

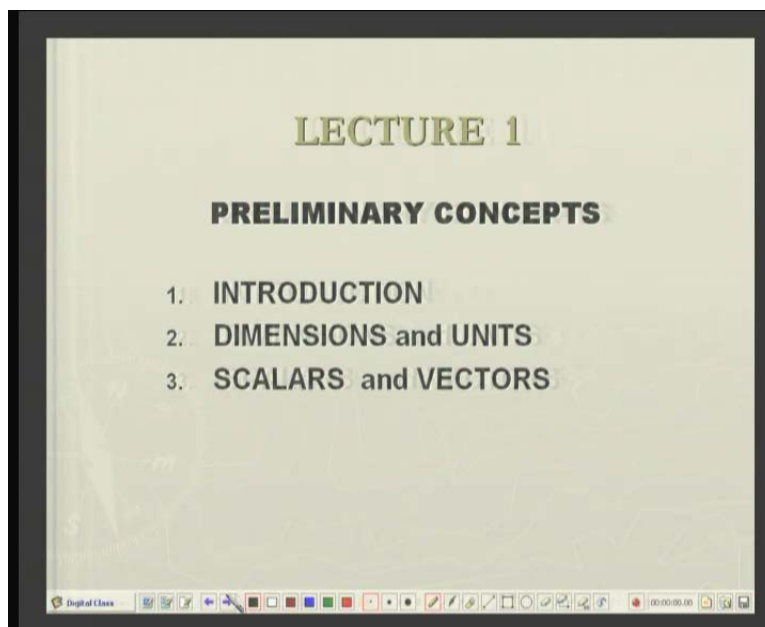
Applied Mechanics
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Lecture - 1
Preliminary Concepts

Welcome to the video course on Mechanics. I am your teacher, Professor R.K.Mittal from the Department of Applied Mechanics, IIT Delhi. I will be taking lectures on Mechanics, in particular a component of Mechanics called Statics.

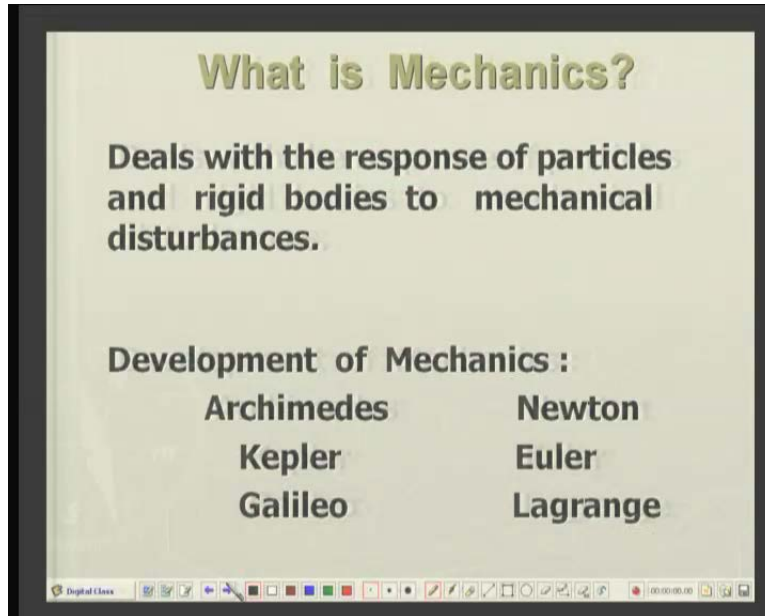
Mechanics is not something new to you. Most of you have learnt part of or detailed courses on mechanics in your physics courses in your high school or elsewhere. But mechanics is much more than a part of physics. It's a full-fledged subject by itself and lies at the core of many engineering disciplines like civil engineering, mechanical engineering and aeronautical engineering and it also plays quite an important role in other disciplines like naval architecture, textile engineering, etcetera,. There is a hybrid branch of engineering called Bio Mechanics. Here Mechanics is a very crucial component and deals with its application in biological systems like humans, etcetera.

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In the first lecture I will be discussing some preliminary concepts. These concepts you have come across in, as I mentioned, earlier courses. But I will be revising those concepts. The first thing is what is Mechanics or rather how do we introduce Mechanics.

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Mechanics is a branch of engineering science or physical sciences which deals with the response of particles and rigid bodies to mechanical disturbances. Here I would like to distinguish mechanical disturbances from other types of disturbances like chemical or thermal or electrical disturbances. Mechanical disturbances in our context mean forces, moments, displacement. So in this subject on Mechanics we will discuss how forces or moments can influence the response of different types of bodies.

Well it will be also very useful to look at the development of this important subject through the ages. The earliest record on mechanics was due to philosophers like Aristotle, Plato, etcetera, who discussed some concepts of mechanics from philosophy or metaphysical point of view.

But Mechanics, as we know today, the foundations of this subject were initially laid by Archimedes. You are all familiar with his Law of Buoyancy and his work on lever and

other machines that was about two hundred and fifty years before Christ and the development was very slow after that, till about middle ages, that is fifteenth or sixteenth century when astronomers like Kepler from Netherlands studied the motion of heavenly bodies, planets and etcetera. This was followed by the work of Galileo who investigated on the motion of falling bodies and he had some important ideas about how the various types of bodies fall from different heights and what is the time taken by them, why it looks to be different from, let's say, a feather like body and solid body like brick but the real progression of mechanics is attributed to Newton, the English mathematician, physicist who formulated the laws of motion and then important contributions are made by Euler Lagrange and other mathematicians. Throughout the development of history of mechanics you will notice that mathematicians have contributed a lot in the development of mechanics and there has been lot of interaction between mathematicians and experts on mechanics. For example, Newton is also credited with the development of differential and integer calculus and similarly Euler and Lagrange also made important contributions in theory of equations, differential equation as well as algebraic equation, etcetera. So the developments of mechanics are very much the work of various scholars and mathematicians.

