Virtual Reality
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Lecture – 04
Geometry of Virtual Worlds

(3D rotations and yaw, pitch, and roll, cont'd)

Let us continue onward, so we have three canonical rotations as I said last time yaw pitch and roll.

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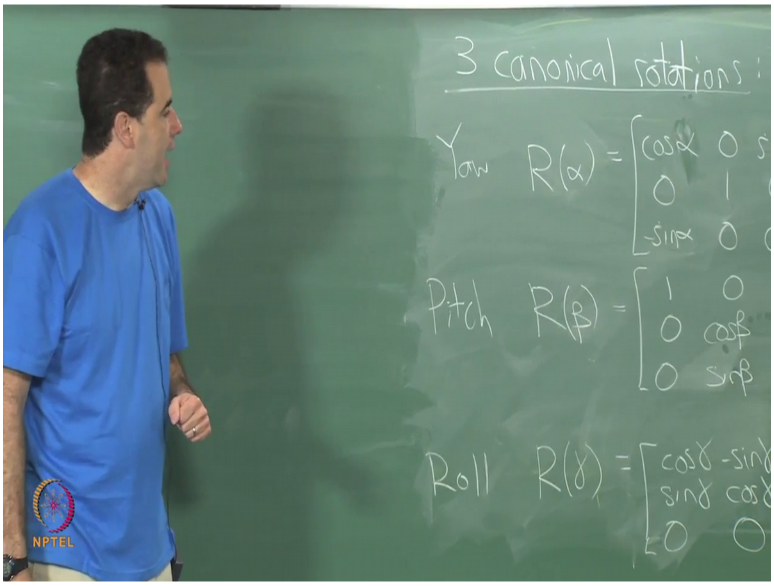

And these are these three dimensional rotation matrices that achieve each of these. So, yaw is rotation about which access though the y axis, so it is rotation about that axis which means that it leaves the y coordinate alone and is a rotation that seems to occur in the x z plane, similarly pitch is rotation about the x axis and roll is rotation about the z axis.

So, with these 3 matrices you can put them together sequentially, to get any rotation you like. So, if you want you just get any 3D rotation R by chaining these together, rotate R alpha, rotate R beta, rotate R gamma.

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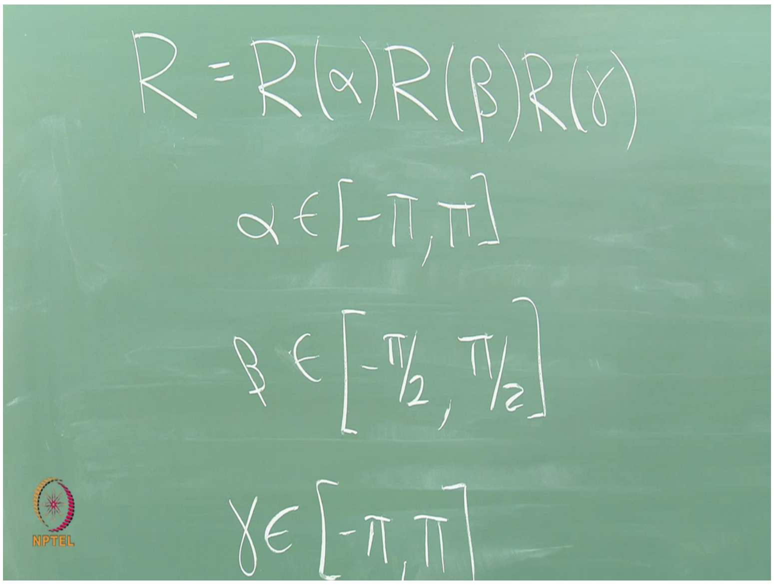

So, you end up with a result, and if we constrain alpha to be between minus pi and pi, alternatively because 0 to 2 pi, if you do not like negative angles and beta to be between minus pi over 2 and pi over 2, and gamma to be between minus pi and pi again.

So, if we impose these constraints then we can still reach every possible 3 D rotation by picking some alpha beta and gamma and applying these rotations together in a chain. So, we can reach all of them. So, this is some kind of parameterization of the space of all rotations and it is very nice because, it is very easy for us to understand what yaw pitch and roll mean. These are also sometimes called Euler angles, there are many other ways to obtain Euler angles there is different variations.

This one's the maybe the simplest and easiest to understand, note that the range for beta is not minus pi to pi, but I cut it in half, because if I allowed it to go from minus pi to pi I will actually add a double representation it is a little bit too much.

So, the furthest extreme I want to go to with pitch is looking let us say straight up or straight down, if the head is the thing that we are rotating, which it is for a head mounted display if we are doing tracking for that. So, this turns out to be enough right. So, now I want to talk about some of the problems or difficulties with 3 D rotation, some of the issues.

