Applied Mechanics
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Lecture No. 11
Properties of Surfaces (Contd.)

Today we will take up lecture eleven which is a continuation of lecture ten. In lecture ten, you may recall that we had discussed properties of surfaces, namely, the first moment of a given area. We were concentrating on plane areas and then from first moments of plane areas about x axis and y axis we could find the centroid of an area. Now towards the end of our last lecture, we had started with theorems of Pappus Guldinus and we were discussing the surfaces of revolution and volume of revolution. Let us try to recap that. The surface of revolution is obtained by rotating a curve about an axis. I will show you a simple example here.

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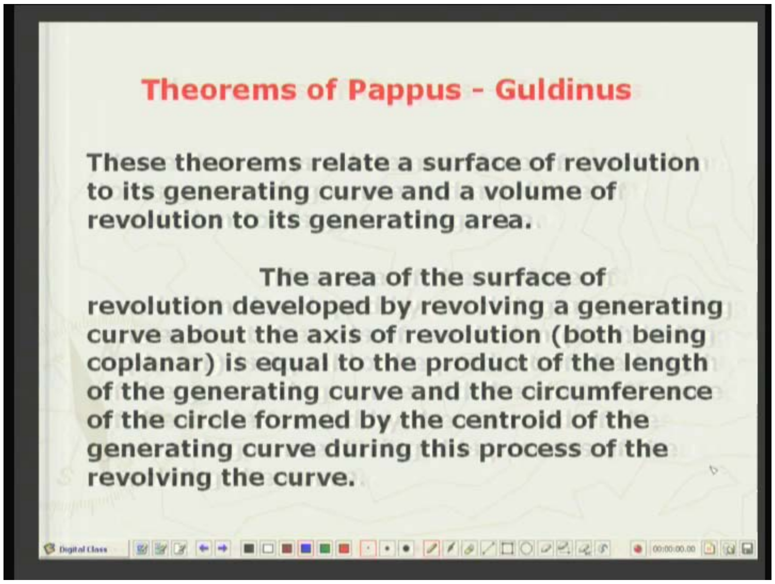

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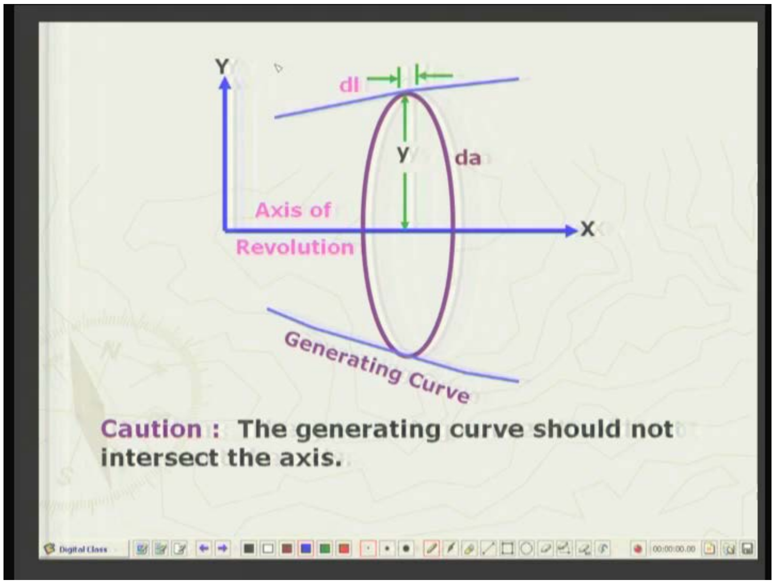

Suppose we have an arbitrary curve and one axis. Now keeping the curve in its position, this is how we will rotate this curve around the axis as shown over here. So this whole curve is going around the axis and in the lowermost position it will be something like this and then it will come back. So in this process we will be generating a surface, a kind of conical surface. This surface is called the surface of revolution and similarly we can talk about volume of revolution. Again, I will show you an example.

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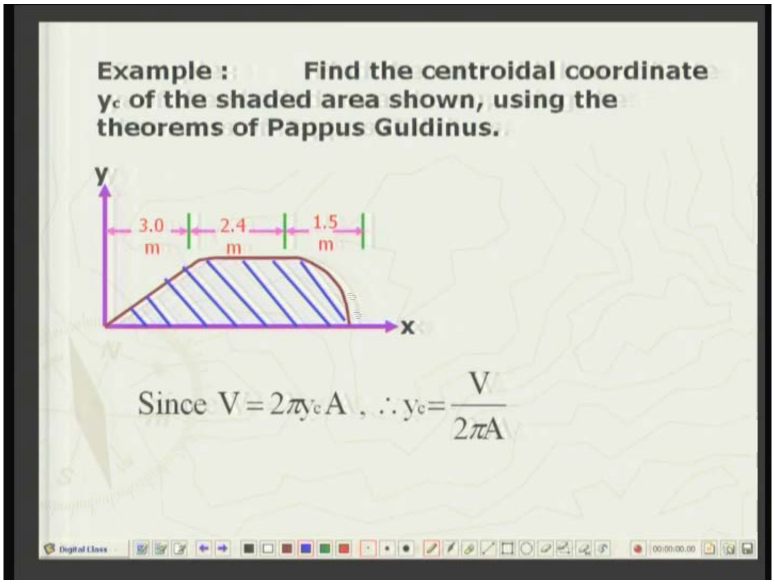

Suppose we have a planar area which consists of the boundaries, this area, a line straight line and a curvilinear portion and about the lower edge of this area, you may imagine that we have cut out a cardboard piece of this shape and at the lower edge of the cardboard piece we have attached a kind of axis or an axle and around that axle this cardboard is being rotated. So the volume generated is the volume of revolution. So after having understood how to generate a surface of revolution and a volume of revolution, the theorems of Pappus and Guldinus help us to find out, number one, the area of the surface of revolution in terms of the centroid of the curve which generated that surface of revolution and in theorem number two, we will be doing a similar thing for the volume of revolution. The volume of revolution will be determined in terms of the centroid of the generating surface of revolution. So let us take these theorem one by one. Well, first I will take up the theorem which relates to the surface of revolution with the centroid of the revolving curve.

The theorem reads like this, the area of the surface of revolution by revolving a generating curve about the axis of revolution, both being coplanar. So once again, we are focusing our attention only on those curves which are in the same plane. They are not leaving this plane and secondly, the axis about which the revolution is being done or it is being rotated and which also lies in the same plane is equal to the product of the length of the generating curve and the circumference of the circle formed by the centroid of the generating curve during the process of revolving the curve.

So let us go to the figure. Here is a generating, arbitrary curve which may be smooth or it may be piecewise smooth and this generating curve is being revolved around this axis, which we are taking on x axis and during the revolution this surface is being generated. Now in choosing the curve of revolution, we have to take some precautions. The generating curve should not intersect the axis. It should not pass or cut the x axis. At the most, it can touch tangentially the x axis, otherwise you will have a positive and negative surfaces and that complication is not necessary at the moment.

Well the proof is also very simple. We consider a small element of the curve of which is being revolved. Arc length of this small element is dl and as it is being rotated, this dl will generate a ring shaped surface. Now to find out the area of this ring so formed, we will have the circumference into the width. Width of this ring is dl, that is, the arc length of the curve and the circumference is two pi r. r is the radial distance from the axis which is simply y, as you can check it again, this is y this is the radius of the circle and the width is dl.

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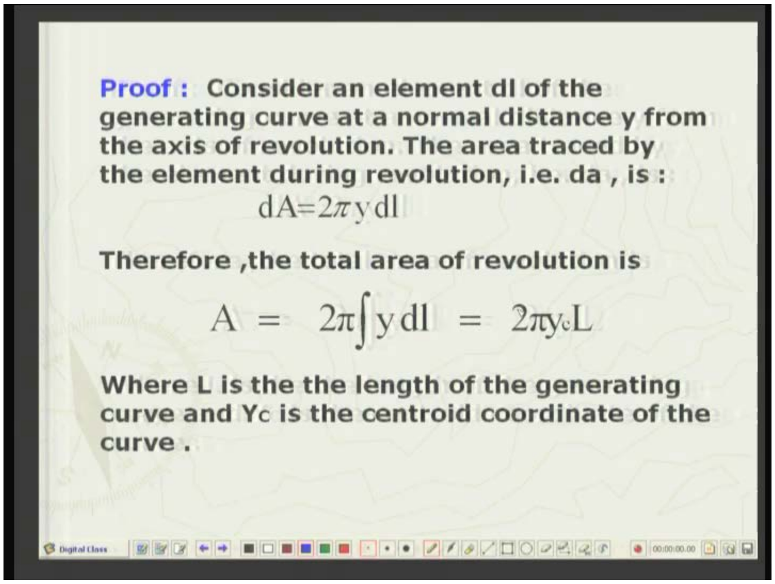

So the surface area of the ring shape surface is equal to two pi y dl and if we integrate it from end to end on the generating curve, then the total area so generated A is equal to two pi y dl in line integral. Now this is the definition, if you recall from lecture ten. Of the y coordinate of the centroid of a centroid of the generating curve so vice versa. So this integral will be equal to yc into L. So the total area will be equal to total area of revolution which will be two pi into yc into L, where L is the length of the generating curve and yc is the y coordinate of the centroid of the curve.

