

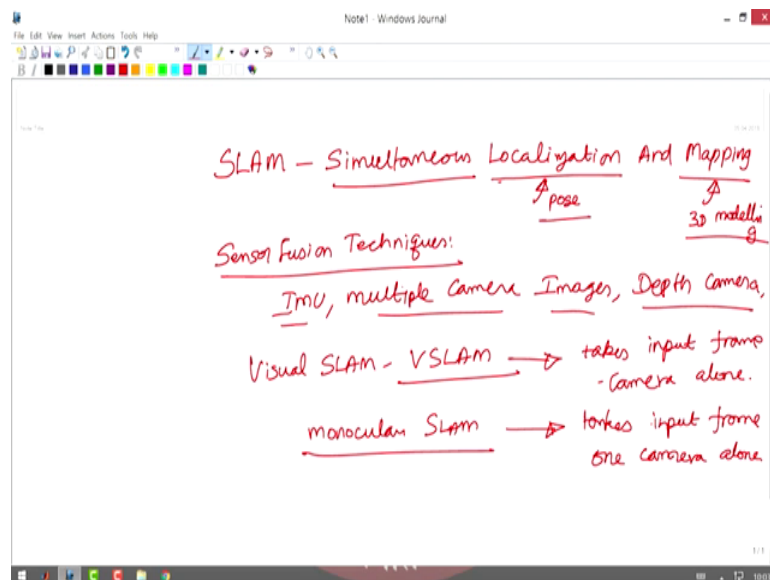
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**Lecture - 87**  
**Introduction to Kalman Filter**

Welcome back. In today's class, we will look at some important concepts which is necessary for augmented reality. The concepts we have been looking at so far in the class is for virtual reality as well as the augmented reality. The concepts can be used both in virtue virtual reality as well as the augmented reality, but there are some concepts which is necessary specifically for the augmented reality.

For example, if you if you look at the augmented reality, we need to augment the existing environment with a virtual objects, we need to know what is the already available room in an environment physical environment and over this physical environment, we should be able to overlay the virtual objects right. So, we should we should be able to measure the physical environment in real time.

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So, there are specific algorithms, which is required for augmented reality called SLAM S L A M SLAM for simultaneous localization and mapping simultaneous localization and mapping. So, there is a there are two things happening localization and mapping and that is simultaneously happening.

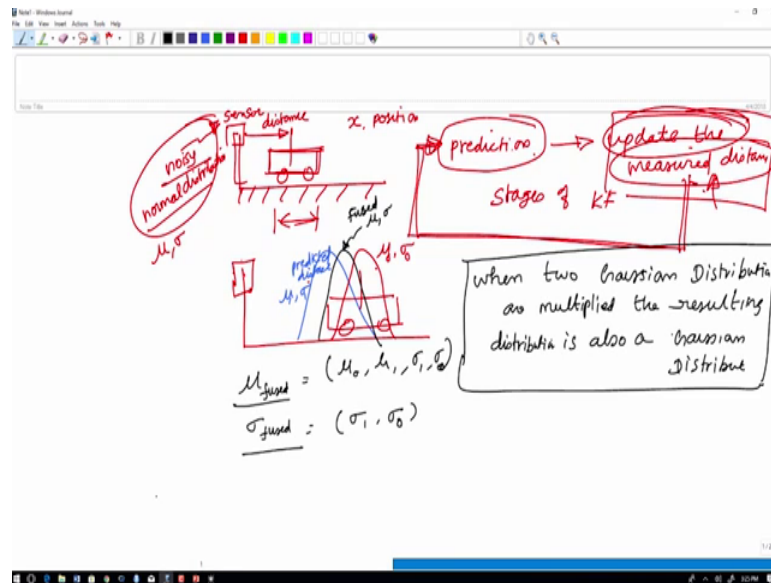
Localization is to estimate the pose of the camera in a augmented reality to estimate the pose of the user where exactly he is and mapping is to have 3D modeling of the environment, so that we can place the virtual object onto the real environment, so that we can and overlay the 3 D objects virtual objects into the physical environment in the camera images.

There are many SLAM techniques which have been proposed in the last 20 years. There are essentially there these are all sensor fusion algorithms, sensor fusion techniques. This techniques SLAM techniques uses mostly IMUs, and multiple cameras multiple camera images, and depth camera and so on. So, the SLAM technique takes input from all those things, and then estimate the pose as well as the mapping of the 3D environment.

In this class, in virtual reality what may be required is a particular classes of a particular class of SLAM technique called a visual SLAM, it is also called the VSLAM. So, in VSLAM, we focus only the algorithms, which take input from the camera. So, this technique takes input takes input from camera alone camera alone. And this VSLAM can have know multiple camera inputs there are even another class of VSLAM which is called the mono plus monocular SLAM which takes input from only one camera, takes input from one camera alone.

So, this would be the very simple VSLAM technique which we will focus in this. And also the SLAM techniques, which is discussed in the many of the reading materials over there are uses many mathematical notations which scares away the students. So, I am go to reduce the uses of the mathematical notations, and then make it as an intuitive introduction to the SLAM technique in this next a few minutes.

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In this example, I am going to introduce a very simple a motor vehicle moving in in the road let us say; this is the road we are talking about. And this motor we need to look at what is its position let us say this a position we need to measure it. And let us say here is the sensor which measures the position. So, when it measures the measurement distance, it measures the distance right.

