Non-conventional Energy Resources
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Lecture – 22
Wind Energy Efficiency

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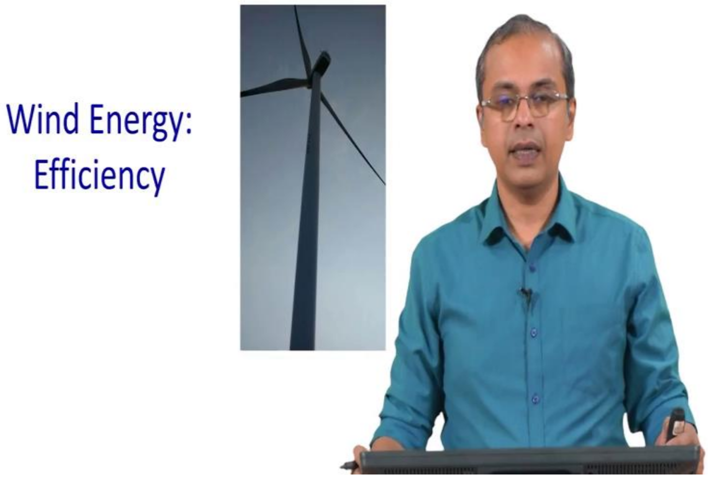

Hello, these past few classes we have been talking about wind energy and we will continue in the same line of discussion. And we started by looking at an overview we looked at you know what is possible with in wind energy, what are these various aspects associated with it, how people laid out, how these windmills are located and all those aspects we looked at. We then looked at you know various energy considerations associated with it, particularly we understood the fact that the velocity of the wind is a very critical factor in determining how much power is there in the wind. And it’s not even a linear function it’s a function where it is the power is associated with the cube of the velocity and therefore, the if you know double the velocity of the wind the power that is available in the wind goes up by a factor of eight and so, therefore, that is very significant, it is not you know normally we talk of an increase which is 10 percent, increase 20 percent increase something like that is what we you know we are looking at.

But here you are looking at a you know 300 percent increase or or more I mean fact more than that you are looking at in this case you know you doubled it and it went up by

400 percent, so based on the cube of that quantity. So, therefore, and therefore, you have to keep that in mind when you make the decision on where to locate the windmill because the wind is a very strong, I mean there is a very strong function with the velocity of the wind. Even though you have many locations where you will have wind if you have to actually do a fairly careful mapping of the location to understand if that is really ideally suited for the location of that windmill.

We also spoke about the fact that you know we do have these days even buildings where on top of the building you can locate a windmill. So, various ideas are considered of course, there is an infrastructure cost associated with it and so on and at the end of the day there is also some economics associated with it, people are always trying to figure out if a particular solution to our energy crisis also fits in in the grand scheme of the economics of it. The factors that are really positive towards the wind energy tapping are the fact that it is clean, there is no real pollution except you know when will you maybe manufacture the windmill blades or something where you have to again take a look at what chemicals are used, but generally it is all very clean and once it's up there it is just you know you are not really burning anything, breeze is anyway blowing you are just tapping the energy. So, a lot of countries are pushing hard to you know increase the amount of capture of wind energy across the length and breadth of the country and India is not an exception we are the you know the 4th largest or such efforts internationally and in that Tamilnadu is the leading state in terms of how much of these wind farms are present.

So, in today's class we are basically going to look at the efficiency aspect of it. You have energy in the wind you have some power in the wind, what is the efficiency associated with the process in capturing this in through using a windmill.

We already mentioned it in one of our previous classes as the Betz limit we were going to actually look at it in much greater detail in this class.

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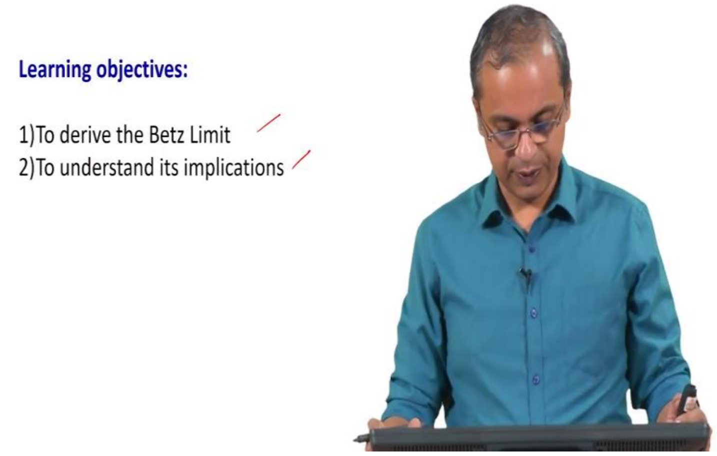

So, our learning objectives for this class are to derive the Betz limit and to also understand its implications.

So, we will go through this derivation that also I think brings together just the way we derived you know how much energy is there in the wind. If you go through this derivation you will have a better sense of I know what are all the parameters that people you need to consider when you are just looking at this kind of a energy process okay. So, this is what we will try to do.

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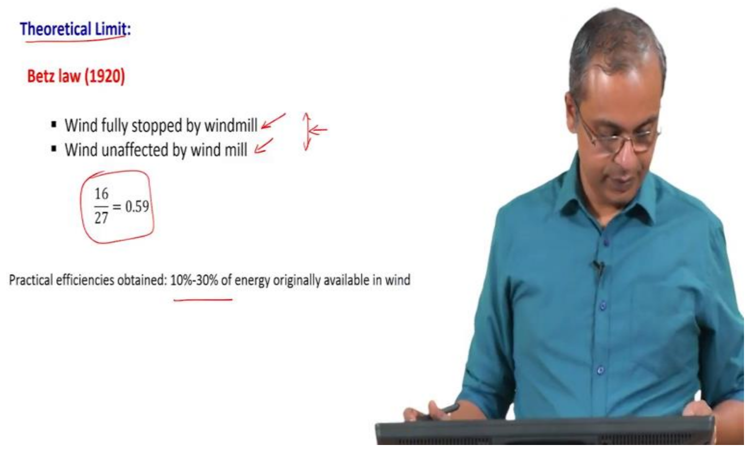

This as I mentioned is some kind of a theoretical limit on what you can extract from you know blowing breeze or blowing wind. Of course, there are some limits to this limit even if you want to look at it that way because like with any other thing there are some assumptions based on which this limit is derived and you can always think of some way in which you can maybe you know violate those assumptions or work around those assumptions and then therefore, this limit may not be you know completely the absolute upper end of it. But it is still considered a very important law because pretty much all the windmills that are out there, a very large fraction of the windmills that are out there will sort of are designed in such a way that they sort of confirm that their design is that inherently they fall within the ambit of this law. So, they have not, they don't have any new design features which are in violation of this law so to speak, but largely they are in this inconsistent with this law.

