Radiation heat transfer
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Lecture - 6
Triangular-enclosure

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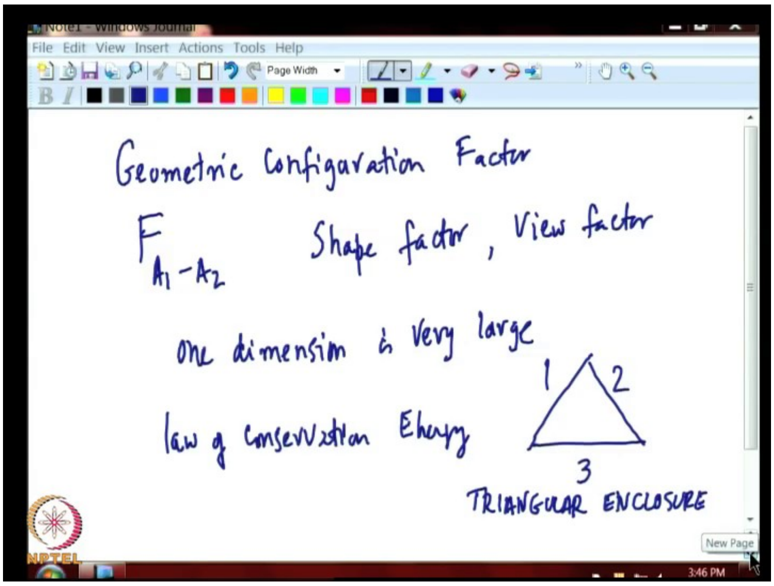

In the last lecture, we defined the geometric configuration factor F_{A1-A2}. This tells us the fraction of radiation that leaves surface A_1 , and reaches surface A_2 . So, the term geometric configuration factor is the correct definition because this depends on the geometry of the problem, the various geometry involved and the relative configuration of the two surfaces. But since this term is long and unwieldy, people use short terms like shape factor or view factor. So, these are words normally are used to indicate this term, which is the geometric configuration factor.

We saw that this can be calculated by the direct integration of the equation over both the areas, but this can sometimes get quite complicated and not easy to evaluate. It is useful to know for practical application, some shortcuts and other techniques to evaluate this for a given situation. We started with one example in the last lecture, which is for situation where one dimension is very large.

When 3 dimensional problem is essentially 2 dimensional, but even in a 2 dimensional problem, we have to integrate over the third dimension, but if the third dimension is very large, we can appeal to law of conservation energy to simplify the problem to some extent. In the last class, we saw that if we have a triangular enclosure, which is the infinite in the third dimension then we can easily calculate the shape factor between 1 and 2 and 1 and 3 and 2 and 3 by just using the law of conservation energy and also the law of reciprocity.

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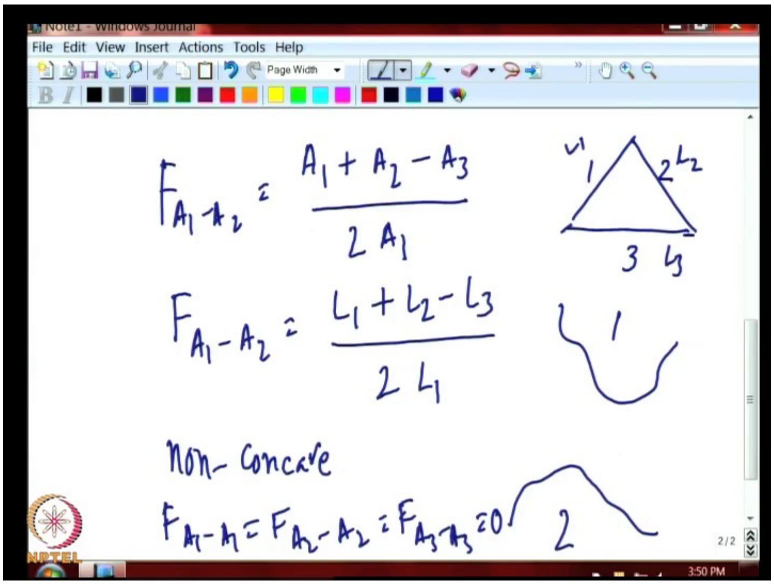

In the last class we derived this equation using both the law of conservation energy and reciprocity and we saw that we could derive an expression for F_{A1-A2} in terms of the area of A_1+A_2-A_3/2A_1. We derived this just by looking at the 6 shape factors involved F_{12}, F_{13}, F_{21}, F_{23}, F_{31} and F_{32}. We have 6 equations, where 3 equations satisfy the first law of the energy for these surfaces and the other 3 equations that involves reciprocity.

