**Radiation Heat Transfer**
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**Lecture - 9**
**Electrical analogy**

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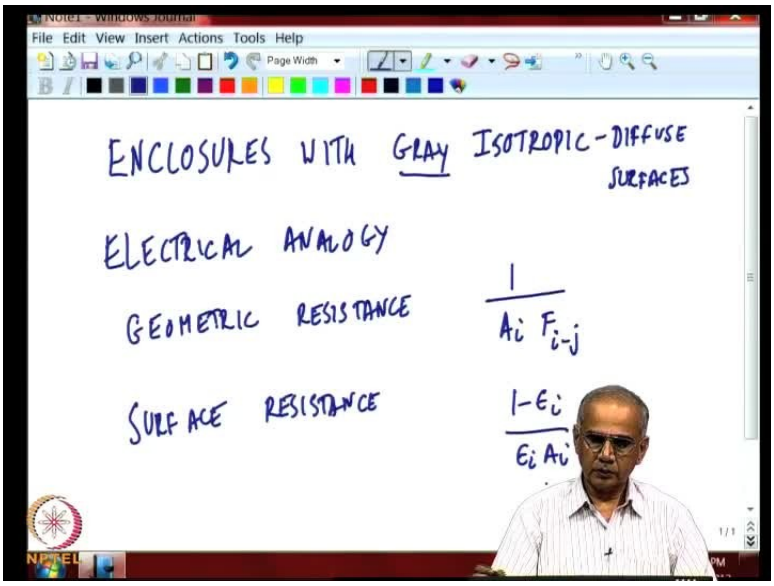


In the last lectures, we were looking at enclosures with gray diffuse isotropic surfaces. In this case because of the assumption of isotropic surface and gray surface, the problem became quite simple and because the surface was gray we were able to avoid the wavelength integration. We got a very simple electrical analogy wherein we saw that the geometric resistance within any two surface is nothing but, $1/A_i F_{i-j}$ and the surface resistance was $1-\epsilon_i / \epsilon_i A_i$. This resistance is occurring because the surface is not a black body, so the resistance to a flow of heat and this the first term. The geometric resistance is coming in because of the relative configuration between two surfaces. Using this idea of geometric and surface resistance we were able to draw typical example of surfaces which involved a battery and surface resistance.

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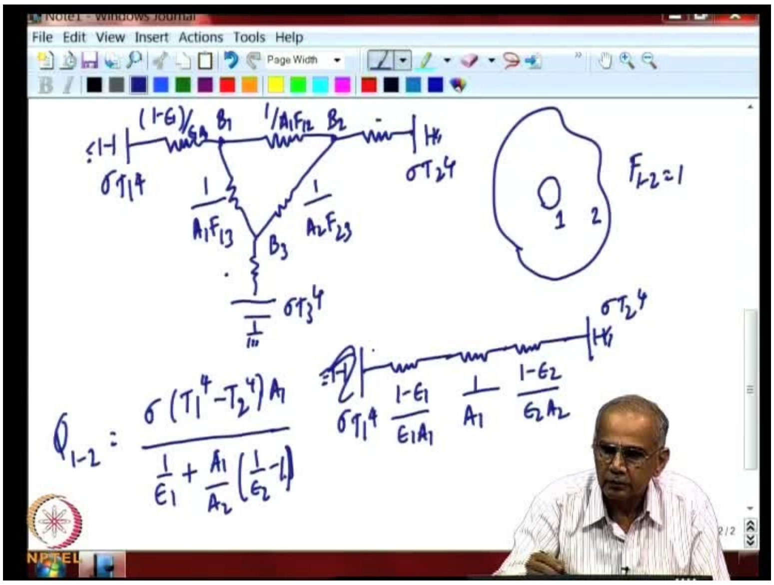


A geometric resistance, another surface resistance and the second battery and if we have three surfaces in the enclosure, we drew two more geometric resistances and then we had another battery here. So, this is radiosity point $B_1$, radiosity point $B_2$ and radiosity point $B_3$. Between the three radiosity points we have the geometric resistance while between these radiosity point and the black body radiation is surface resistance.

We also applied this to a few simple examples and now we will illustrate one example which is of great relevance to many situations in engineering. Imagine a convex object, a cylinder surrounded by a general object with surfaces 1 and 2 as shown above. We can draw our electrical circuit for this case as one battery $A_1$ $\sigma T_1^4$. $1-\epsilon_1 / \epsilon_1 A_1$ is 1 because surface one is convex and is surrounded completely by surface 2. Hence any radiation leaving surface 1 reach surface 2. $F_{1-2}$ is 1. Surface two is geometric resistance and finally we have battery here. To simply this further we can write this energy transfer from 1 to 2 as $\sigma[T_1^4 - T_2^4]$ $A_1$.