So, we also mentioned that you know you can consider two extremes you have a windmill and you have wind blowing towards the windmill. So, you have two extreme possibilities one is that the wind is completely stopped by the windmill and therefore, it would appear inherently that you know; that means, all the energy in the wind got transferred to the windmill and therefore, got captured as energy that you could use somewhere. So, that seems like the best possible way in which you would want to do it because hundred percent of the energy is captured.

But in practical situation that is not what happens because when you say the wind is 100 percent stopped it is not disappearing anywhere it is stopped there I mean. So, it is just basically come and stopped in front of the windmill. So, it is not gone past the windmill. So, a subsequent wind that is coming there is not in a position to reach the windmill because the wind that came there first is parked there all those molecules that arrived that are parked right there in front of the windmill they have not gone anywhere. So, the wind that's coming behind cannot reach the windmill and therefore, even if you think of this as a process hypothetically it can it can at most happen at one instant at that at the beginning moment it can happen after that by itself it will self destruct it will basically stop the whole process from occurring. So, therefore, a wind fully stopped by the windmill is not a way in which the windmill can you know effectively capture the energy for us and it represents one extreme of how the wind might interact with the windmill. It is coming and it's completely stopped wherever.

The other extreme is the fact that the wind is effect unaffected by the windmill. So, in the first case the wind is fully affected by the windmill it comes to a full halt. The second case that hypothec a hypothetical case that we are considering the wind is completely unaffected by the windmill and just goes past the windmill completely unaffected in any manner. When you say unaffected it primarily means that no energy has been transferred from the wind to the windmill right. So, absolutely no energy has been transferred from the wind to the windmill.

So, again the energy that will be captured in the version 2 is 0 basically there is no energy being captured version 1 instantaneously something might be captured, but from that moment onwards 0 is captured. So, these two represent two extremes of how the wind might interact with the windmill and in both cases the energy captured is 0. So, naturally you can you know visualize that there will be a range of possibilities in between these two which is where the amount of energy that is transferred from the wind to the windmill is steadily increasing and reaches some kind of a maximum and then goes back down to the other extreme of the process.

So, these are the two extremes in between them is where you can exit somewhere in between, we don't exactly we are somewhere in the middle not exactly them it is somewhere in between these two you can expect some maximum being attained by this during this transfer process. So, we would like to get a sense of what that is. It is also seen that through this law that the limit seems to be about roughly 59.3 percent. So, that number we will see if we can arrive at 0.593 that seems to be the limit that you can extract from the wind which is not bad I mean if you are extracting 60 percent of something that's of the energy that is anyway coming to you free that seems to be a pretty good thing to do. Practical efficiencies are already in the 10 to 30 percent of the energy that's available in the wind. So, if you take into account any other you know in inefficiency in the process. So, you are still getting about 30 percent, which again as I said is excellent because you are doing nothing. I mean I mean either wind this anywhere blowing you are not doing anything about it you just put a windmill and keeps generating that electricity for you right.

So, so his is the framework of the Betz law and this is what we will try to derive in our class today.

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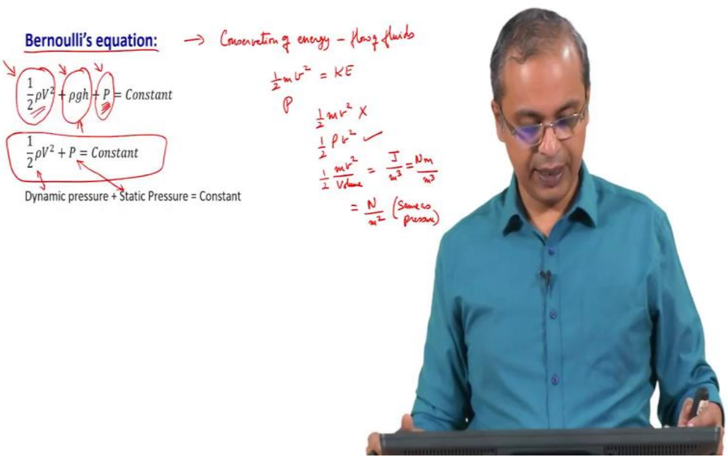

So, to derive the Betz law we need to start off with an equation which many of you may have heard of and may have differing levels of familiarity with it, it's basically the Bernoulli's equation. So, the Bernoulli's equation in many ways I mean, again based on your background if you are in mechanical engineering maybe you have spent more time reading about this equation and so on. Basically it is simply equation that represents conservation of energy, but the scheme of you know parameters within which we are looking at this conservation of energy is in the flow of fluids.

So, we always have you know kinetic energy, potential energies. So, we have various forms of energy we say energy is conserved which means you start off with the system that has a total sum total energy which consists of potential energy kinetic energy any other form of energy that might be there in the system. So, with passage of time one form of energy will get converted to another form of energy based on the circumstances. So, maybe potential energy goes up, maybe kinetic energy comes down and something else happens all those things might happen at the end of the day the total energy it will still remain the same we are not lost energy anywhere it will still remain the same okay. So, that is the, if you take the system as a complete you know all entities are involved in that system.

So, this is this is how we look at it we are more familiar with it, in the form of you know day to day objects we will typically would have encountered problems where there is let's say a ball traveling with some velocity and then it reaches some height. So, based on the initial velocity you can decide what height it has reached because whatever kinetic energy was that gets converted to potential energy. So, many such problems we are we are likely to have become familiar with.

