Radiation Heat Transfer
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Lecture - 7
Evaluations of shape factors

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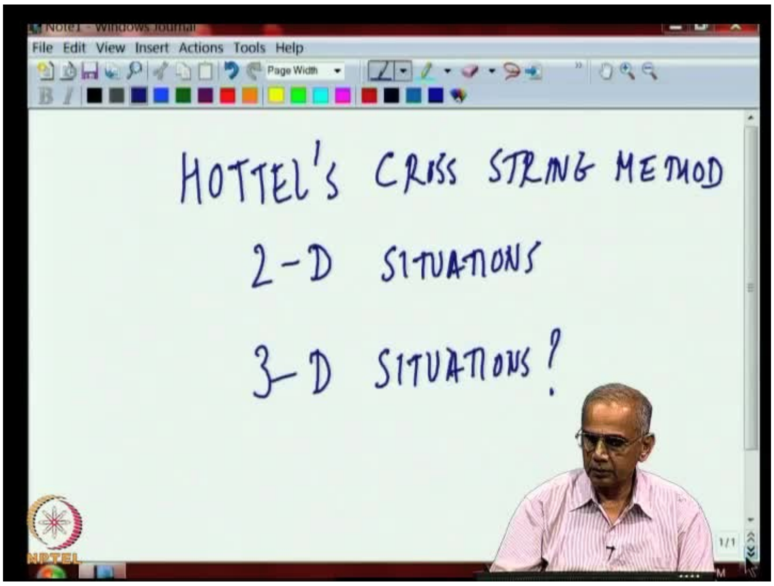

The last class we covered the Hottel's Cross String Method for evaluating shape factors, which is valid only for 2 D situations. In situations where one dimension is very large compared to other dimensions this method works very well. But there are many examples in real world, where the problems are inherently 3 dimensional. We will see few examples in this lecture as to how to handle problems in 3 dimensions where one dimension is not very large compared to the other dimensions.

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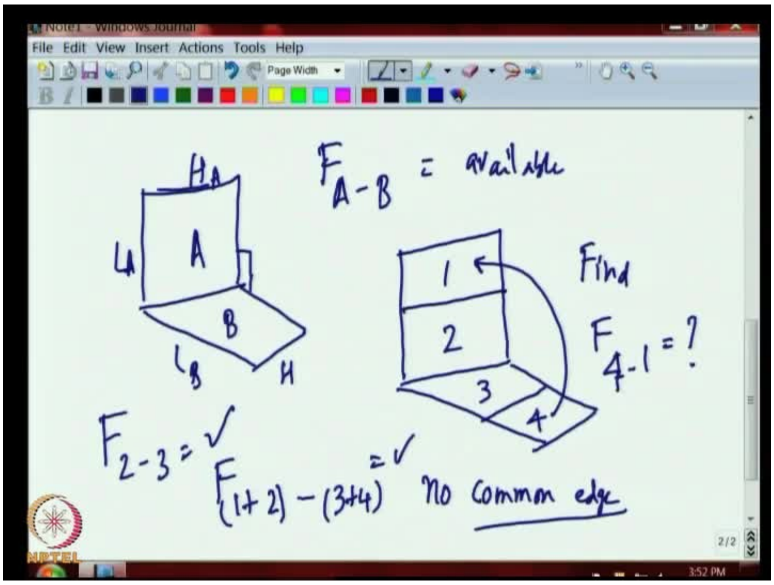

Let us take the first example, a rectangular object let us say A and B and we have to find F_{A-B}. Now, this is available in standard text books that for 2 rectangular areas perpendicular to each other with the common edge, is standard configuration. In most text books on radiation this integration has been done.

This quantity is available in terms of the H. This one L and the dimensional H_A L_A L_B H_B. If all these dimensions are known, then we can calculate this quantity. Now suppose given this we want to evaluate between 2 surfaces. Let us say 1,2,3,4 but, we want to find F_{4-1} this quantity. Now, the term here is that these 2 rectangular surfaces are perpendicular to each other but, there is no common edge.

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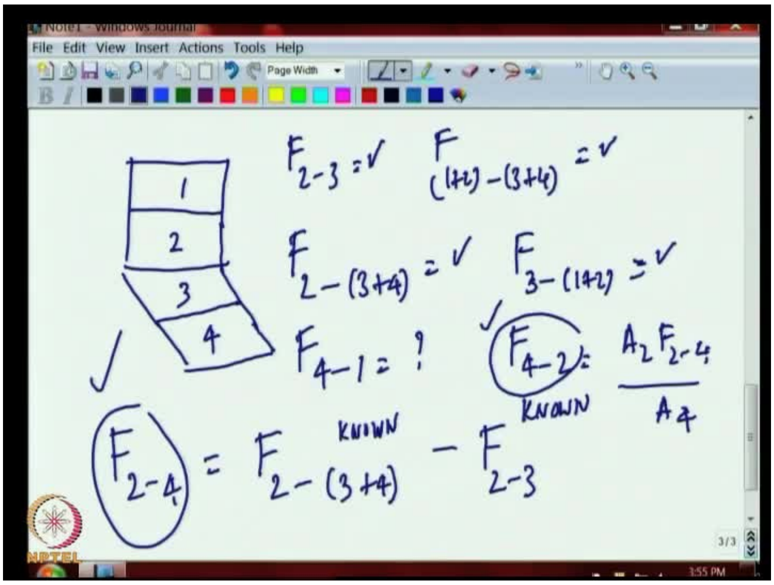

If we are given this quantity F_{A-B} we need to calculate F_{1-4}. The 1st thing one may recognize that, if F_{AB} is known then F_{23} is known because rectangular areas 2 and 3 have a common edge and they are perpendicular. Similarly, $F_{(1+2)-(3+4)}$ is also known. We have again that F_{2-3} which can be calculated and $F_{(1-2)+(3-4)}$ that can also be calculated. $F_{2-(3+4)}$ can also be calculated because there is common edge. $F_{3-(1+2)}$ can also be calculated because these are all 2 rectangles perpendicular to each other with a common edge.