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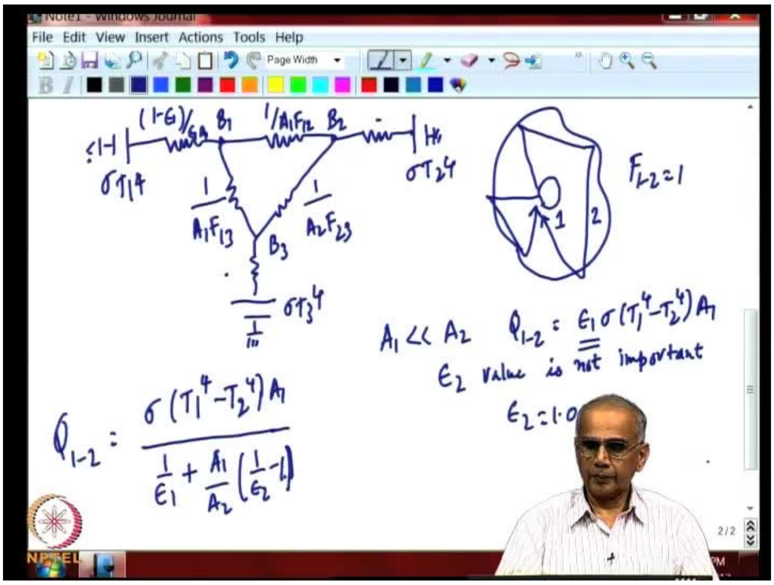


We divide this term on electron by $1/\epsilon_1 + A_1/A_2(1/\epsilon_2 - 1)$ Let us look at some limiting cases of this to understand what it implies. Now, suppose the object 1 has a much smaller area, than surface 2. This is typical in engineering. Imagine a steel pipe going through a room, the pipe area is very small compared to the room area. So, A 1 is much less than A 2 and hence we neglect the second term. Then $Q_{1-2}$ becomes epsilon 1 sigma into the difference in the temperature. It means if the room area is much larger the area of the pipe area then epsilon 2 value is not important. Whatever the value of epsilon 2 the heat transfer rate will not change. This is because if this surface A2 is much larger than surface 1 any radiation leaving surface 1 after multiple reflection, will reach surface 2. Because of that, it does not matter what the reflectivity of the surface 2 is (that is 0 or 1) because ultimately radiation leaving 1 will be reflected and comes back to 1 by multiple reflections.

When surfaces are large , radiation ultimately will, so the rate of radiate heat transfer, between surface 1 and 2 is not depended on the reflectivity or emissivity of 2, but only on the emissivity of surface 1. This means that we need not bother really about the exact value of epsilon 2 or the emissivity of the room and essentially it means we can treat the room as a black body. This because any radiation leaving 1, will ultimately be absorbed by 2 at some point out of 1 multiple reflection. So, as far as surface 1 is concerned, surface 2 is a effectively a black body.

These are very important concepts and we will encounter this concept little later when we derive effective emissivity of certain objects. This result is same as if epsilon 2 is equal to 1 Already, 1 this term drops out of and we are getting the same result. So, although surface 2 is not black body, if its surface is very large it is behaving like black body and the rate of heat transfer by radiation from 1 to 2 is independent of the emissivity of the surface 2. So, this both practical relevant as well as providing an understanding of the role played by the area ratios. If some surfaces large area compared another surface, then the second surface really is in a way behaving like a black body.

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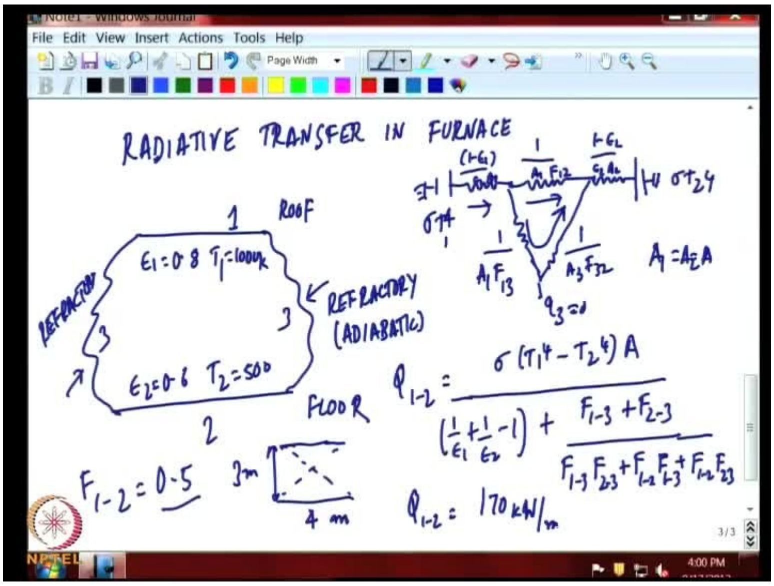


Now, let us take another example now a more practical example of relevance to mechanical engineering, which is radiative transfer. In furnaces a typical furnace and here we talk of metallurgical furnace not used in power plant has a roof and a floor and some side walls, which are refractories whose job is to really reflect the radiation and avoid leakage of radiation. These are the refractories, so when you call the roof surface 1 floor as surface 2 and a side wall refractory as a surface 3 and now generally assume that these are insulating adiabatic surfaces, no heat is transferred through the surface or the heat leakage is small. Let us take example where the emissivity of this is 0.8.

Let us say this temperature just as an example as 1000 Kelvin, surface 2 emissivity is 0.6 temperature is 500 K and we want to know the rate at which heat is transferred from the roof to the floor now. Since, surface 3 is adiabatic, when we draw our electrical analogy , although we come to surface 3, but since surface 3 is adiabatic no heat flows through it and so q3 is 0. Any heat going from 1 to 2 will have to go directly or goes by reflection through refractory back here. This is nothing but a simple series of parallel circuit, and this is surface resistance, geometric,, surface, geometric, and geometric. Assume A1=A2=A .