The same concept is now used in the in in the case of fluids. So, what we are saying is there are 3, broadly there are 3 parameters to look at something associated with the kinetic energy of that fluid, something associated with the potential energy of the fluid and something associated with pressure of the fluid and the sum of these 3 is a constant is basically what we are saying there are some assumptions here. So, we will at least briefly consider those assumptions. But before we even do that we have quantity here pressure right and somehow we have on this side something that looks like energy right. So, we say half mv square and we associate this with kinetic energy. And you have something like half rho V square here, here you have pressure. So, how come we have pressure? And kinetic energy in the same equation right. So, it looks like there's something wrong with the equation, but actually it's not. If you actually look at what is here you don't have m you have rho. So, we don't have, we don't have half mv square. So, this is not what we have in the equation we have half rho V square this is what is there in the equation.

So, now, let us look at the dimensions of half rho V square you will find that interestingly you will find that we are going to find value which is basically which will show you why it can be on the same side as the pressure. So, what is rho? Rho is simply mass per unit volume right. So, it's the same as half mv square divided by volume correct. So, it's just the rho density is mass per unit volume that's basically what I have put here that is all I have done.

So, that half mv square is kinetic energy. So, that is in Joules, energy in Joules and volume is in meter cube and Joules is nothing, but Newton. So, some force which is in Newton's times the distance that the force has been used to move something right. So, Newton into meter, so so much Newton's you apply, so many meters you push something using that force. So, the work done is that force into distance, so Newton into meter Newton into meter by meter cube. So, this is simply Newton per meter square which is the same unit as pressure right. So, this is the same as pressure okay. So, that is why we have half rho V square.

So, this, you can think of it as energy per unit volume. So, this is basically kinetic energy per unit volume this is potential energy per unit volume and this is pressure and they have the same dimensions. And, so that is how this equation comes together and therefore, they are consistent. So, there's nothing you know suspiciously wrong about the equation or any such thing. So, this is the equation. So, this will be our starting point. So, the kind of assumptions here are that this is this flow is considered laminar. So, the streamlines are you know smooth you don’t have you know liquid running in all different directions and we are in neglecting things like friction that the liquid might face on the surface of any you know pipe that it is flowing through and that it is steady.

So, in other words the flow is steady means if you take one location in that fluid and you observe that location over a period of time then at that location it will always have the same velocity v it will always have the same density etcetera we are in fact, assuming density is constant, but things like that the various parameters associated with that fluid will be constant at that location with time. So, you go to some other location it will be different, but that location also you wait long enough you will see everything being constant okay. So, there are some assumptions here. So, within the context of that assumptions we will do this calculation. So, we will also assume that you know in many of the situations associated with fluid flow and certainly in the case that we are trying to consider there is no variation in height we have a windmill that is standing and there is a breeze that is blowing towards the windmill will assume that the breeze is horizontal and it comes to the windmill and does some does something to the windmill or the windmill has interacts with the breeze in some way.

So, it is horizontal. So, there is no change in height of the of the wind that we are considering of that the mass associated with the wind whatever height it was coming in it continues to maintain that height. So, this factor does not change from time to time. So, what we write on the left hand side would be the same thing that we will write on the right hand side. So, we can neglect it we will just cancel it out and it is irrelevant to us. So, for us that is already a constant there is nothing varying there by itself it is a constant rho g h by itself will be a constant in our case because h will remain a constant. So, this term the second term is irrelevant. So, really the equation that we are working with is this one, half rho V square plus P is a constant okay. So, and in fact, the way they described it in the of fluid flow is that they call the second term as the dynamic pressure and the first term as the I am sorry the first term there is the dynamic pressure and the second term here is referred to as the static pressure.

So, pressure in the context that we typically associate pressure with is the P that you see here, but we are also saying that when the when the when the fluid is flowing there is a because of the fluid flow you can think of a sort of another pressure which is the dynamic pressure which builds because of the fact that the fluid is fluid. So, this is the constant. So, when the velocity of the fluid picks up the since the sum is a constant the static pressure is coming down the dynamic pressure is going up is the way in which we interpret this equation okay. So, this is what we will keep in mind as we go forward and we try to derive the Betz equation.

So, as I said the way we will do it is we will consider that there is some windmill standing here, so some windmills. So, I just draw the we will assume that this represents a windmill that is you know facing the breeze and then, so there is some region. So, there is there is wind that is blowing this way and it goes past the windmill interacts in the windmill and then goes away.

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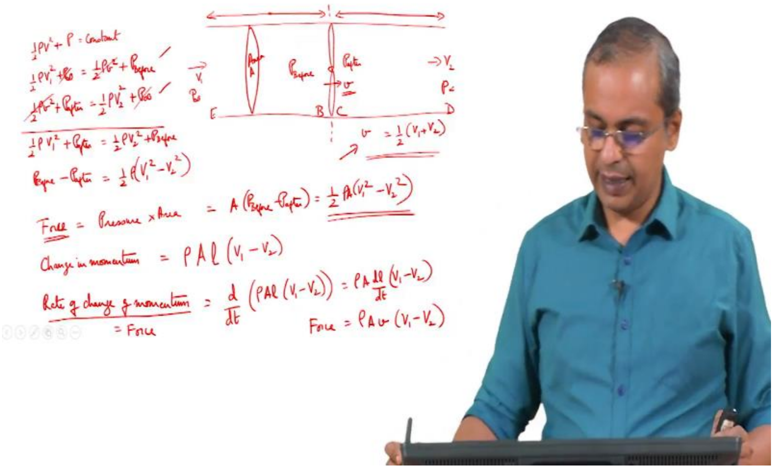

So, coming in we have, the wind is coming in with a velocity V 1 okay and we will say that this is far enough away from the windmill that it is the pressure there is unaffected by the static pressure there is unaffected by the presence of the windmill. So, we will call that P infinity far away from the windmill. And then when you come out on the on this side again let's go far away from the windmill let us say now the velocity has stabilized at a value V 2 okay.

So, it is passed the windmill somehow some interaction happened and then after that also something else might have happened to the wind it eventually stabilized at a value V 2 and then again it is far enough away from the windmill that the pressure is back at P infinity. So, this is what we are assuming has happened to the breeze P infinity. So, as it has come from one side of the windmill to the other. So, as it crosses the windmill let's say at that instant it has a velocity small v okay. So, at that instant it has some velocity small v and we will say that there is a pressure before just before it interacts with the windmill and there is a pressure after just after it interacts with the windmill.

