Applied Mechanics

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Lecture No. 10

Properties of Surfaces

Today, we will take up lecture ten, which deals with properties of surfaces. So far we have been talking about forces and moments and today, we will be slightly deviating from those concepts and we will be discussing surfaces. Now, in earlier lectures, may be lecture five or six, we were talking about equivalent force systems and also in some of the problems we discussed triangular distribution of force or rectangular distribution of force in context of beams, etcetera. These are also some kind of surfaces and we replace those distribution with their equivalent single force concentrated at a point. So all these facts emerge from the properties of surfaces. Now whenever you want to buy a plot of land or in some other context, you know that the size of a surface is given by its area and then the shape is given by whether it is a triangular or circular or trapezium, etcetera, orientation. You can describe that this piece of land is south facing, etcetera, but to put these concepts in a quantitative fashion, we need some analytical properties which we will be discussing now.

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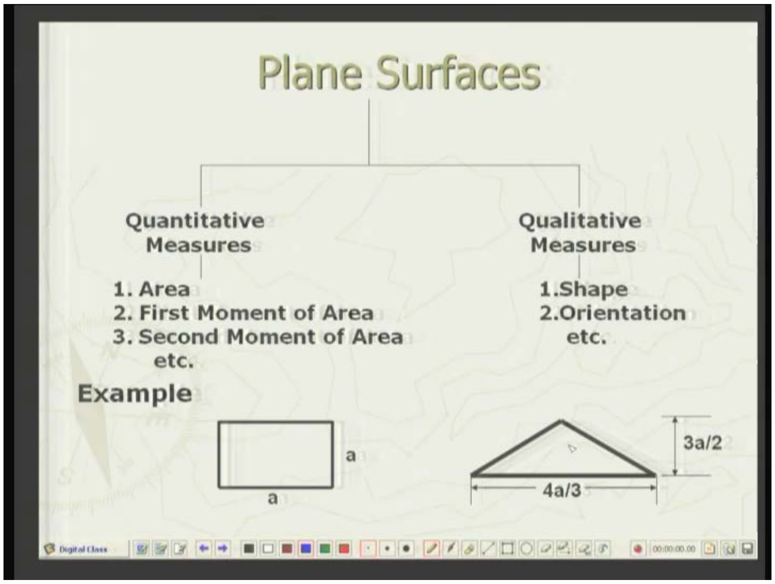

Well I have given here some measures. Some of them are quantitative measures that they can be expressed in number. The others are qualitative measures. We have already talked about area which is a measure of size of a figure or a plot of land, etcetera, and then in the qualitative measures we had shape. Again, we can talk of a circular area or a triangular area and to put it in a quantitative manner, different shapes or orientations, we can use or rather get some information by using first moment of area, second moment of area, etcetera. Let us look at this example. Here we have a rectangular area or a square area where each side is a. So its area is a square. On the right hand side, we have a triangular area. The base is four a by three and height is three a by two. Again, if you calculate its area, it will again come out to be a square. So both these areas are of the same size but quite obviously they are different. If we take the first moment of these two areas, they will come out to be different. There may be a case that two areas have the same figures, have the same area as well as same first moments. Then the second moments may be different if the size first moment and second moment are also equal. Then the third moments will be different and so on and so forth. So the higher we go in taking the moments, more and more distinction we can make between the given two areas. So first we will start with the first moment of an area.

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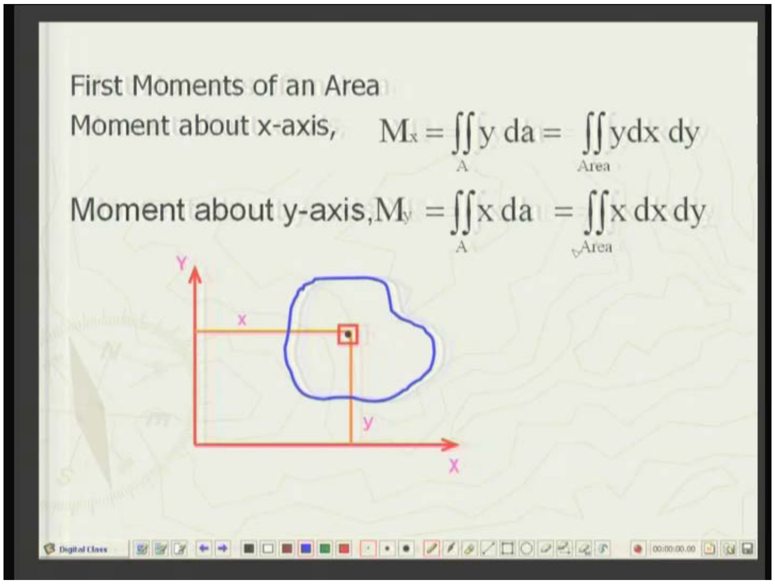

Here is an arbitrary figure. Well, we can calculate its area graphically or if the figure is well described by an equation, we can, with the help of area integrals, find out its area to get the first moment of this area or of this figure. Suppose we consider a small element of this area, of this size dx along x axis,, dy along y axis. So that small area is dx by dy. Then from the centre of this rectangular area, the distance of the y axis is x and distance of the x axis is obviously y. So we will define the first moment or simply moment about x axis which will designate as Mx which is equal to the area integral y times da. da is that small area. So we multiply this small area with the distance from the x axis. That is, this is y and it can be also written as, for rectangular Cartesian coordinates area, integral of y dx dy. Similarly, we can define the moment about y axis which is My and this will be the da, the infinitesimal area da times x integrated over the entire figure and it can be written again as x dx dy integration over the area. Now, immediately connected with the moments of area, we have the concept of centroid of an area and here I would like to draw an analogy between centroid of an area and something we have already learnt, that is, the centre of pressure.

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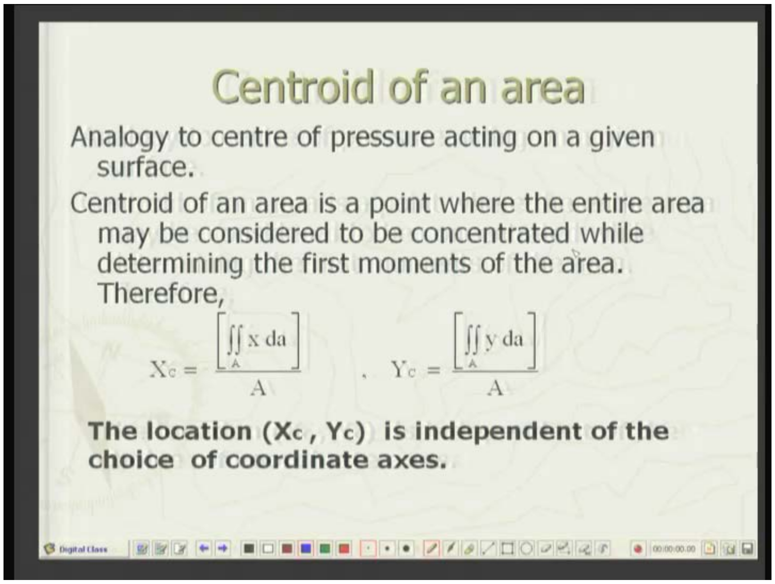

Suppose there is a given area and some fluid pressure is acting on it and it is acting in a normal direction to the area. Then you remember we replace that distributed force system with an equivalent force, a single concentrated force acting through a point which we call centre of pressure. Similarly, if we consider any body under the gravitational field, then the whole weight can be considered to be acting through a single point, namely, the centre of gravity. So similarly, over here, we have the given figure. Then we can concentrate the entire area of that figure through a single point called the centroid of an area for the purpose of calculating the first moments of that area. So the x coordinate of the centroid x, c is equal to the first moment about y axis divided by the entire area or you can say that the first moment about y axis of the figure is equal to the total area times distance Xc from the y axis and similarly for y coordinate of the centroid namely Yc, we have the first moment about x axis divided by the entire area of the figure. Now this property of centroid, that is, it is described by Xc, Yc is independent of the choice of coordinate axis. To illustrate this let me show you the next figure.