We have $(1/\epsilon_1 + 1/\epsilon_2)-1 + [F_{1-3}+F_{2-3}/F_{1-3}F_{2-3}+F_{1-2}F_{1-3}+F_{1-2}F_{2-3}]$. We have all possible combinations here because we have reflection from the adiabatic side walls and we can substitute numbers and get an answer, which we will do presently. Let us understand whatever role played by refractory. Suppose $F_{1-2}$ is very small compared to $F_{1-3}$, where $F_{1-3}$ and $F_{2-3}$ are very large.

If we only retained this term compared to these two terms, then we will get a simple expression in terms of $1/F_{2-3}$ and $1/F_{4-3}$ in this clearly showing that essentially, resistance is very small and $F_{1-3}$ is 1. $F_{1-3}$ and $F_{3-2}$ are quite small. Then the resistance is very large. The other extreme case is where $F_{1-2}$ is large, where this a open circuit and the flow actually go through both these are possible in this situation and both do occur in real world.

Both situations occur, if the two plates are very close to each other. These two very far apart if it is very small and is an open circuit. We need to calculate $F_{1-2}$ and by the Hottel's cross string method We will get 0.5 as shown in the above figure because the distance between the two plates is 3 meters and this is 4 meters and this is 3 meters. The gap, will be 5 meters. We apply the cross string method.

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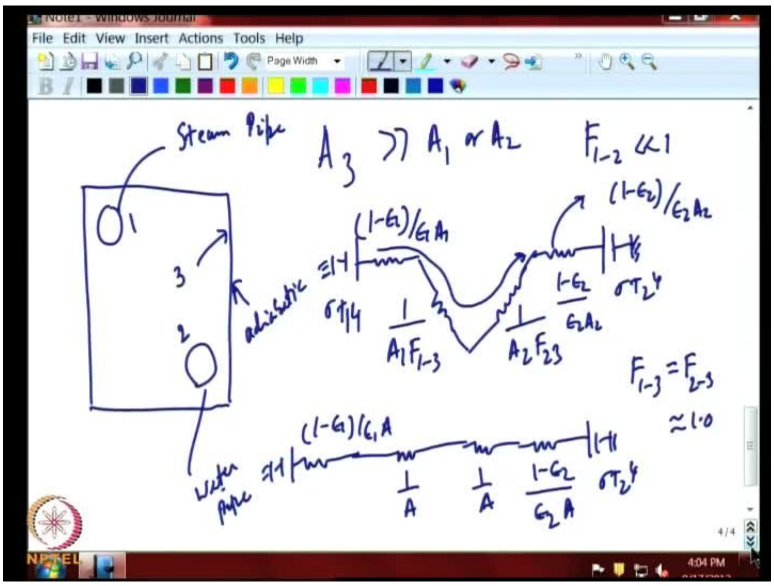

Sum of the cross strings, sum of the parallel strings by 2 we get 0.5 putting this into this equation along with the values of emissivity and temperatures you should get a number like $Q_{1-2}$ is around 170 kilowatt per unit depth. We can use same formulae to look at a different example. Suppose we have a large room containing a steam pipe and a water pipe. The surface of the steam pipe is 1, water pipe is 2 and the inside of the room is surface 3 and the main requirement here is that A3 is to be very large.

Typical in many situation in industry A3 is much large than $A_1$ or $A_3$. There is a 3 surface enclosure and we can assume this as adiabatic and since A3 is much greater than A2 all of you can visualize that $F_{1-2}$ is much less than 1, because the these two parts fall apart. Thus from the exercise, we have done we will get a very simple electrical circuit.

This is $(1-\epsilon_1)/\epsilon_1 A_1$, $1-\epsilon_2)/\epsilon_2 A_2$, $1/A_1F_{1-3}$, $1/A_2F_{2-3}$. We have made the open circuit because $F_{1-2}$ is very small so that the flow is really occurring through the walls to the other side. We assume $F_{1-3} = F_{2-3} = 1$. We neglect the radiation exchange between 1 and 2, assume most of the radiation from 1 reaches 3 and not 2. We assume that the two areas are same so that $1/\epsilon_1 A$ and $1/\epsilon_2 A$. 1/A cancels out.

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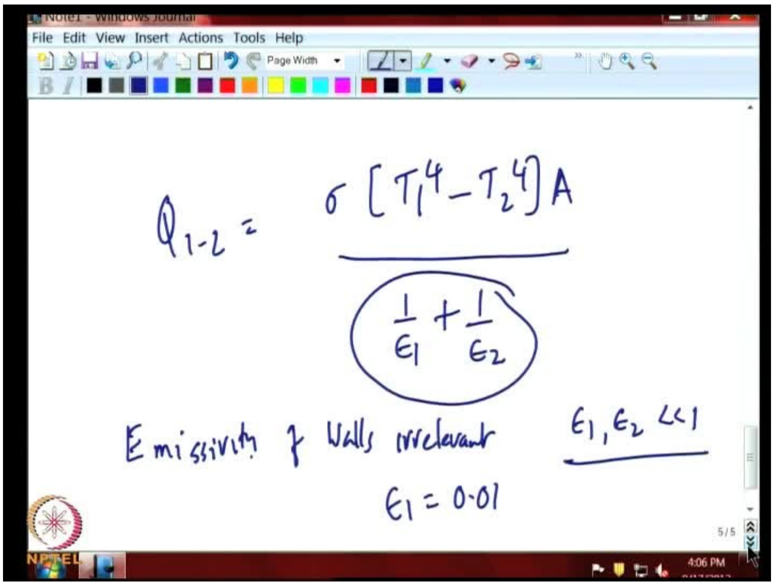


We have very simple expression for the radiation between two pipes, which will be $Q_{1-2} = \sigma[T_1^4 - T_2^4] A/ (1/\epsilon_1+1/\epsilon_2)$. The answer is determined by the emissivity of surface 2. The emissivity of the wall is irrelevant, which is not surprising because the room is very large.