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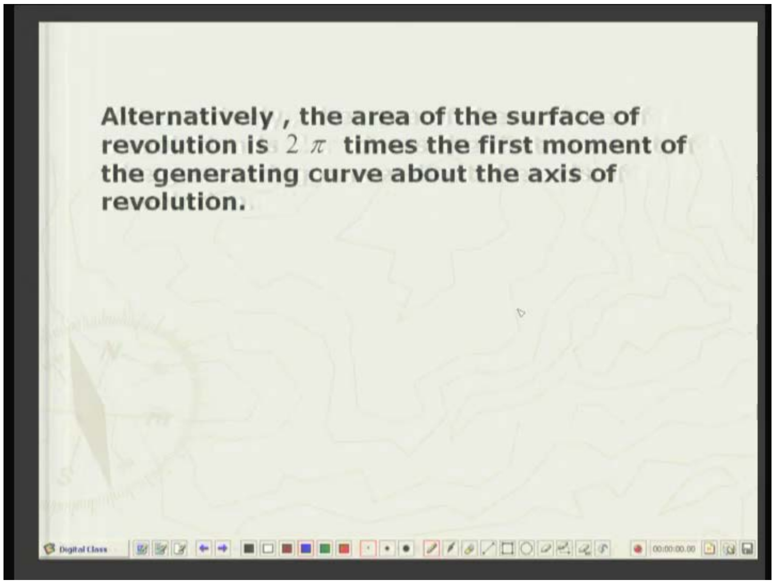

Alternately, we can state that the area of the surface of revolution is two pi times the first moment, that yc into capital L or line integral of y into dl is the first moment of the generating curve about the axis of revolution. So both ways we can understand this theorem. Now let us move to the second theorem of Pappus Guldinus.

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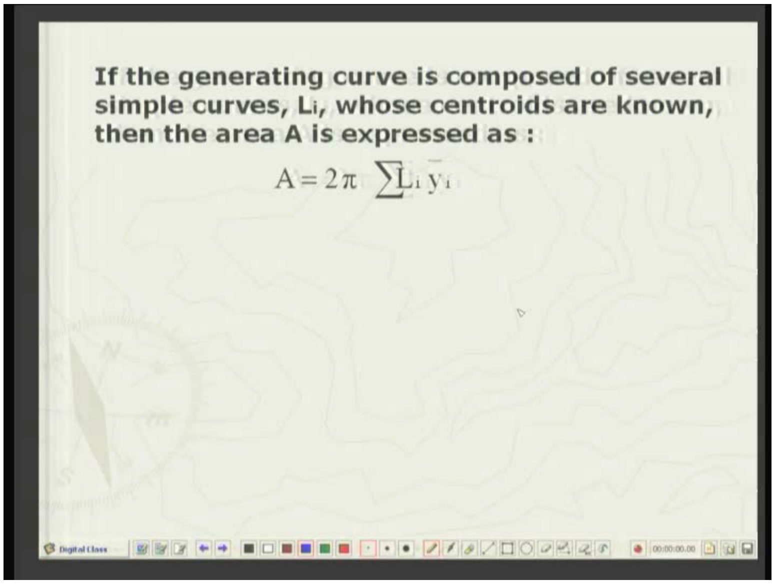

Before I do that, let me mention that if your generating curve consists of several segments of different lengths or different orientations, then the surface of revolution can be obtained. Area of the surface of revolution A will be equal to two pi times the summation. We will conduct the summation on all the segments LY, that is, the length of the segment times the centroid of that length at this moment. I can show you an example which we will be doing later. Suppose I consider this example which consists of a generating curve which has three segments.

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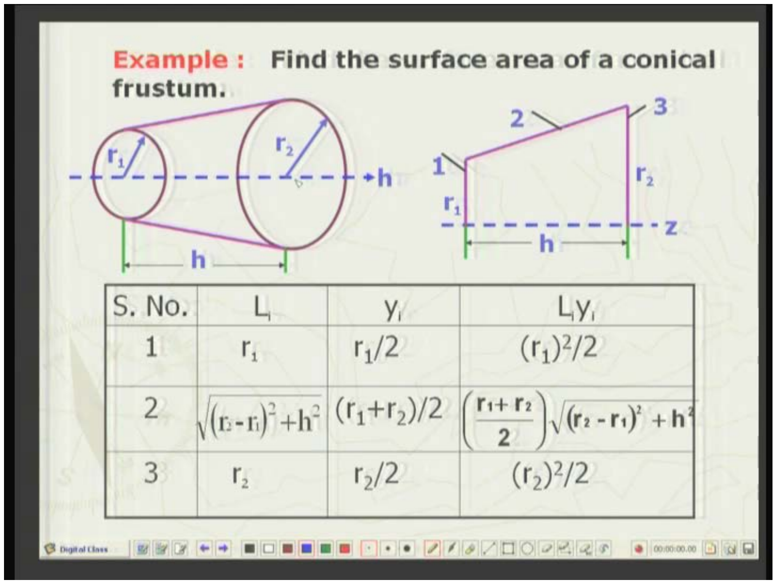

Segment number one which is a vertical line, segment number two which is a slant line and segment number three which is the vertical line. So what we will do if you rotate this C shaped body or wire bent in the form of a C. Then if you will rotate it about this z axis, then it will generate the frustum of a cone, that is, it will have one surface over here, second surface over here and the third surface will be this slant surface. Now the radius of the left hand surface or end will be equal to r one. Right hand end will be r two and the length of the slant surface will be this length. Now the calculation of the surface area can be very conveniently carried out in a tabular form. One, two, three are the three segments. Now the length of segment number one is r one. The centroid will be at the mid height r one by two and the length into that height of the centroid will be giving me r one into r one by two. So this will be r one square by two. Then we come to the slant length which will be by Pythagoras theorem r two minus r one square plus the dimension h. So plus h square whole under root.

So this will be the hypotenuse of this triangle and the centroid will be at the middle point. So the height of the centroid will be easily obtained r one plus r two by two and then you multiply this length by this height of the centroid. You get this expression r one plus r two over two whole in to square root of the hypotenuse into the length of the hypotenuse and similarly, for the third segment, segment number three, length is r two middle height, for the centroid r two by two and then r two square by two. So all the three lengths, all the three L into y have been determined.

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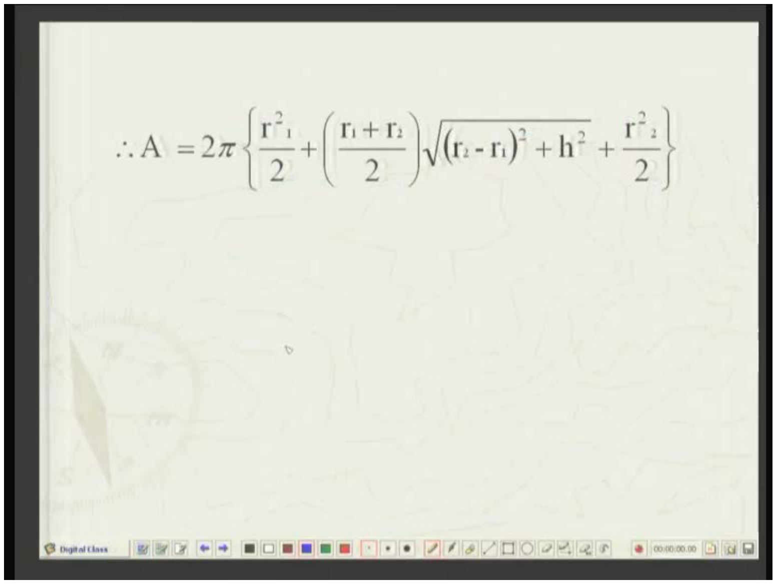

And if we add up, we will get the total area which will be two pi times the addition of the last column. So this is a very simple way of calculating the total surface area of the frustum. You can do it otherwise from geometry. It will be much more complicated. Now let us go back to the second part of Pappus Guldinus theorem which will relate the volume of revolution with the area of the generating surface.

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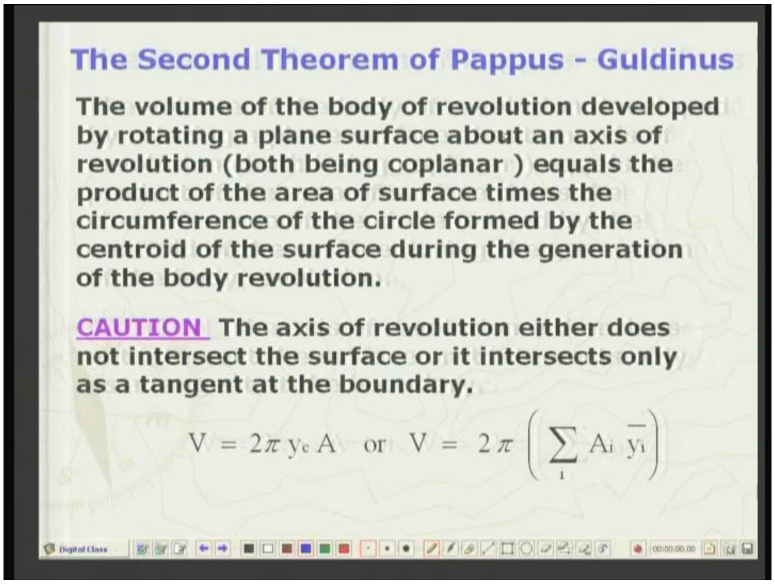

Well, let us see the statement of the theorem. The volume of body of revolution developed by rotating a plane surface about an axis of revolution. Both of them, the surface and the axis are in lying in the same plane, is equal to the product of the area of the surface times the circumference of the circle formed by the centroid of the surface. Area times the circumference of the centroid generated by the centroid during the generation of the body of revolution. Again the precaution is similar precaution. As we have for the first theorem, the axis of revolution either does not intersect the surface or it intersects only as a tangent to the boundary. So the statement is that the volume of revolution is equal to two pi times the y coordinate of the centroid times the area of the generating surface and if this is in its piecewise segments, then we can do again the same thing as we did for the first theorem, two pi times the summation of the each of the areas times the corresponding centroid height of the corresponding centroid from the axis of revolution. Again, best way to illustrate the use of this theorem is to take up an example.