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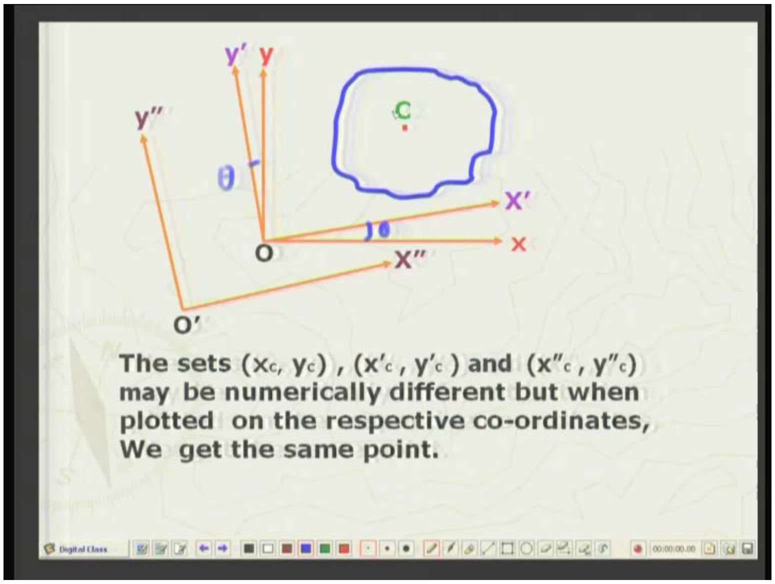

Here is a given figure and using the definition of centroid in the coordinate axis, x and y and z is perpendicular to the plane of the screen. We have located that the centroid is at point c. Now let us say, we rotate the coordinates through an angle theta. So this is theta. So the new coordinate system is x dash y dash and of course z dash is parallel to or is coinciding with the z and it is normal to the plane of the screen. Now, if we use the definitions of the first moments, we can again calculate the centroid in the new coordinate system. That is, Xc dash and Yc dash.

Although numerically, they may look different but if I plot them, these two coordinates, x dash x axis Yc dash along y. Again, the same point will be obtained, that is, same point c. So it means that during rotation or as a result of rotation, the centroid, the absolute location of point c has not changed. In this example, the origin was same but only rotation was given. Suppose now I change the origin also from o to o dash. Then again I go through the same definitions and calculate the x double dash c y double dash c. Again they may look different but when I plot them in this coordinate system, I will get the same point c. So what does it mean? It means that whatever coordinate system you may use, as long as they are rectangular Cartesian coordinates, the location of point c will be invariant. It will be unchanged. Although the individual values of the x coordinate or y coordinate will be different, after plotting you will get absolutely the same point of course. You are not making any calculation errors. So this is a very important result. That is, the centroid does not depend upon the choice of coordinate system. Let us consider an example.

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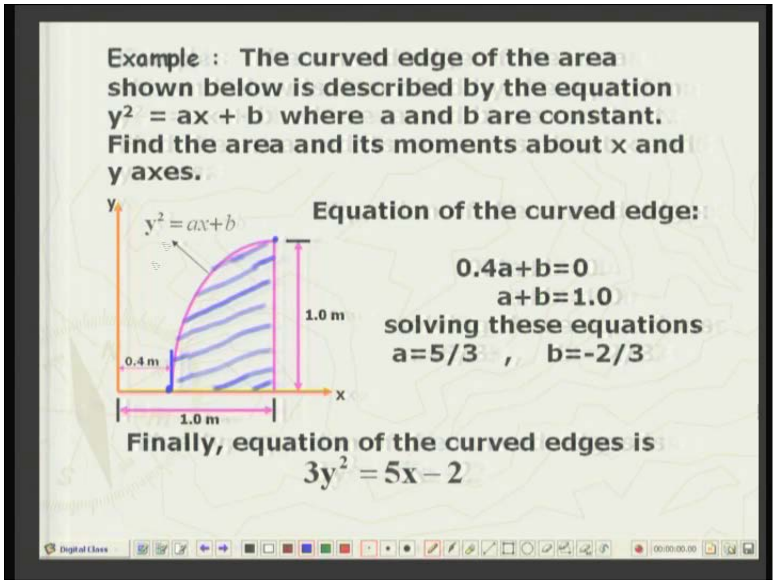

Here is a given area which I can show you over here. This is the area and on one side, there is a vertical line, horizontal line and the other line is a curve given by an equation, y square is equal to ax plus b. So it is a kind of curvilinear triangle. Now, it is also told to us that the curve side is passing through a point on x axis, at a distance point four meters from the origin and also it is intercepting the height at a distance of one meter from a line along a line at a distance of one meter along x axis. So these two points are given on this curve. So the curved edge of the area shown below is described by the equation where a and b are constant. Find the area and its moments about x and y axis. That is what we have to determine. First of all, we have to determine the constants a and b so that we know the equation completely and since this point four lies on this curve, if we substitute x for point four and y is zero, at this point. So zero is equal to point four a plus b. This is first equation and then I go to this point whose coordinates are, you can easily see, one comma one. So if I substitute x is equal to one and y is equal to one, I will get a plus b is equal to one. So these are two simple equations from which I can obtain two unknowns. So they are found to be a is equal to five by three b is equal to minus two by three. Substituting back the values of a and b into the equation, we will get three y square is equal to five x minus two. So this is the equation of this curvilinear edge.

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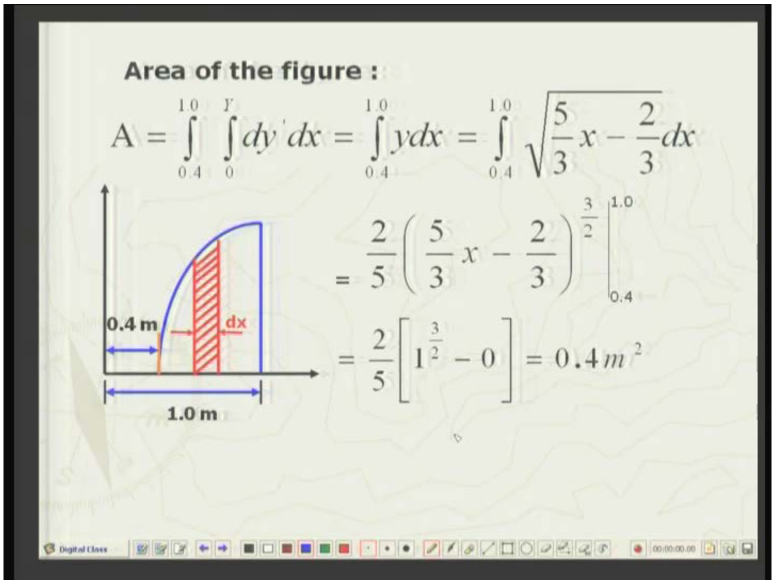

Now to obtain the area of the figure, we have to use the area infinitesimal area. You can imagine we have already done dx into dy. Now I am using it instead of y dash because y is the upper limit of the integration on y dash. So to make a distinction between the variable and the upper limit, I am using y dash. So first I integrate the inner integral. So this will give me y dash into dx from limits zero to y and on substitution I will get y dx from point four two one. That is simple and this has a very interesting interpretation. This y dx is the area of a strip of thickness or rather width dx and height is equal to y. So you can alternately say that the total area consists of various strips of thickness dx and height y depending upon where we are along this curvilinear edge.