We could solve for all the shape factors and so, we got this result. Now, this result can be made simpler because all the dimensions of this enclosure are infinite into the page. So, we can replace all the areas with lengths L_1, L_2 and L_3 (L_1+L_2-L_3/2L_1). We just have to measure the lengths of these three sides of the triangle and we will get the shape factor. This is the very convenient and simple method to calculate shape factor without actually, integrating those equations. Although it is a nice result you might wonder how many real life situation we will encounter, which has triangles with straight sides like these. There are too many such cases.

The real world is more complicated as it has curved surfaces and it may not be convex actually. For example suppose, we want to shape factor between these two surfaces 1 and 2, this is not two straight objects, but a slight change at triangle enclosure. That was shown by Professor Hottel from MIT that even if the real problem does not have straight surfaces suppose, they where concave surfaces. That is F_{A1-A1}, F_{A2-A2} and F_{A3-A3} were equal to 0. In many real life situations we are going to encounter surfaces, which are concave that is which see themselves and in which case F_{A1-A1} will not be equal to 0. In such a situation we need to extend what we had obtained just now, for a triangular enclosure to a more general case.

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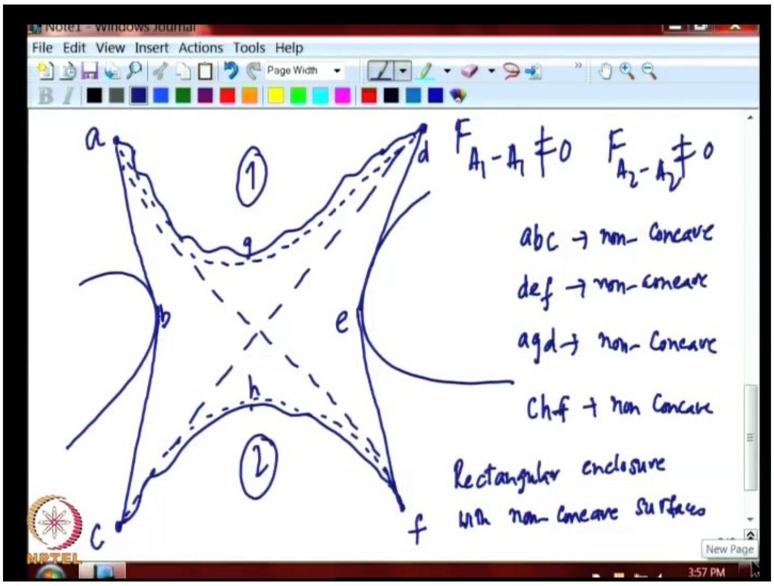

Let us derive a more general technique available. Let us say this surface 1 and this is surface 2 both have concave features. Then F_{A1-A1} and F_{A2-A2} are not equal to 0. The solution suggested by Professor Hottel of MIT, was to create two triangular enclosures within this one. Let us take the edge of this surface that is a from the above figure and the other surface as d. The other two edges of the surfaces are c and f.

Now we need to create artificial surfaces, which are not concave. We will take the edges of this protrusion and draw a straight lines as shown above. So, a, b, c, is non concave because it is made up of two straight lines, which are tangent to the point b. Similarly, we choose the extreme point e here and draw a tangent tangible to join f. We get d, e, f which is also non concave virtual surface.

Now, both the surfaces have a concave portion. We now will draw an imaginary surface a, g, d which is always convex. Similarly, we will draw another imaginary surface c, h, f. Now we have created a rectangular enclosure with non concave surfaces that is surface where in a self-view factor that is F_{A1-A1}, F_{A2-A2} and F_{A3-A3}. They are all zero's because these surfaces do not have concave features because ultimately we have to find F_{12}.