Similar to what we saw in the case of the furnace if the walls are large the emissivity of the walls are not relevant, same thing here.

The room is very large it does not matter what emissivity or reflectivity, it ultimately behaves effective like a black body and so the heat transfer between 1 and 2 depends probably on the emissivity of the two surfaces. If we want to minimize heat transfer from the steam pipe to the water pipe by radiation, we will just ensure that the emissivity is very, very small or keep the outer surface of the two piping as highly reflective. This is what is normally done all these steam pipes and the power plants and industries are usually insulated and then further covered with the reflective film. Its effective emissivity is pretty small and we can see that it can be controlled. For example, if we take emissivity of the order of let's say 0.01. We reduced a transfer between the two pipes by a factor of 200 compared to if they were black body. This shows us how one can control the heat transfer between the surfaces by adopting surface properties. This is an of how providing a shield to a thermo couple also reduces the error due to heat transfer.

These are the examples illustrating the use of electrical analogy and simple approximations, to simplify the problem that we encounter in solving problems, but we notice that if the number of surface enclosure becomes very large, then there could be some problem in handling such cases. Now let us take a few examples relevant in the case of shield. We had looked at thermos flask shield earlier.

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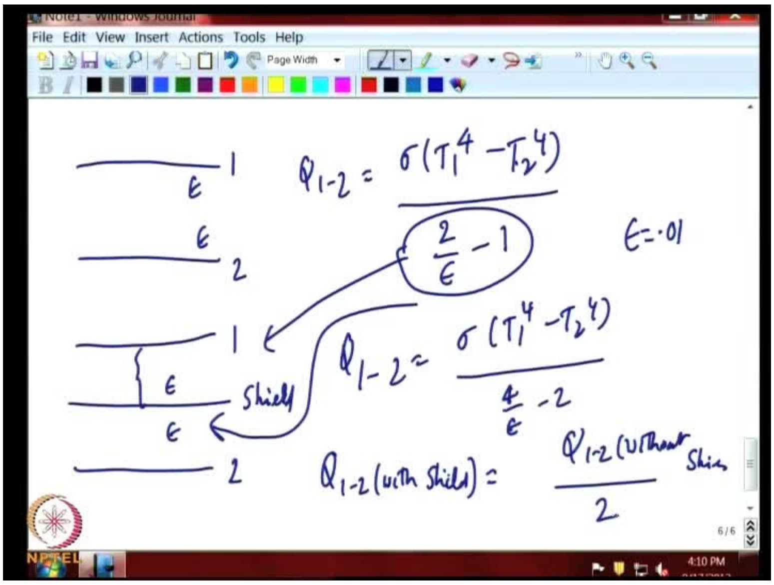

Suppose, there are two parallel plates 1 and 2 both having the same emissivity epsilon, and these are infinite parallel plates. We know that heat transfer between the two infinite parallel plates $Q_{1-2}$ is nothing but $\sigma (T_1^4 - T_2^4) / (2/\epsilon) - 1$ One can see that if suppose we had kept the epsilon very low at 0.01, that will be 200-1=199. We can reduce the heat transfer by a factor of 200 almost compared to the heat transfer between two black plates.

But suppose we want to decrease further the rate heat transfer, between the two plates we can put a shield. Now we put a shield. The role of the shield is to reduce the heat transfer on this plate. Let us say the plate is also a shield and emissivity is epsilon on both sides of the shield. Now, we can draw the electrical analogy here. It can easily shown that $Q_{1-2} = \sigma (T_1^4 - T_2^4) / (4/\epsilon) - 2$.

Now, this is coming in because between these two parallel plates, is 2 epsilon minus 1 between these two parallel plates. Therefore clearly $Q_{1-2}$ with shield is equal to $Q_{1-2}$ without shield divided by 2. Essentially we put one shield between two parallel plates with the same emissivity and shield introduced is also of same emissivity. Then it cuts down the radiation transfer by half because of the same logic.

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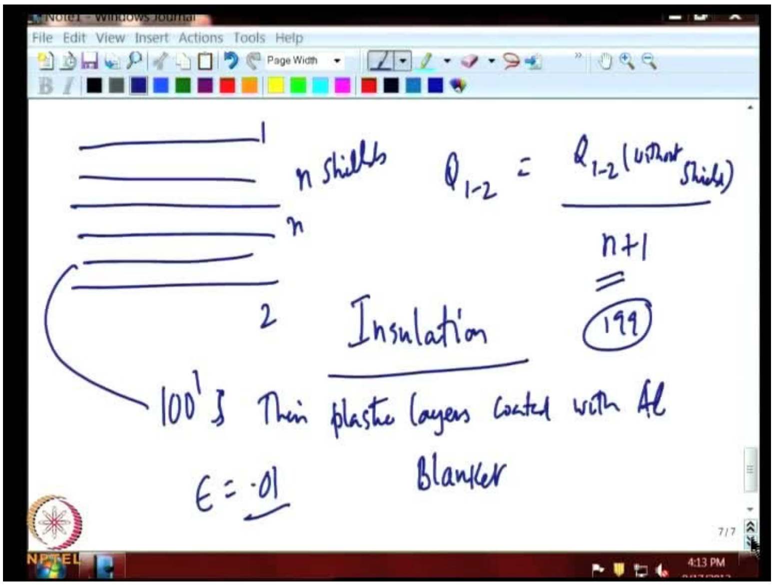