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Newtonian Mechanics
Compared to :

- Relativistic mechanic
- Quantum mechanics

Idealization of Mechanics

1. Particles
2. Finite bodies
 - a). Non-deformable (Rigid)
 - b). Deformable (solids, fluids etc.)

Mechanics of particles and rigid bodies:

1. Statics
2. Dynamics

Next is the development in the last century or century and a half where the domain of mechanics was enlarged tremendously by the introduction of relativistic mechanics by Albert Einstein and quantum mechanics by Max Planck, etcetera. We will be, in this course, mostly concentrating on Newtonian mechanics and this is sufficient for most of the engineering applications but it's useful to know what are the zones of application or domains of applications of relativistic mechanics and quantum mechanics.

When the speed of moving bodies is very large, that is comparable to the speed of light, then Newtonian mechanics breaks down and we have to resort to relativistic mechanics. On the other extreme when the size of the bodies is very, very small, let's say, on the size of molecules or atoms or even subatomic particles, etcetera, then again Newtonian mechanics breaks down and we take help from quantum mechanics. We will not be discussing both relativistic mechanics and quantum mechanics in our present course on Newtonian mechanics. Now let's, from now onward, concentrate on Newtonian mechanics.

Before I go further let me tell you that it is useful or very helpful to idealize certain concepts in mechanics and I will be discussing what are the various idealizations which are made in investigations on mechanics. One is particles. Particle theoretically means a point, that is, an entity which has no dimensions or very, very negligible dimensions, length, breadth, height, etcetera. So geometrically it is just a point although we know that every body, every substance in this universe has some size but from the mathematical point of view we neglect the size and we concentrate all its movement to a point.

When it is to be done, when the distances traveled by the body are much larger than the size of the body itself then we can approximate that body as a particle. Sometimes even heavenly bodies like earth or other planets are also treated like particles although we know that their size is very, very large but the distance traveled by, let's say, earth around sun is much, much larger than the diameter or circumference of our planet earth. Similarly when this size is comparable to the distances traveled then we will take them as finite bodies and again, within the realm of finite bodies, we will distinguish them as

non-deformable or rigid bodies and second category is deformable bodies like solid, fluids, gas, elastic materials, etcetera. Now, well first, I will quickly talk about deformable bodies like solids and fluids. Normally they are discussed under solid mechanics, fluid mechanics or continuum mechanics and these are separate courses which you most of you will be taking in your higher classes. So we will be in this course concentrating on non-deformable or rigid bodies.

Again there is an idealization involved in it. The idealization is that everybody under the action of large enough force deforms to some extent. Some bodies deform to a great extent like a rubber, for example. A common experience that we can squeeze a ball of rubber whereas we cannot squeeze a ball of steel or wood or something like that because although there is a deformation, the deformation is so small that you have to observe it under some magnification. So ignoring that we will treat if the deformations are much smaller than the size of the body, we will treat the body as a rigid body.

So in our course on mechanics we will be discussing the mechanics of particles and rigid bodies, that is, we will completely ignore or exclude deformable bodies in this course. Well, this mechanics we will divide into two components. One is statics, that is, the study of bodies at rest and the other is dynamics, when the bodies are in motion. Now after this introduction to mechanics I will revise or revisit another important aspect which I am sure you have learnt earlier also, that is, the dimensions. Dimensions are properties assigned to every physical quantity depending on its role and nature.

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DIMENSIONS

Dimensions: Property assigned to every physical quantity depending on its role and nature.

Two types of Dimensions:

a) Basic or Primary Dimensions

- 1) Length (L)
- 2) Time (T)
- 3) Mass (M)

b) Derived or Secondary Dimension

Velocity (V) = $\frac{L}{T}$ Area (A) = L^2

Mass density = $\frac{M}{L^3}$

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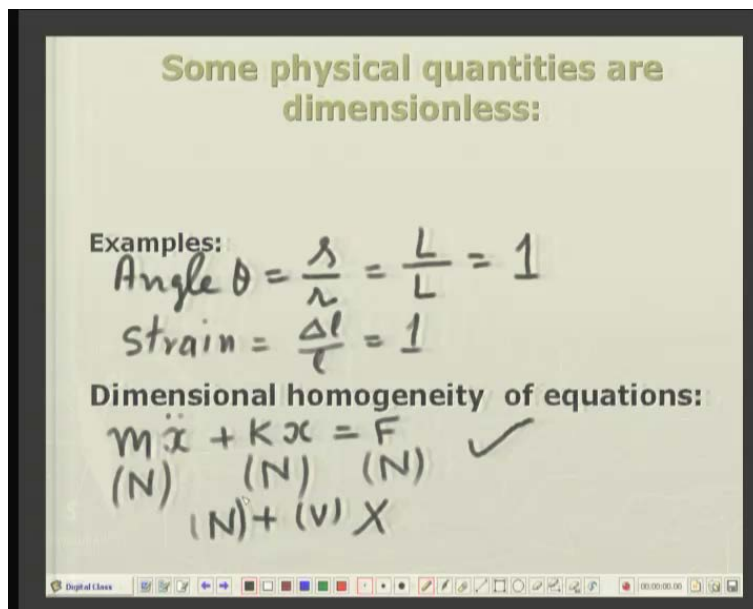
Well some properties are related to the size, some are related to the difference between the two events, etcetera. So we will study them as dimensions and basically we can categorize various dimensions into two types. One is basic or primary dimensions. For example, in basic dimensions we have length which I will abbreviate as L. Okay. Length, well for that matter, breadth, thickness, they will be treated of the same type. They indicate to us the size of the body. So the first basic dimension is representative of the size of the body. Number two is time, which I will abbreviate as T. Well, time is very difficult to define but we are all aware of it right from our childhood. Time means the ordering of event. We say that if one event has happened earlier than the second event, we say that the occurrence of the first event was at an earlier time as compared to the occurrence of the second event. So you can say time is the ordering of the occurrence.