But what happens is this sensor is noisy. The output of the measurement and of the sensor itself is noisy. So, we are not very confident about what the sensor is. Most of the sensors are like this; it is uncertain, the measurement is a little bit of uncertain. So, in the case of the uncertain measurements, how are we going to how are you going to make better measurement of our or our objects in the virtual reality environment is a question.

So, we are talking about how do we improve the noisy measurement process. What Kalman filter does it, it uses some prediction algorithms and then uses this prediction algorithms to update the measured measured are now distance for example here. And then the measured distance is used to rotate there are the two stages in the Kalman filter.

So, in a normal process, what we will do is we will use only the you know measured distance without worrying about the prediction. Measured distance and using it directly is a little risky because it the data is noisy. So, to avoid this Kalman filter does it is that it uses some special prediction algorithms and uses this prediction algorithms to update them measure distance. So, the measurement distance is not directly used, but it is

updated based on the prediction as well as from the measurement data. Once we update the measured data, then we use it to predict again ok. There are the two stages of the stages of Kalman filter right.

So, in this example, suppose we have the vehicle mood little ahead then again we have this sensor. And the measurement it gives you, since it is noisy it is modeled as a simple normal distribution instead of a having only one value as in the Kalman Kaman filter in as in the complementary filter. In the Kalman filter, we the measured data is not one single value, but one single value that is the mean of the data and also a normal distribution. As you know the normal distribution has a mean as well as the variance. So, the measurement comes with the mean and the variance.

So, in this case, let this example let us say we have this  $v$  and variance. Now, let us say we have an algorithm to predict a position depending upon the previous data. Let us say this is a predictor data. Let me take another one, another color and let us say this is a predicted, predicted distance. The predicted distance also has a been under variance. Let us say this is one, and this is one, and the measured one is the, this one.

Now, combining these two things we need to do the updating. So, we have the measurement distribution, we have the prediction distribution, comparing those two things we can update it which can be yet another distribution let me make this as a probably green yellow (Refer Time: 11:12) that that looks like let me let us a black. So, it can be another distribution that is a, this is the updated one or updated or fused let us say fused. This can be comma 0.

So, this is one of the very nice properties when two Gaussians distributions. When two Gaussian distributions or multiply the resulting Gaussian distribution is also yeah resulting we can say resulting and distribution is also a Gaussian distribution. This is a very nice property it is guaranteed to have the resulting Gaussian distribution. So, the predicted distance itself is a distribution, the measured distance is also a distribution Gaussian distribution. When these two things are together multiplied say let us say fused, the resulting itself is a again a Gaussian distributions.

So, we can and write equations for or what is the  $\mu$  of the fuse two data, what is the, these variants of the fuse two data is a function of know  $\mu$  naught and  $\mu$  1, and the

sigma 1 and sigma naught right. Similarly, this also sigma 1 and sigma naught we can write an equation and for the, what is the fume used or what is a sigma mused ok.

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The image shows a handwritten slide with the following content:

$$G_1(x, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

I.D

$$\mu_{fused} = \mu_0 + \frac{\sigma_0^2 (\mu_1 - \mu_0)}{\sigma_0^2 + \sigma_1^2} k$$

$$\sigma_{fused}^2 = \sigma_0^2 - \frac{\sigma_0^2}{\sigma_0^2 + \sigma_1^2} k \sigma_0^2$$

Kalman Gain

$$\mu_{fused} = \mu_0 + k (\mu_1 - \mu_0)$$

$$\sigma_{fused}^2 = \sigma_0^2 - k \sigma_0^2$$

So, essentially if you look at the, if you take a Gaussian distribution, x is a measurement distance, we are talking about mu is the mean and sigma is the variance. Then this is a standard equation for the Gaussian distribution Gaussian bell shaped curve; it has e power minus as x minus mu whole square divided by 2 into sigma square right. This is the standard bell shaped Gaussian distribution. If you look at mu 1, mu fused then it is going to be mu naught plus sigma naught square into mu 1 minus mu naught divided by mu naught squared plus mu 1 square.

Similarly, we can write sigma fused is going to be sigma naught square minus sigma naught squared divided by sigma naught square plus sigma 1 square. This is the equation for the fused distribution. In this, if you observe, if you consider the whole thing as some constant k, then and this also is going to be k, therefore, we can write again the same equation, if you fused is going to be mu naught plus k times mu 1 minus mu naught sigma fused is equal to sigma naught squared minus is k times k times sigma naught square yeah.

So, here there is one naught it is actually it is going to be yeah k times this is going to be sigma naught squared again. So, this is going to be sigma naught square ok. So, this k we can consider this as the Kalman gain. So, this again is a Kalman gain. You can compare

this with a complimentary filter are which makes it a little more easier yeah. In the complimentary filter, we use a complimentary gain filter gain right. Similarly, we have this Kalman gain. Only thing is here we need to add the variance also into picture because the Kalman filter is based on the distribution probability distribution in order to handle the uncertainty.

This is for the 1D whatever we have seen it right. If there are many parameters as or involved more than one measured variable a same were not only  $x$ , but also velocity then and the calculation becomes much more difficult. So, usually the Kalman filter are discussion or the derivatives in the textbooks and other materials, you will see is in the now matrix notation.

So, I am going to introduce the Kalman filter in the matrix notation. And again without much of the now mathematical notation and so that you are not you are familiar with the Kalman filter when you come across in your textbooks. So, essentially we have as we have seen in the earlier slides, there are two stages of the Kalman filter; one is the prediction stage, and then the update stage right. We will stop here.

Thank you.