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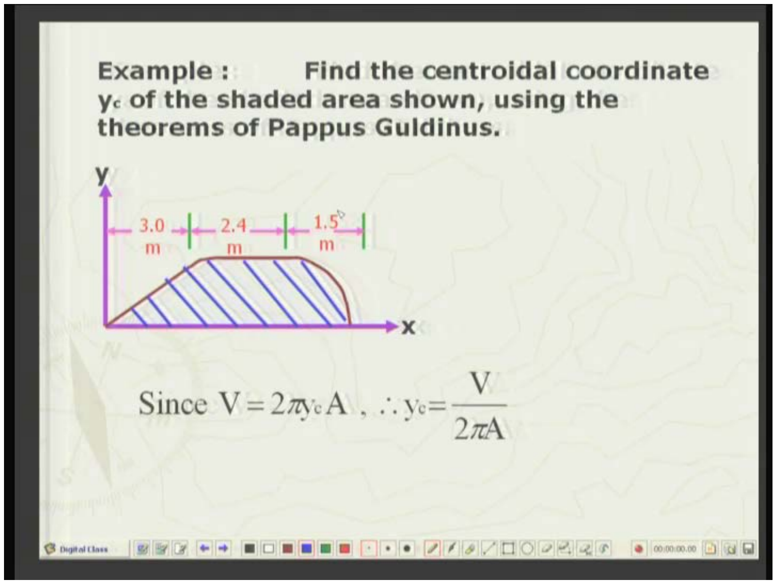

This example shows an planar area which consists of a straight line slant, a horizontal straight line and the quadrant of a circle. This is the quadrant of a circle. This shaded area is being revolved around this x axis. The various dimensions are given. This is three meters. This should be two point five. I believe this will be two point five meters. Let me check. No. Two point four meters and again, this is one point five. So the obviously the height of this will be also one point five. So this dimension is because this is part of a circle, first quadrant of the circle. So to find the centroidal coordinates, we have been asked to do the reverse thing yc of the shaded area using the theorem of Pappus Guldinus. So since by the statement of this theorem, the total volume is equal to two pi times yc times area. So yc will be equal to v divided by two pi times the area of the surface of revolution. Let us do again. Very simple. I will go back to the figure. When it is revolved, this slant length will generate a cone and this segment will generate a cylinder and this segment will generate a hemisphere. So we have to find out the volumes of these three geometrical figures.

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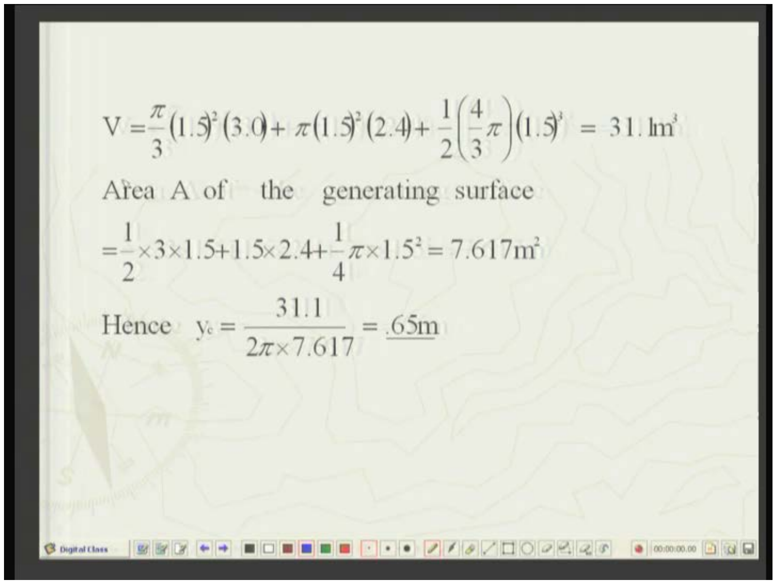

For the cone, you know, the volume is equal to one third of the base area pi into one point five whole square into the length or height of the cone, that is, three meters. Then for cylinder, pi r square into length of the cylinder. So again that is easy and then finally for the hemisphere, you will have, because it is half of the spherical body, one half four by three into pi into the radius cubed. So after calculation, this gives me thirty-one point one meter cubed cubic meters.

The area of the generating surface which consists of a triangle a rectangle and a quadrant of a circle. So it will be one by two into a base into altitude. Base is a three meters, altitude is one point five. So this is the area of the triangle, this is the area of the rectangle one point five into two point four and this is the area of the quadrant of a circle one fourth pi into r square. So this comes out to be seven point six one seven meter square and if I divide the volume by this two pi times the area, then I will get the height of the centroid of the given generating surface. So that will be point six five meter. So we have seen that with the help of the theorems due to Pappus and Guldinus, we can correlate one type of figure with another. Next, hierarchy of the figure provided is generated through revolution.

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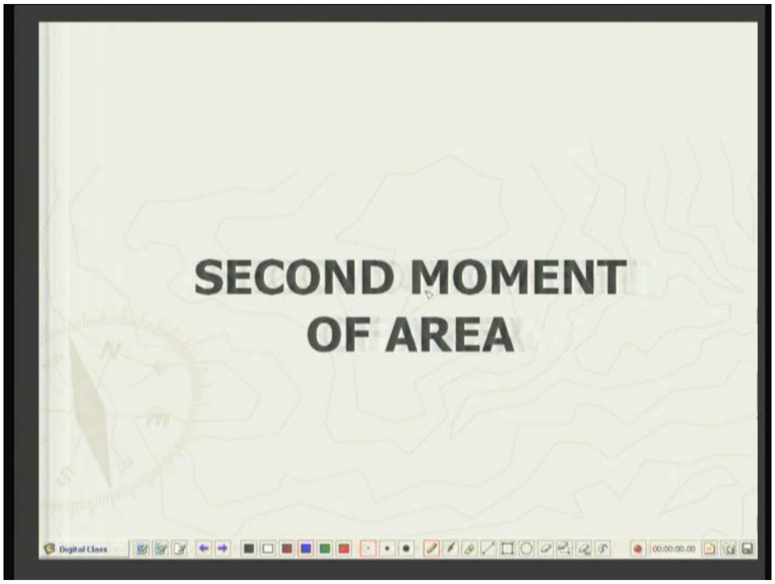

Now, so far, we have discussed the first moments of the area and from that, we determine the centroid of the area and then came the theorems of Pappus and Guldinus. Now, we will go to the next quantitative measure of the properties of a given surface, that is, the second moment of area.

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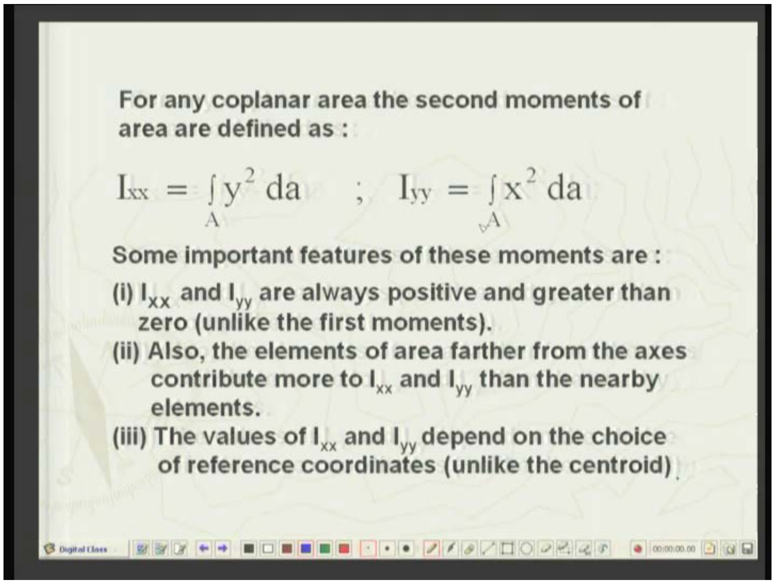

So let us take up by the definition of the second moment of any given area. Again, we will focus our attention on to coplanar areas, that is, the area which lie in the single plane. No shell like surface will be considered here. So we define the second moment of area about x axis, that is, Ixx is equal to y square. Remember Ixx will have y square area integral over the entire given area. So y square da integrated over the entire. This is the area integral. Similarly, Iyy, that is the second moment about y axis, will be the integral x square da, that is, the distance from the y axis square integrated over the entire area. Now, some important properties or features. These moments are, number one, Ixx and Iyy are always positive and greater than zero because they involve square terms. So whether it is a negative quantity or a positive quantity, square will be always be positive and unless the area is zero, that is, it is just a point. The zero dimensional surface the quantity will be nonzero. So remember, in the case of first moments, the quantity could be positive, negative or even zero. That is, first moment about its axis of symmetry came out to be zero but in the second moment, you will not get a zero as the second moment of area. You will only get positive numbers. So that is a very important result to understand. The first moment is entirely different in that aspect.

Secondly, the elements of area farther from the axis about which we are taking the moment, whether it is x axis or y axis, they contribute much more than the nearby elements. As you go farther and farther, the contribution of that element of the area will be more and more, not even linearly but typically more and the third is the values of I_{xx} and I_{yy}. They are very sensitive to the choice of the reference coordinate. You recall something from the first moments. There we showed that whether the coordinates had been rotated or translated to another point, the coordinate of the centroid was unchanged or the location of the centroid was unchanged. Although the numbers may be different, when you plot those values of x_c and y_c for different choices of coordinate frames, the location of the centroid will be invariant. It will be the same location but here the values will be different for different choices of reference coordinates. So these three features, I have to be thoroughly understood and compared with those of the first moments. Next definition is about the radii of gyration of a given area.