If we have 'n' number shields then it can be shown that $Q_{1-2}$ is equal to $Q_{1-2}$ without shield divided by n plus 1. With no shield. of course, we get back to the old result, this one shield we get which is what we saw with two or three shields. It is quite clear that we can bring down the radiation transfer between two surfaces by a quite a large margin, by having a large number of shields.

Now, this is what is done in the case of insulation. We can have hundreds of these thin plastic layers coated with aluminum. We have coating aluminum on both sides, which makes it highly reflective. The epsilon is pretty low and in addition we can bring down the heat transfer by the factor of 100. Previously with the single shield or any other shield the epsilon was very low or reflectivity was very high, we can cut down the heat transfer compared to two black bodies by factor of 200.

We can have 199 shields. Then we further reduce it by a factor of 200. This is an effective method, of reducing the surface heat loss and is used routinely in both space applications and other applications. It is just called an insulation blanket and is essentially hundreds of plastic layers, which are highly reflective and which are very inexpensive as well as a low weight insulation because plastics are light and thus are not very heavy.

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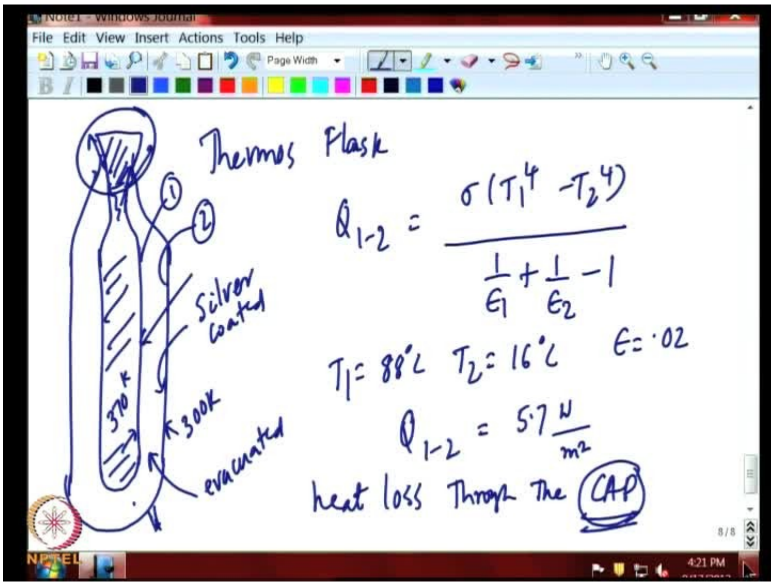


This is widely used and one must have seen it in many applications. This is widely effective technique to control heat transfer. Now, we may give example of another insulation with which all of us are familiar with thermos flask. In a thermos flask, We have an inner cylinder and an outer cylinder shown in a better figure here. Let us assume that the emissivity of the two surfaces is kept low by coating. They are highly reflective. The outside may be a temperature of 300 degrees Kelvin room temperature and inside can be hot coffee may be 370 Kelvin. We essentially minimize the convection and the conduction losses and then the only loss left is radiation. It can be shown that heat transfer from 1 to 2 can be treated as between two parallel plates and then we assume that the surfaces are grey diffuse isotropic.

This is one result, but we will they are result more rigorously in a subsequent lecture, when we deal with highly reflecting coating. Now, if we treat this as a diffuse isotropic surface we are not really making the right assumption because we must recognize that the surfaces are highly reflective. Hence, it cannot be treated as diffuse isotropic, but we will show later that the answer we get by assuming diffuse isotropic is not that different from actual accounting for the mirror like reflection that occurs in this case. Let us look at some numbers for this case and let us see what it looks like for the thermos flask situation.

We neglect the heat transfer by conduction and the convection because we have essentially reduced them by evacuation. But the key issue to should remember is that ultimately we control conduction control totally and have only radiation. Let us put some typical numbers here and see what are the numbers, suppose we assume surface 1 is at 80 degree centigrade. Then we see surface 1 and we assume surface 2, which is the essential ambient to be on a cold winter 16 degree centigrade and we are assumed the emissivity of the order of 0.2, which is achievable with silver surfaces.

Assume all that we will find that $Q_{1-2}$ is the order of 5.7 watts per meter square. It is quite small and if we actually assume this is the only heat loss and we can add little bit of heat loss due to conduction. We will find for this heat loss, the coffee should remain quite warm here for many, many days. But in reality most of us have the experience that in a good thermos flask the coffee remains only hot for may be 8, 9 hours. The main reason why our calculation is too optimistic is that the heat loss to the top heat loss to the cap this is very large.