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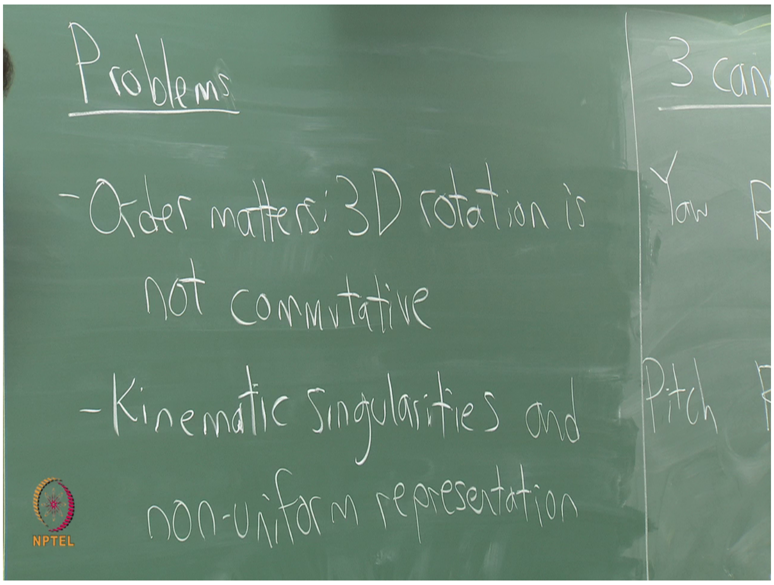

So, even though I can reach any rotation like this, the order in which I apply these affects the outcome.

So, that is one of the first problems that order matters, in other words, 3 D rotation is not commutative. 2 D rotation is commutative right; all we are doing is combining a bunch of rotations let us say by theta 1, plus theta 2, plus theta 3 it does not matter you can put your thetas in any order you like for a 2 dimensional rotation, they are all rotating about the same axis it does not matter. Now we are rotating about potentially different axes. 3 D rotation is commutative if all of your rotations are about the same axis, but because they can be rotating about different axes you end up with some kind of trouble.

Let me give a very simple illustration with this blockhead, this by the way I have a sentimental value for me I bought this in Finland, when I was doing a head tracking for oculus in the very early days; when the company was only a couple of months old and we did not quite have a full head set yet.

So, I just had the 1 of the original sensor boards taped to this and then I would perform rotations with my hand to try to see if the tracking software was working correctly. And then eventually we got a headset and was they were able to do more and more. So, let me try this, let us try to do 2 rotations in 1 order and then do it in another order and see I will do rotations by 90 degrees in each one of these; if I perform 1 of this rotations, I sort of imagined that, I am grabbing some kind of skewer or metal rod that goes through the objects and I grab onto it and twist.

So, a pitch will be like this, a yaw would be like this, and a roll would be like this alright. So, let us see. So let us first do a pitch by 90 degrees. So, I pitch by 90 degrees the face is looking up and now I which one is this roll, I roll by 90 degrees alright. So, in this case see where the face is. So, it is looking to the side the top of the head is towards the board right. So, now I want to do a roll first. So, I grab here and I roll by 90 degrees and then I pitch by 90 degrees and now the face is pointing up right, in the top of the heads that way. Very easy illustration right, of non commutatively of 3 D rotation I got 2 different results by applying these two essentially these two matrices right I applied a pitch and a roll in different orders and I got different results.

So, everyone agrees you do not have commutatively, if you mix these things up, if you start changing the orders of your matrices around, you are not quite sure what you are doing you will end up with a mess somewhere at the end trying to debug your code with all kinds of matrices in the wrong order and it ends up being very frustrating. So, if you understand it and get it right the first time you will be in good shape. There is another problem which is called kinematic singularities, and a little more generally non uniform representation.

So, this is what I was talking about before in 2 dimensions, you can vary theta by some amount and the amount of rotation of the object in physical space will vary by a consistent amount. It will be the same regardless of you know how you rotate you vary theta by some amount, and it does not matter what the original orientation was of the object, that you are rotating it will vary by the same amount. If you start varying Euler angles you will not necessarily end up with a consistent amount of rotation that corresponds proportionally to the amount of variation that you did in alpha, beta or gamma.

Let me see if I can give a very simple explanation of this, the first thing I want to explain is what do I mean by this kind of non uniform representation; well we have a very simple example that you may be familiar with if we look at the top of the earth and I imagine lines of longitude right.