So, if you actually look at the derivation of the Betz law and you look at various you know sources where they have discussed this in some detail many of them take different approaches. So, there is differing levels of detail in the approach we will try to put this together in some reasonable detail which captures many of the salient features you will see some variations as I said based on the source in which you learn about this the first time or you look up at other sources. But generally the flow will be similar, so of the derivation. So, you can decide for yourself the level of comfort you have with specific steps and and the way in which the derivation is it done.

So, right now this is the way we have set it up. So, for example, there are various books which will simply tell you that this small v which is somewhere in the middle between V 1 and V 2 can be assumed to be the average of V 1 and V 2. So, they will just say that we have an inlet velocity of V 1 outlet velocity far away, I mean inlet velocity far away from the the windmill which is V 1 and an outlet velocity again far away from the windmill which is V 2. So, we will assume that the V which is next to the wind I will just write down the windmills location is the average of V 1 and V 2. So, this is the way in which some books will describe it.

We will just do a little bit of derivation starting from the Bernoulli's equation that will actually get us this as a result. So, so we need not you know blindly assume this result we can actually arrive at this result.

So, as we said the Bernoulli's equation simply has half rho V square. So, where V is the velocity plus P is a constant. So, this is the equation. So, what we will do is we will just consider this as two halves. So, this side we will write the equation and this side also will write the equation okay. So, or rather you know, so this entire stretch for this stretch we will write the equation and for this stretch also we will write the equation and then if you look at that those two equations and compare them we will actually arrive at this result. So, what do we have here? We have the, we are assuming that the density of this of the air is not changing so that we will make an assumption. So, we have half rho V 1 square plus P infinity equals half rho small v square plus P before okay. So, this is just exactly the Bernoulli's equation which is conservation of energy for fluids I have written from two different locations far away from the windmill and at the windmill.

Same thing will write for the second section. So, we have here half rho small v square plus P after is equal to half rho capital V 2 square plus P infinity okay. So, that is all we have done written Bernoulli's equation from you know point A B C and D. So, comparing point A and B we arrived at this first equation, comparing points C and D we arrived at second equation. So, this is what we did.

So, clearly you see there are enough terms here which if you subtract you can cancel or in this case if you add you can cancel because they are on opposite sides of the equation. So, we basically have if you just add these equations or yeah in the way it is written you can just directly add it and in that case this term here will cancel with this term here and similarly this P infinity will cancel with this term here with the in the final summation once you complete the final summation.

So, after you add them. So, this cancellation will happen after you add them. So, you basically have half rho V 1 square plus P after equals half rho V 2 square plus P before okay and so if you simply rearrange it a little bit, I mean moving the velocities to one direction and the pressures to the other side of the equation you will simply have P before minus P after equals half rho V 1 square minus V 2 square okay. So, this is what we have got, P before minus P after equals half rho V 1 square minus V 2 square.

So, if you want to look at force or thrust, force is simply pressure into area. So, here we are assuming this area of cross section is A. So, I should have just marked this is something else, but, so this area of cross section happens to be A area. So, don't confuse it with this other A that I have put down here.

So, if you look at the area A. So, we will just remove this here to avoid confusion. So, for now I will call this B C D E whatever okay, so area A. So, if you look at the A force which is pressure into area that is basically A into P before minus P after equals half rho V 1 square minus V 2 square okay. So, this is how we got the force. So, this is one way in which you can arrive at force we have basically looked at this energy conservation and from there we have arrived at the force for the because from there we were able to get some equation that showed us the difference in pressure from there we arrived at the force. The other way is to look at momentum and then look at rate of change of momentum. So, let’s now look at momentum, change in momentum right.

So, change in momentum is simply the momentum before, but minus the momentum after and so for the momentum we need mass into velocity right. So, that is we have rho which is the density, we take a cross sectional area L, A and we will assume some distance l that is the amount of gas that has gone past the windmill. So, A times L is the amount of gas that has passed gone past the windmill. So, what is the momentum that it had that it delivered to the windmill or what was the change in momentum of the gas? So, it arrived as rho Al V 1 and then it came out as V 2 okay. So, when as it came in it was coming in with a velocity V L. So, that the momentum it had was that that AL volume of gas which had a mass of rho AL had a momentum of rho AL V 1 right. And then it same mass of gas later on it had a velocity V 2. So, it had a momentum rho AL V 2. So, that is the momentum. So, if you want to change of momentum the initial momentum minus final momentum rho AL V 1 minus V 2.

So, if you want rate of change of momentum that is simply the d by dt of this you differentiate this with respect to time of rho AL V 1 minus V 2. And in this case rho is a constant, A is a constant we are assuming V is V 1 is far even far enough away from the windmill that it is a constant. V 2 is far and far away far enough away from the windmill that that is also constant. So, the only thing that can change with time is the is associated with the flow or associated with the length. So, that is simply rho AL rho A, dl by dt V 1 minus V 2 and this dl by dt is this is an interaction we are talking of at the windmill. So, at that point the velocity which is the dl by dt is the small v right. So, that is the velocity. So, this is equal to rho A small v V 1 minus V 2. Okay so, that is the velocity at that point.

So, now, we have rate of change of momentum which is again equal to force, this is equal to force. So, force is equal to rho a small v V 1 minus V 2 by just looking at change in momentum and rate of change of momentum and we also looked at force another expression was force, force we got through the energy conservation which is half we have a here half rho A V 1 square minus V 2 square right. So, that is the other expression for force. So, we have two expressions for force, one from the energy conservation the other from the rate of change of momentum. So, we now just equate these two and we will find that we will arrive at this result. So, if we write this down here we have half rho A V 1 square minus V 2 square is equal to rho A small v V 1 minus V 2 okay.