So if we integrate now, this y is substituted from the equation of the edge, that is, three y square is equal to five x minus two. So from here y can be easily obtained. You divide throughout by three and take the square root. So which we have done five by three x minus two by three whole square root times dx. Now this square root function can be again integrated and limits can be substituted to give us, finally, the result as the total area which comes out to be point four meter square. So either you can use the strip integration or you can use infinitesimal rectangle dx by dy and do the integration. Both are equivalent. Then we come to the moments of this area.

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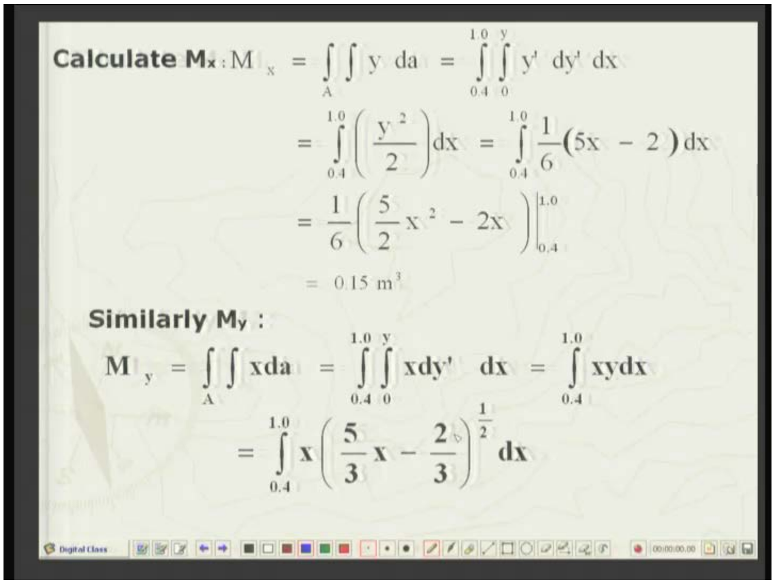

First of all, I will take the moment about x axis. So that will be area integral da. da stands for dx dy. So again, you can carry out the integration. So again y dash dy dash integral from zero to y and x integration from point four two one. First, carry out this integration on y y square by two and then substitute for y square from the equation of the curvilinear edge. You will get this integral and carry out the integration on x substitute, the lower limit and upper limit. You will get the moment as point one five meter cubed area, was meter square obviously and moment will be meter cubed. Well finally, we go to the moment about y axis. So by definition it is x da area integral and in the same fashion we come up to this integration. After carrying out the y integration and substituting for y square, we come to this integral.

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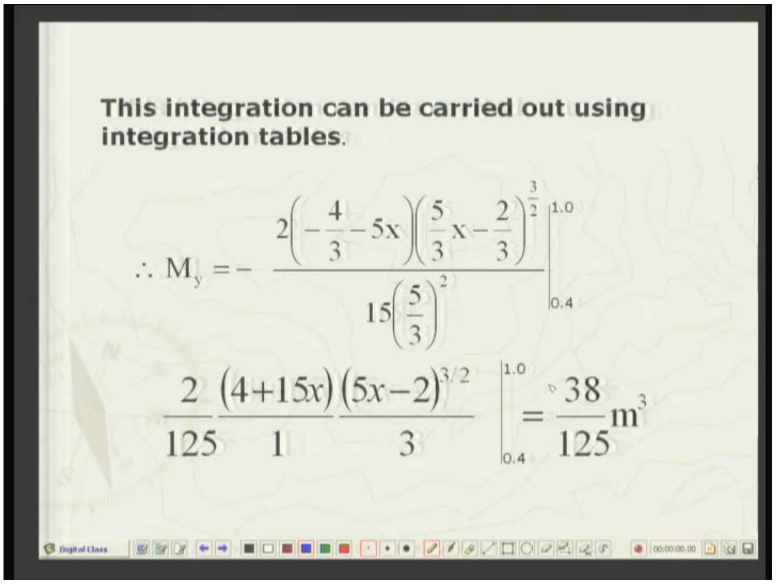

This integration integral is slightly complicated and we take the help of integration tables. They are easily available in mathematical handbooks or sometimes in the subroutines. So you can get the result from the integration tables and this will be the integration and again we substitute the lower limit and upper limit and after little calculations, we get the moment about y axis. First moment about y axis as thirty-eight by one twenty-five meter cubed. So we have calculated area, first moment of the area and the first moment of the area about x axis, first moment of area about y axis. Though in all these area integrals or moment in integrals, we are using only plane areas, if the areas are curvilinear shell type like a paraboloid or hyperbolized, etcetera, then the similar definitions will be applicable but now the integration will be in the curvilinear coordinates which is quite complex and that is why, we are not considering shell like areas. Now, let me make a very interesting observation. Sometimes we find areas which are symmetric about an axis, that is, if i place a mirror vertical to the area then the right half of the area will be a mirror image of the left half or vice versa.

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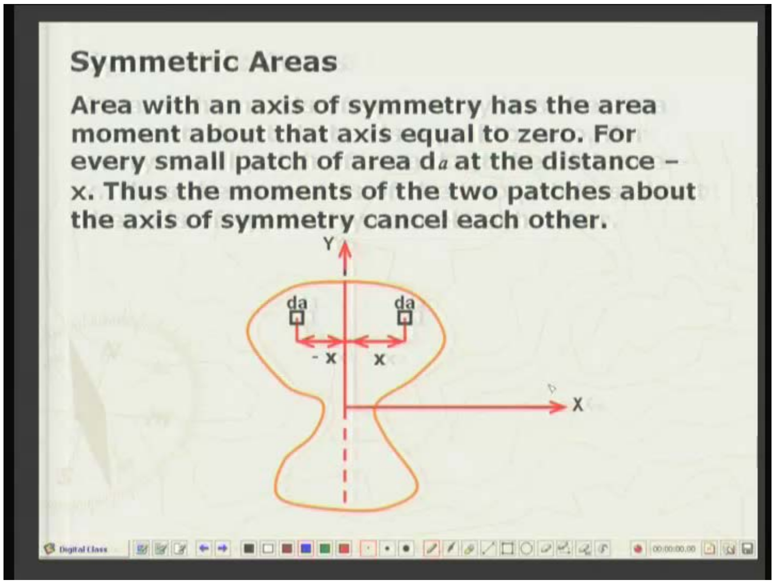

For example, here this is an area, a kind of figures eight and about the y axis, if I put a mirror, this part is mirror image of this part, sometimes you may have two axis of symmetric. That is x axis and y axis. both are symmetrical. So in those two cases our determination of this centroids can be very easy or rather, first we will determine the first moments and then we will come to the centroid. For example, I want to find out the first moment about y axis and y axis happens to be the axis of symmetry. Suppose I choose a small infinitesimal area, da, then as I have said that due to its mirror image, there will be corresponding area da on the left side. If the distance of the right side area is x, then the distance of the left side infinitesimal area will be minus x from the same y axis. so da times x will cancel out da times minus x and that can be said for any other areas. So for every small infinitesimal area on the right side, there will be a corresponding area on the left side but at a distance negative of the former one. So there will be a pair wise cancellation of all these moments and we will find that if y axis is the axis of symmetry, then my will be zero. If the x axis is the axis of symmetry, then Mx is equal to zero and accordingly, if y axis is the axis of symmetry, then Yc, that is, y coordinate of the centroid rather x coordinate of the centroid Xc will be zero and the y coordinate will lie anywhere along the y axis.

If x axis is the axis of symmetry, then Yc will be zero and Xc will lie along anywhere and if the figure is such that it has two axis of symmetry and these two axis are mutually orthogonal to each other, you can choose them as x coordinate and y coordinate.