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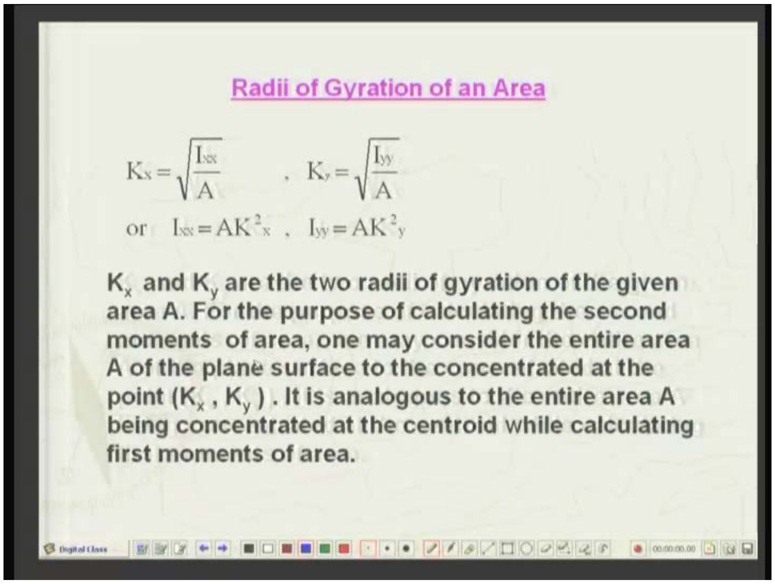

It is again a new feature. We will define the x radius of gyration, that is, Kx is equal to the second moment about the x axis divided by the total area, taken square root of and similarly, for Ky, that is, radius of gyration about y axis is equal to Iyy divided by A whole under root or alternatively, I can write that Ixx, the second moment about x axis, the total area times the radius of gyration about x axis, that is, Kx whole square. Similarly. Iyy will be A into Ky square. Now, what is the physical significance of Kx and Ky for the purpose of calculating the second moments of area? You can treat it as if the whole, given area is concentrated at a point whose coordinates are Kx and Ky. So, if you shrink or degenerate the whole area to a single point whose coordinates in the given coordinate system are Kx and Ky, then area times the distance along x axis square will give me Ixx area times distance along the radius of gyration about the y axis Ky square. That will give me Iyy. So just like the centroid which played the role that the whole area can be concentrated at the location of the centroid for the purpose of calculating the first moment. So the role of Kx and Ky is counterpart for the second moment and then third thing which is of importance for the area is the product of area.

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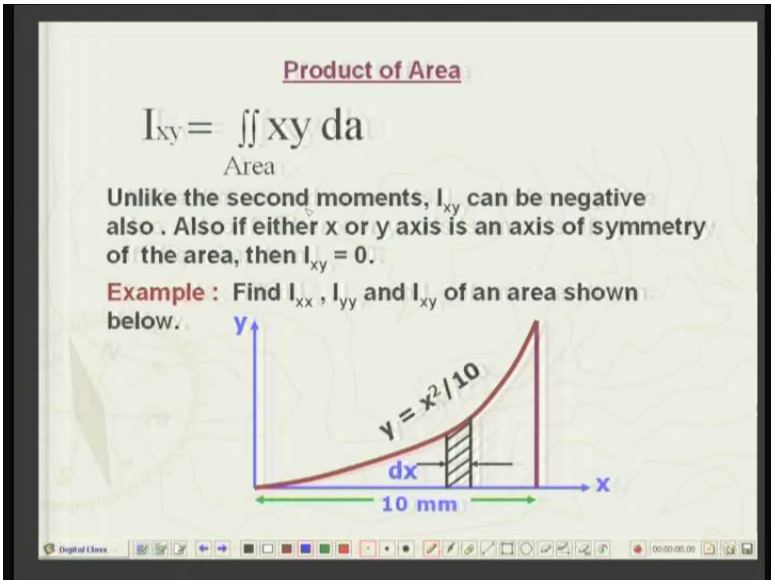

Here it is a mixed quantity, x into y integrated with the area. Now this quantity because there is no square involved is just a product which can be positive or negative in value or when you integrate, partly it is negative, partly, positive and both these positive components and the negative balance out each other. Then the value of a Ixy will be zero. So this product of area, in contrast, with the second moment of area, can be positive negative or even zero. Whereas at the second moment of the area, it was always positive and nonzero. So please remember this distinction. Now let me take up an example. An area is given. The base of this area is ten millimeter and it is along the y axis. The height of this area is obtained by intercepting with this curve and the equation of the curve is y is equal x square divided by ten. So two straight lines and a curvilinear line. You can call it a curvilinear triangle. This area is given for which we have to find out Ixx, Iyy and Ixy.

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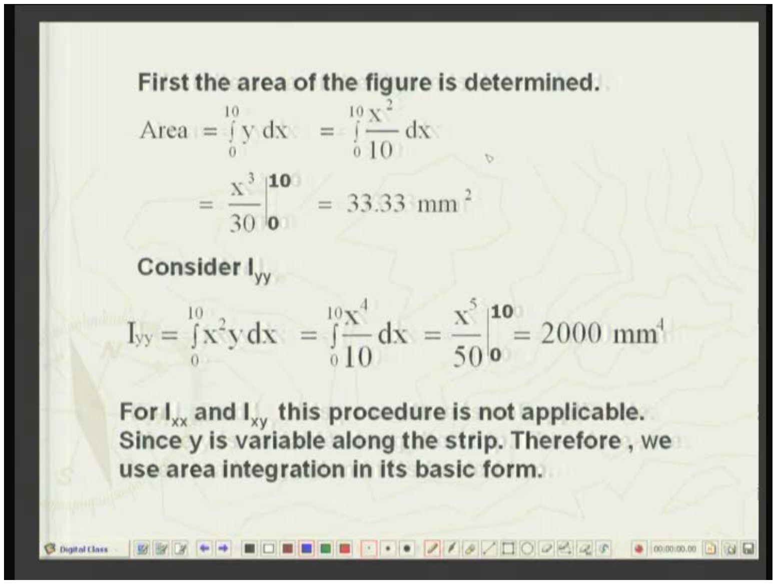

First of all, let us find out what the area itself for this curve is. You can easily see that I take a thin strip. Although it is shown as a bit thick, it has a very small thickness. dx is a very small quantity. So it is a thin strip going from the base to the intersection with this curve. So the area of this shaded area of this strip will be the height times the width dx. So height is y and then we integrate it. y dx is the area of the strip integrated from the left hand end to the right hand end, that is, from zero to ten and for y, we substitute the equation in terms of x. So x square by ten is equal to y. It was the integral of x square by ten dx zero to ten. Very simple. You get the x cube by three into ten. So zero to ten, we will get thirty-three point three three millimeter cubed.

So the dimensions of the area are millimeter square. Now we will consider Iyy, that is, the second moment about y axis. So we will again have da which will be y into dx into the distance from the y axis, that is, x coordinate and this x coordinate is uniform, same for all the height of the strip. So x square into ydx. So we will have x square into ydx. This is the area of the strip. This is distance square integrated from zero to ten. Again, substitute for y is equal to x square by ten. So you will have x four by ten integration from zero to ten and very simply, you get two thousand meter four millimeter four power four. Now next, we have to find out Ixx and Ixy, that is, Ixx will be area of the strip into the distance from the x axis, that is, y itself and this distance is varying along the strip. So the strip cannot be used because this distance from the x axis, the normal distance from the x axis, is varying along the location in the strip.

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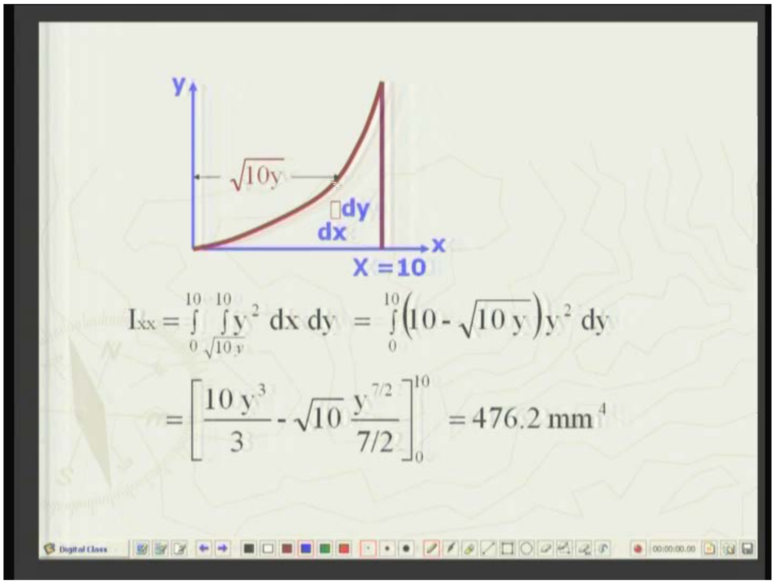

So we have to use the more fundamental definition of the area integral, that is, in terms of a small element of width dx and height dy. So then it will be y square. da will be replaced by dx into dy and what will be the limits for, let us say x. Suppose I take up any point on this point. This will be varying along x axis from this point to this point. On this, the distance x is equal to ten and here it is obtained by the equation, that is, ten into y square root. So the limits of integration are very crucial, in this case. So it will be ten y under root to ten, that is, from this point to this point and then after having conducting the integration for x, then we will have the integration for y. So first, integration on x will give me simply x. So we will substitute the upper limit and the lower limit. So ten minus ten root ten y into y square an integrate on y. So again not a very difficult integration which you can carry out and we will get four seventy-six point two millimeter power four.