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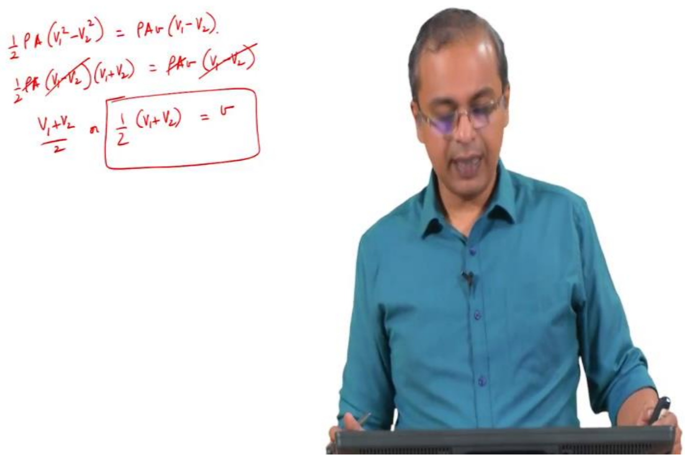

So, if you simplify this, this is simply half rho A V 1 minus V 2 into V 1 plus V 2 equals rho A v V 1 minus V 2. So, you can cancel this out and the rho is rho s will go and therefore, you have V 1 plus V 2 by 2 or half of V 1 plus V 2 equals V.

So, in other words. So, this is the result. So, I told you that you know in some some books they simply tell you that we can assume that the velocity in the middle is the average, but we have now been able to prove it using the Bernoulli's equation. So, it basically means that if you measure the velocity of the breeze far enough away from the windmill before it reaches the windmill and far enough away from the windmill after it has crossed the windmill then the velocity that the windmill that the breeze has or the wind has as it just crosses the windmill is the average of these two right. So, that is basically what we have here.

Okay so, now that we have this let's see what is the energy that we can get out of it what is the power that we can get out of it.

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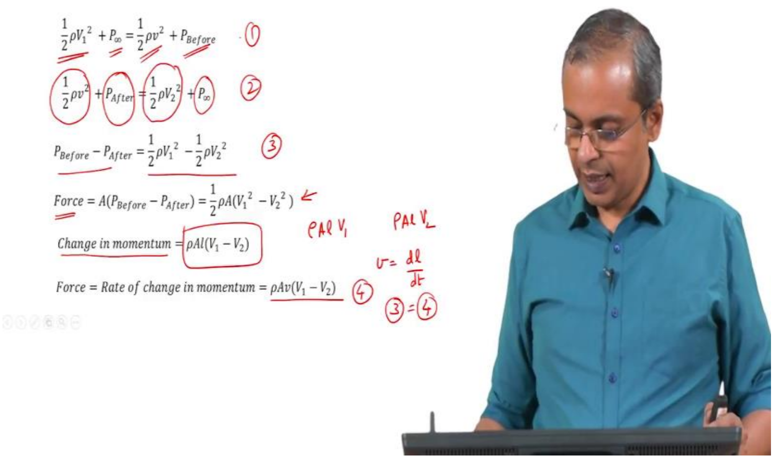

So, this is basically what we just discussed which I have just put down as equations before we start looking at the power. So, as I told you that this is the know kinetic energy per unit volume, this is the pressure at infinity, kinetic energy at unit volume close to the windmill, this is the pressure just before the windmill and. So, those are the terms that you see there the same thing we wrote as a second equation with this is the kinetic energy per unit volume just after the windmill and this is the pressure just after the windmill and this is the kinetic energy per unit volume far away from the windmill and this is the pressure at the pathway from the amendment.

So, when you subtract those two equations or you know manipulate these two equations 1 and 2 appropriately in this case you can just add them and cancel out the terms this is what you will get P before minus P after equals half rho V 1 square minus half rho V 2 square and the force is simply area into P before minus P after. So, that is half rho A V 1 square minus V 2 square.

So, that’s the equation that we have here and we also said we can look at the force as a change in momentum or rate of change of in momentum. So, we start by change in momentum. So, the momentum before came as rho AL V 1 and the momentum afterwards is rho AL V 2. So, you subtract them that is this term here rho AL V 1 minus V 2 and therefore, rate of change in momentum is simply rho A v which is dL by dt V is small v is dL by dt and, if you differentiate this change in momentum with respect to time you arrive at rho Av V 1 minus V 2. So, this equation this okay, this is we will treat this as 3 and this is 4, 3 is equal to 4 and that’s what we are doing here 3 is equal to 4 or rather in this case 4 is equal to 3, equation 4 is equal to equation 3 and therefore, we arrive at this equation okay.

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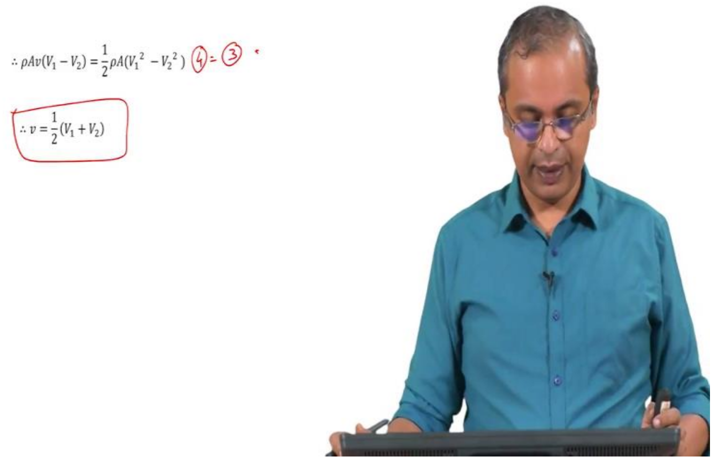

So, this is the way in which we arrive at this fact that the velocity is you know basically the average of the two velocities far away before the windmill and far away after the windmill right. So, now, from this let’s look at the power possibilities right. So, again we go back to the kinetic energy.