The key thing to remember is that from this exercise is that the difference between a good thermos flask and an average thermos flask is the design of the cap because one is able to control very closely the heat loss through the side by controlling conduction convention as well as radiation. The main leakage of the heat is now through the cap and the cap design becomes critical. So, between a good design and a bad design is, how well the cap is designed to prevent evaporated heat loss through the cap here. For the good thermos flask, the heat loss through the cap is being controlled by a very careful design of the cap, which is the major heat loss path. This also gives us a very good idea as how sometimes one may miss out when one looks at the design of thermos flask, that the cap design is the most critical. One may think that keeping the evacuated gap is major problem. The cap is so designed as to reduce the heat loss. This kind of calculation helps us to highlight the role the dominant parameter.

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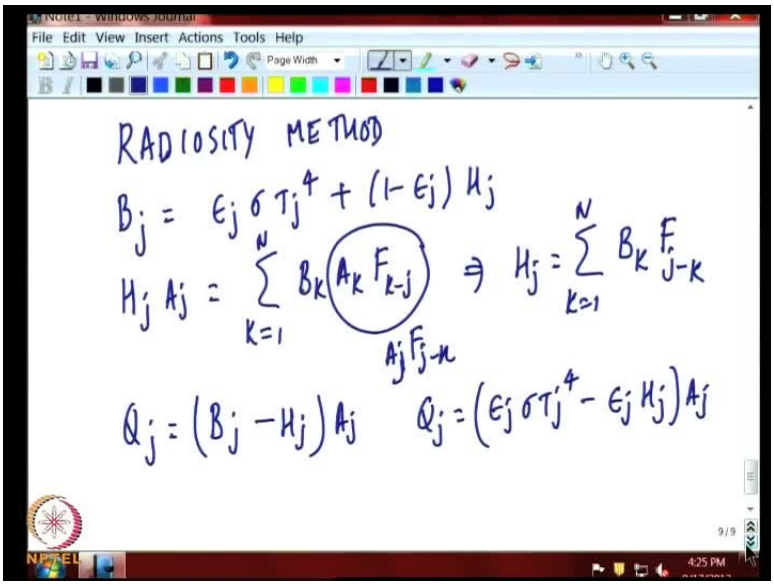


Now suppose we have an enclosure consisting of large number of surfaces not just 2, 3 or 4 that we discussed in the last two lectures. Then we cannot solve it using through electrical analogy but, solve it on the computer using the power of matrix algebra. This method is called the radiosity method.

We solve radiosity by matrix inversion, we set up the whole equation in terms of the radio what the radiosity of surface j, B j is nothing but emissivity of surface j , emissive power plus what is arriving in surface H j we also know that H j while arriving at surface j is what left at surface k. How it arrives surface j, summed over all surfaces k equal 1 to n, but by using reciprocity , this can be made look simple H j. We write this as Aj H j F j k using reciprocity. So, A j is not being summed over.

It cancels out, and we have simple expressions in terms of B k F jk. We can write Q j heat to be removed as radiation leaving surface minus the arriving times area of surface. We can also write it as radiation emitted minus radiation absorbed for a gray diffuse isotropic surface alpha j is equal epsilon j.

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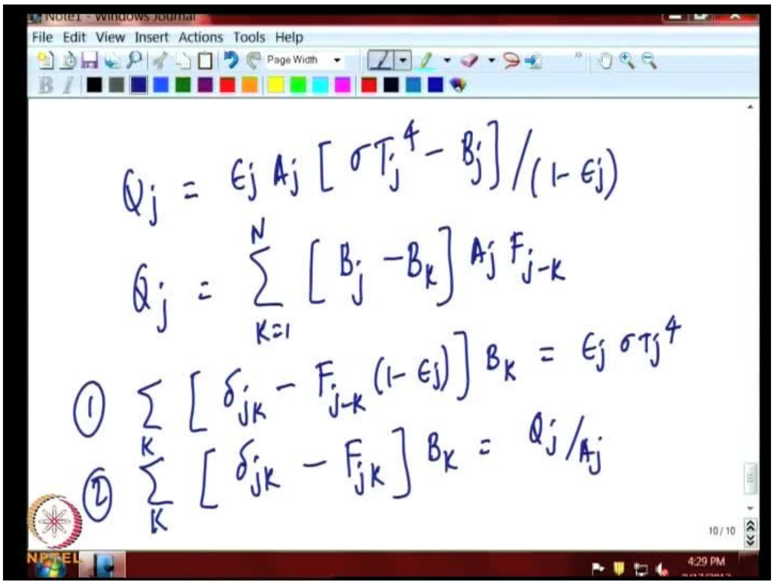


This is actually absorbed radiation by emitted radiation. Now, what we would like to do is essentially eliminate radiation row all this expression, because that is unknown quantity. We write everything in terms of radiosity. We can do by elimination of Qj from the equation we get two equations. Now, which is Q j is equal to epsilon j A j sigma T j to power of 4 minus B j divide by 1 minus epsilon j. This relates the heat transfer to the temperature surface and radiosity. Similarly we can write Qj as sum over all k surfaces 1to n radiosity of surface j minus radiosity of surface k Aj F j to k.

This we derived when talking about electrical analogy. Suppose a true radiosity is unknown problem, and we have N surface enclosures. In some surfaces temperature is known, but heat flux is not known. In other surfaces heat flux is known and temperature is not known. If that is the case we can rewrite our formulation as follows: sigma over k, which is correlated data, that is when j is not equal to j 0 and j equals to k equal to 1, F j k 1 minus epsilon j into B k is that rewriting the expression by substituting k v A and getting everything in terms of B.