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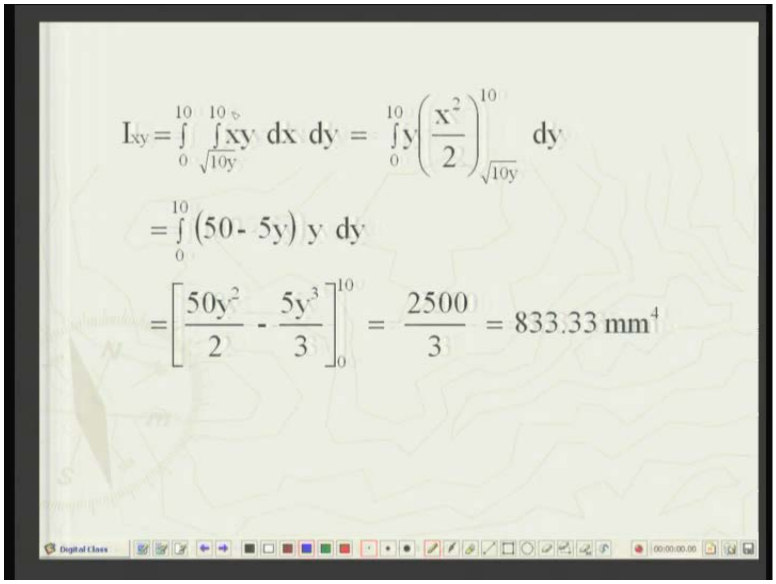

And last of all, we will have the product of a the given area, curvilinear triangle again. We cannot use this strip method because x is varying. So we will use the small element. Again, dx dy into xy and again, you have the lower limit, upper limit, lower limit, upper limit for y and without spending too much time, you can get the final answer as eight thirty-three point three three millimeter four. Here it is positive but for some other axis, it can be negative. So we have seen this second moments and the product of an area can be obtained very conveniently with the help of integration. Now let us go to another very important part of this lesson.

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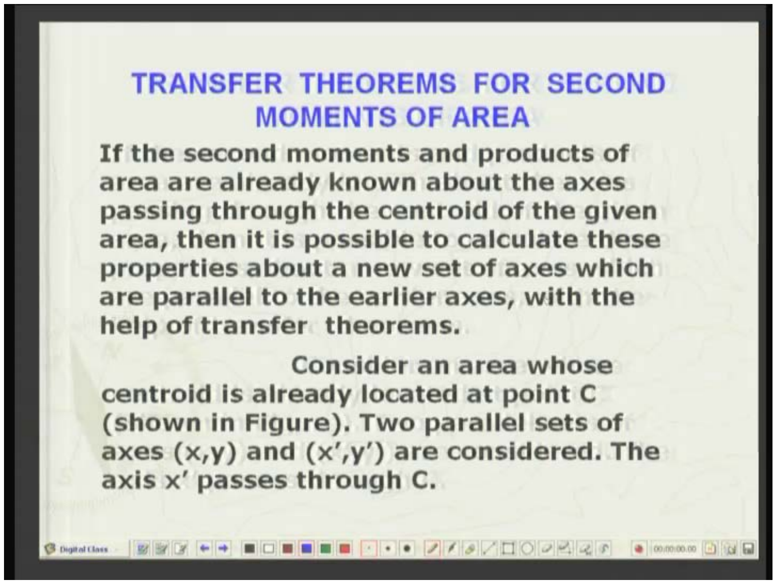

Transfer theorems for second moment of an area. Suppose you know the second moments, that is Ixx, Iyy and Ixy, about a given frame of reference, that is, xy coordinates, z being perpendicular to the plane of the area. We are working on plane areas only and you choose another coordinate axis and the question is, do we again go through a fresh integration process to calculate the second moments about the new coordinate system or with the help of the old coordinate system or earlier coordinate system can we get some results? The answer is yes. It is possible, if you know the second moments of a given area about the axis, which is passing through the centroid. Then you can find out the second moment about a parallel axis. One is the given axis passing through the centroid. Second is the parallel axis in terms of the second moments about the first axis. So it can correlate the two. This theorem will tell us how to correlate between two parallel axis, provided, one of them is passing through the centroid of the area. So, let me read out. If the second moments and products of area are already known about the axes passing through the centroid of the given area, then it is possible to calculate these properties about the new set of axis.

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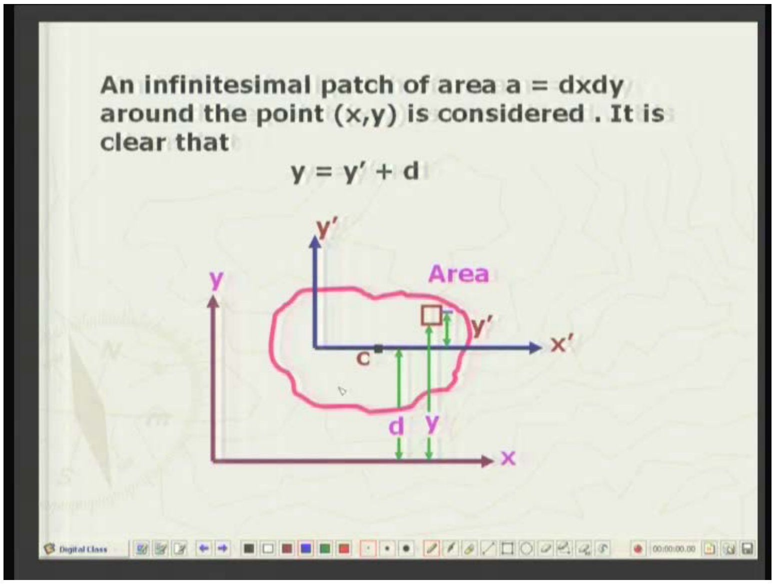

Let us say this is the given area and one set of axis, this blue one. X dash y dash and z dash is coming out of the plane of the book screen is passing through the centroid C of this area and then a fresh set x y z. Again, z axis, coming out of the screen, is considered. So Ixx, about this new axis can be related to Ix dash x dash, about the old axis, in terms of the distance between the two and the values of the I x dash x dash. So for this purpose, we will consider a small area, as shown over here and its dimensions are dx and dy. This area is at a height y along the y axis from the x axis and y dash from the x dash axis. So, from the figure, it is easy to see that y is equal to y dash plus the normal distance between the two axis, namely, d so y is equal to y dash plus d.

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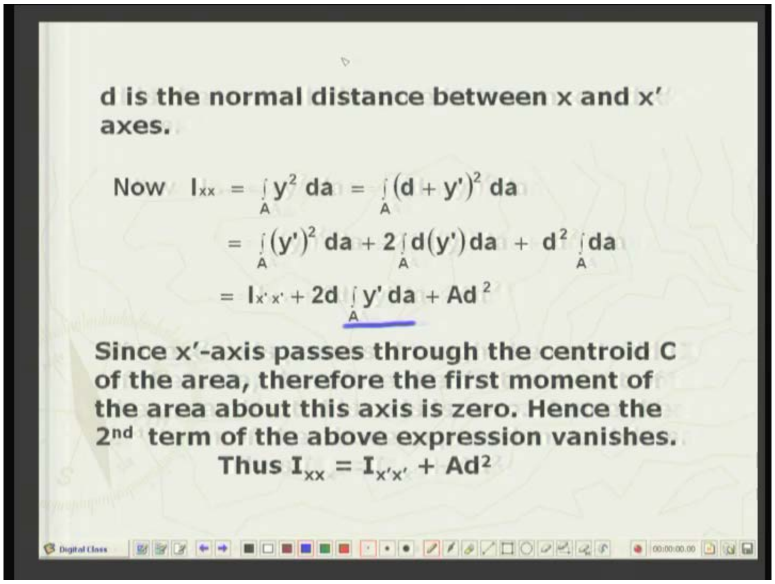

Definition of Ixx, namely, area integral of y square and for y square, we substitute d plus y dash and expand this bracket, we will get area integral of y dash square two times the area integral of d times y dash and d square d being constant, has been taken out d square time da integral area integral. Now I can easily recognize two of the terms, that is, this will be Ix dash x dash, that is, the second moment in the old coordinate system about x dash axis and the last term is d square being constant as such, this is the area of the given figure. So capital A times d square. Now the middle term or the second term, two into d, d being constant taken out into y dash area integral.

Now this quantity is known to us. Why is yc, that is, the y coordinate of the centroid? In the old coordinate system that is in the x dash y dash coordinate. So the centroid in the x dash y dash since x axis x dash axis lies on the centroid. So the y coordinate will be zero obviously. So this integral is this integral. It will be zero being yc dash, you can say. So collecting the terms Ixx is equal to Ix dash x dash, that is, in the old coordinate system plus A times the normal distance between the two parallel axis squared.