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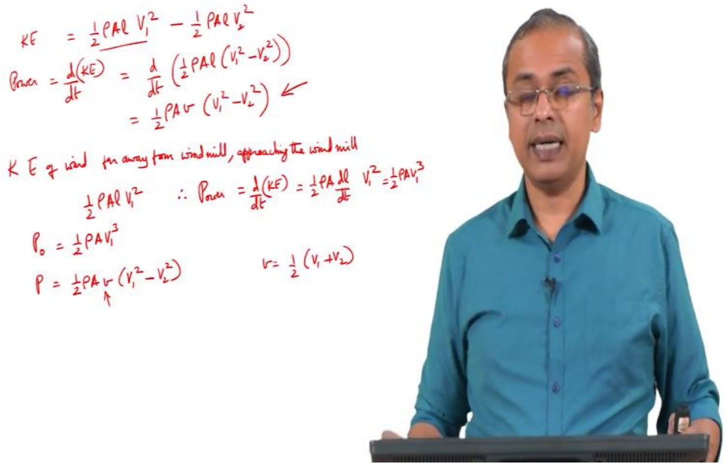

So, kinetic energy, change in kinetic energy let's just look at kinetic energy, the change in kinetic energy is half rho AL, okay so this is the mass V 1 square minus half rho AL V 2 square. So, this is the change in kinetic energy. So, the power is simply d by dt of kinetic energy okay. So, power simply rate of change of energy. So, energy per unit time is power. So, this is the d by dt of this. So, if you do that this is simply d by dt of half rho AL V 1 square minus V 2 square okay. So, this is what we will get. So, if you just you know to complete this process again everything these are all constants rho and A are all constants. So, only dL by dt is that velocity associated with that wind as it crosses the windmill. So, that is V and P 1 square minus P 2 square. So, this is power.

Now, if you again look at the, if you only consider the breeze far away from the windmill that is what is heading towards the windmill then the kinetic energy in the original breeze kinetic energy of wind far away from windmill. So, kinetic energy of the wind far away from the windmill as it is approaching the windmill. So, as it is coming towards the windmill is simply half rho AL V 1 square that's the first term that we had up here right that's the term. We already saw this and I am simply deriving it here again for you therefore, the power that is there in that wind the original wind that is heading towards the windmill is simply d by dt of this and in this case this is simply half rho A again dL by dt and V 1 square, and here this is still far away from the windmill it has not reach the windmill. So, the dL by dt here is V 1 itself there is no other V here that is the only way that is available. So, that is half rho A V L cube and this is how we saw that the power in the wind is proportional to the cube of the velocity of the wind.

So, this is the original power that is available in the wind. So, we can call this as P naught. So, we will call this as P naught okay. So, this is P naught. Now, we have expression for P which is the power that you know got delivered to the windmill. So, we looked at the change in kinetic energy as it crossed the windmill and based on the change of kinetic energy and rate of change of the kinetic energy we found a power. So, that change in kinetic energy is the energy that it delivered to the windmill right. So, that it came with some kinetic energy it is eventually found with some other kinetic energy that difference in kinetic energy is the energy that it lost the wind lost and therefore, we assume that that's the energy that has been gained by the windmill. So, whatever we came up with here this, this equation here is the energy that the wind has delivered to the windmill in this case we have looked at the rate at which it has done it. So, therefore, this is the power that it has delivered to the windmill.

So, P the power delivered to the windmill is simply half rho A v V 1 square minus V 2 square okay. So, we have basically caught actually most of it here we already saw that v is equal to half of V 1 plus V 2 right. So, this v we have some expression in terms of V 1 and V 2. So, using this we are going to now do some simple manipulation to arrive at some equation that tells us what we can expect from the windmill okay. So, we will do that here I just put the same two equations down here.

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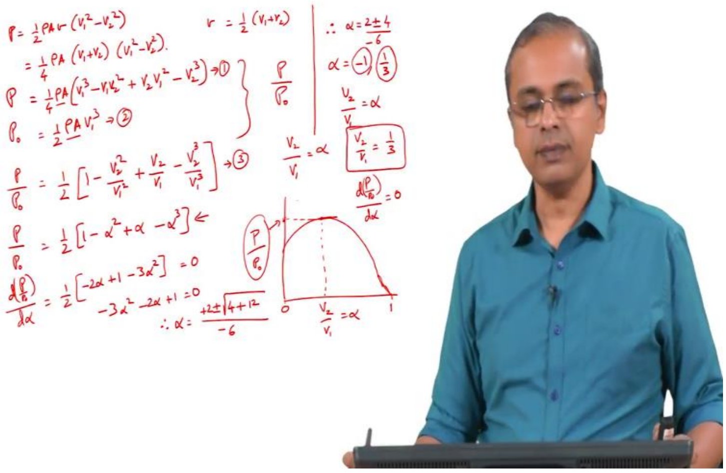

We have P equals half rho A v V 1 square minus V 2 square and I also had V equals half V 1 plus V 2. So, I will substitute this here. So, we simply have 1 by 4 rho A V 1 plus V 2 into V 1 square minus V 2 square. So, we just complete this mathematical steps here. So, this is half rho A you can open the bracket here. So, this is V 1 cube minus V 1 V 2 square plus V 2, V 1 square minus V 2 cube okay. So, this is the total power that has been extracted from the wind. We also know that the original wind had a energy of half rho A V 1 cube.

So, we would like to know what fraction of that original power that that was there this P naught, what fraction of that P naught did the windmill manage to extract that is the point that we would like to understand. So, we we are really interested in using these two relationships and trying to find out what is the value of P by P naught and to use that to understand whether we are you know understanding something about what’s the best that we can do in this case. So, to do that we will simply we will just divide these two equations because that is exactly what that is, so P by P naught equals you can see here that the, we you simply have a factor half here because the rho a here will remove the rho a there because they are in the numerator and denominator in both cases in there it is there both in the numerator as well as the denominator. So, the rho A will be removed and this factor 1 by 4 and 1 by 2 will combine to give you this factor 1 by 2 okay.

So, that is basically what we have here. So, this is half and you have V 1 cube divided by V 1 cube, so that is 1, minus you will have V 2 square by V 1 square plus you will have V 2 by V 1 minus V 2 cube by V 1 cube. I got this simply by dividing the equation here let us say whatever this is in this page this is equation 1 and this is equation 2. So, I just got it by dividing equation 1 by equation 2 in this page. So, that's all I have done and I have arrived at this. So, we will just for simplicity sake we would call V 2 by V 1 as some value alpha. So, that we don't have to write you know so many terms here. So, this is simply P by P naught equals half of 1 minus alpha square plus alpha minus alpha cube where alpha is simply the ratio of V 2 by V 1 velocity far away before after the windmill divided by velocity far away before the windmill. So, this is all we have got.