And the third dimension is mass. We will abbreviate it as M. It's very difficult to define mass as such. Well we can say that mass is the amount of matter in a body. It is different from the volume as well as from weight although to some extent we can say that it depends upon the response of a body to, when some forces are applied to it. So in that sense, weight is the response of a mass to gravitational force. So there is some connection between mass and weight but not an absolute connection. The second type of dimensions are derived and are secondary dimensions they are expressed in terms of

basic dimensions. For example, again, velocity, you know, is distance traveled by unit time. So which I will write it as V. Velocity 'V' is equal to distance which has a dimension of L and time as dimension T. So velocity has been expressed in terms of two basic dimensions and it is written as L over T.

Let's take a mass density, another example. Mass density is mass per unit volume. Well, we can write it as mass M divided by volume is length into breadth into height, so it has L cube. Okay. So this is another secondary or derived dimension. You can say, another example, area A is length into breadth, so L square. It has a dimension of L square. So in this way there are a very large number of dimensions which can be expressed in terms of basic or primary dimensions. Well there are some physical quantities which don't have any dimension. They are dimensionless.

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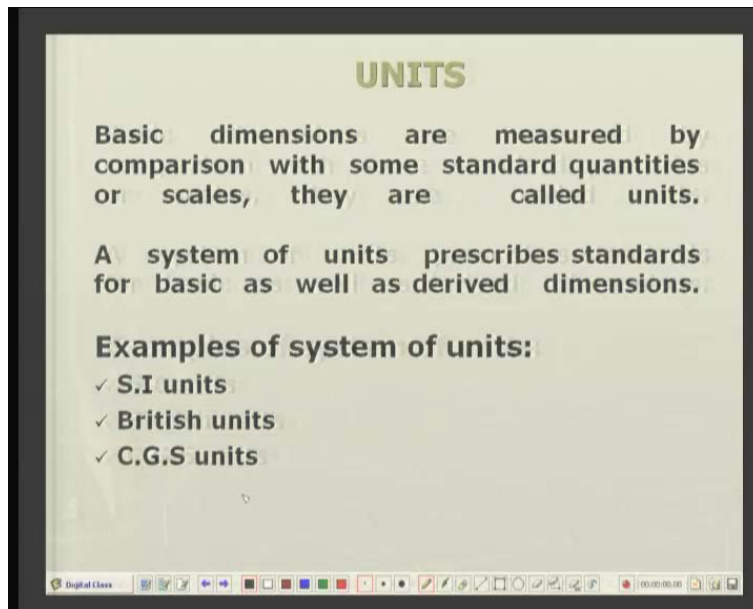
For example, angle, let's say, theta, well you know, that angle theta is arc length s divided by the radius r which is arc length is also having dimension L and radius r also having dimension L. So it means it is dimensionless. Another example, we can take strain, for example. Strain is change in length per unit length, so delta L over L. So both have dimensions of length. So again it is dimensionless. There is one important aspect

about dimensions which you have to always remember, that is, if whenever you write any question it has to be dimensionally homogeneous. For example, in an equation, the right hand side has two terms and left hand side has, for example, one term. Let me take the case of motion of a mass under the action of spring forces. So, for example, mass 'm' times its acceleration x double dot. We will learn this acceleration, plus k times x which is the force due to stretch or compression of the spring is equal to the applied force F . Now this equation on the left hand side has two terms, on the right hand side it has one term. Mass times acceleration, as you will learn in Dynamics is having the dimensions of force. It is a derived dimension. You will learn very soon its dimensions are N. Okay.

Newton. K times x . K is the spring constant, that is, force per unit deflection into x is deflection. Again it will be having dimensions of N and of course on the right hand side there is applied force or forcing function. So it has also dimension N. So in this equation all the terms involved have the same dimension, so it is dimensionally homogeneous.

For example, you cannot let me give you an example when the equation is not dimensionally homogeneous. Let me say that you have one term which has dimensions of Newton's force dimension and if you add another term which is proportional to velocity then it will have dimensions of velocity. So such an equation is dimensionally non homogeneous. So you can be showed that this is a wrong equation when there is no dimensional consistency. You should look for some error in it. Okay. Now let's come to the next important topic, Units.