But we want to know what is F_{4-1}. We need to find the shape factor algebra. For example, suppose we want to calculate F_{2-4} by using shape factor algebra, we obtain $F_{2-(3+4)}$ - F_{2-3}. Now, the right hand side of both of these shape factors are shape factors between two rectangular objects perpendicular to each other with a common edge which are known.

Based on this known quantities one can calculate F_{2-4} which is the shape factor between two rectangles perpendicular to each other but, without a common edge. We have been able to extrapolate from information available for rectangles with the common edge, two rectangles without a common edge. Once F_{2-4} is gone we can find F_{4-2} because F_{4-2} is nothing but, A_2F_{2-4}/A_4. This is reciprocity, so F_{2-4} is known. Once more we exploit the use of angle factor algebra to calculate F_{4-1}.

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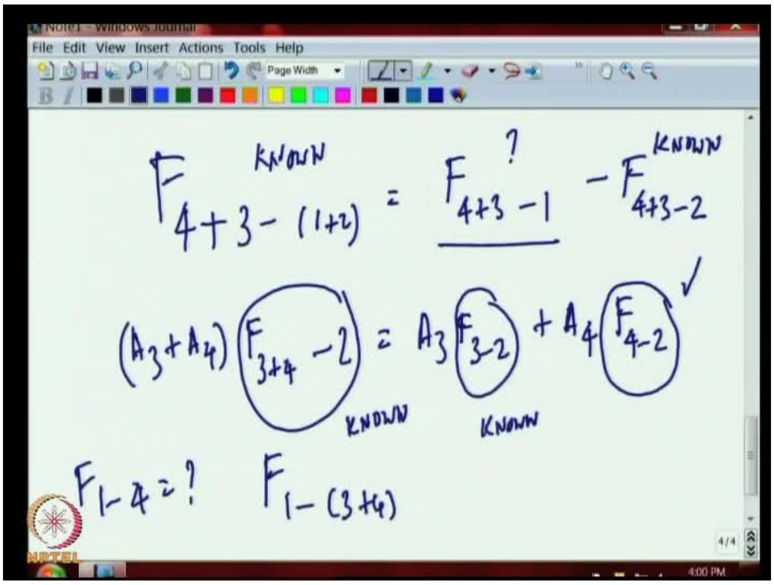

Let us write $F_{4+3-(1+2)} = F_{4+3-1} - F_{4+3-2}$. Again, we see that the left hand side is known. F_{4+3-1} is not known but, we know F_{4+3-2} because it has a common edge. Once more we are able to obtain shape factor between two rectangles perpendicular to each other without a common edge by using data on rectangles with the common edge.

We are ultimately interested in F_{4-2}. From applying the law of conservation of energy, total energy leaving F_{3+4} and going to 2 has been equal to energy leaving 3 going to 2, and energy leaving 4 and going to 2.

Now, we notice that when we are trying to split the first term of the shape factors that is the leaving area we need to be very careful and incorporate energy conservation. When we are trying to split the receiving area we can just split it like that because the other area is common. Now, here let us look at this quantity F_{3+4-2} which is known to us. We can calculate F_{4-2}. Here we are using the law of conservation energy which state that total energy leaving surface is 4 to 2 and reaching 2, will be same as energy leaving 3 as in 2 and energy leaving 4 as leaving 2. We are able to get F_{4-2} by merely exploiting the concept of 1st law. If we want to find F_{1-4} we apply the 1st law again. We start with $F_{(1+2)-(3+4)}$ which is a known quantity. Then we apply first law.

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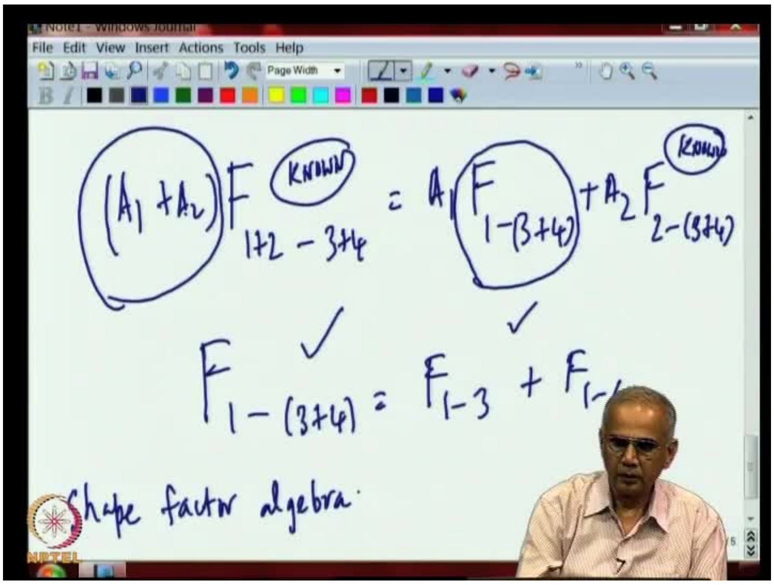

We can write $(A_1+A_2)\ F_{(1+2)\text{-}(3+4)}$. This is again using the law of conservation energy and here $F_{1\text{-}3+4}$ is not known. But $F_{(1+4)\text{-}(3+4)}$ is known, because it is a common edge $F_{(2+3)}$ is known which is a common edge. We can find all the above quantities.

We want this quantity now which is known and $F_{1\text{-}3}$ and $F_{1\text{-}4}$ are not known, but, we already found out $F_{4\text{-}2}$ by using the first law. Similarly we can find $F_{1\text{-}4}$. We can see that we have to rewrite the quantity that is unknown in terms of known quantities by splitting either the receiving area or the sending area. While splitting the receiving area we can do it immediately because at the receiving area it is the sum of two shape factors. But if we want split the sending area we have to always ensure to include the receiving area because you cannot split this thing casually. We can split the receiving area but, not the sending area. Let us summarize what the shape factor algebra is all about.