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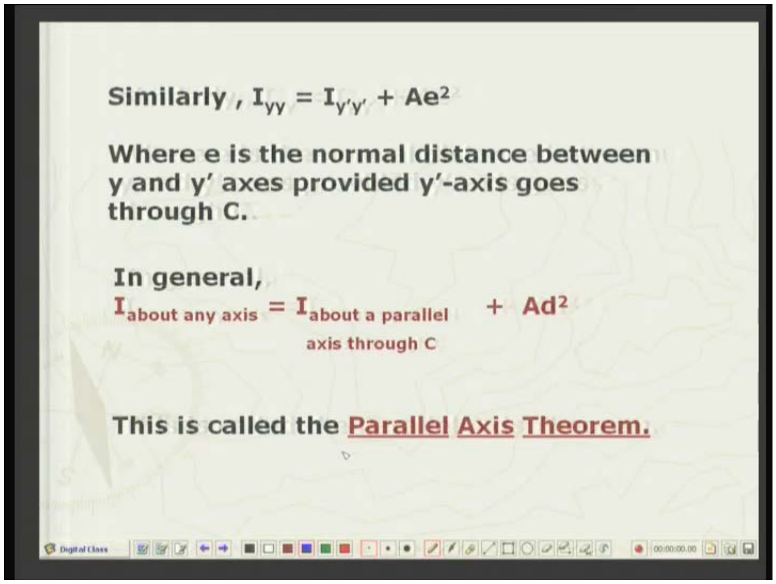

Similarly, for y coordinates Iyy can be written as Iy dash y dash, that is, the second moment about yx y dash axis in the old coordinate system plus the total area times the normal distance between y and y dash axis. Let it be e, the normal distance between the two parallel axis is e. Then this quantity will be Ae square. Now, in general, we can say that the I about any axis is equal to I about a parallel axis passing through the centroid plus the total area times the normal distance square this is called the parallel axis theorem for the second moments of areas.

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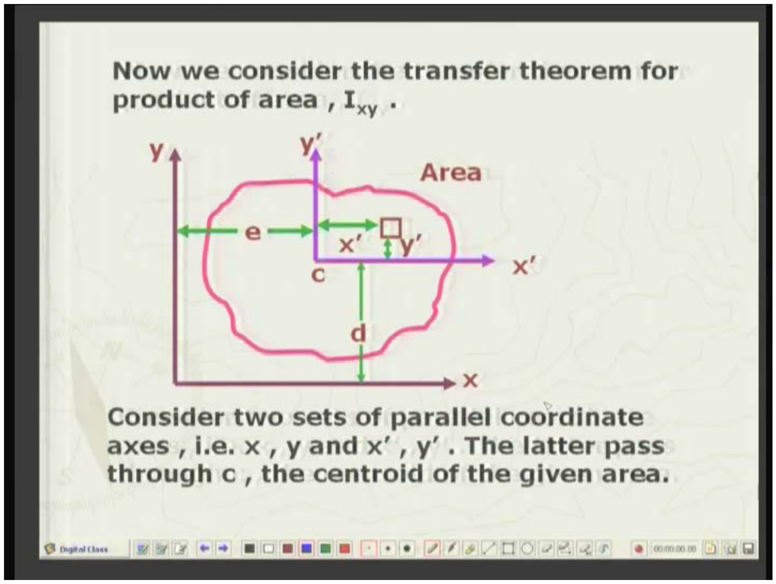

We want to do similar thing for the product of an area. Again, I am taking an arbitrary area. The coordinates x dash y dash are both passing through the centroid. So you can say the centroid is the origin of this old coordinate system. X dash, y dash and z dash coming out of the plane of the screen and the new coordinate system. So, that x is parallel to x dash y is parallel to y dash and this may be having any origin let's say o okay the origin of this is centroid but this can be arbitrary origin. So you see the normal distance between y and y dash is e. Normal distance between x and x dash is d.

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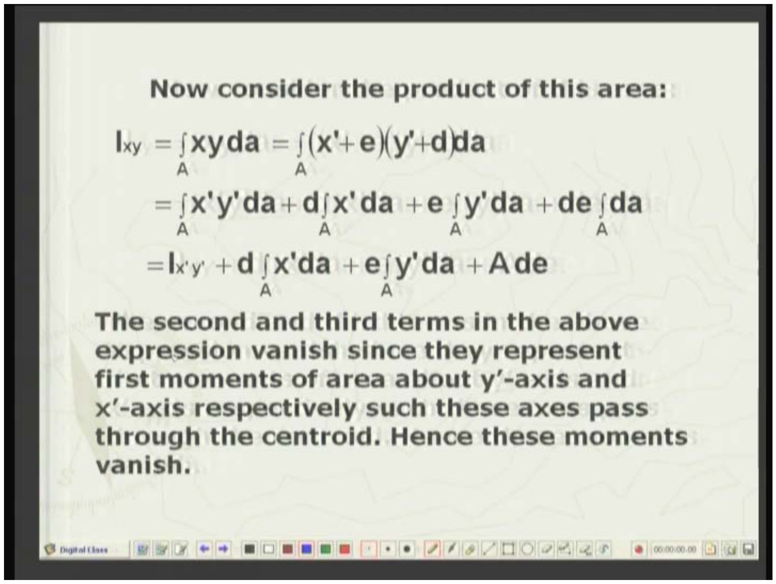

The product of the given area Ixy, by definition, is equal to area integral xy o da and this again, we can substitute in terms of the old coordinates x dash plus the normal distance between the two y axis into y dash plus the normal distance between the two x axis integrated over the entire area. Again, simplify this bracket and we will get this quantity, the product of area in the old coordinate system, that is, x dash y dash and area times d into e. Then these two middle terms which will be x dash area integral, y dash area integral and these are nothing but the centroidal coordinates in the old coordinate system. That is, x dash c and y dash c since the origin, that is, zero zero point is located at the centroid. So this quantity as well as this quantity is equal to zero. So these two middle terms are zero quantities. So we can say that these contributions vanish.

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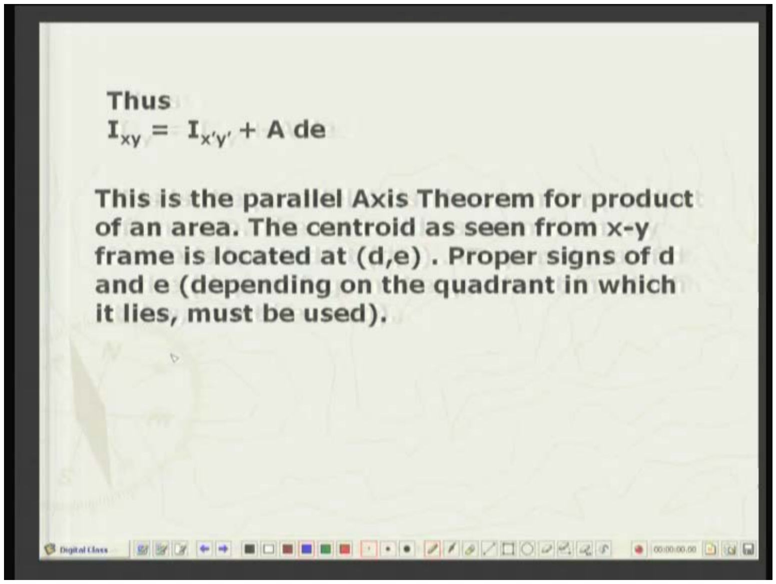

Finally, Ixy is equal to the corresponding term in the old coordinate system, that is, Ix dash y dash plus an additional term A times the product of the normal distances between the corresponding coordinates axis. So, with the help of these two theorems or these two formulas, we go back and forth between the two sets of second moments and product of areas, provided, one set of coordinate axis passes through the centroid. That is very essential. You cannot take just two any two arbitrary parallel axis. One of them must go through the centroid.

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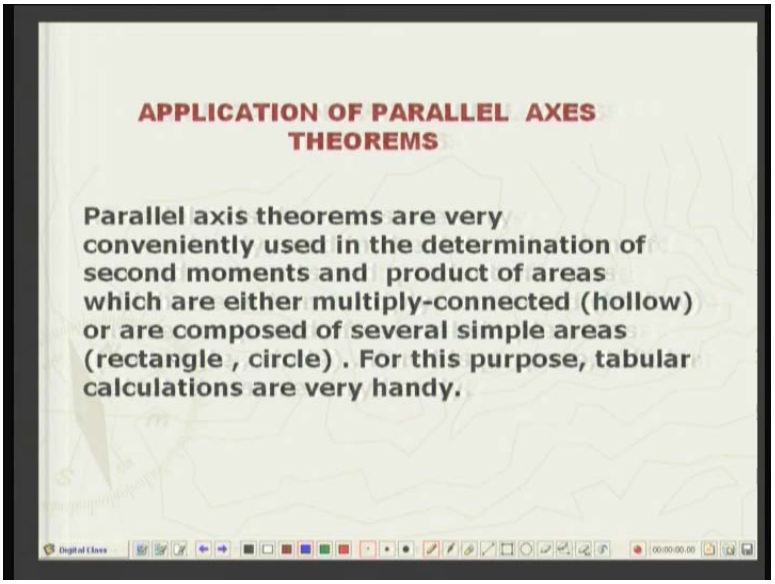

Let us see how we can apply this theorem of parallel axis. This can be very useful. Suppose the geometry of the area is quite complex and the integration will be very complicated. It will take lot of time and sometimes you may have to resolve to numerical integration. So in such cases if we can use the parallel axis theorem, as I will be illustrating, then it will be very convenient.