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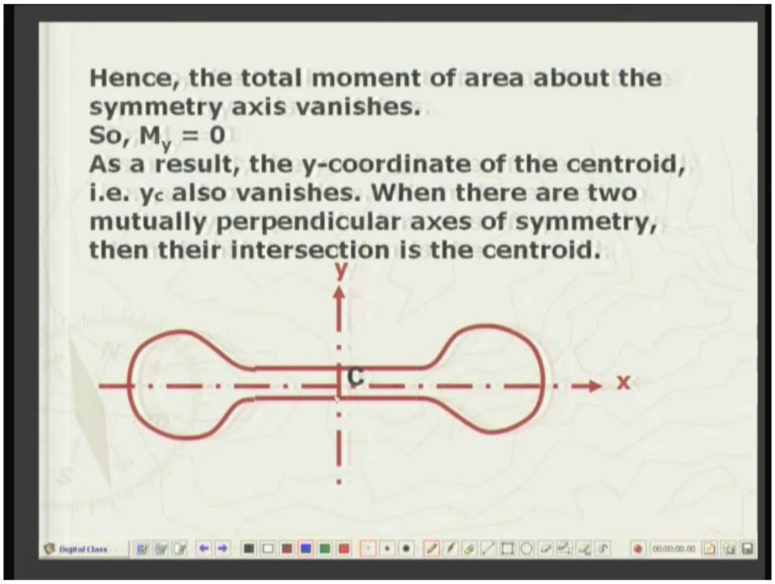

Then you will find, that the centroid lies at the intersection of these two coordinates, that is, it will be at zero zero. So symmetric figures are very convenient to work with.

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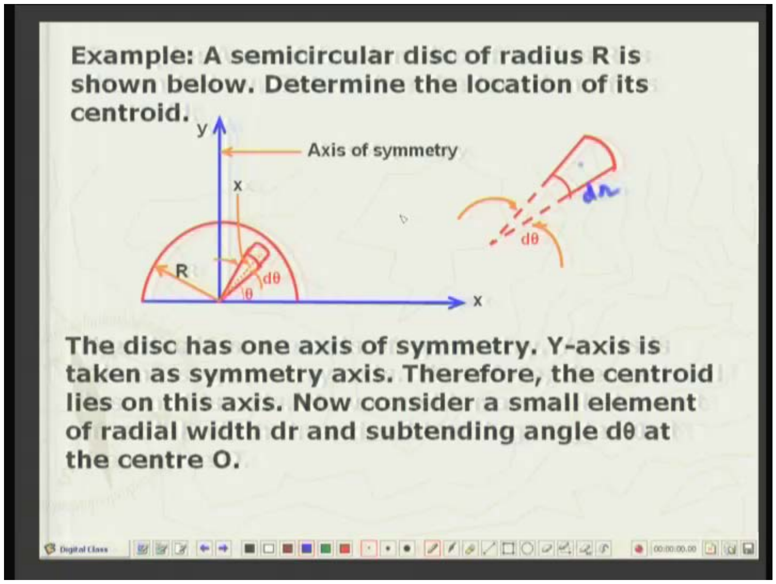

Now let us take up an example to illustrate this feature. Suppose there is a semicircular disk of radius r, shown over here. Determine the location of the centroid. Well by the very definition of a circular area, two perpendicular diameters or rather any diameter is an axis of symmetry and we will choose one diameter as the base of the semicircle and the other diameter perpendicular to it passing through the center. So this y axis is the axis of the symmetric. So we know that the centroid has to lie on this y axis. So the x coordinate of the centroid is automatically known to be zero. Only we have to find out the y coordinate. Well, to find out the y coordinate, here we will take instead of a rectangular infinitesimal area, a curvilinear infinitesimal area. How do we get it?

We will take any two radii and subtending a very small angle d theta at the centre. Here I am showing it in an exaggerated manner. This sector like shape is chosen. So it subtends an angle d theta at the center and its inclination , now of the sector like body, is theta is to the x axis. This side is dr and the center of this sector is at a distance of radius r from the center.

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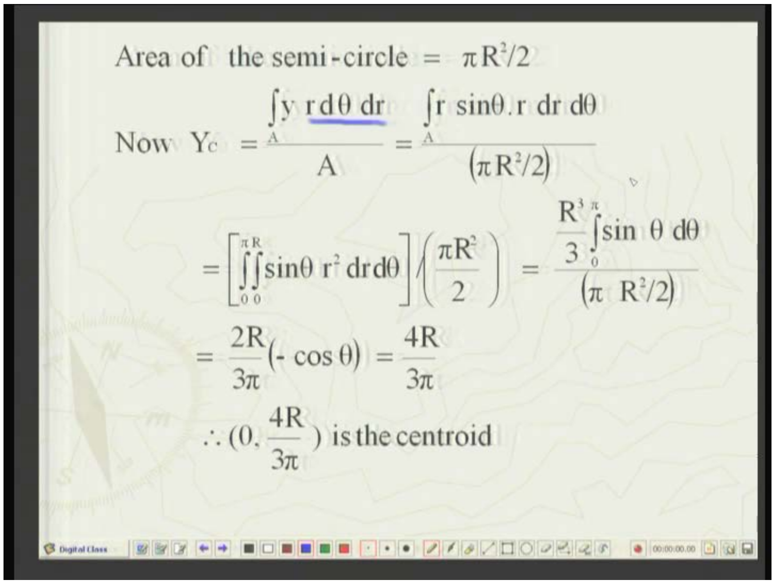

So we find out the area of the semicircle. For the circle, the area is pi r square. So semicircle will be pi r square by two and then to determine Yc, we want to determine this rd theta. dr is the area of this shape and into y because y is the height. So this y is equal to r sin theta. Again this y is equal to r times sin theta. So y r d theta dr divide by the total area of the semicircle. So you can see that it is the area integral r sin theta r dr d theta divided by pi r square by two. Now to calculate the area integral, it is not difficult. We will have the radius going from zero, that is, from the center to the outer periphery, that is, the radius capital R and theta angle going from positive x axis to the negative x axis, that is, zero to hundred eighty degrees or zero to pi.

So sign theta R square dr d theta divided by pi R square by two. So first we will carry out the integration on R square dr which will give me R cubed. So if I substitute the lower limit and the upper limit, capital R cubed divided by three into the theta integration of sin theta d theta and that will give me minus cosine theta and finally we will have this as four R by three pi. So what are the coordinates of the centroid? We have already seen that it lies on along the y axis. So x coordinate is zero and the y coordinate is four R by three pi. So this is how you will get the centroid of any figure, provided, you know the shape of the figure or equation of the periphery or circumference of the figure. Now, we will come to slightly more practical problems.

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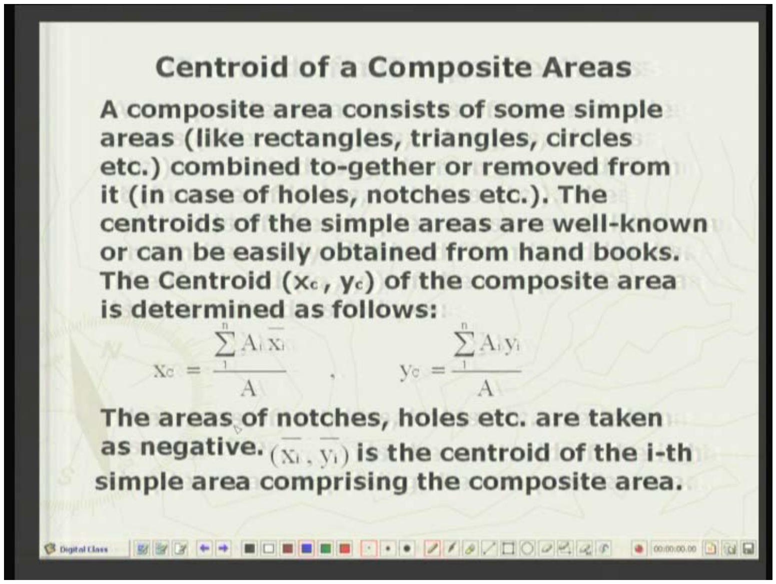