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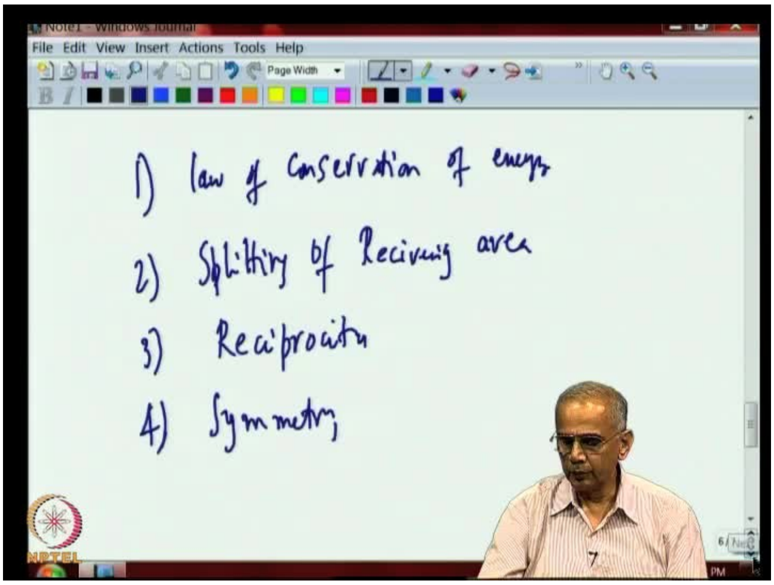

It involves 1) law of conservation energy, 2) splitting of receiving area which receives the radiation 3) reciprocity and 4) is cause like symmetry which we will use now. So, these four concepts can be used together or separately to get the answer we want. This can be illustrated in a couple of other ways that means one more example to show about symmetry.

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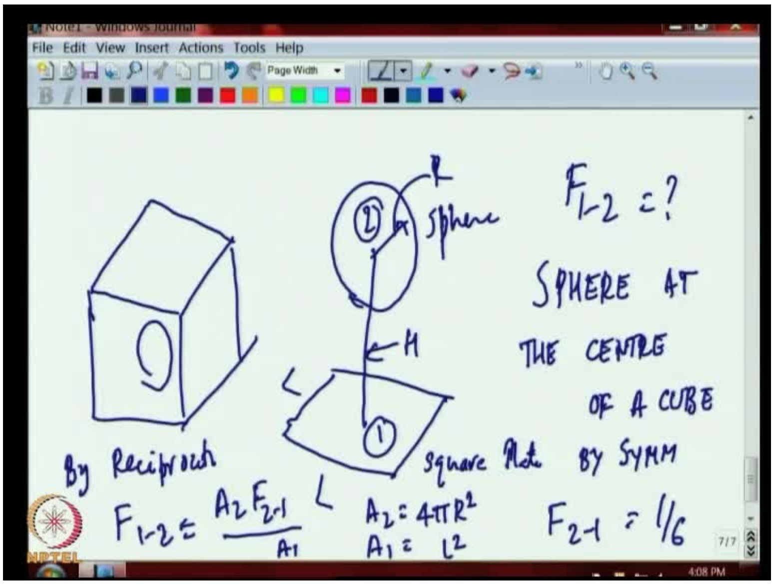

Suppose we have a sphere fig2 and a square fig1 under this figure. Let us say the center of this figure 2 sphere to the square 1 is around a distance H and the square is L by L and surfaces 1 and 2. We want to find F_{1-2}. One can do integration but that will be quite tedious and time consuming. But if we look at this problem we can see clearly that one can now draw a square box more like a cube. By symmetry we imagine a sphere at the center of the cube.

If the sphere is located at the center of a cube which has 6 sides then, we can see by symmetry F_{2-1}=1/6. The radiation leaving the sphere has reached the 6 sides in equal way 1 by 6. Once we know that by reciprocity $F_{1-2} = A_2F_{2-1}/A_1$. In the present case $A_2 = 4\pi R^2$ which is radius of this sphere and the area A_1 is nothing but L^2. We just rotate the area ratio between the area or of the sphere and area of the all the square plate. We will get our answer for F_{1-2}. Now, in this we have exploited the symmetry of the system.

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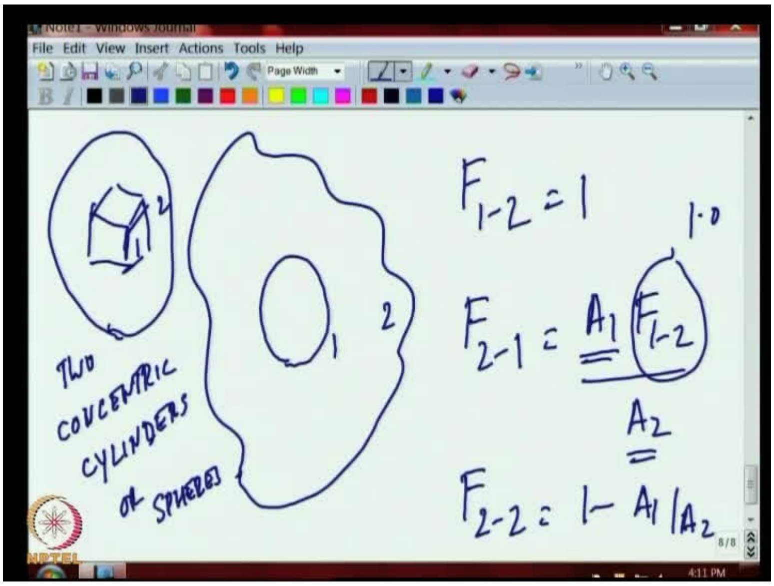

This can also be seen in another context, if we have a complicated shape of object surrounding the sphere. When we redraw it we imagine a sphere which has surface 1 now or a cylinder pattern surrounded by a complex object with surface 2. The surface 2 is quite complicated as it has various wiggles and curves and so on. We know from energy conservation F_1 has to be 1 because surface 2 completely covers surface 1. So, any radiation has got reach to 1. Here we can exploit symmetry and reciprocity let us say, F_{2-1} is A_1F_{1-2}/A_2. Still this is equal to 1. All we need is either ratio A_1 by a 2 surface by ratio.

We also know F_{2-2} radiation leaving to reach 2 is $1-A_1/A_2$. We got 2 shape factors by just appealing to the fact that radiation leaving 1, all of it has reached 2. F_{1-2} is 1 then we appeal to reciprocity and find that is $F_{2-1}=A_1/A_2$. Once we have done that by first law of thermodynamics then sum of $F_{2-1}+F_{2-1}=1$. So, $F_{2-1}=1$. Now, we can find F_{2-2}. This is true for two concentric cylinders or spheres. As a matter of fact they are also valid, if we have square object.

