point B where x is equal to two point five meter. So rho is equal to one kilogram per meter. So one point zero is equal to A plus plus two point five B. So from these two equations, I can solve for two unknowns. A will come out to be point eight and B will come out to be zero point zero eight. Hence the density rho is equal to point eight plus zero point zero eight x. So, we have determined the function which describes the variable density of the material of the rod.

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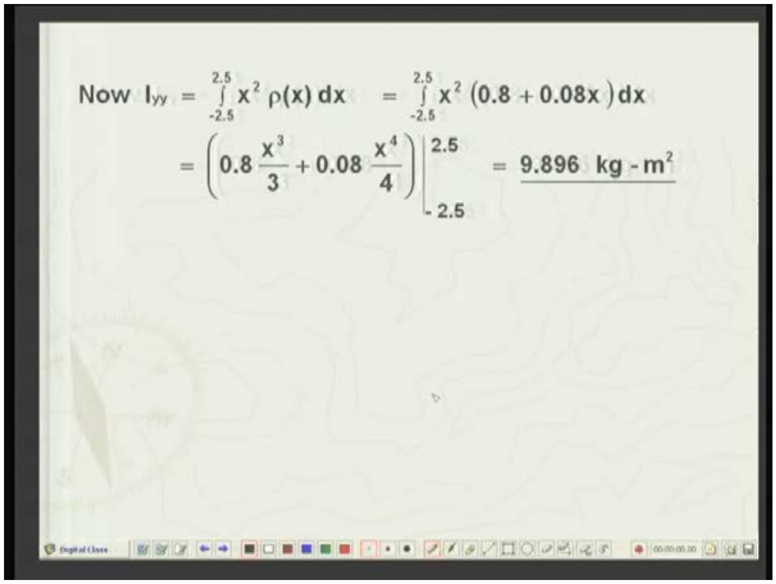

Then I_{yy} is to be obtained. So this is y axis at the middle. So we have to find out the distance of any element of the rod from the y axis. That is the distance x. x square times rho x d x from minus two point five to plus two point five and this is very simple integration since rho is a function of x, as we have determined. So x square into point eight plus zero point zero eight x integrated along x from minus two point five to plus two point five integration is very simple and substituting the upper limit and lower limit, we find that I_{yy} is equal to nine point eight nine six kilogram meter square. So this is a simple one dimensional case but here the density was variable density.

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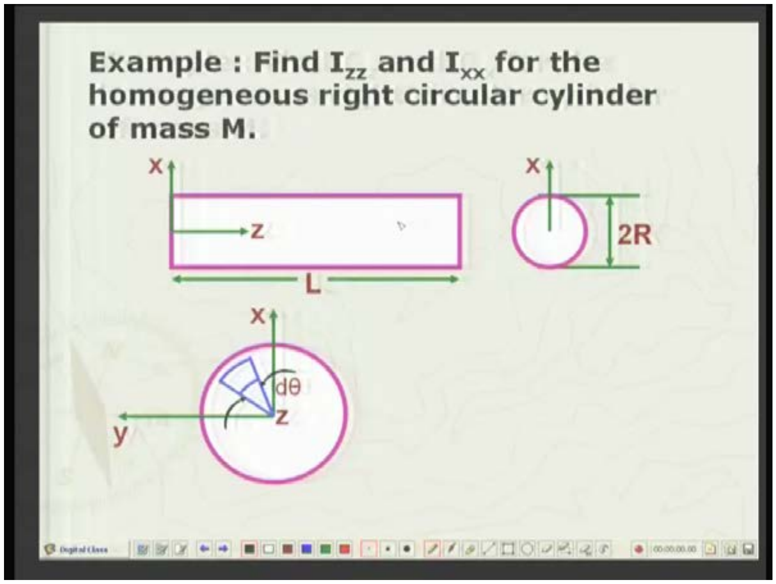

Let us take up another example. Here we are asked to find out I_{zz} and I_{xx} for a right circular cylinder of mass M. So you can see that these axis are at the end. Z axis is along the length of the cylinder. The length of the cylinder is L from end to end and the radius is R. So the diameter is two R along x axis and the third axis is the y axis. Again, we can consider a small element which I have shown on a larger scale. The small element of the body which is subtending an angle of delta theta is a narrow element and the radial distance is the R and the distance along the z axis axial distance is dz. So let us consider this problem. The arc length of that sector like body is rd theta. Radial distance, I have shown already told is dr and the axial distance is dz. So the volume of this element is rd theta dr dz.

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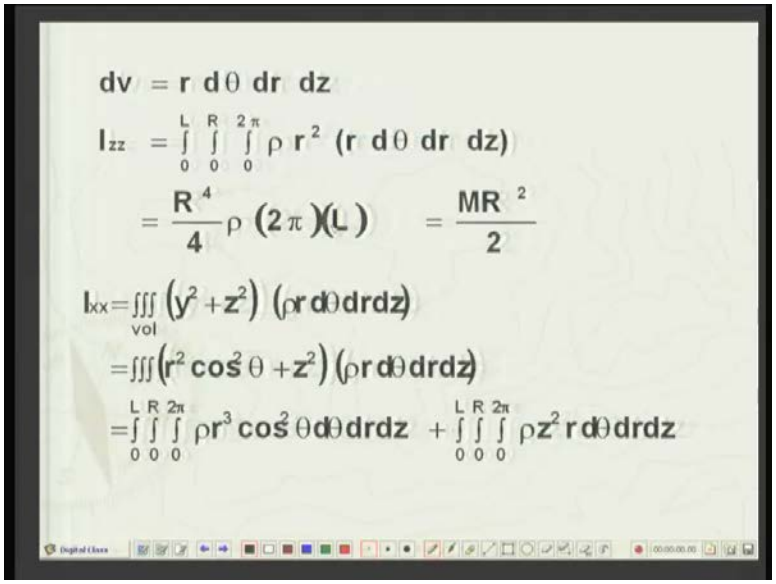