Centroid of a composite area in many engineering application. The areas may not be simply a rectangle or a circle or a triangle or a trapezium, etcetera but the areas may have some notches, some holes or it is made up of two different rectangles, one rectangle, one circle, etcetera. So all these areas are called composite area. Now, these areas can be treated as a combination of, as I said, different areas and for holes and notches, we have to subtract those areas, that is, we will treat those areas as negative areas. How do we calculate the centroid of such areas? For example, let a given composite area consisting of n number of simple areas, either positive or negative. Then the x coordinate of the centroid of the composite Xc is equal to summation of the individual simple area times the corresponding x coordinate of the centroid. So after taking the summation you divide by the total area. Similarly for the y coordinates, that is, you take the individual simple areas multiply by the corresponding y coordinate of the centroid, sum over all the areas and divide by the total area. These concepts are very useful. The centroid of the composite area is very useful in a later course of solid mechanics. In theory or analysis of beams, the centroid of the area of cross action is a very important concept and it is very useful to know how to calculate the centroid. The centroid calculation for composite areas can be very conveniently handled with the help of tables. That is, we will do is calculations in a tabular manner.

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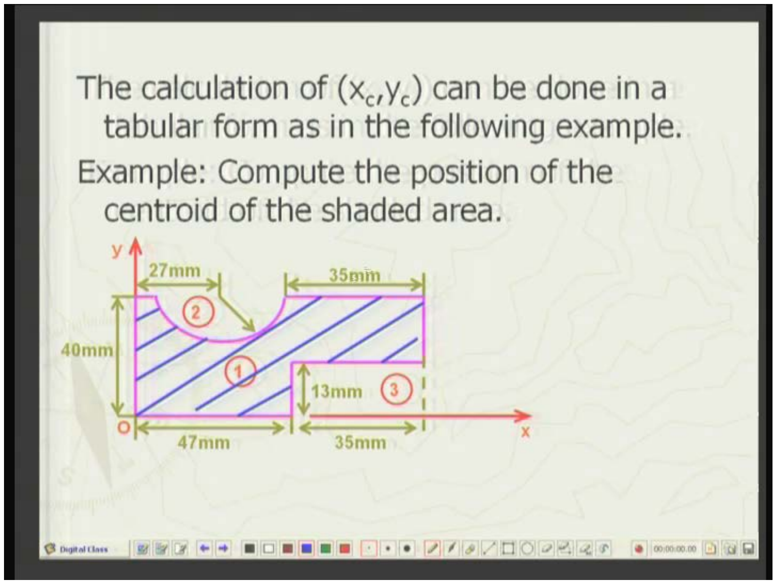

To illustrate this, I have selected an example. Here is a composite area. What it consists of? You can say that there is an outside rectangle of size forty mm in one direction and forty-seven plus thirty-five, that is eighty-two mm in the perpendicular direction. So the length is eighty-two mm, height is forty mm out of this rectangle, its semicircular area has been cut out and the rectangular area has been cut out. So this hatched area is the final figure for which we want to calculate the centroid.

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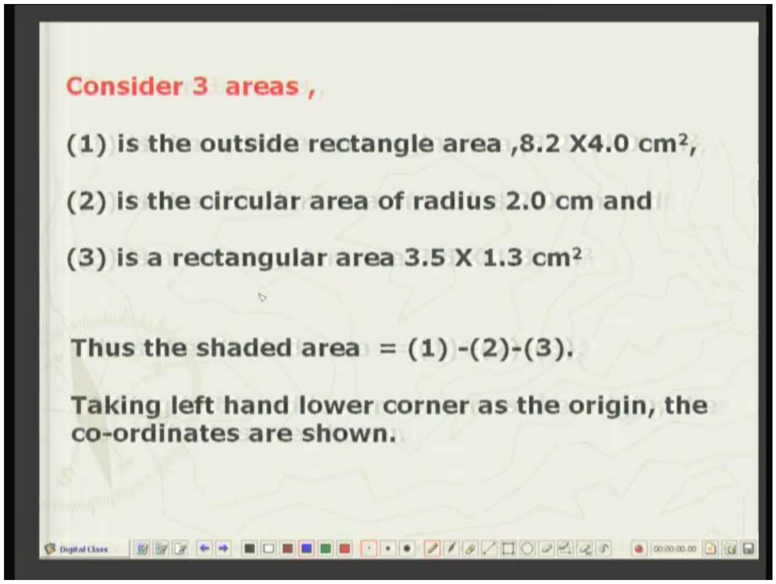

So we will consider three areas. First is the area number one. It is the outside rectangle, as I said, of dimensions eight point two into four centimeter square. Then there is a circular or rather semicircular area of radius two point zeros and centimeter and then there is rectangular area of three point five that is thirty-five mm and thirteen mm. So this is three point five into one point three centimeter square. So the shaded or the area of interest is one minus two minus three areas and we will take the origin for our calculation as the left hand lower corner of the figure.

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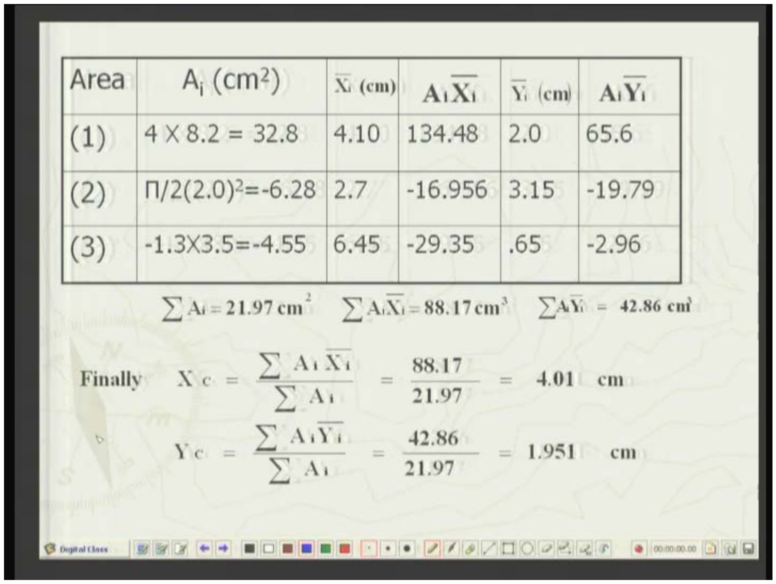

Area	A_i (cm^2)	$\bar{X}_i$ (cm)	$A_i\bar{X}_i$	$\bar{Y}_i$ (cm)	$A_i\bar{Y}_i$
(1)	4 X 8.2 = 32.8	4.10	134.48	2.0	65.6
(2)	$\Pi/2(2.0)^2$=-6.28	2.7	-16.956	3.15	-19.79
(3)	-1.3X3.5=-4.55	6.45	-29.35	.65	-2.96

$\sum A_i = 21.97\ cm^2$ $\quad \sum A_i\bar{X}_i = 88.17\ cm^3$ $\quad \sum A_i\bar{Y}_i = 42.86\ cm^3$

Finally $X_c = \dfrac{\sum A_i\bar{X}_i}{\sum A_i} = \dfrac{88.17}{21.97} = 4.01$ cm

$Y_c = \dfrac{\sum A_i\bar{Y}_i}{\sum A_i} = \dfrac{42.86}{21.97} = 1.951$ cm

Now area one, two, three. We have first the outside area. If you look at the figure, this outside area of dimensions eighty-two by forty and its centroid will be, because of the symmetry, at the inter section of the two symmetry axis. So it will be at the center. So we will see, first area is, instead of millimeter we have calculating everything in centimeters, four into eight point two which comes out to be thirty-two point eight centimeter square and this x coordinate of the centroid of figure number one is half of the length, that is, four point one area into x coordinate we have here. Similarly, the y coordinate of the centroid is this half of the height, that is, two centimeters and this is a times y. For that figure number two, which is actually a semicircular notch, it will be a negative area. So negative pi by two into so pi R square by two. So it will be minus six point two eight. Now we have already calculated the centroid of a semicircle. So using that formula, we will have two point seven centimeter along your y axis and this will come out to be this minus sixteen point nine five six and y axis of the coordinate of the centroid of the semicircle will be three point one five and this will give me the moment as minus nineteen point seven nine. Third figure is again a rectangle. Dimensions are one point three and three point five. Again, it is a notch. So minus and you will have the contribution along the x and y coordinates.