Our aim is to find F_{12} given this geometry. We are going to construct two more surfaces. One is joining a and f and c and d. Both these surfaces are straight lines. What we have done essentially is we have constructed two triangles a, b, c, c, h, f, a and a, f, h. The two triangles we have constructed would be such that there is a common base. We calculate the shape factor using the formula for a triangle., We calculate with a, g, d as the base.

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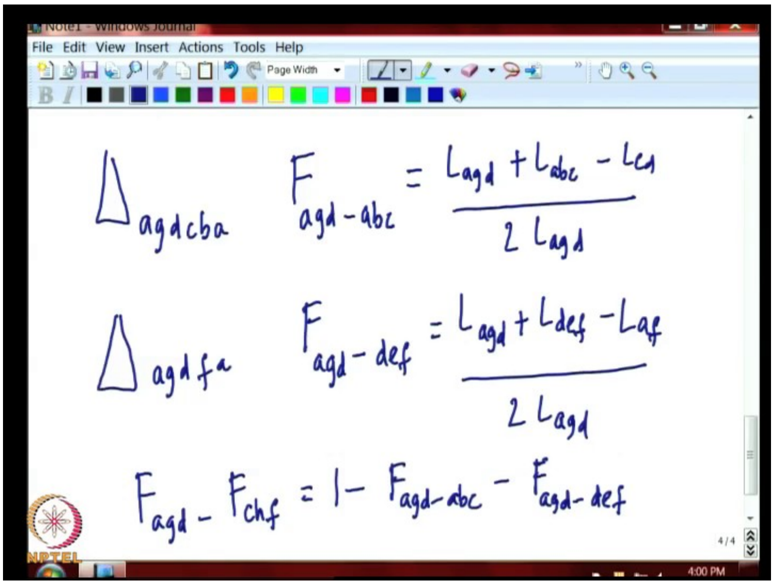

We consider triangle a, g, d, c, b, a. From the formula for a triangle we can write $F_{agd-abc}$ both are concave surfaces. According to the triangle law, we have derived this will be equal to $L_{agd} + L_{abc} - L_{cd} / 2L_{agd}$. Thus by applying the triangular formula for non concave surfaces, the triangle is a, g, d, c, b, a.

Similarly, consider triangle a, g, d, f, a. Based on the above derivation, we can say $F_{agd-def} = L_{agd} + L_{def} - L_{af} / 2L_{agd}$. We have taken two triangles with the common base for each of this triangle, which contain by construction non concave surfaces. We apply the formula for a triangular enclosure with non concave surfaces and arrive at the result. We can now write down by applying the first law that $F_{agd} - F_{chf} = 1 - F_{agd-abc} - F_{agd-def}$.

We calculate the shape factor from base surfaces of the a, g, d to the two side surfaces as above. The remaining unknown is a, g, d to c, h which is what we need, but we knows that this sum of shape factor a, g, d to a, b, c, a, g, d to d, f and a, c, h f is equal to 1 and hence knowing these two, we can get the third because the sum of the three is equal to 1., We now need F 1 2. We have managed to get F_{agd} - F_{chf}. To get F 1 2 we have two more steps.

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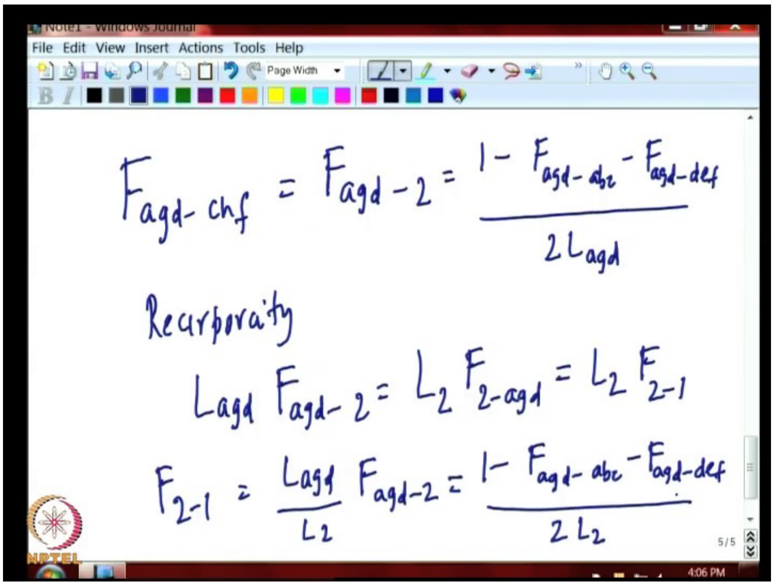