If we know the temperature with the known quantity we write the everything in terms of unknown quantity and these are known shape factor and surface temperature. On the other hand in another surface, where the flux is known, we will write this expression. So the two equations we have, one where in the temperature is known and we use equation two where the heat flux is known. On any given surface we expect that either the temperature is given or the heat flux is given and in surfaces in which temperature is given we want to know the unknown heat  flux to be supplied to keep the temperature at that value and in surfaces where heat flux is given, we want to know what temperature that surface will attain.

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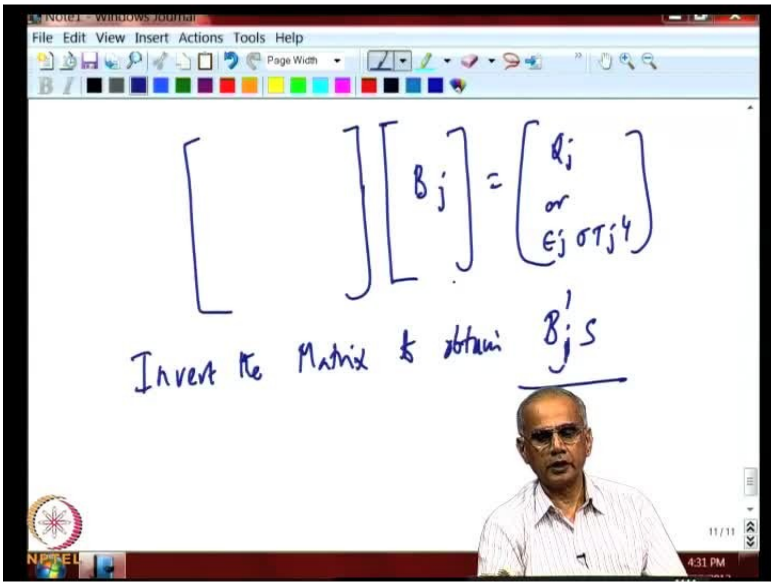


In the first case the unknown is Q j in this case the known is T j, but we could solve for the two unknown. But then we prefer to solve it in terms of unknown radiosity and once the radiosity are known we can go back to these equation and if B j is known we can calculate Q j. If B j and d j are known Q j and B j and Q j are known, so this equation can be used as A T j or Q j, whichever is unknown once you know B j, so this is the matrix inversion  technique, very simple methodology  today with computer available.

Broadly we have a matrix of unknown and these are the B j's these is are either Qj or epsilon j sigma d depending on which is specified and accordingly the matrix module will change, so invert the matrix to obtain B j. So, that is the main spirit of the radiosity method.   We have shape factor and  emissivity's are known and in those surfaces where heat flux is specified we put 1 matrix element and where in temperature is specified we have another matrix element we know all the matrix element invert the matrix and get the radiosity.

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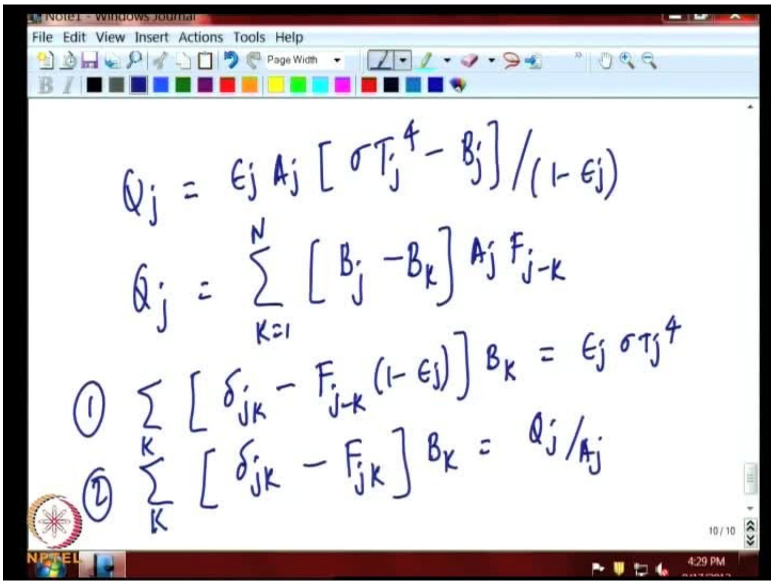


Once we know the radiosity, you can go back to the previous. The radiosity where temperature is specified and have estimated the radiosity by inverting matrix we get the flux where flux is specified and radiosity is estimate from inversion we get the temperature. So that there is the equation used to surface either B j or Q j, given that B j from the inversion matter. These are the matrix elements here in thus equation and this can be completely be automated.

There are standard software available which will solve for this and the advantage we can realize is today computers an inversion technique very, very quickly. We can solve this kind of problems with 100 or 200 surface or even thousands.