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How do I measure, let's say, the length of this table? I will take a meter rod and place it against the edge of the table and see how many times I have to shift this standard length and that will determine the length of the table. So what I want to say is that the basic dimensions like length, etcetera, are measured by comparison with some standard quantities or scales and this comparison gives us units. If you use different scales or different standards you will get different units. Now for all the basic dimensions, engineers and thesis have evolved certain scales or standards and the collection of these scales are called system of units. There are basically three prevalent systems of units. One is SI units, that is, international standard units or standard international units, British units and CGS units. In India we have been using SI units and all over the world also they are accepted. British units have limited appeal, that is, in Britain or in America and few other countries they are still prevalent and CGS units are mostly used in physics topics. Let me compare two types of units.

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Dimensions	S.I.	C.G.S.
Length	meter (m)	Centimeter (cm)
Mass	Kilogram (kg)	gram (gm)
Time	second (s)	second (s)
Force	Newton (N)	dyne

One is SI units, the other is CGS units. In SI units, length is measured as meter and it is generally written as m. In CGS units it is written as centimeter and it is abbreviated as cm. Mass in SI units is designated as kilogram or kg and in the CGS units, it is gram or gm. Time in both the SI as well as CGS units is second and is generally abbreviated as s. So same here second and s. Although force is not a basic unit we have known that it is a derived unit or secondary unit but because of its importance we will write in this table. Force in SI units is designated as Newton, after the great mathematician and physicist Sir Isaac Newton, who laid the important contributions to the foundations of mechanics and this is abbreviated as N and in the CGS unit the unit of force is dyne. Okay. So these are the various units under the two systems, the international standard system and CGS system.

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CONVERSION OF UNITS

A basic dimension like length(L) may be expressed in two different units:

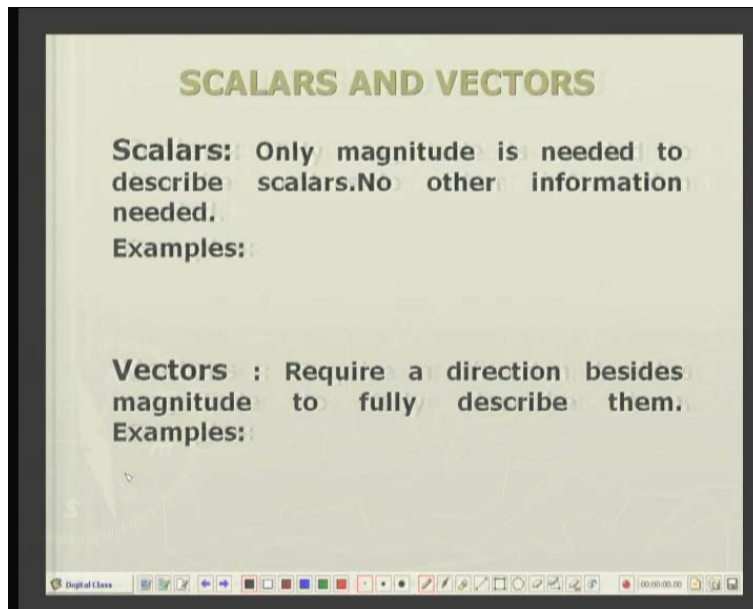
$$L = n_1 u_1 = n_2 u_2$$
$$\Rightarrow n_1 / n_2 = u_2 / u_1$$

$u_2 / u_1 =$ conversion factor.
meter/centimeter = 100;
meter/foot = 3.281;
dyne/Newton = 10^{-5} ;

Now very often we may be required to convert from one system of units to another system. Let's say from CGS to SI or vice versa or SI to British or vice versa. Now whenever I write force is hundred Newton, it has two parts. Hundred is just a number and N is the unit. So similarly length may be one meter or one point five meter. So there is number n_1 , time the unit and the same length, let's say, it is hundred centimeters, one meter or hundred centimeters. So n_2 will be hundred, that is, the number and u_2 is the centimeter. So looking at the equation $n_1 u_1$ is equal to $n_2 u_2$. This implies that n_1 over n_2 is u_2 over u_1 . Okay.

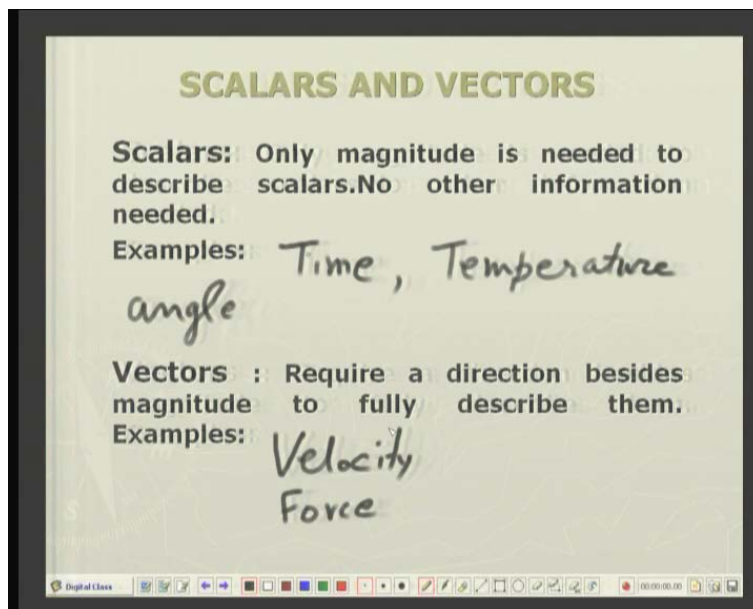
So the ratio of the numbers is equal to the inverse ratio of the corresponding units and this u_2 over u_1 is called the conversion factor. For example, the even converting from the SI units to CGS units, so meter over centimeter has a ratio of hundred or if I am converting from a meter to SI unit to British units, meter over foot is three point eight one. And similarly, here for the force. So it is always possible to go from one system of unit to the other through the help of conversion factors and many books and many tables are available which will give you the conversion factor from between any pair of units for almost all physical or other quantities.