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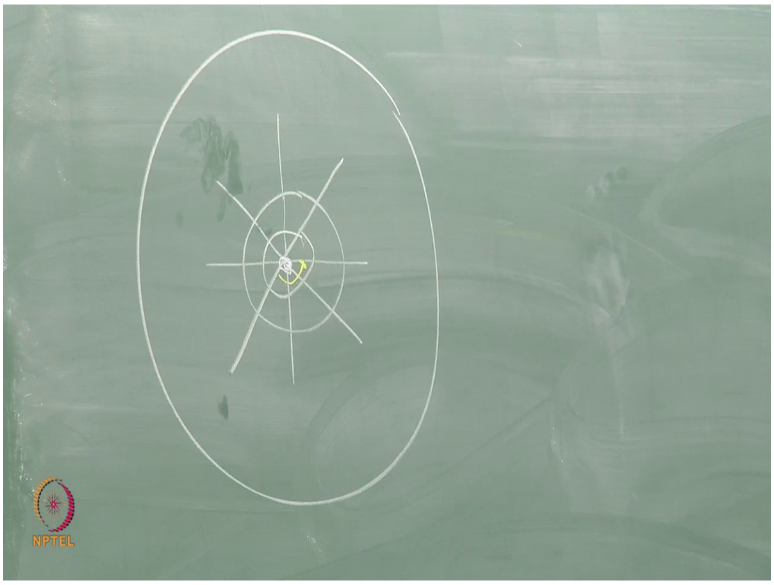

So, suppose we are at the North Pole looking down on the top of the earth, and then I have lines of latitude that go around like this right. So, today we are all located very close to the equator right.

So, if I use latitude and longitude coordinates, for our position around here on the earth it feels very much like being in the plane right; yours using x y coordinates right we could say the y coordinates latitude, the x coordinates the y coordinate is latitude and the x coordinate is longitude, and everything seems fine. If I get all the way up near the North Pole; now if I just very you know I start moving a tiny bit around the north pole my latitude is changing by a lot correct? No whoops got the wrong 1, the longitude is changing by a lot my latitude maybe is not changing at all if I am walking in a tiny circle around the North Pole.

So, along one direction you might think in terms of coordinates because the longitude is changing very quickly, that I am moving a lot, but you know you are actually not it is a distortion because of the way that you have parameterized your position on the earth. So, the same kind of thing is what is happening when you pitch and you get very close to these boundaries. When you get very close to plus or minus 90 degrees, it turns out that very large changes in the parameters may correspond to little or no change in the orientation and that is quite confusing, that makes sense. But it is one dimension higher and very hard to visualize, I think I will do a very quick algebraic demonstration of it, but beyond that it is going to be hard to visualize because, this is a 3 dimensional surface that essentially lives in 9 dimensional space right; because we started with 3 by 3 matrices and we applied a bunch of constraints.

So, there is 3 dimensions left some kind of strange surface and somewhere in that strange surface if I choose to parameterize it badly, I end up with a bad point that is a lot like the North Pole. Let me see if I can show you algebraically what is happening. So, let me imagine chaining these matrices together in some order right. So, I will just expand these out it is very closely related to the problem of gimbals lock you can look that up, if you like as well which is known by people in aerospace.

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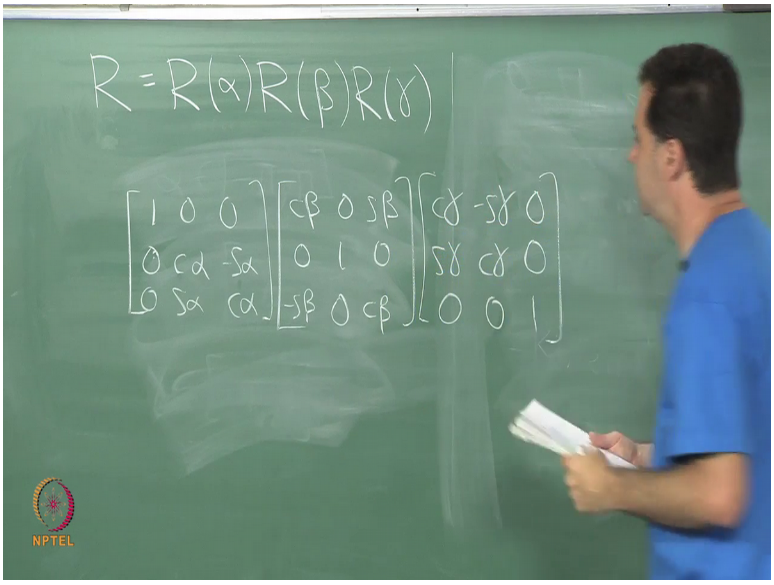

It is a mechanical manifestation of this problem alright. So, with my example let us suppose I multiply this out 1 0 cosine alpha minus sin alpha 0 sin alpha cosine alpha, I am going to use a very short hand cut hand convention here and just write c for cosine and as for sin, otherwise it will take too much time to write them all out. So, I am just trying to give you a very quick example. So, if I multiply these matrices out here is the beta, and now the gamma matrix alright. So, I have this chain of matrices right and we have said that, we can reach any 3 dimensional rotation by doing this sequence of rotations of the canonical rotations of your pitch and roll.