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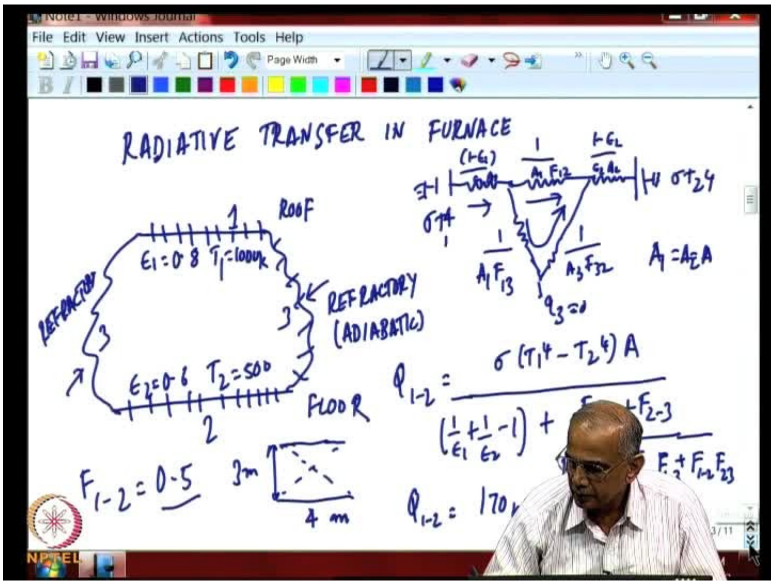


All though there are three surfaces like in a furnace, it can turn out that the temperature of the three surfaces is not uniform. They are non adiabatic , if the non-isothermal , but all are dividing into many surfaces may be 10 or 20 as many as you would like to do so that within each element you can assume.

Similarly, here is a non-isothermal element so you divide into many parts, so we divide into 100 parts, a 300 parts. We have a 300 surface enclosure not a difficult one. We know the shape factor and we know the surface properties we can easily invert the matrix to get the 300 radiosity and given the radiosity, get temperature or heat flux given the methodology as the quantity. This is now fairly routine method in which terms observed for radiosity and these are the methods, which is a preferred method today. Although we discussed the electrical analogy that is more for our attempt, to illustrate the physical insight in drawing those electrical residences, but as soon as we get into a situation, where the number for surfaces both 2 or 3, the electrical energy becomes much more very useful. It is much more useful to just use the radiosity method and just inverter matrix now. Just so that we call that this method is a useful technique.

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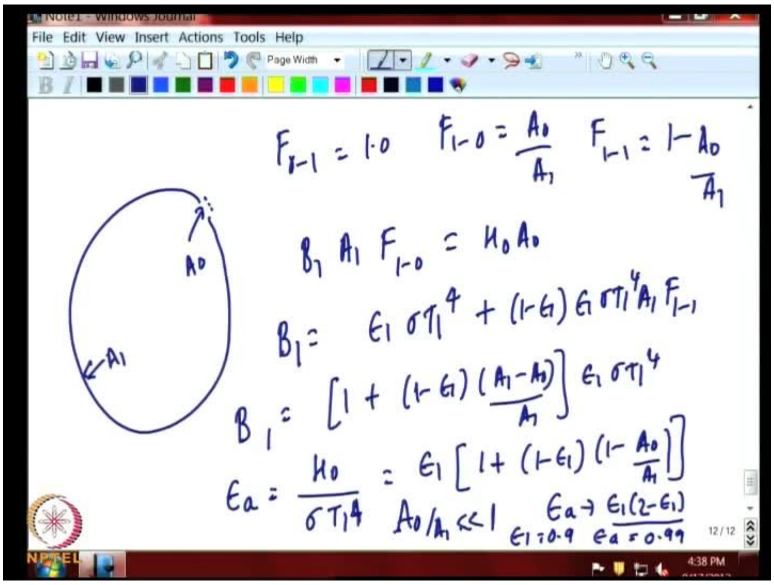


We will now go on to illustrate through the radiosity method the emissivity of a surface. If we have a large sphere with small opening, let us call this area as A1 and the opening area is a A0. This shape factor and F0-1 is 1 because surface A0 is flat and it is only seeing the other surface. The shape factor is 1, so at F10 is nothing but A0 by A1 by reciprocity. The radiation leaving surface 1 and going through the gap is the radiation leaving is 1, the surface but we also know that radiosity 1 is the nothing but emission plus reflection and the reflection is of radiation that is emitted by 1 and A0 is an imaginary surface.

So, only radiation emitted by surface 1 and it is seeing itself. Since, we know F10, we also know F11. F11 is nothing but 1 minus A0 by A1. Therefore, this is substituted here will get B1 as 1 plus 1 minus epsilon1 A1 minus A0 by A1 into epsilon 1 sigma T1 to the power 4. If we define the apparent emissivity of the cavity epsilon a upper emissivity is radiation leaving the cavity 0 divide by what it would leave a black body at temperature Td and this can be calculated and comes out neatly as epsilon 1 into 1 plus 1 minus epsilon 1 into 1 minus is A0 by A 1. Now, we are doing all this exercise because to illustrate the point as the hole in this cavity becomes small and smaller that is as A 0 by A 1 becomes externally small. This term goes to 0, so we get 2 minus epsilon 1, so the upper emissivity cavity reaches the limit this epsilon 1 into 2 minus epsilon 1, very interesting result because we find that if epsilon1 is 0.9, the apparent emissivity of the cavity is 0.99.

If epsilon is large, so we can make the apparent emissivity of the cavity approach 1 and so this is the standard practice by which we obtain black body in the laboratory by having the small hole in a large sphere, which is at a uniform temperature, which can be a copper sphere. That is small hole emits radiation like a black body because all the actual emissivity of the cavity may be 0.99. What is coming out of the cavity will be approach 1 as closely as we can arrange area of the hole to be much smaller than the area of the cavity. So, this the standard method of creating black body in the laboratory. So, with that we come to the conclusion of the discussion of gray diffuse isotropic enclosures. In the next lecture will tackle more complex problem of non-gray enclosure.