Imagine a square object with surface 1 inside an enclosure with surface 2 then the same method can be applied. This is only area of this square object. The cube is different from that of a sphere or a cylinder. The only requirement here is that the inside object has to be non concave, that is inside object cannot see itself. If the inside object cannot see itself then F_{1-2} is = 1.

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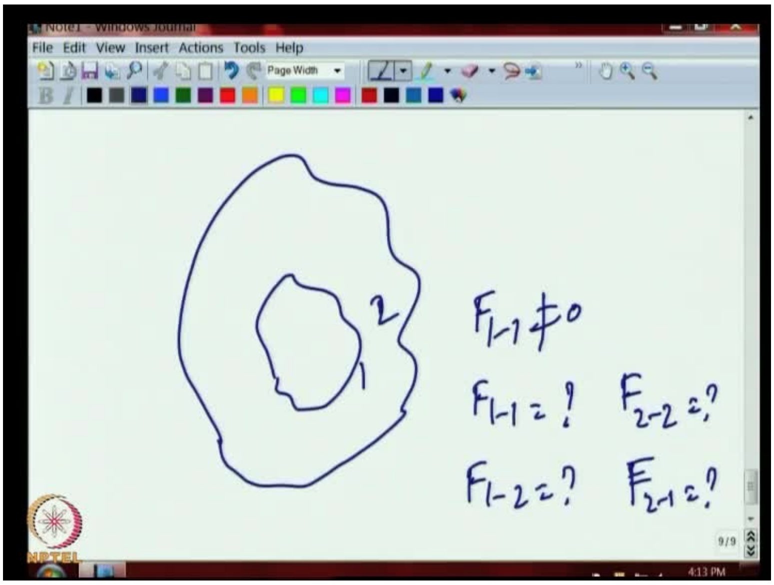

If F_{1-1} is not equal to 0 then in this situation, there too many unknowns. There are 4 unknowns F_{1-1}, F_{2-2}, F_{1-2}, F_{2-1} and we have 3 equations. We have to accept the fact that in this situation simple method is not possible. Let us take one more example.

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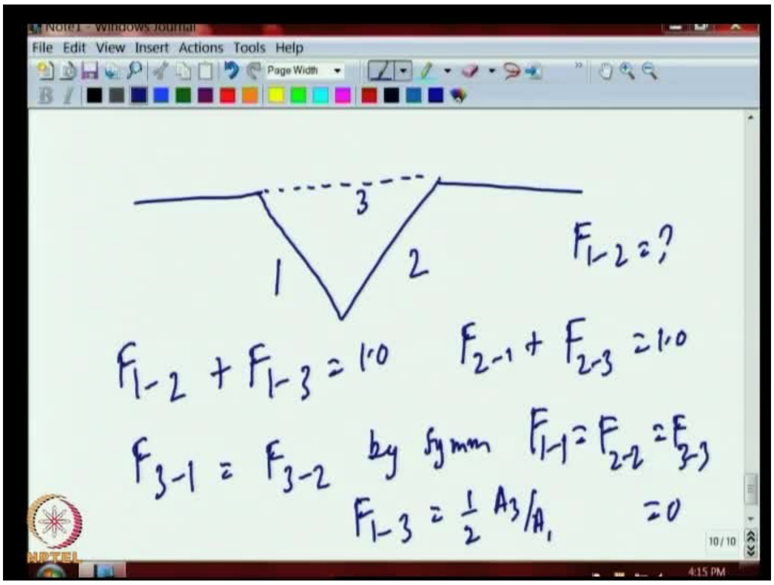

Suppose, we have a wedge like this and this surface is 1, surface 2 and, we want to find $F_{1\text{-}2}$. What we can do this case is construct an imaginary third surface, which is flat. We can write down that $F_{1\text{-}2} + F_{1\text{-}3} = 1$. $F_{2\text{-}1} + F_{2\text{-}3} = 1$ and by symmetry $F_{3\text{-}1} = F_{3\text{-}2.}$ $F_{1\text{-}1} = F_{2\text{-}2} = F_{3\text{-}3} = 0$. We can show that $F_{1\text{-}3} = 1/2\ A_3/A_1$. We just have to find the area A_3 and A_1 and we are going to get the answer. Here is an example where we have used first law and symmetry considerations, to get the answer that we want. Now let us give one more example of this and then we will want to move to other situations.

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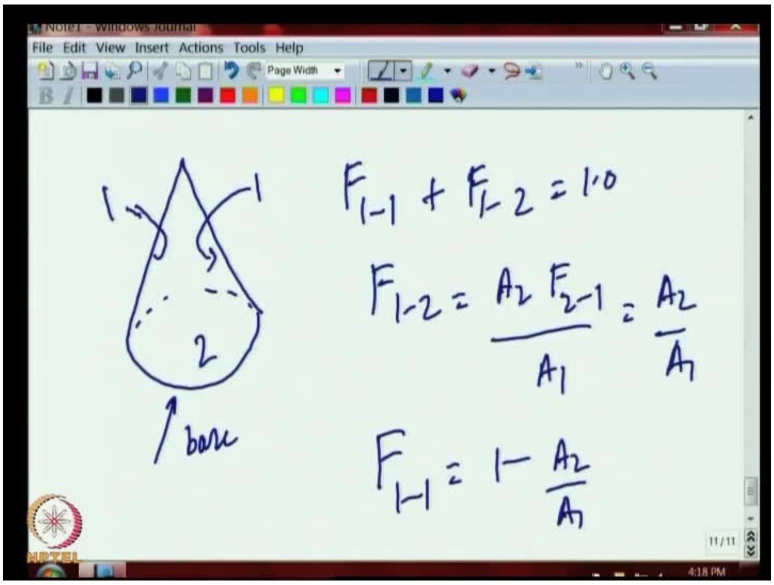

We imagine a cone with surface 1, and surface 2 as shown above. We know that F_{1-1} plus F_{1-2} equal to 1 that is the first law. Then F_{1-2} is A_2 $F_{2-1}/A_1F_{2-1} = 1$. Because it is a flat surface. This is a kind of example where one can exploit symmetry to one's advantage. Now, the best example about symmetry and exploiting symmetry is, it is illustrated by the application of spherical symmetry and utilizing it to obtain shape factors between two discs.