So what if I add up all these three areas, taking into account the negative signs? Then we will have the net area as a area of twenty-one point nine seven centimeter square and the sum of these moments of this x moments of this area are eighty-eight point one seven. Sum of the y moments are forty-two point eight six centimeters cube. So by formula for the Xc and Yc, when we substitute, we will get four point zero one and one point nine five one centimeter as the coordinates of the centroid of the shaded area. So this is how you can calculate for composite areas the centroid. Well, the concept of the centroid of area and the moments of the area, etcetera, was for the two dimensional figures. We can go to three dimensional figures. That will be volume, naturally, or one dimensional figure which will be curves or lines and these are also sometimes very useful in solid mechanics or in dynamics problems.

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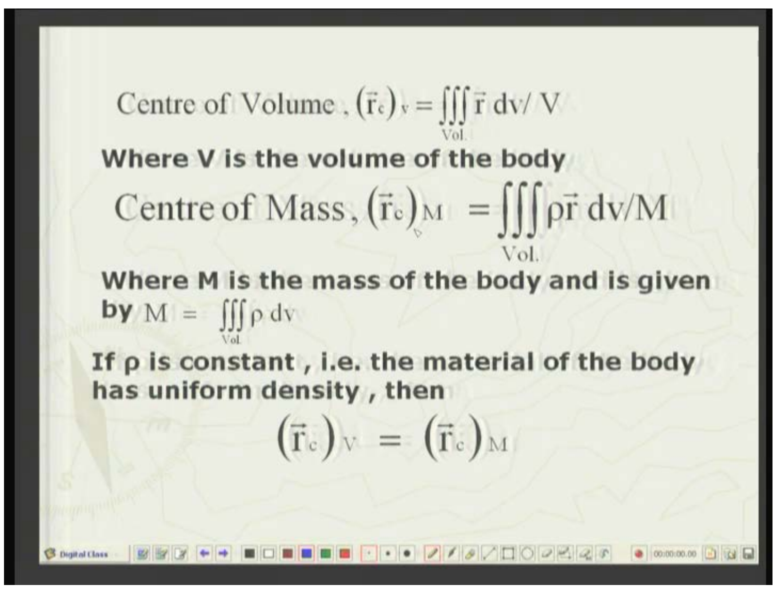

So as an extension of what we have done about areas, let us do the similar thing for volume. So center of volume, since volume is a three dimension concept, we will be using the three dimensional coordinate system xyz. So the position vector of the center of a given volume is obtained by an infinitesimal volume of dimension of, let us say, in the rectangular Cartesian coordinates, dx times dy times dz. So this will be written as dv. The position vector of that, the center of that infinitesimal volume, is R vector and you integrate this over the entire volume and divide by the entire volume. That will be the center of the volume where v is the entire volume of the body. Now this is for volume. You know that volume into the mass density gives you the mass. Here we have the center of mass which is position vector Rc m. This is position vector for volume. It was rc volume. So the definition is exactly similar to the center of volume except that now we will have rho, that is, the mass density which can be a function of the position. That is, the density is varying from point to point. If the material is non-homogenous or a mixture, let us say, mixture of two or three types of metals. So depending on whether you have metal one or metal two or metal three. The densities will be different. So for non-homogenous densities, you will have this definition as center of mass. The total mass is very easy to understand, rho times the infinitesimal volume integrated over the entire body and obviously if rho is constant, that is, materials is now homogenous material, that is, every point has the same density, then this rho can be taken out and you can see that the center of volume will be equal to the center of mass.

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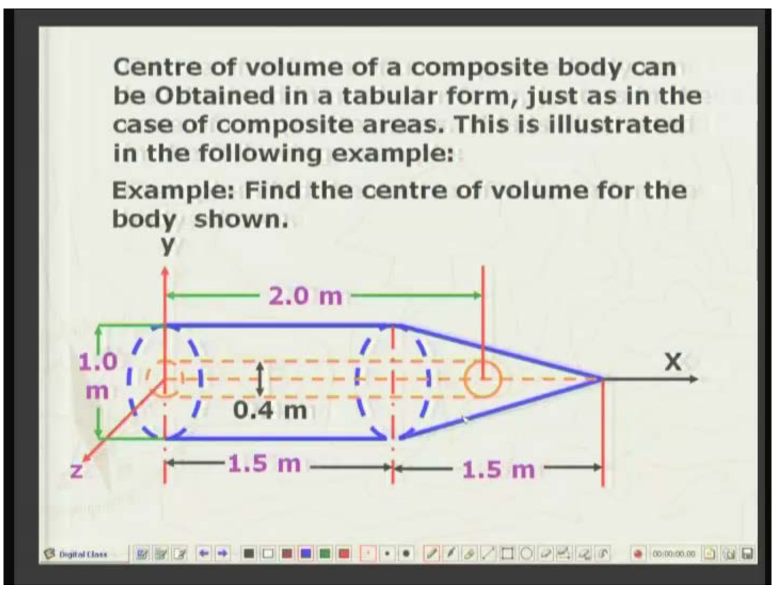

Let us consider an example. To obtain the center of volume for this three dimensional figure which consists of a cylindrical part and a conical part and a cylindrical cavity has been drilled out of it. Again, it is an example of composite volume. The simple volumes are a cylinder plus a cone minus a cylinder. Various dimensions are given, that is, the length of the cylinder is one point five meters diameters one meter, the height of the cone or the length of the cone is one point five meter and base circle is one point zero diameter and for the cavity, the diameter is point four meters and the length of the cavity is two meters. So out of a total of three meter length of the bullet shaped body, up to two meter, there is a cavity. So we have to find the center of the volume for the body shown.

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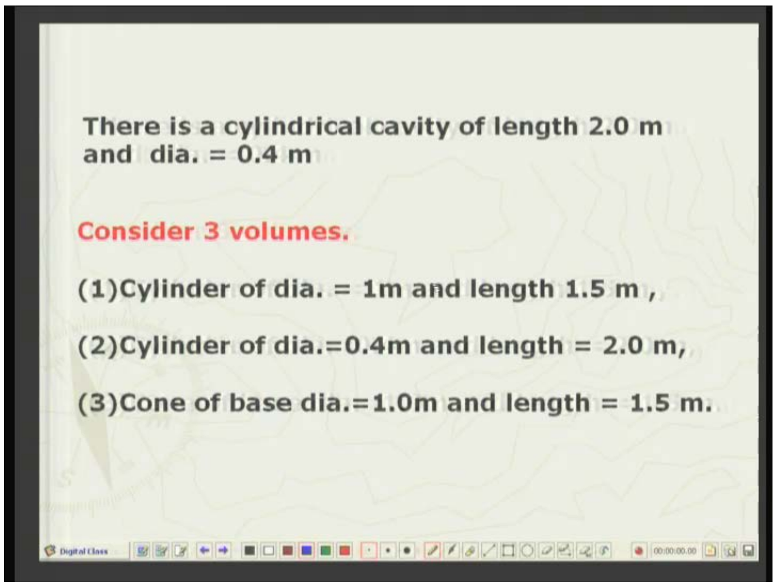

Now, consider three volumes: cylinder of diameter one meter and length one point five meters, cylinder of diameter point four meter, with a negative volume and length two meters, the cone of base diameter of one meter and length one point five meters.