We will now write F_{agd} - F_{chf} = F_{agd-2} because whatever radiation leaves a, g, d and reaches c, h, f must also reach 2. Once it crosses c, h, f there is no other place to go except surface 2 and with these constructions, we have come to the end of the derivation. All these are what is called as taut strings, that is why the method we had derived just now is called the Hottel's cross string method. We are drawing two cross strings here and two parallel strings and all these strings are so, drawn that the triangle that emerges from this constructions is a triangle containing non concave surfaces and hence self viewing factor is 0.

One can apply the triangle formula to get the result in terms of the length of the strings and so, the final step here is to calculate F_{agd} - F_{chf} = 1 - $F_{agd-abc}$ - $F_{agd-def}$ / $2L_{agd}$. Now, to get F 1 2 we use reciprocity which says that L_{agd} F_{agd-2} = = L_2 F_{2-1}. This is always valid for all surfaces. Now, when we calculate F_{2-agd} whichever radiation reach as a, g, d from 2 has to reach 1. Once, they cross this imaginary surface it has to reach 1. Therefore, we have reached our final goal that is F 2 1.

We will write F_{2-1} = (L_{agd} / L_2) F_{agd-2}. By substitution we rewrite this as F_{2-1} = 1 - $F_{agd-abc}$ - $F_{agd-def}$ / 2 L_2. When we say L_2 we mean the actual length that has to be measured along this crooked surface c, h, f. Now, we can substitute expression for a, g, d and a, g, d, f.

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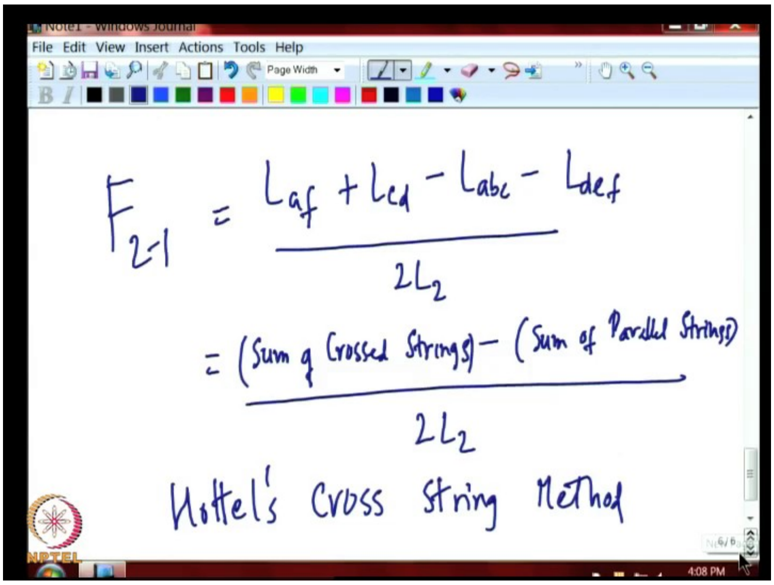

The final expression we get for $F_{2-1} = L_{af} + L_{cd} - L_{abc} - L_{def} / 2L_2$. We can remember the result as the sum of crossed string, minus sum of parallel strings divided by 2 L 2. This is very simple and clean result and the only challenge is for one to correctly identify, the cross strings and the parallel strings to be drawn between the two surfaces. In drawing these strings, one must take special care to ensure that one creates two triangular enclosures each of them having only non concave surfaces. Then we can apply the laws that we obtained for non concave triangular enclosure.

We use what is known as the Hottel's cross strings method. We are now going to give a series of examples to show how to apply, the Hottel's cross strings method to a number of practical situations.