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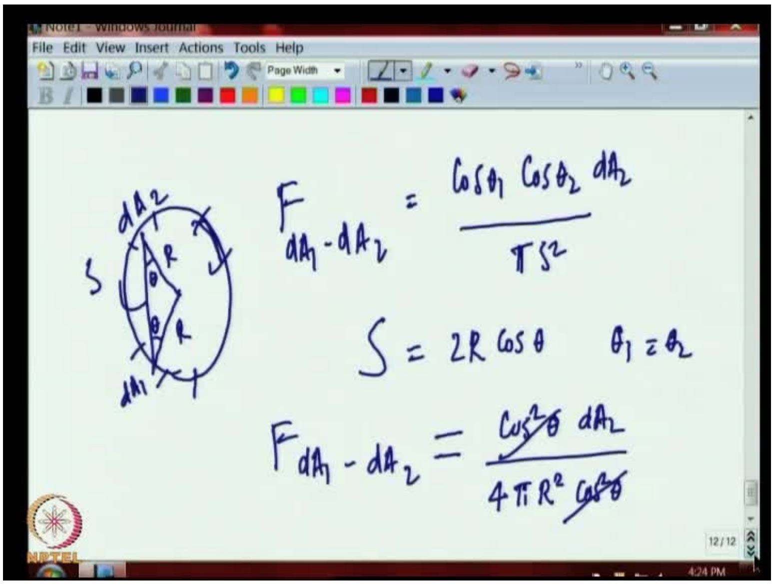

Let us consider spherical enclosure here. We look at two surfaces in enclosure d A 2 and d A 1. We can draw a line a chord between them and join the ends of the chord at the center as shown above. Inside this sphere, both these chords make an angle θ. We write $F_{dA1\text{-}dA2}$ from the basic definition of shape factors. This is $F_{dA1\text{-}dA2} = \cos\theta_1 + \cos\theta_2\, dA_2 / \pi S^2$, where S is the chord between the 2 surfaces R and R. We can show that S is nothing but, $2R\cos\theta$ and $\theta_1 = \theta_2$ for a sphere. $F_{dA1\text{-}dA2}$ becomes $\cos^2\theta\, dA_2 / 4\pi\, R^2 \cos^2\theta$. $\text{Cos}^2\theta$ cancels out. The result says that the shape factor between two areas in a sphere is nothing but the area of that receiving surface d A 2.

Read by the area of the sphere and is independent of the area of the surface d A 1 and independent of the angle between them. This is a unique feature of a spherical enclosure, and this means that $F_{dA1\text{-}dA2}$ is known once we know the radius of the sphere. The relative locations of d A 1 and d A 2 are relevant, they are located in the opposite sides of this sphere or adjacent to each other. They all have the same shape factor. Here the surface is the distance between surfaces is small but it is not viewing much of this surface. Hence the two effects can surround exactly in the case so that the shape factor d A 1 d A 2 does not depend upon the distance between the two objects. They depend only in the area of the receiving surface which is the surface area of that sphere. Now, we are going to exploit this unusual feature in another example.

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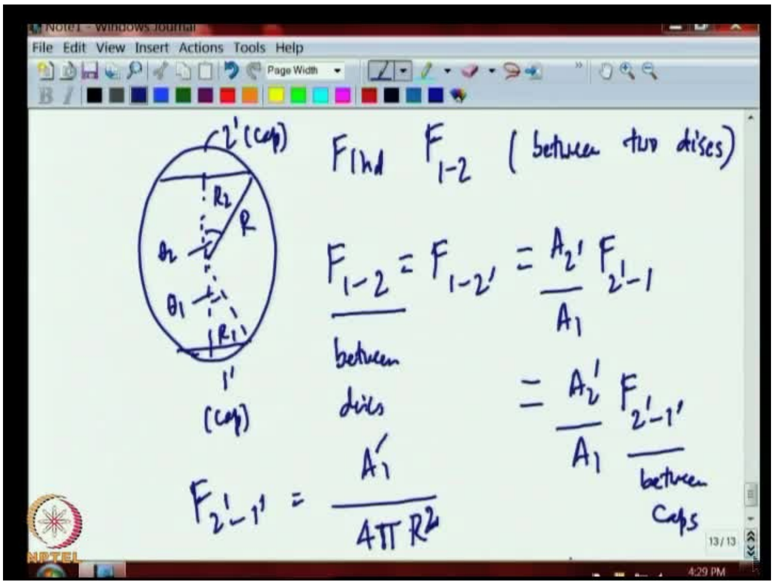

We take a sphere and draw two discs with radius is R as shown above. We can draw the line joining the centers let us call this angle as θ_1 and θ_2. We have to find F_{1-2} which is the surface between the two discs. It may look surprising that we are trying to find shape factor between two cylindrical objects, two discs by putting them inside an enclosure which is spherical. But we will see how this is done. We call the gap between surface 1 and circle as 1' and surface 2 with enclosure as 2'. Our logic is F_{1-2} will be same as F_{1-2}' because any radiation which leaves 1 and reaches 2 will also reach 2'. Any radiation which leaves 1 and arrives at 2 will also arrive it 2' same as the first logic.

We exploit reciprocity where this as $A_2'/A_1\ F_{2'-1'}$, there is a reciprocity. Then any radiation which leaves 2' reaching 1 will also reach 1'. Again we are exploiting the fact that radiation leaving 2' and crossing 1 will also cross 1', because 1 completely covers 1'. We notice that starting with shape factor between discs we have arrived at shape factor between gaps.

We have reduced a problem of finding shape factor between 2 discs to that of finding shape factor between 2 gaps within a spherical enclosure. This we already know from the previous discussion that $F_{2'-1'}$ is nothing but the area $A_1'/4\pi R^2$. The shape factor between 2 gaps in a spherical enclosure depends only on the area of the receiving surface; and the surface of this area of the spherical enclosure. Now, we have completed the derivation. We can easy calculate A_1', A_2' and A_1 and we got F_{1-2} shape factors between 2 discs in terms of area of the gaps. Hence this is large simplification of the problem.