This is a typical infinitesimal element in cylindrical polar coordinates. So you must have learnt it in coordinate geometry or advanced calculus. So I_{zz} from the axis is the radial distance r square times rho into the volume element, that is, rd theta dr dz and the integration is from zero to two pi along theta zero to R capital R along the radial direction and zero to L along the axial direction. Well, this integration is very simple. First of all, r cubed integrated from zero to R will give me R four by four and theta d theta integrated from zero two pi will give me two pi and dz will give me integration to simply L. So we will have R square by four into rho into two pi into L. Now, the mass of the cylinder area of cross section is pi r square. So into L will be the length where L is the length. So that will give me the volume times density. So if you simplify this, it will be the total mass M time R square by two. So MR square by two. That is the moment of inertia of a right circular cylinder about its axial direction, the axis z I_{xx}, that is, you can recall x was along the diameter direction. So about any diameter, if you want to find the moment of inertia, then Y square plus Z square into rho times db rd theta dr dz. So, once again if I go back to the figure, if I have to find out this is the radial direction R, the distance will be R cosine theta because this angle is angle theta. So R cosine theta will be replacing Y. Sorry. Plus z square as such times the mass of the elementary body. So if you simplify and carry out the integration, you will have two terms. First, the r cosine square theta term and the second, z square term. So these are the two terms and now, I have to integrate them.

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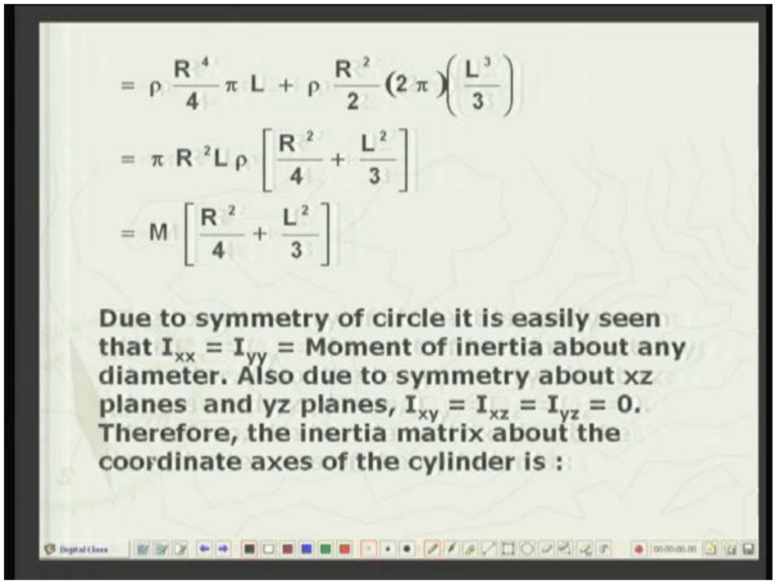

We will find that the first term will give me rho times R four by four pi L plus the second term will give me rho times R square by two into two pi into L cube by three. That is the z square term integrated. So if I again collect these terms, take out the common thing, pi R square into L into rho, pi R square is the area of cross section times length. That will give me the volume times density. So the total mass times R square by four plus L square by three. So, we have seen $I_{zz,}$ $I_{xx,}$ and I_{yy} will be equal to I_{xx} because the body is symmetric about any diameter. Whether x diameter or y diameter, it will not be different. So Ixx and Iyy are zero and the product of inertia terms I_{xy} I_{yz} I_{zx} will be also zero because of the fact that the body is symmetric about, let us say, the middle plane. If I consider a plane over here, that is, the zy plane, the body is symmetric in the zy plane body. Symmetric in xy plane body is symmetric in the xz plane. So all the three orthogonal planes are symmetric. So it means that the product of inertia terms will be also zero.

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So if I write the inertia matrix for this body, right circular cylinder, then the terms are I_{ij}. So, I going from one two three, j going from one two three. So I one one, that is I_{xx} will be M R square by four plus L square and similarly, I_{yy} same quantity but I_{zz} is equal to MR square by two. So this is the final expression for the matrix of inertia and the product of inertia terms are, as I have seen, zero. So there are only three diagonal terms. Off diagonal terms are vanishing. So, I think we can close today's lecture over here. So we have learnt in this lecture, the moments of inertia, products of inertia and what is their correlation with the second moment of area and product of area, which we have learnt in pervious lecture. So their correlation for a thin uniform, homogenous body is very simple. That is, rho times thickness, which is the area and which is the density of the material per unit area. So the the correlation lies through that. So a second moment of area about x axis is equal to the mass moment of area about x axis divided by rho t or vice versa. This mass moment of area about x axis is equal to rho t times the corresponding area moment of area. Same, similar thing can be said about the products of area verses the product of inertia. So in the next lecture, we will be again looking at the parallel axis theorem and the rotation theorem for the mass moments of inertia and product of inertia. Thank you very much for your attention.