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Now let's go to the aspect or another preliminary concept which you must have learnt in your earlier courses and I will try to revise it and elaborate it. One is scalars and vectors. Now scalars, they require only magnitude to describe the quantity, no other information is needed. Example of scalar is time. Okay.

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Well I don't have to say the time is in this direction or that direction. Just the number one hour or thirty seconds or two point five second or micro second is sufficient to give you the full information about what I am talking about. The other thing is temperature. Again, just see the number and the unit or scale, like a ten degree Celsius or twenty degree height, etcetera. No other information is needed to convey the full meaning and similarly angles, whether degrees or radian, it's sufficient to understand. On the other hand, vectors, just specifying the magnitude are not sufficient. You have to give further information, that is, the direction. So a vector needs both magnitude and direction to fully explain it.

Example is, let's say, velocity. We can say that the velocity of a car is hundred meters per second or something like that. Well, what does it mean? Okay. You know how much distance the car is traveling in, let's say, ten seconds or so. But that's not sufficient. I want to know whether it is going towards east or west or south west or north east, etcetera. So if I don't give that information my knowledge about the movement of car is incomplete. So I will say a given car is moving at a 'so and so' velocity from this direction to that direction. Okay. From this point to the other point. So that will define me, let's say, towards east or towards west, so that will give me the complete information about the movement of the car. Another is force, okay. Whether I am applying force upward or downward or sideways, etcetera. So besides specifying that the applied force is hundred Newtons, I have to also say which direction or which way it is acting. So velocity, force, acceleration, etcetera, they are incomplete, if I don't prescribe their direction or vection. Now how do we specify the vectors while writing them? There are several possibilities.

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Representation of Vectors

a) **Written as :**

$\vec{V}, \bar{V}, \underset{\sim}{V}$ or **V**

b) **Graphically:**

c) **Using reference co-ordinates:**
Most commonly used and simplest co-ordinates are rectangular cartesian co-ordinates.

One is, you write a symbol, let's say, **V** and then put an arrow, which will distinguish it from a scalar quantity. Sometimes instead of an arrow some people would like to put a bar or a tilda below it or in books when it's a question of printing it, the vectors are distinguished from the scalar quantities by putting vectors in a bold or capital font. For example, here capital **V** is quite bold as compared to others. So this will be distinguished as a vector quantity. Graphically, we can prescribe vectors as a line. The length of the line is to be chosen properly. You choose a scale, let's say, one centimeter represents ten Newton's and if the force of fifty Newton's is to be written up, then you draw a line of five centimeters and then to indicate the direction you draw an arrow so that now the description of the graphical description is complete. Here is an arrow whose length is proportional to the magnitude of the quantity and the direction of the arrow is in the same sense as the actual applied force or velocity. Incidentally this is called the tip of the arrow and this is the tail of the arrow. So the distance between tip and tail is the length of the arrow which is proportional to the magnitude.

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Vector Components

$$\vec{V} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

=> v_x, v_y and v_z are Cartesian Components of \vec{V} .

$$A'B' = \sqrt{v_x^2 + v_y^2}$$

$$AB = \sqrt{(A'B')^2 + v_z^2} = \sqrt{v_x^2 + v_y^2 + v_z^2} = |\vec{V}|$$

$|\vec{V}|$ = Magnitude of \vec{V} .

Now the third way of describing a vector quantity is to use reference coordinates. For example, let's say, I will draw three mutually perpendicular directions. These directions describe the three dimensional Euclidian space in which we'll live, that is, one direction which I will label as x, the other direction is y and the third direction is z.

Now the choice of xyz is not arbitrary. It follows what is known as the right handed screw system for labeling the coordinate axis. Incidentally xyz, the three directions are called the coordinate axis. Rectangular Cartesian coordinates, these are. Now the labeling of xyz is like this from x to y, when I rotate, then the z direction will point in the same sense as a right handed screw will move. For example, if I rotate a screw driver, if it is a right handed screw, if it is moving a right handed screw then the screw will go up. A left handed screw, if I give a same rotation it will go down but internationally the convention used is the right handed screw system and then a point P in this three dimensional space will be labeled as, let's say, point P xyz. What it means is if I go along x axis through a distance x, then I go a distance y parallel to y axis and then I go upward from this point to this. This is x, y, z. Then I am starting from O, I have reached the point P. So first I'll go to x axis, second y axis and third is z axis. So these are the three coordinates.

So this is how the coordinates have to be understood and this is how the labeling has to be done according to the right handed screw system. Now suppose, there is a vector which I have represented as AB . It represents the vector V . Then this vector V can be written as v_x into unit vector 'i'. I will explain. Plus v_y into unit vector 'j', v_z into unit vector 'k'. Now what are these vectors. xyz is the rectangular Cartesian coordinate system, right handed, as we have observed. Then along x axis I consider a vector of magnitude one. Let's say, this is the vector of magnitude one. Okay.