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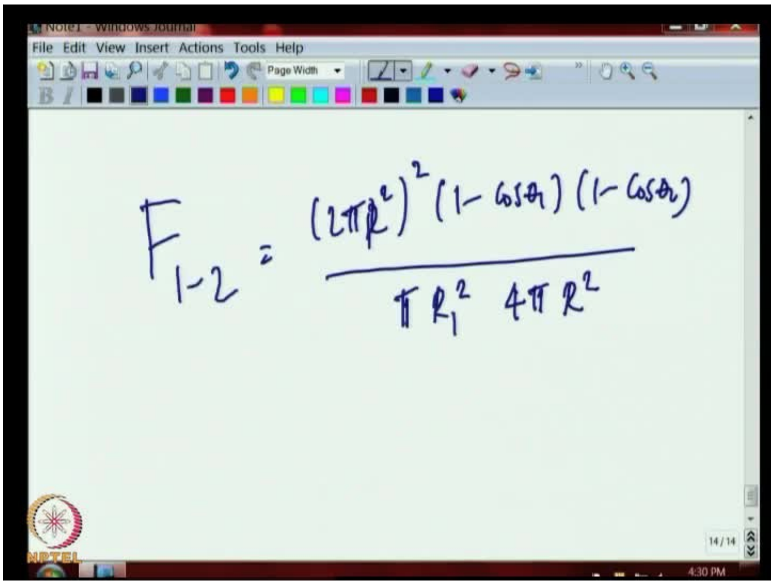

The final result will come out as $(2\pi R^2)^2 (1-\cos\theta_1) (1-\cos\theta_2) / \pi R_1^2 4\pi R^2$. This is obtained by just geometry of the problem. This clearly shows the power of arguments of related symmetry which helps us to derive new shape factors based on old shape factors. We should be able to look at the basic definition of shape factor and see that there are some interesting factors.

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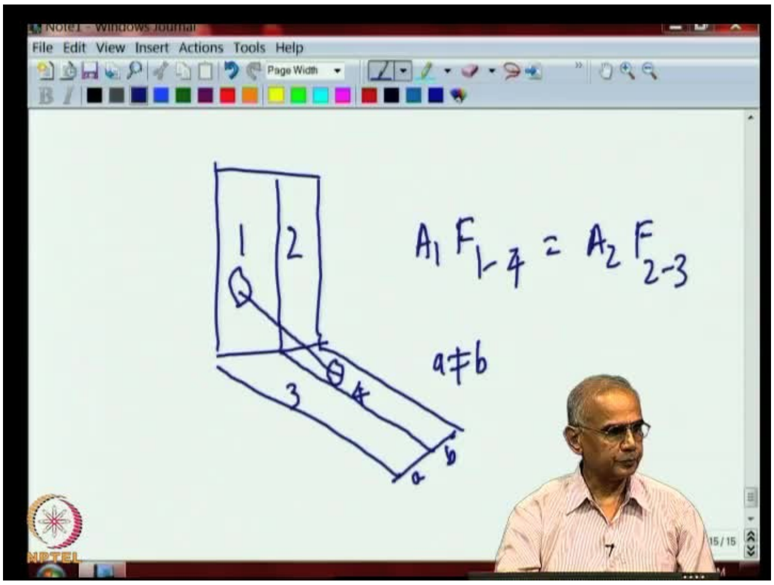

For example, suppose these are surfaces 1, 2, 3 and 4. We can show that $A_1 F_{1\text{-}4} = A_2 F_{2\text{-}3}$. When we write down actually, integrals we will take areas dA_1 and dA_2 we find they are identical. We draw a line as shown in the above figure calculate the integral factors. We can write down the full double integral of these two surfaces 1,2,3 and 4. We see that the expression can be identical based on the identity of the expressions $A_1 F_{1\text{-}4}$ and $A_2 F_{2\text{-}3}$.

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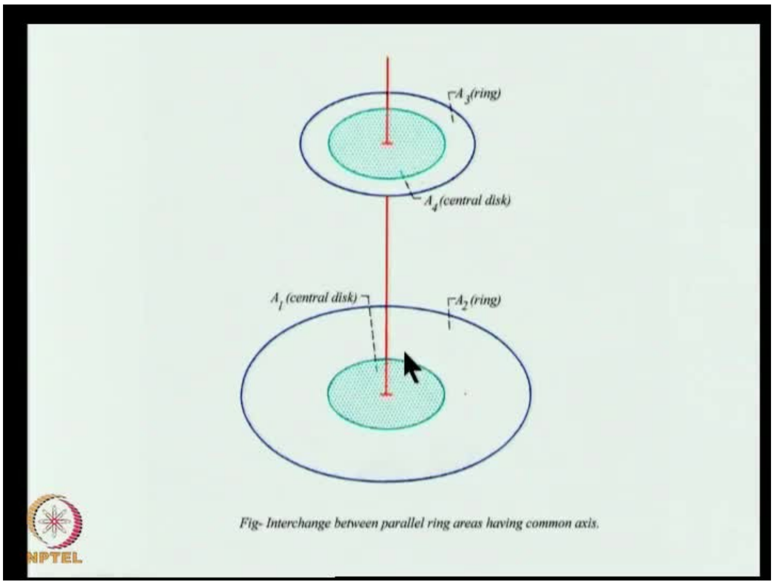

We show one more example of calculating shape factor using shape factor algebra. Now, we are interested in calculating shape factor between two rings as shown above. Given the shape factor between two discs, we want to find , shape factor between two rings. We know A_3 is a ring and A_4 is the handle. We now have 3 plus 4 to 1 plus 2. So, we can write it has $F_{(3+4)-1} + F_{(3+4)-2}$. We use reciprocity each time keep splitting the receiving area. By doing so finally we will emerge with the result that is similar to what we did for the case of a rectangular object. This methodology works quite well whatever is shape of the object whether it is rectangular, spherical or any shape.