So in fact, all you have to do is plot P by P naught. So, now, that we have this equation let's say I let me call this equation 3 in this page. So, I just have to make a plot of this equation P by P naught versus V 2 by V 1 for all various values of V 2 by V 1 the maximum V 2 could be I mean the minimum V 2 could be is 0, maximum V 2 could be is V 1 or we can assume something like that and then you plot it between 0 and 1 right. So, if you do that you will see that the P by P naught varies and at some point you will have a maximum and that we will say that that is that combination of V 2 and V 1 at which maximum power is drawn. So, that's a simple way to do it. So, you simply have to make a plot off and that is what I am going to show you also shortly P by P naught versus V 2 by V 1 which is nothing but alpha. So, if you do the plot you will see something. So, this is we can say you know as I said V 2 can be 0 or it can be equal to V 1. So, this is 1. So, if you plot it you will see some some curve, something like that whatever some curve we will get and so there will be some value which is the maximum.

So, that is the ratio of V 2 is and V 1 at which we can extract the maximum power from that windmill and we will get some value here that is that fraction which is the maximum fraction that we can get and hopefully you have given the way we are trying to derive this that will be our Betz limit I mean so, if you have gotten this right. So, this is one way we can do it we can just plot it and come up with it or we can simply differentiate this equation because that is exactly what it is we want this maximum value. So, at that maximum value we will simply have d P by P naught by d alpha is equal to 0. So, if you solve that equation also you will get the maximum. So, we can just solve that equation.

So, d P by P naught d alpha is simply half of you will have minus 2 alpha plus 1 minus 3 alpha square.

So, therefore, we simply have to set this to be 0 to get the maximum. So, that is the quadratic equation is minus 3 alpha square minus 2 alpha plus 1 is equal to 0. So, the solutions are minus B. So, it's simply, so that's therefore, alpha is equal to plus 2 plus or minus square root of B square is 4 in this case minus 4 A C. So, minus 4 into A is 12 into 1 is 12, so minus 4 into minus 3 is plus 12. So, you get plus 12 here by 2 A, so which is minus 6. So, you have 2 plus or minus square root of 16 by 6. So, this is simply we will put it right here we have therefore, alpha is equal to 2 plus or minus 4 by 6. So, alpha can be pi minus 6 here. So, alpha can be minus 1 or it can be 1 by 3. So, if it is 2 plus 4 you have a 6 by minus 6 which is minus 1 and if it is 2 minus 4 you have minus 2 by minus 6 which is 1 by 3. So, now, let's look at it.

So, this is alpha is as I said V 2 by V 1, right. So, that is what is alpha. So, alpha being minus 1 basically means that the wind came hit the windmill and actually reversed back the same velocity with which it came okay. So, that seems very interesting, but actually that is not possible because the wind has two will hit itself right it will it will turn back and they are still wind coming. So, this is not something that can be sustained because you are not really distributing it anywhere the wind comes and it's trying to head back the same direction in which it came we are not allowing density to change nothing is going to change. So, it cannot really do this. So, even though the equation gives us a minus one it is actually not meaningful it is not physically possible. So, alpha equal to 1 by 3 is the only one that seems possible. So, V 2 by V 1 equals 1 by 3 is the more meaningful result that we will have through this process and that is where we will get the maximum efficiency.

So, we can actually substitute that back here and if you substitute that back here you will get a value. So, we will just see that in a moment.

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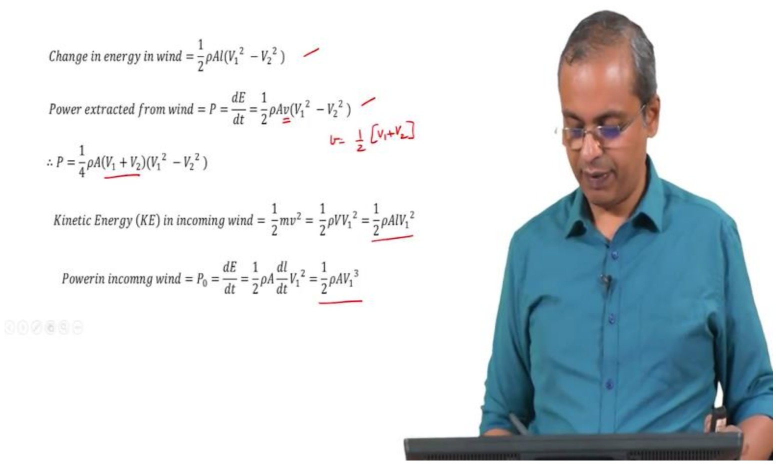

So, the same equations I have put here what I just wrote down on the screen. So, the change in energy in the wind is rho AL V 1 square minus V 2 square that is what we have here. Power extracted from the wind is simply dE by dt which is this rho A small v V 1 square minus V 2 square and this V 1 the small v is half of V 1 plus V 2 and that is what has been put here and because of that half this half here becomes 1 by 4 here. Kinetic energy in the incoming wind is this and therefore, power in the incoming wind is this value which is all what we did in our previous screen.

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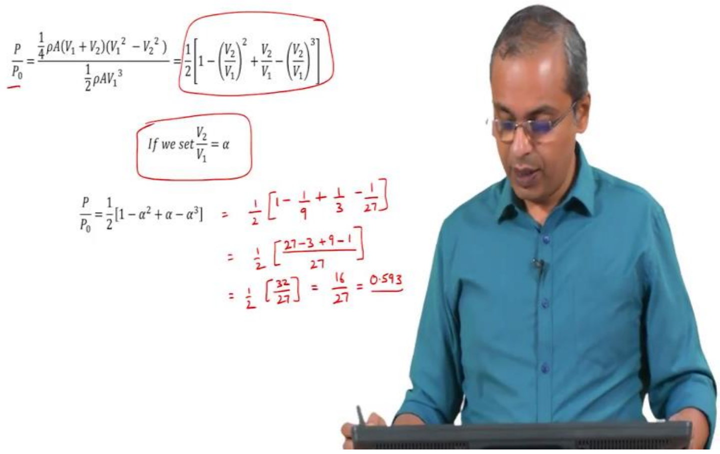

And therefore, when we do a P by P naught this is the value that you will get and that that's how you arrive at this equation and then we set V 2 by V 1 is alpha and we arrived at this P by P naught equals one minus alpha square plus alpha minus alpha cube right.