This, we label it as vector 'i' and the unit vector, to distinguish from others, I will write it as 'i hat'. Hat will be always reserved for unit vector. Similarly along y axis I will have a unit magnitude, vector 'j' hat and along z axis again a vector of length one unit, which I will label as 'k' hat. Okay. So i hat, j hat, k hat are the unit vectors. Then, let's say, this vector AB , I will draw projections from point A and point B on to the xy plain. That is, I draw perpendiculars. And similarly from B , I will draw a perpendicular and this will be the projection of vector V on to the xy plain and further this projection I will label it as A dash B dash, okay. And then I can draw projections of A dash B dash on the x axis and y axis. For example, again I will have similarly on the y axis. Then this projection is v_x and this projection is v_y . Okay.

So I have one projection v_x on the x axis, the other projection v_y on the y axis. These are the components of vector V along x axis and component along y axis. Similarly, to draw the projection of AB on the z axis, you will have all. I have to extend this direction for this will be v_z . So this is how all the three components of a given vector V are obtained and it's very easy to see that the length of the projection A dash B dash is by Pythagoras theorem, obtained from v_x and v_y by using the Pythagoras theorem, v_x square plus v_y square whole under root and ones you have got A dash B dash then it can be combined with the component v_z . So A dash B dash square plus v_z square will give me the final vector AB . So, in other words, the length of vector AB is v_x square plus v_y square plus v_z square whole under root and this is labeled as the absolute value or magnitude of vector V . Okay. So magnitude of V is the sum of the squares of all the three components whole under root.

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Vector Algebra

Deals with mathematical operations (addition, subtraction, multiplication etc.) for vector.

Consider two vectors

$$\vec{V} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$
$$\vec{U} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$$

Geometrical representation of various operations is also given.

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Now we start with another important aspect of vector analysis namely vector algebra. Vector algebra deals with some mathematical operations like addition, subtraction, multiplication, etcetera. For vectors, for this purpose, we will consider two vectors. One is vector V, its components are v_x , v_y and v_z . So accordingly it is written as v_x i unit vector, v_y times j unit vector, v_z times k unit vector. The second vector is U vector and it's written as u_x i unit vector, u_y j unit vector, u_z k unit vector. Now we will be discussing these mathematical operations both analytically and from geometrical representation point of view.

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Addition of Vectors

$$\vec{W} = \vec{U} + \vec{V} = (u_x \hat{i} + u_y \hat{j} + u_z \hat{k}) + (v_x \hat{i} + v_y \hat{j} + v_z \hat{k})$$
$$= (u_x + v_x) \hat{i} + (u_y + v_y) \hat{j} + (u_z + v_z) \hat{k}$$

$\therefore w_x = u_x + v_x$

$w_y = u_y + v_y$

$w_z = u_z + v_z$

Some Properties of Vector Addition

i) $\vec{U} + \vec{V} = \vec{V} + \vec{U}$ **Commutative law**

ii) $\vec{U} + \vec{V} + \vec{W} = (\vec{U} + \vec{V}) + \vec{W}$
 $= \vec{U} + (\vec{V} + \vec{W})$ **Associative law**

Let's first take up vector algebra. Addition of vectors:- Two vectors U plus vector V, the sum is vector W and this is written as $u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$, plus $u_x \hat{i} + v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$. You add up their corresponding components, that is, $u_x + v_x$. This is $v_x \hat{i}$ unit vector plus $u_y + v_y$, \hat{j} unit vector plus $u_z + v_z$, \hat{k} unit vector, that is, the individual components are added so that the sum vector will have the components w_x which is equal to $u_x + v_x$, w_y is equal to $u_y + v_y$, w_z is equal to $u_z + v_z$. So the sum vector is obtained very easily as seen above.

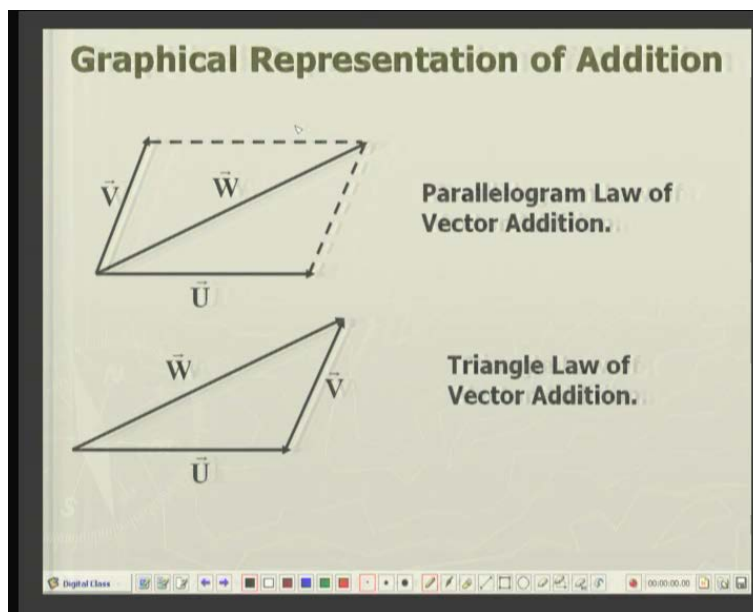
Now, let's have a look at some of the properties of the vector addition operation. Number one is that the addition operation is commutative, that is, the order of addition does not matter. U plus V is equal to V plus U. And second is associative law which means that U plus V plus W vectors is equal to first you add the first two vectors U plus V and their sum is added to W or you can add the first vector plus the sum of the next two vectors U plus the sum of V plus W.