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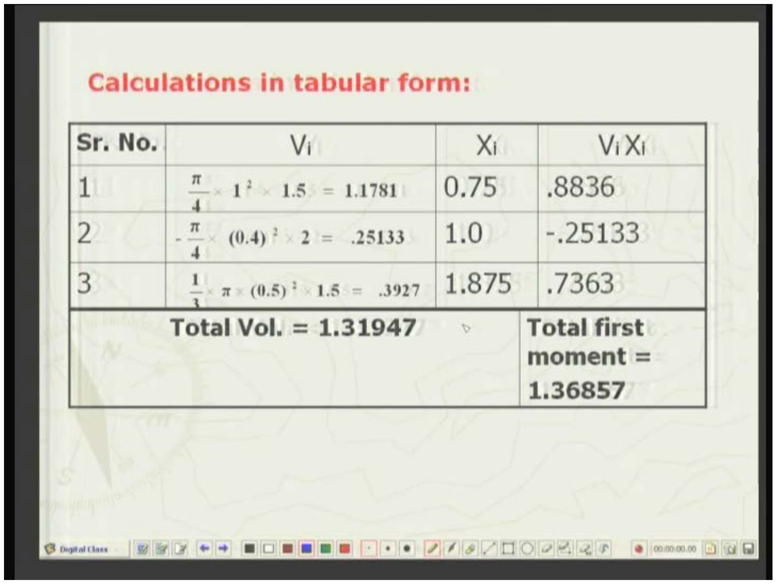

Calculations in tabular form:

Sr. No.	V_i	X_i	$V_i X_i$
1	$\frac{\pi}{4} \times 1^2 \times 1.5 = 1.1781$	0.75	.8836
2	$-\frac{\pi}{4} \times (0.4)^2 \times 2 = .25133$	1.0	-.25133
3	$\frac{1}{3} \times \pi \times (0.5)^2 \times 1.5 = .3927$	1.875	.7363
	Total Vol. = 1.31947		Total first moment = 1.36857

So calculations again are very easy to do in tabular form. So three volumes, one which is a cylinder, two is the cavity and three is the cone. So its individual volumes are calculated or cylinder the base area times the length of the cylinder. So pi by four into one square into one point five. This gives me one point one seven eight one for the cavity. Again because it is a cavity minus pi by four. Again length is two meters, the diameter is point four. So you will get minus. Minus has been missed out. So this is minus two point two five one three three and this is for the cone. Cone is one third of the enclosing volume of the cylinder. So one by three into point five pi R square into length. That is it. So you can observe one thing very clearly that this entire composite body is symmetrical about this x axis. So x axis is the axis of circular symmetry. So the centroid must lie along x axis. So we don't have to calculate the y coordinates or z coordinates. We have to be only concerned with the x coordinates or the centroid. That is why this problem is very much simplified. So you take the centroid of the cylinder, that of the cavity and that of the conical portion. So these are the values obtained from the hand book. Well, for cylinders, you know very easily, half the actual length. So one point five divided by two and similarly two meter is the length of the cavity. So one point zero and for cone, you can get it easily from the hand books and then you take the moments. So total first moment, that is, you add up this column, you get these values.

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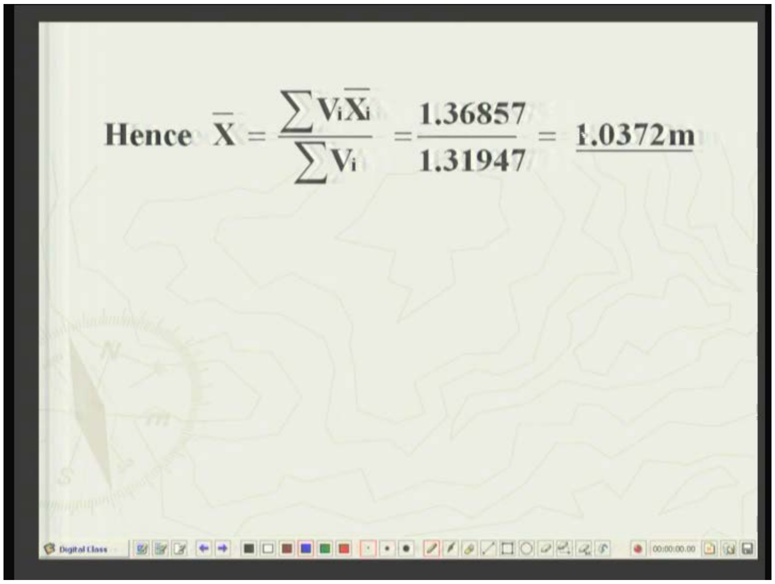

Finally we get the x coordinate of the centroid of this composite figure as the first volume moments divided by three total volume and it will come out to be one point zero three seven two meters. So you have seen how we can, in a systematic manner, calculate various centroids, etcetera. That was for the three dimensional case starting from area, that is two dimension. We went to three dimension and sometimes we have do the similar exercise for a one dimensional case or a curvilinear body or a simple curve.

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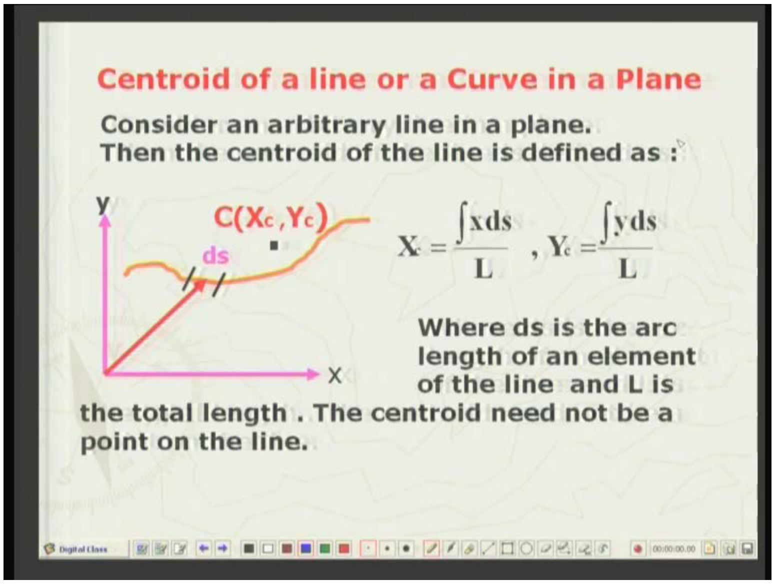

So that will be centroid of a line or a curve in a plane. This line or curve is laying in a plane of course. If it is a helical type of thing, then it a three dimensional curve which we are not including over here but it can be easily extended to three dimensions. So at the moment, we will be concerned with the two dimensional curve, either a straight line or segmental line or a curvilinear figure. So here, for example, we have a curve and a small arc length. Of this curve of length ds is shown over here. The position vector of the center of this small length ds is vector R.

So this is vector R whose coordinates are x and y and this is the origin o. Now by definition and by analogy to the other centroids, we will have Xc as x into ds and this is the line integral divided by the total line length and similarly, Yc will be y times ds line integral divided by total length L. Now one interesting thing is that whereas in the area integral and volume integral, the centroid was inside the area or inside the volume, in the case of centroid of a line, the centroid can be need not be always on the line. It can be outside the line over here or over here or over here. It can be on the line also but that is not always the case. Again I will illustrate this with the help of a simple example.