So, this is the manner in which we got this and if you actually look at this value of P by P naught. So, we will have actually this is equal to if you set alpha equal to 1 by 3 you will have 1 by 2, 1 minus you will have 1 by 9, plus 1 by 3, minus 1 by 27. This is kind of what you will get right. So, on this basis we can look at it. So, this is equal to 27 and then we will have 27 minus 3 plus 9 minus 1. So, you will have 27 minus 3 plus 9 minus 1. Yeah so, we will get half of this value. So, P by P naught if you do this calculation. So, we will get half of. So, if you complete this calculation it is 27 plus 9 is 36, minus 3 minus 1 so that is minus 4. So, this is simply 32 by 27 and so if you complete this you will get 16 by 27 okay. So 16 by 27 is this P by P naught and this is that value we saw in our Betz equation. So, this is 0.59. So, this comes to about 0.593.

So, this is the derivation for the Betz equation we are able to get this 0.593 and that directly comes from you know the starting from the Bernoulli's equation looking at the energy per unit volume essentially you know incoming what is outgoing and equating them and then are arriving at this value 0.593.

So, as I said you can also do this as a plot. So, that is essentially what I have done here.

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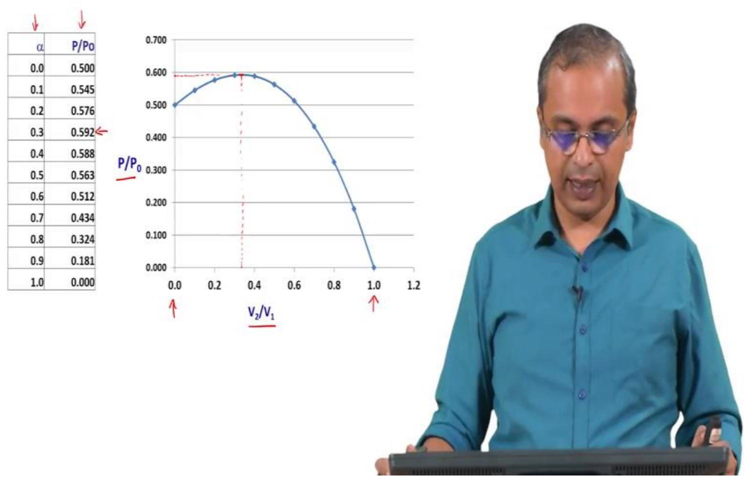

So, if you see here this value P by P naught I have plotted as a function of V 2 by V 1. So, you can easily do this you simply take any spreadsheet. So, I have just used excel you can use any other spreadsheet or any other you know plotting software that you have and you make a plot of various values of V 2 by V 1 as I said for us it is meaningful to look at values between 0 and 1 which means V 2 is either 0 if the wind has been fully stopped or V 2 is the same as V 1 in which case you know it is basically taking the I mean which is going un undisturbed by the windmill.

So, if you do that and plot it as a function of P by P naught or P by P naught as a function of V 2 by V 1 you see this curve and at this curve you see a maximum. So, this maximum as you can see is very close to 0.6, 0.593 is the value we got if you go down this, this is some 0.333. So, that is the value at which we get it you can see here I have just marked all the values of various values of alpha and for that the various values of P by P naught. So, you can see here at 0.3 we have reached a value of 0.592 and it is only marginally more than that 0.593 is the maximum that we can get. So, that is how the Betz limit comes about and it’s a calculation done in the nineteen twenties and as I said you can see various versions of this and the calculation works out as we described.

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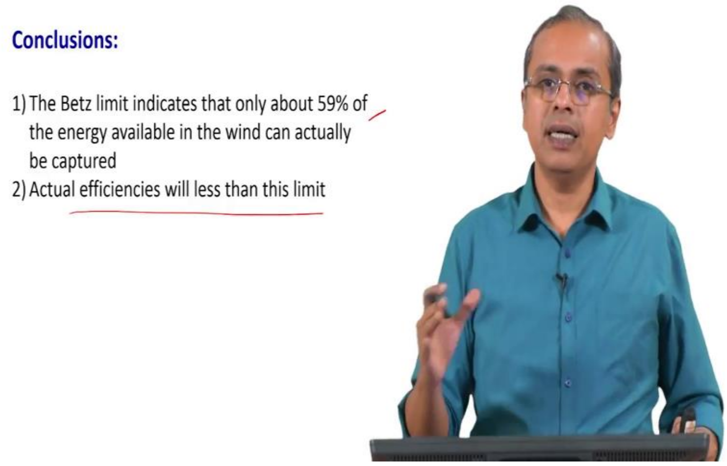

So, in conclusion the Betz limit indicates that only about 59 percent of the energy available in the wind can actually be captured this has some assumptions. So, there are ways in which you can think of you know going around these assumptions and there are some versions of the windmill which may not conform to this assumption. For example, we have assumed that the windmill is a plane right. So, a single plane across which this velocity variation is happening some of the vertical axis windmills actually occupy some you know horizontal direction as well right. So, horizontal extent they have that is some places are closer to the wind that is incoming and some are away from the incoming wind. So, that changes the dynamics of this equation how the equation is set up and so strictly if you just look at this equation and directly apply this equation there it is not really a valid way to do it some variations are there and that can change the way in which we look at the efficiency.

In general the actual efficiencies would be less than this limit because this basically only sets the upper limit for a certain type of wind. So, in summary in this class we have tried to understand how the derivation for the efficiency of the windmill comes about and the idea that there is this limit which is the 0.593 or roughly 59 percent, that's sort of the maximum that we can extract the maximum power that we can extract from the wind that is in coming towards the windmill.

So, I think that concludes our class it gives us a good background to understand what is possible with the windmill and you know what are some limits of this whole process.

Thank you.

KEYWORDS:

Betz Limit; Bernoulli's equation; Efficiency of Wind mill

LECTURE:

The efficiency of the wind mill is defined and measure using the using Betz limit. The Betz limit is derived and its implications on the process are weighed.