Now they look to be very simple laws but sometimes the commutative law is not applicable. For example, consider finite rotation. The magnitude of rotation may be one radian or half a radian or sixty degree and the direction may be the axis of rotation. So, as

such, finite rotation looks to be a vector quantity because it has both magnitude and direction but it is not actually a vector quantity because the sum of two rotations is not commutative. For example, I take this book, let's say, this book has x axis, y axis and z axis. First I give a rotation, anti clockwise rotation of ninety degree about x axis which will bring the book in this position and then I give a ninety degree rotation about y axis, so it will bring the book in this position. So the final position of these two rotations is like this. So if I bring it back to the original position and now I give the rotation first about the y axis, ninety degrees, anti-clockwise. So it will be something like this and then I give a rotation of ninety degrees about the x axis. It will be like this.

So the two final positions are not congruent. So it means the order of operations of rotations is very crucial. So the finite rotation is not a vector quantity although it appears to be a vector quantity. However, later on, we can show that very small rotations, infinite infinitesimal rotations. They add commutatively and hence their vector quantity and as a result the angular velocity which is an infinitesimal rotation is divided by infinitesimal time. That is a vector quantity.

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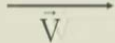
Next we look at the addition operation graphically. Suppose a vector U is given here and a vector V is given like this. Now the sum of these two vectors can be obtained by first drawing the vector U and vector V and completing the parallelogram, that is, this line is parallel to the V vector line and this line is parallel to the U vector line and you join the diagonal that represents the sum vector W . This is called the parallelogram law of vector addition.

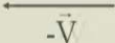
Alternatively, the vector addition can be depicted. Here is vector U , that is, magnitude wise and direction wise and at the tip of vector U you draw the vector V , again, direction wise and magnitude wise. And the third side of the triangle to complete the triangle will give me the sum vector W and this is called the triangle law of vector addition. So analytically as well as graphically we can obtain the sum of two vectors.

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Subtraction of Vectors

Negative vector : Same magnitude but opposite sense.

\vec{V}


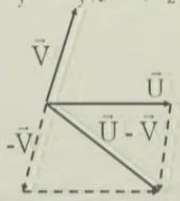
$-\vec{V}$


is the difference of two vectors.

$$\vec{W} = \vec{U} - \vec{V} = \vec{U} + (-\vec{V})$$

$$\therefore \vec{W} = (u_x - v_x)\hat{i} + (u_y - v_y)\hat{j} + (u_z - v_z)\hat{k}$$

Graphically:



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Now I come to subtraction of vectors. Well it is very easy. First of all, we will consider what the negative vector is. Suppose vector V is given. Over here magnitude and direction wise, then minus V vector, that is, the negative of V vector is the vector having the same magnitude and the same line of action but the sense of operation of this vector is reversed. If it is to the right, minus V vector is towards left, then the difference

between two vectors, U and V, which we will represent as the W vector is given by the sum of U vector and the negative V vector obviously.

So we can write that the vector W is equal to component wise u_x minus v_x i unit vector minus u_y minus v_y times j unit vector plus u_z minus v_z times k unit vector. So the components of the difference vector W are obtained respectively by taking the difference of the corresponding components, that is, u_x minus v_x , u_y minus v_y , u_z minus v_z . Graphically again we can look at it. Here is the U vector, here is the V vector, both direction wise and magnitude wise and the reverse by extending the line in the opposition direction we will get the minus V vector.

So you continue this line and then you complete the parallelogram and the diagonal of this parallelogram shown here, will give me the difference vector, U minus V vector. So both addition and subtraction operations are quite easily understood.

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Multiplication of Vectors

Three operations are possible

- a) Scalar Multiplication by a scalar.
- b) Scalar or Dot Product.
- c) Vector or Cross Product.

Multiplication by a scalar ($c\vec{V}$):

\vec{V} $c\vec{V}$

Property:

$$(a + b)\vec{V} = a\vec{V} + b\vec{V}$$

Now next we come to the multiplication of vectors. In vector analysis, three types of multiplication operations are possible. One is the scalar multiplication or multiplication by a scalar quantity, second is dot product or also called scalar product and the third

operation is cross product also called the vector product. First I will take up the scalar multiplication. Suppose a vector V is given ground, according to the magnitude and direction here then c , which is, c is just a number, a scalar quantity, no direction assigned to it. So c times V , that is, the scalar product of c with V is just that the magnitude is multiplied by c but the direction remains the original direction, that is, the direction of vector V .

So the length of the vector is increased or decreased depending upon the value of c . Suppose c is two, then the length is double, c is, let's say, one by four, then the length is made one fourth. So obviously you can check that a plus b into vector V is equal to, first you multiply V vector with a and then you multiply V vector V vector with b and then you add up the two vectors so obtained and to get the resultant.

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Scalar or Dot Product

$$\vec{U} \cdot \vec{V} = |\vec{U}| |\vec{V}| \cos \theta$$

θ is taken as the smaller angle between the two vectors.

Special cases of Dot Product:

If $\vec{U} \perp \vec{V}$, then $\vec{U} \cdot \vec{V} = 0$.