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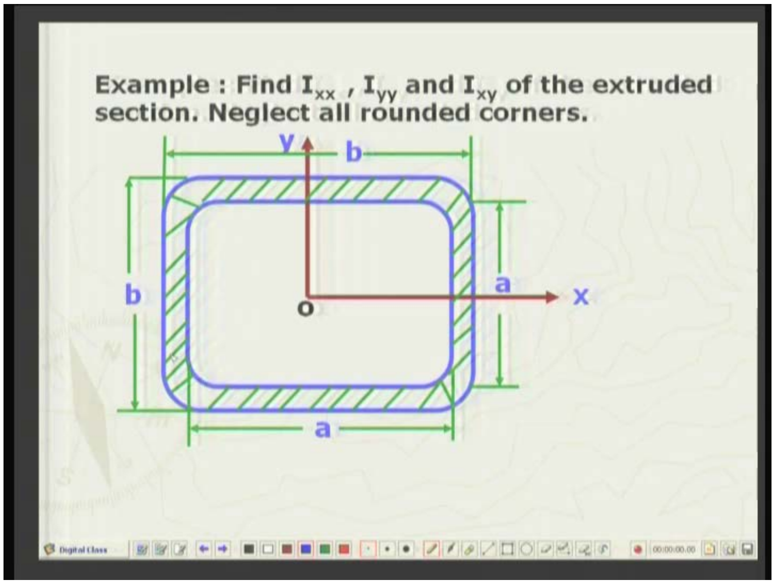

I take up an example to illustrate. Suppose here is a kind of rectangular annulus. That is, this is the outer boundary and this is the inner boundary. So the area of concern or in question, is this shaded area. Well, technically such an area is a box action in air craft industry and in many other applications, a box like structure is quite conveniently or quite usefully employed. So you have a box like this and for the sake of simplicity, these corners we will neglect. We will say that it is an outside rectangle and inside rectangle. So the outside rectangle has the dimensions b and b, that is a square, and inside rectangle has again a and a.

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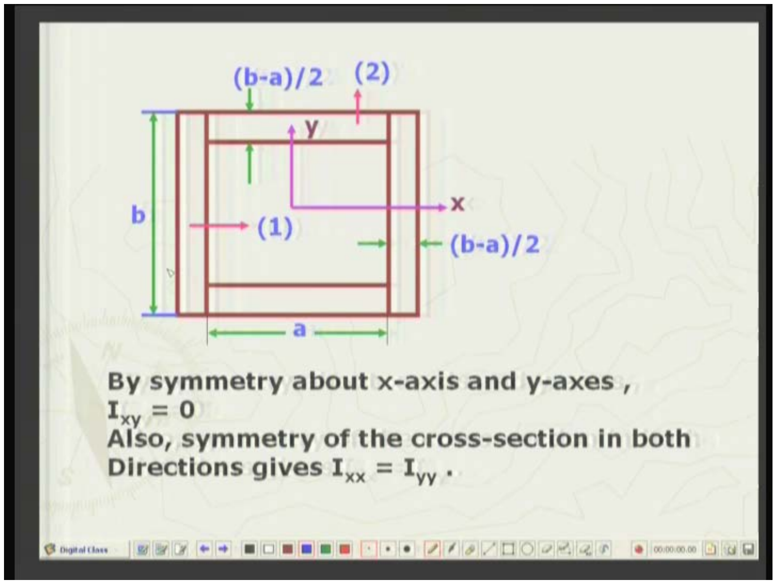

This is simplified like this. That is, you have two rectangles verticals and two horizontal rectangles and you can easily see that this rectangle is symmetrically placed to this rectangle and this rectangle is symmetrically placed with this rectangle. So whatever the contribution of this rectangle about the middle axis, will be the contribution of this and similarly, whatever the contribution to the second moments of this same will be over here. So I need to consider only one pair. The other pair will be just a repetition. Also, as I said, it is a square body from inside as well as outside. So whatever I get Ixx and if I rotate it through ninety degree, I will get yy. So there will not be any difference because of the symmetry through ninety degree. So my task is very simple, much simplified, namely, we will have Ixx is equal to Iyy and about the symmetry axis, the product of area is zero. We have already shown that if the axis is a symmetry axis, then the product of area term is zero. So, let us say, Ixy will be zero about the symmetry axis I x axis and y axis and Ixx is equal to Iyy. Now we will call the vertical rectangles as area one. This is area one. This vertical and the horizontal rectangle is area two.

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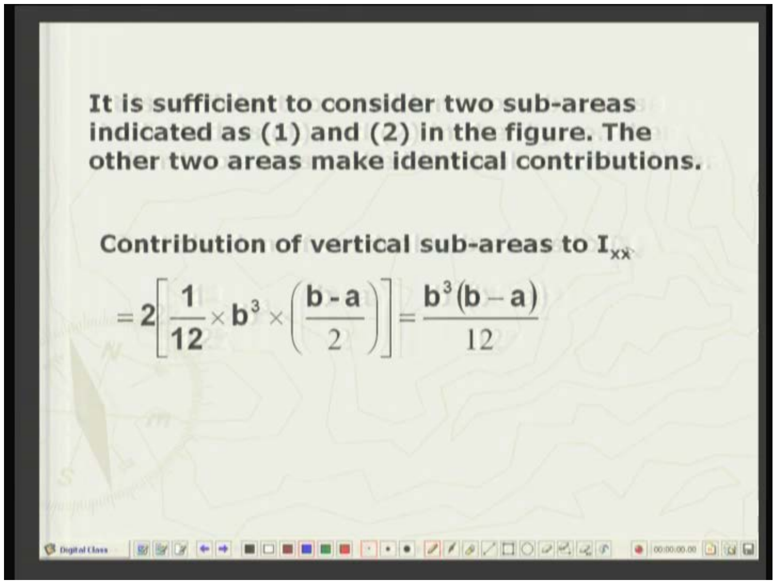

Contribution of the verticals of area to Ixx. So first, we are calculating Ixx. Now, this is the base into height cubed. This calculation, either you can find out the second moment of area of a rectangular body from hand books or you can easily integrate it. You will find that it will be b. If you have a rectangle of width b and depth h, then bh cubed, that is, width into height cubed or depth cubed divided by twelve. So this is exactly what we are doing. b is the depth. This is b and this is b minus a divided by two. So b minus a divided by two. This is the width into depth cube divided by twelve and since one rectangle on the left, second rectangle on the right, symmetrically placed. That contribution is equal. So adding up the contribution means twice. So twice into one by twelve b cubed into the thickness of the rectangle, that is, b minus a by two and if you simplify this, you will have, very easily, b cubed into b minus a divided by twelve.

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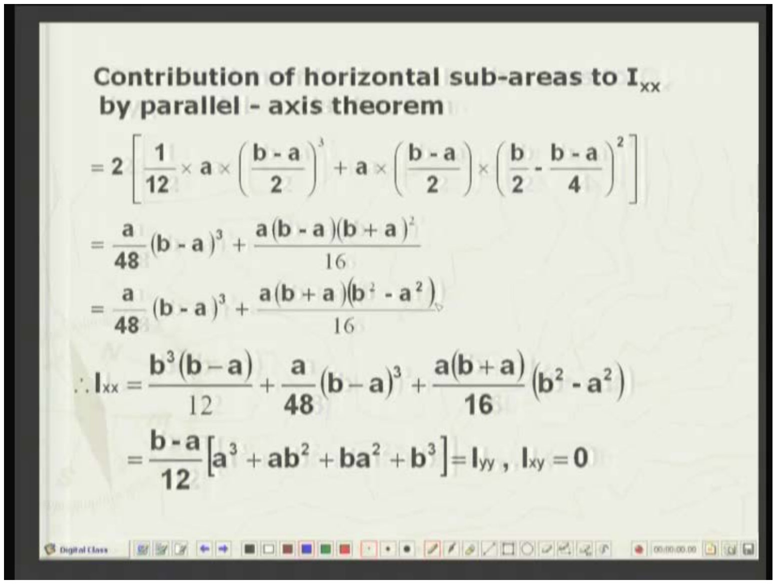

Then we come to the horizontal rectangle. This rectangle's width is now what? How much it will be? Simply, a and the depth is b minus a divided by two. So about this. Its central axis, that is, going through the mid thickness, it will be a into b minus a by two whole cubed by twelve and then, with the help of parallel axis, we will transfer it over here to the x axis. So that is exactly what we are doing. Well, two factor of two one horizontal rectangle above the middle plane and one below contribution being, say, same. So we are taking twice one by twelve into a, that is, width of the rectangle into the depth cubed. So this is the Ixx dash about the middle axis and then by parallel axis theorem, we will have a times d square. a is the area of the rectangle, that is, width is a depth is b minus a by two. So this product gives me the area of the rectangle times the distance of the middle of the rectangle from the x axis over here.

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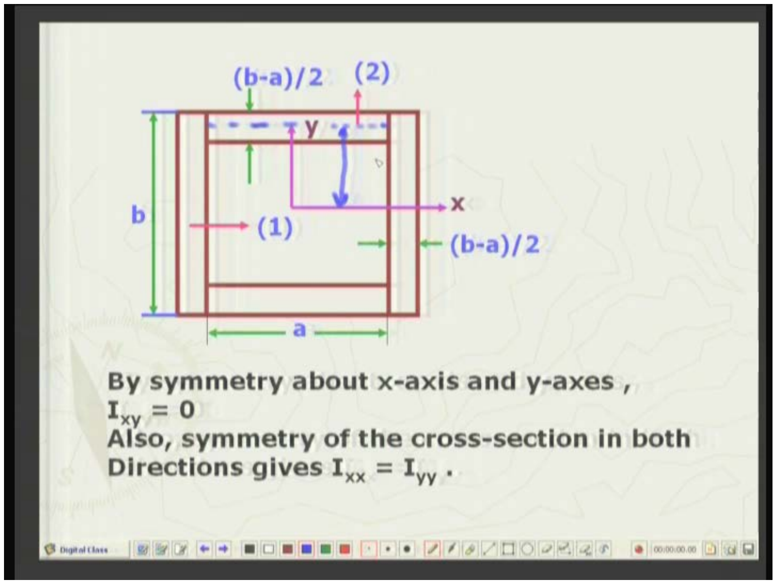

This distance is the middle of this rectangle. So, when you do this, you will get, very easily, this area. Rest is some simplification of these terms and when you do this, you will get a by forty-eight b minus a cubed plus a into b plus a into b square minus a square by sixteen. So, this is the contribution of the horizontal sub area of the rectangle and in the previous slide, we have seen the contribution of the vertical sub area. You add up the two and you will get this as my final answer and this, I have already shown. This Ix is equal to Iyy because the body is symmetric when you give a rotation of ninety degrees. So x axis becomes y axis. It remains unchanged and then the product of area due to symmetry is equal to zero because the body is symmetrical about x axis as well as y axis. So the product of area has to be zero and that is the final answer. So we have seen that a box like area. If you have to do it by integration, that will be quite a complex phenomenon because the limits of integration, etcetera, will not be so easy to calculate. Even if you can do it, it will take a lot of effort. Here, with the use of parallel axis theorem and availability of some easy results, that is, Ixx of a rectangle or a triangle or a circle, they can be easily calculated or they can be found out in any standard book. You can calculate the Ixx, Iyy and Ixy of complex geometries.