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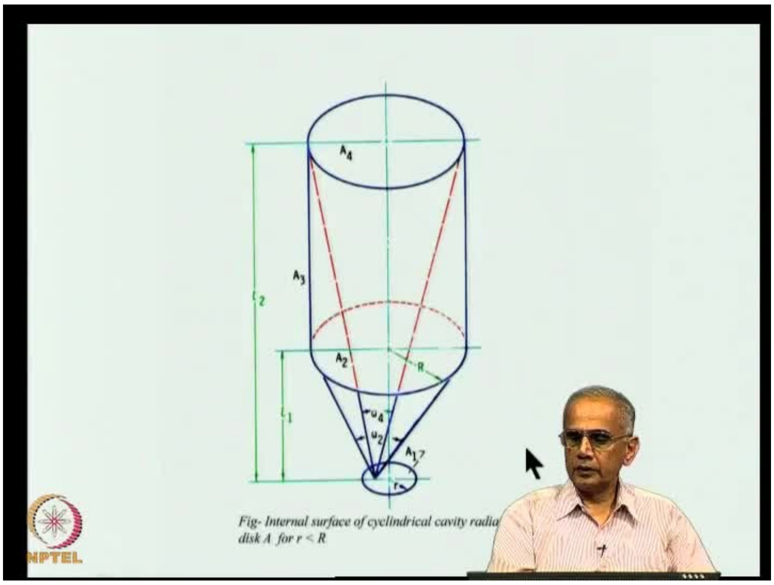

Above is one more example. Here the interest is calculated as A 1 to A 3 based on disc to disc shape factor. For example, we know F_{A1-A2} because it is shape factor between two discs, that is usually available in most text books on radiation. Similarly, F_{A1-A4} is also available in most text books. We know by the first law that the radiation that leaves at 1 and arise at 2 ultimately reaches 3 or 4. That is $F_{A1-A2} = F_{A1-A3} + F_{A1-A4}$. We know F_{A1-A2} and F_{A1-A4}. Therefore, F_{A1-A3} is F_{A1-A2} - F_{A1-A4}. We are able to calculate shape factor from a disc to inside of a cylinder by using shape factor between two discs. This is again a simple use of common sense and the first law of thermodynamics.

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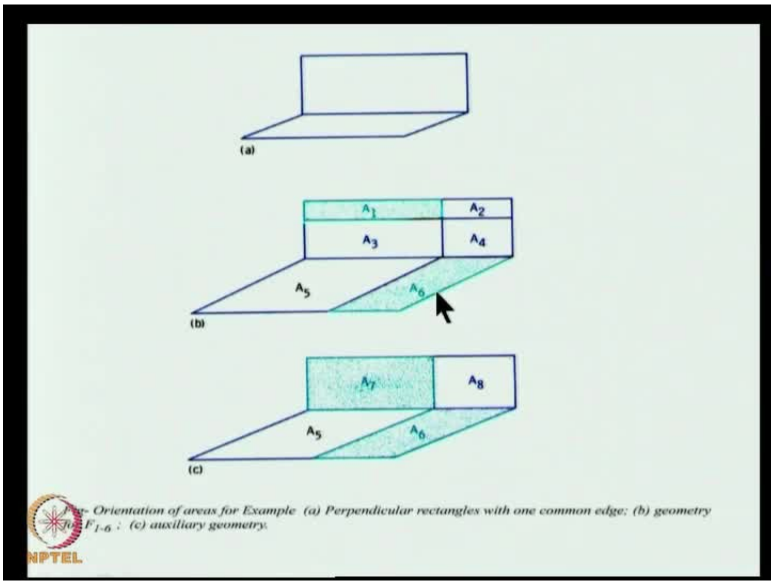
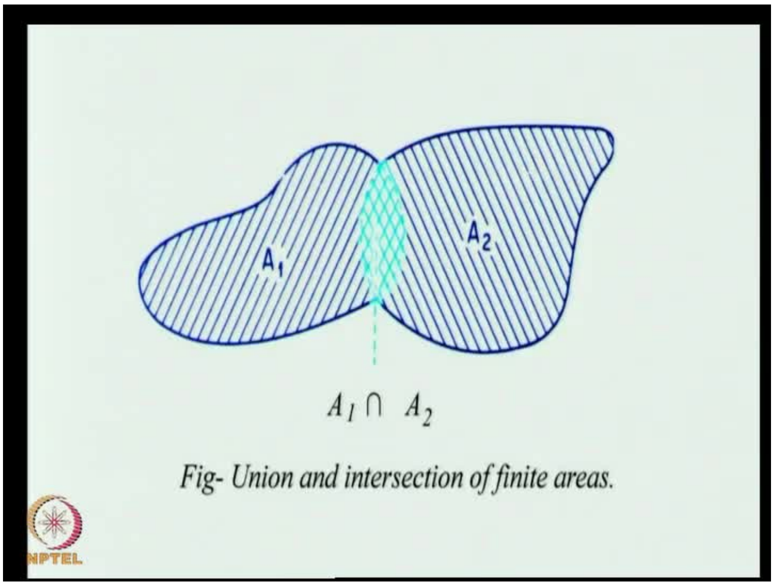


Now, similarly for example given the shape factor between two rectangles with the common edge, we can calculate the shape factor between two rectangles without the common edge. This is an extension of what we discussed earlier in this lecture. We start with the fact that we know 1 plus 2 plus 3 plus 4 to 5 plus 6. To split that we know 1 plus 2 plus 3 plus 4 to 5 and 1 plus 2 is equal to 6. We then use reciprocity and the first law of thermodynamics; ultimately we will get this quantity. (Refer Slide Time: 48:32)



We can also exploit concepts of set theory. Suppose, we want to find the shape factor from some surface to a common surface which is composed of two sub surfaces with the common area. Then F_{A3-A1} + F_{A3-A2} - F_{A3} to this common area can be removed as this is the overlapping region which is recorded twice very easily.