If $\vec{U} \parallel \vec{V}$, then $\vec{U} \cdot \vec{V} = |\vec{U}| |\vec{V}|$

Therefore

$$\vec{U} \cdot \vec{U} = |\vec{U}|^2 \text{ or } |\vec{U}| = \sqrt{\vec{U} \cdot \vec{U}}$$

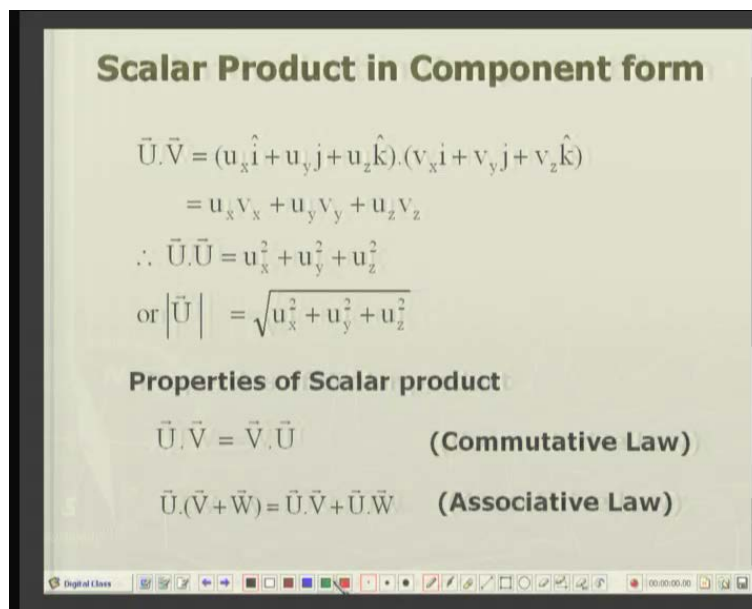
Next multiplication operation is the scalar or dot product. Well it is defined like this. Suppose two vectors are given, vector U and vector V . First you take the magnitude of U vector and then the magnitude of V vector, multiply the two and then this product is multiplied by the cosine of an angle theta. Theta is taken as the smaller angle between the two vectors. See here is vector U , here is vector V . Now between these two vectors, one

angle is less than ninety degrees, the other angle is greater than ninety degree. So we take the angle which is less than ninety degree and take its cos. So $\vec{U} \cdot \vec{V}$ is the product of three quantities, magnitude of \vec{U} , magnitude of \vec{V} and cosine of the smaller angle. And since it's a product of three numbers, the result is also a scalar quantity, just a magnitude. Hence the word scalar product. Now let's look at some special cases of dot product or scalar product. When two vectors are mutually perpendicular to each other, that is, the angle between the two vectors is ninety degree, and then the cosine of that angle is zero. So you can easily see $\vec{U} \cdot \vec{V}$, that is, the dot product of \vec{U} and \vec{V} is equal to zero.

Suppose \vec{U} and \vec{V} are parallel to each other. All right then. The angle between the two is zero degree, so cosine of zero is one. So obviously $\vec{U} \cdot \vec{V}$ is equal to the product of the two magnitudes because the third term is just one. A special case of this is that \vec{U} is dotted with itself. So $\vec{U} \cdot \vec{U}$. Again it's a case of product of two parallel vectors. So the result is the magnitude of \vec{U} squared or inversely we can say that the magnitude of \vec{U} is equal to the dot product of \vec{U} with itself taken square root of.

So that also gives the definition of or a way to calculate the magnitude of a given vector.

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Scalar Product in Component form

$$\vec{U} \cdot \vec{V} = (u_x \hat{i} + u_y \hat{j} + u_z \hat{k}) \cdot (v_x \hat{i} + v_y \hat{j} + v_z \hat{k})$$

$$= u_x v_x + u_y v_y + u_z v_z$$

$$\therefore \vec{U} \cdot \vec{U} = u_x^2 + u_y^2 + u_z^2$$

$$\text{or } |\vec{U}| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

Properties of Scalar product

$$\vec{U} \cdot \vec{V} = \vec{V} \cdot \vec{U} \quad \text{(Commutative Law)}$$

$$\vec{U} \cdot (\vec{V} + \vec{W}) = \vec{U} \cdot \vec{V} + \vec{U} \cdot \vec{W} \quad \text{(Associative Law)}$$

Now let's have a look at the scalar product in component form. Again same U and V vectors. Two of them. They are represented as u_x multiplied by i unit vector, plus u_y multiplied by j unit vector, plus u_z multiplied by k unit vector and similarly for vector V , what you do is, you multiply each term, one by one, so $u_x i$ times v_x . So here will be the product of the magnitudes u_x, v_x , then $i \cdot i$. So since these are parallel vectors, it is equal to magnitude of the i vector, that is, one. So $u_x v_x$ plus $u_y v_y$, $u_z v_z$ because i multiplied by j dotted with j or i multiplied dotted with k will give me zero.

So, only the similar terms will multiply and non-similar terms will be equal to zero. You can say that U dotted with b is equal to the sum of the products of the corresponding components $u_x v_x$ plus $u_y v_y$ plus $u_z v_z$. As a simple corollary to it, u dotted with the u itself is u_x^2 plus u_y^2 plus u_z^2 . Hence the magnitude of u vector is equal to the square root of u_x^2 plus u_y^2 plus u_z^2 . As before, let's have a look at the properties of scalar products. Well, first of all, the order again does not matter. U dotted with V is equal to V dotted with U , that is, the commutative law for vectors is holds and similarly the associative law, U dotted with the sum of two vectors V plus W vectors is equal to the sum of the product of U with V and U with W . That U dotted with V plus U dotted with W . So this is called the associative law. So very useful and very simple properties of the dot product. In this lecture we will close the discussion on vector algebra. At this stage we have already discussed two types of vector multiplications and the third type we will take up the in the next lecture.