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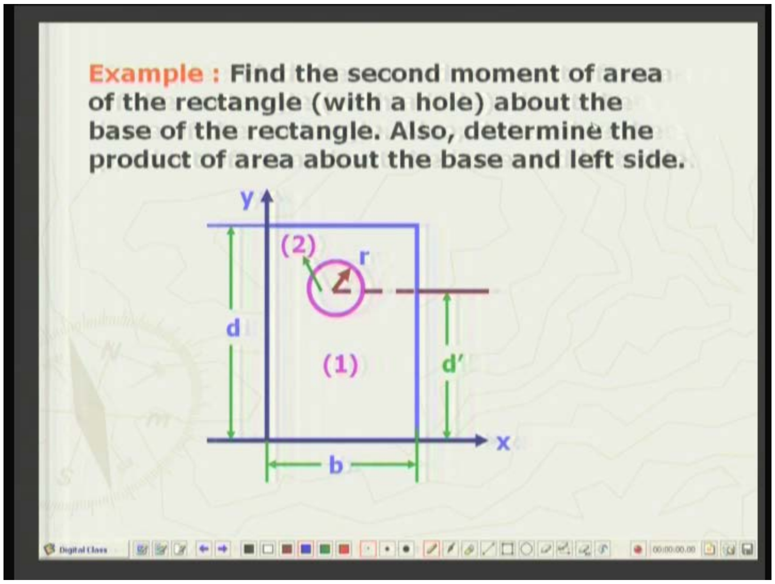

We will take up another example. You have to find the second moment of area of a rectangle out of which this whole has been removed, that is, the net area is consisting of this as the outer boundary and this as the inner boundary. So a rectangle and a circle. So find the second moment of area of the rectangle with the whole about the base about x axis of the rectangle. Also, determine the product of area about base and the left end about x and y coordinates. So I have to find out Ixx and Ixy. Again, we will use the parallel axis theorem.

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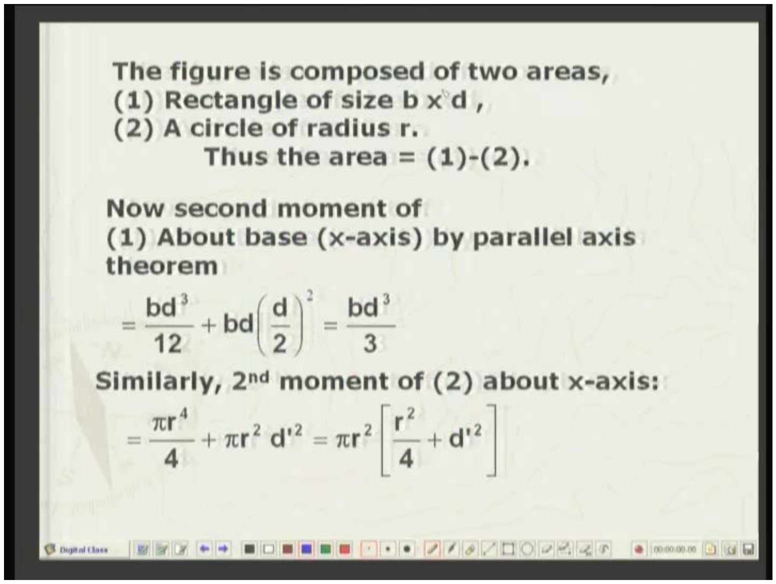

First of all, the net area consists of the rectangle of the dimension b into d. b is the width and d is the depth and a circle of radius r. So the net area is one area one minus area two and the second moment of a rectangular area, we have already found out is bd cube by twelve but now we have to shift it to the base x axis. So area into the distance of the center of the rectangle from the base, that is. d by two square. Again, see it over here. The middle of this rectangle will be somewhere here and this will be d by two. So you can calculate it. It will come out as bd cube by three and then for the circle, let us go to the circle. For the circle, if you want to determine I axis about its diameter, then pi r four by four. Again, it is available in hand books or you can easily integrate it and find out. So it is pi radius four divided by four. This is the quantity I axis and since it is passing through this centroid of the circle, namely, the center itself, I can use the parallel axis theorem between this and this and the height of this will be d dash which will be equal to this. d dash is given. So pi r square into d dash square. If you simplify it, pi r square r square by four plus d dash square.

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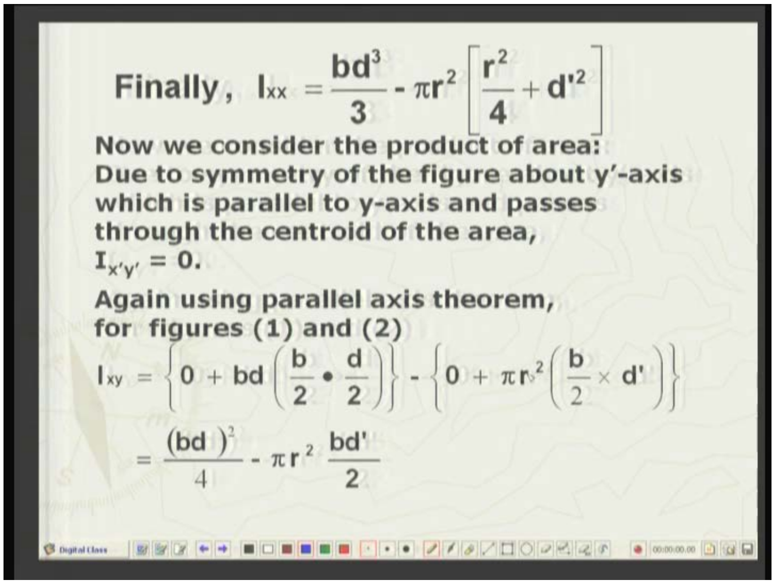

When you subtract the contribution of the circular area from the rectangular area, you get the Ixx of the net figure and to find out the Ixy, the product of area, you can again do or use the symmetry of the problem. So about the new coordinates. You can easily see, this axis is the axis of symmetry for the net figure. So about this axis of symmetry. The product of area is zero, we have already shown and then I have to transfer this to y axis by parallel axis theorem because the centroid of the net figure will lie on this axis of symmetry. So I can use parallel axis theorem and you can again see this Ixy will be equal to zero for the I x dash y dash and this is the b into d into the location of the centroid. So, this is the contribution of the rectangle. This is the contribution of the circle. So finally we have b by two minus d by two. This is Ixy the product of area about the centroid.

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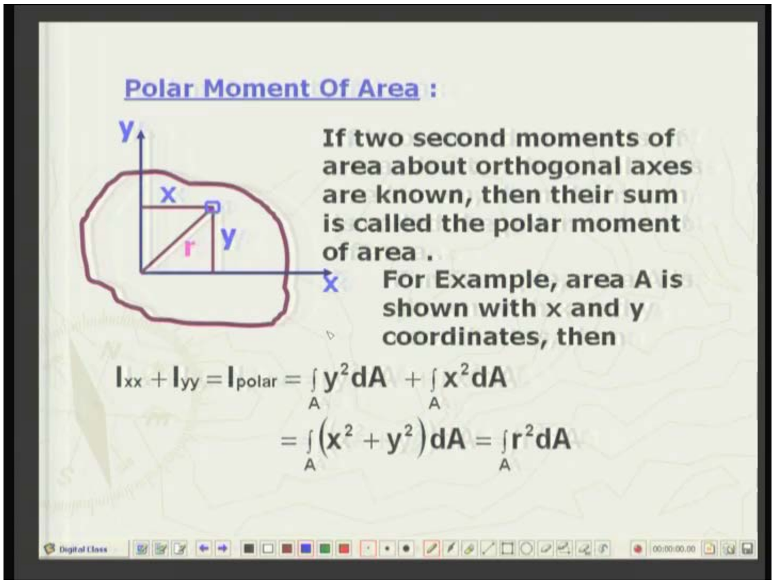

One last thing, polar moment of area, that is, if you add up the Ixx and Iyy, then we will get I polar, that is, the polar moment of area and by definition of Ixx and yy, you will have y square area integral plus x square area integral. When you add up x square plus y square da, this becomes r square area integral. What is r? r is the radial distance of the element of the area from the center. This is the called the polar moment of area. So the only thing is that Ix and Ixx and Iyy should be the second moments of area about the centroid about the two orthogonal axis passing through the origin arbitrary origin O. So today we have seen the contributions made by the second moments of area and how to use the parallel axis theorem. In the next lectures, we will now start with the moment of inertia and product of inertia which is quite similar in concept to the second moments of area and the product of an area. Thank you very much.