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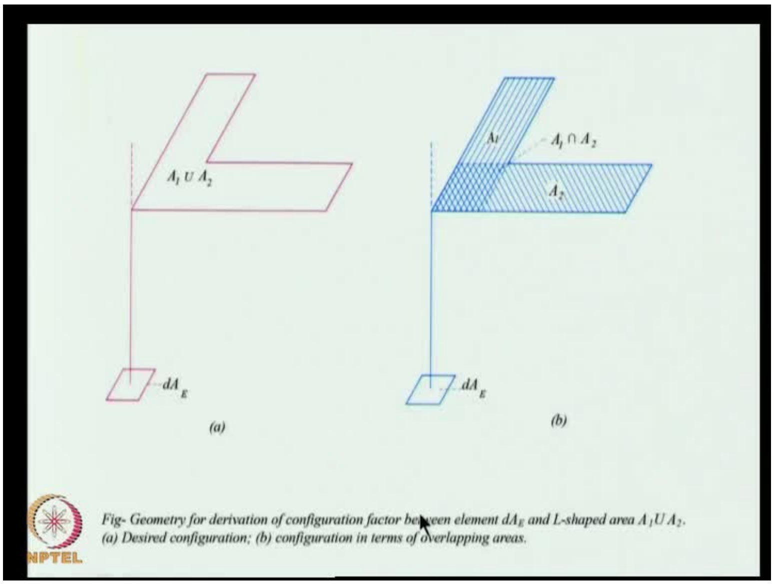

Here is an example where we know this shape factor from the rectangle to another rectangle with the common edge as shown above. We can calculate F_{AE} to this rectangle twice, so we subtract this common area. Hence F_{AE} to this quantity is $F_{dAE-1} + F_{dEA-2} - F_{dAE}$ to this common rectangle. So, this is exploiting again common sense, we take little more complicated sense.

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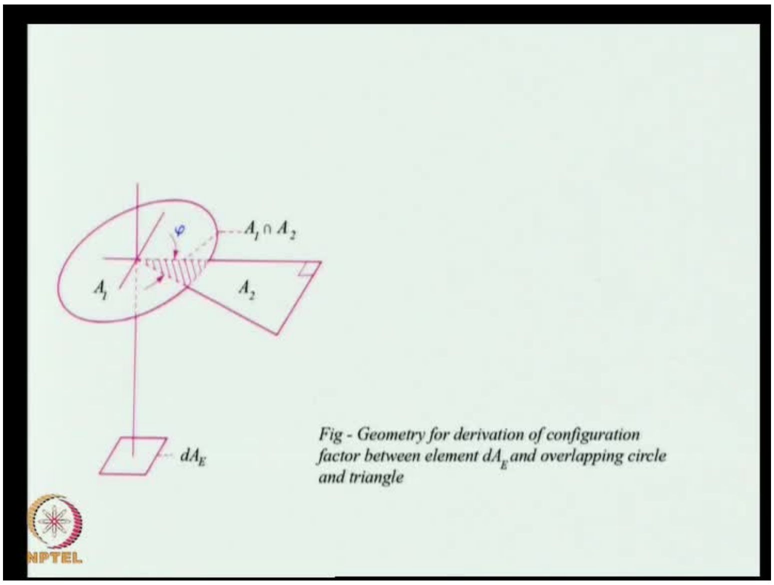

Suppose, we want to find from this area d A E to this key shaped object. The key shape object looks a bit non standard, for which we will not have standard shape factor given in the books. But we do have shape factor from the rectangle to a circle; a rectangle to a triangle; these are available. Given this, we calculate d A E to A 1, d A E to A 2. But in doing so we have counted this part twice. So, we say the total answer is d AE 1 plus d AE 2 minus d AE to this sector of a circle which is nothing, but the total area divided by chord line by 2πR. So, here again we are able to get a shape factor for a complex shape by resolving into elementary standard shapes and ensuring that any place that which has been counted twice has been removed in the calculation.

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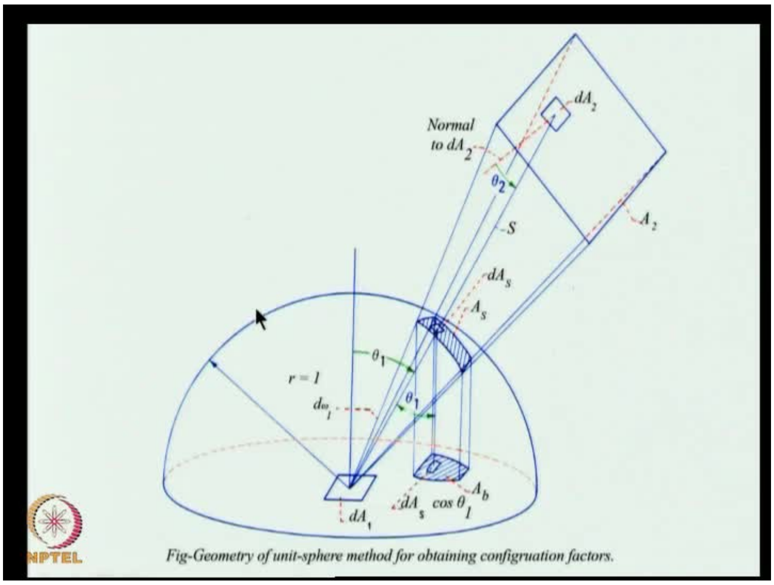

Now, finally we will end this lecture by pointing out that if all other methods cannot be used to calculate shape factors then to calculate shape factor between complex objects like inside of furnace. We can always use completely Empirical Method. In this method we imagine an imaginary sphere which could be a translucent surface and, we create this condition in the laboratory, where we have this object A 2 and object A 1. We want to calculate at $F_{dA\text{-}dA1}$ and for doing that we use completely, empirical technique without actual calculation. We can do that by looking at the image above of the surface. From the top, we take a photograph which will be $dA_s \cos\theta_1$ which has a radius of unity. By geometry the shape factor is nothing but $dA_s \cos\theta_1$. We should measure this area from a camera located on top and once the area is measured we will immediately get the shape factor by purely empirical method without actual calculation. This method was used a long time ago when people did not have access to access to computers. They calculated the area of the image divided by pie time, the unit radius sphere; and they got this all they wanted. So, this is the last resort; we can use in case we are not able to integrate those functions or not able to find shape factor in a text book or any standard database. Then, we can go to lab to create a condition similar to what we want; and take a photograph of that object on a mirror surface and that area will be the area, divided by pi R square assuming 1 as the shape factor. We have completed discussion on calculating shape factor for variety of situations. In the next lecture, we use the information about creativity property of surfaces like emissivity, absorptivity of reflectivity and the shape factor data. We have so far discussed to actually calculate ready to transfer in enclosures.

Thank you.