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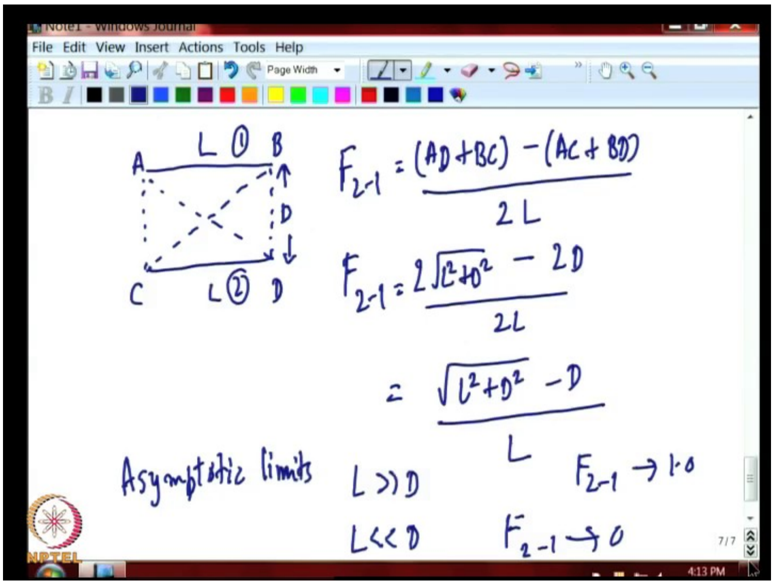

Suppose, we have 2 parallel plates of lengths L_1 and L_2 and distance D and we want to calculate, let us say F_{2-1}. Let us say the lengths are A B and C D. Using the Hottel's crossing method, we can see that we must draw two parallel strings and then two cross strings. The Hottel's method says that F_{2-1} = (AD+BC) - (AC+BD) / 2L. Now, this example is very simple because the basic surfaces are flat and easy to visualize what the strings are. We know that AD is $\sqrt{L^2+D^2}$ and BC is another $\sqrt{L^2+D^2}$. We rewrite the derivation as F_{2-1} = 2 $\sqrt{L^2+D^2}$ - 2D / 2L. We get F_{2-1} = $\sqrt{L^2+D^2}$ - D / L.

The length is much larger than D, we can see that as the gap between the two plates becomes smaller and smaller and not much of the radiation escapes from the sides. Hence, the shape factor F_{2-1} must end to 1 and we will neglect D compared to L and again take this F_{2-1} l as tending to 1 . The other limit L if is very, very small compare to D, the two plates are very far apart, then the chances of radiation from one plate never reach the other plate as chances are very, very low and in this asymptotic limit the value of F_{2-1} will tend to 0. Both these are easily seen in this result. In the other case we neglect D compare to L and we got 1. So, both limits are satisfied. We are quite sure that this derivation was right and we did get that answer. This of course, was a simple problem involving very simple geometries. Now, let us do something little more complicated.

Now, we will take a few more examples in which we can apply the crossing method. We take an example where there are obstructions that come between the two surfaces. To draw the parallel cross strings becomes an issue.

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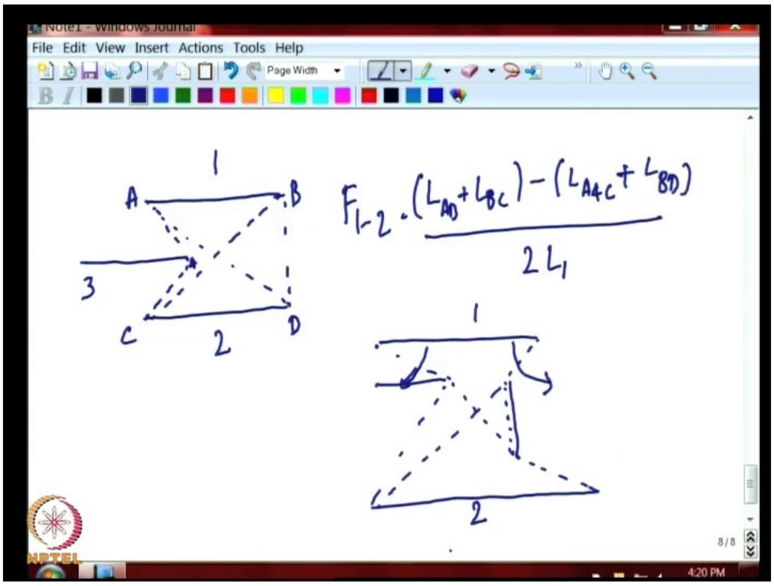

Let us extend the previous example suppose, we have two surfaces 1 and 2 and an obstruction 3 coming. This obstruction reduces the view of 1 from 2 and vice versa. To apply the cross string method, we draw parallel strings as shown in the above diagram. We now have two triangular enclosures with common base.