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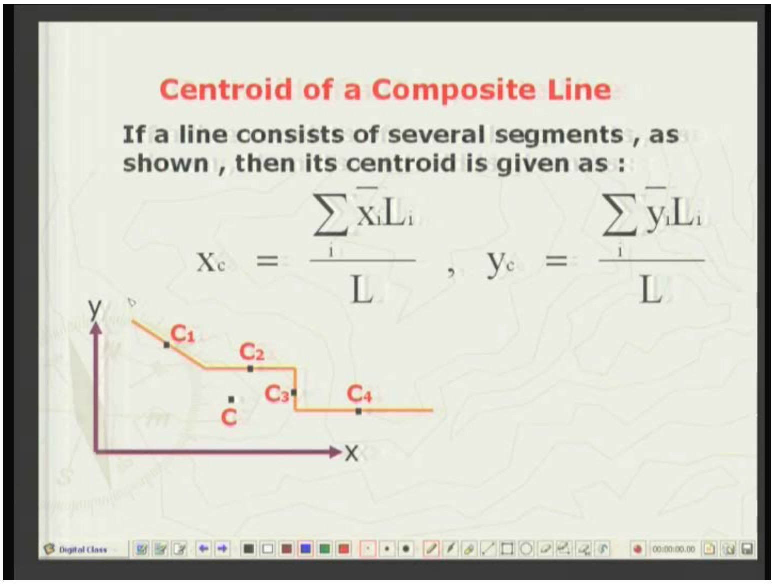

Suppose our line consists of three or four pc's of straight lines or segments. So C one, C two, C three, C four, where each segment centroid is half the length. Then you can find out the centroid of the entire composite line. This is at point C. How to calculate that? You use the formula over here, that is, for example, here is length C first segment whose centroid is over here. So I can calculate the x coordinate and y coordinate centroid substitute over here. Second length again. Third. Fourth and then you add up, divide by the total length of the line and that will give me Xc and similarly Yc. as I said, may be a point somewhere here or over here or over here. So that does not matter.

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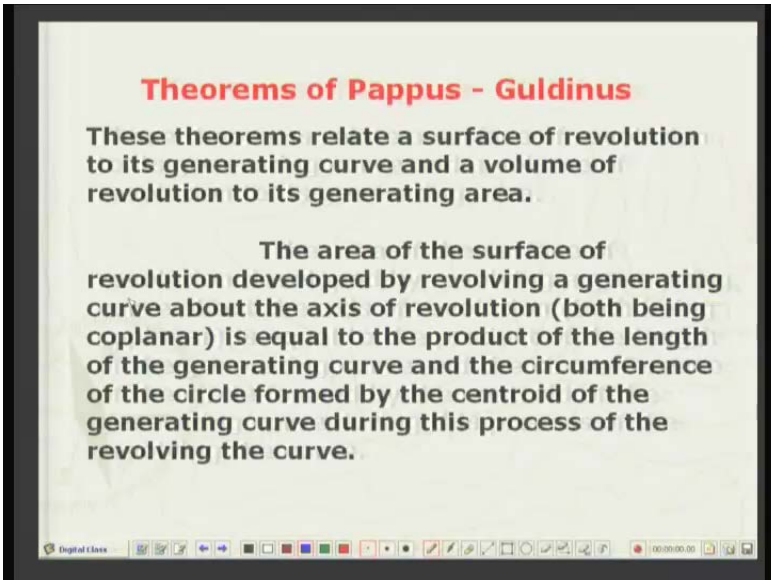

The next thing we will do is some important theorems using these concepts of centroid and in these theorems we can correlate two dimensional figures with the corresponding three dimensional volumes or one dimensional figure with the corresponding two dimensional area. These are for the surfaces or volumes of revolution. Now let me define what is, first of all, I will consider the surface of revolution. Suppose there is a curve. Any arbitrary curve in one axis. Now let us say it is maintained at a particular distance from the x axis and then it is rotated about x axis. Then the curve will generate a surface which will be called the surface of revolution.

You look at this. Suppose this is your x axis and here I am using a straight line as my curve. So if I rotate it about x axis, this will generate a cylindrical surface. So this is called the surface of revolution. Now theorems of Pappus Guldinus. They relate the area of the surface of revolution with the circumference on which the centroid of the curve generating curve moves and similarly we can go to two dimensional case. Suppose here is a, as given, surface and I rotate it about the one of its edges like this. So this will generate a volume called the volume of revolution. So again, this volume of revolution can be obtained in terms of the area of the generating surface and the length of the curve on which this centroid of this area is moving. So these two very useful theorems will correlate to a one dimensional body with the area of the surface. Surface is two dimensional and similarly a two dimensional figure or geometry, that is, an area, with the volume of the generated body.

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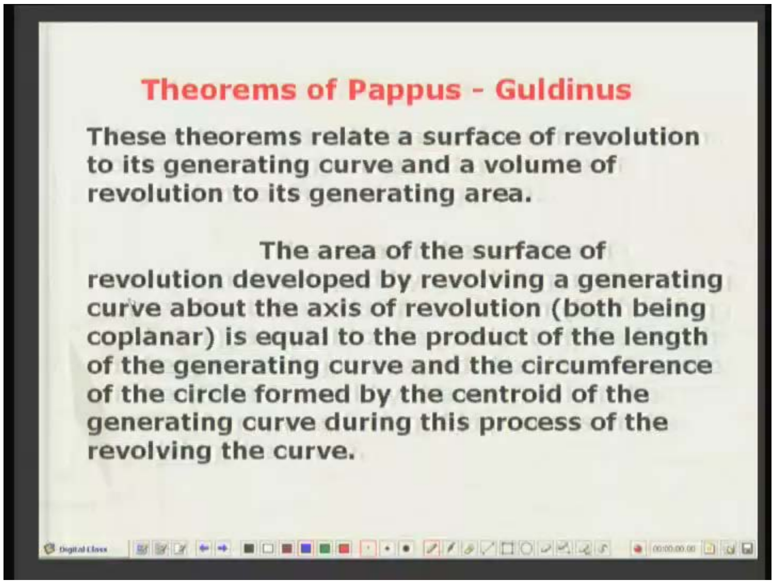

After discussing the centroid of volumes and centroid of lines, we will come to theorems of Pappus and Guldinus. These theorems relate a surface of revolution. I will explain what is surface of revolution towards generating curve and a volume of revolution towards generating area. Remember, surface of revolution is a two dimensional configuration. So surface of revolution is correlated to the generating curve which is a one dimensional manifold.

Similarly volume of revolution is related to the generating area which is a two dimensional manifold. Now what is a surface of revolution? For example, suppose there are x axis and a curve. They are co planer. They are lying in one plane. So now here in this example, I am taking this curve as a straight line but it can be any curvilinear line. Suppose keeping the x axis fixed, this is rotated as if by a machine, keeping the distance same. So then this curve will generate a surface. This surface is a surface of revolution. I give you another example. Suppose this curve is a generating curve, is a slant line. Again a straight line but at an angle. So if I rotate it, I will be generating a conical surface. So suppose, this is a semicircle, then if I rotate it, I will be generating a spherical body. So in this way, these are the surfaces of revolution. On the other hand, if I consider an area, let us say, rectangular area like this page and take this as the axis of revolution and if I rotate it about this axis of revolution, then I am generating a cylinder, a three dimensional volume. So this is the generated volume and the generating area is a rectangle, a two dimensional manifold generates a three dimensional manifold. So the theorems of Pappus and Guldinus will give us a very simple way to correlate the volume of revolution with the area of the generating surface and on the other hand, the surface area of the generated surface with the length of the generating curves. We will take up these discussions in a detailed manner in the next lecture and for the time being, we will close this lecture. Thank you very much for your attention.