Let us call this edge 4, as A, B, C, D. If we really see that F_{1-2} involve having the two cross strings, minus 2 parallel strings which are the length BC, then $F_{1-2} = (L_{AD} + L_{BC}) - (L_{A4C} + L_{BD}) / 2 L_1$.

In this problem since an obstruction appeared from this side, we have to draw the parallel strings in such a way as to ensure that the obstruction was kept out. As a matter of fact there is a simple thumb rule to decide weather, we have drawn the parallel strings and the crossing correctly The parallel strings will be so, drawn that any radiation leaving 1 and reaching 2 will not cross the parallel strings.

That is any ray which leaves 1 and arrives at 2 should not cross the parallel strings, and any ray, which leaves 1 and arrives the 2 must cross the cross strings. After drawing the parallel strings and cross strings, we draw a few extreme rays to see whether, the strings drawn is such that the parallel strings do not allow radiation which should reach 2 to cross the parallel string. Every ray which leaves 1 arrives at 2 and crosses both cross strings.

Now, let us take a few more examples to illustrate this point of view, which is a little more complicated one. Let us take little more inward example and apply the same logic as before. We now have two obstructions on surface 1 and 2. We have to draw the one parallel string same as before to keep the radiation out, and then draw the cross strings and the other parallel string as shown in the figure (Refer Slide Time: 37:31). This is little more complicated than the other example, but still one can draw various rays like this one to make sure that this ray which is a part of a parallel string is a ray, which does not reach 2. We have to guarantee that it does not reach 2.

Then we can calculate the length of these 2 strings and get the answer. We can see that as the problem get's more and more complicated with obstructions we got to be little careful about how we construct the strings, and do the few trial checks to ensure that the triangles are drawn properly.

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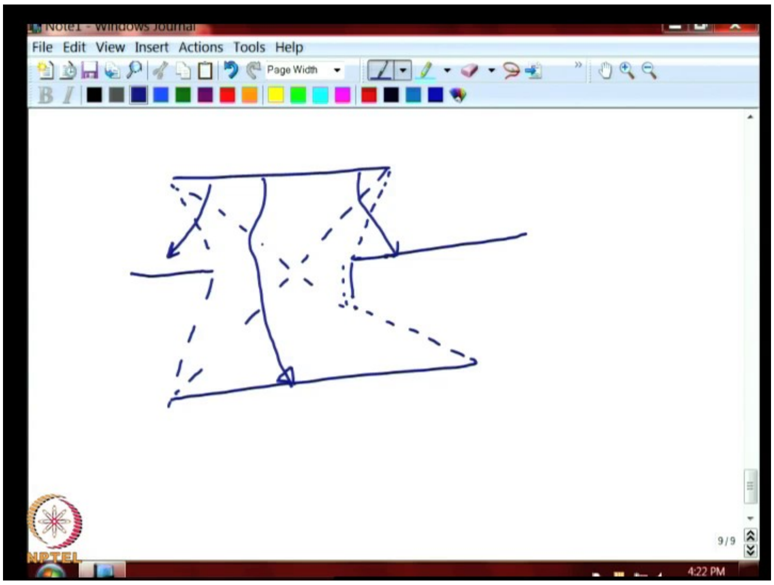

Suppose, there are two obstructions as shown above. To calculate, there should be one parallel string. The second parallel string should be drawn in such a way to keep out obstructions. We draw cross and parallel strings such that, we have ensured that the ray which crosses the parallel strings never reaches the other surface, and the one which crosses both cross strings must reach other surface.

We can draw various rays and make sure that the construction of the cross and parallel strings is accurate, and that it satisfies the rule that any ray leaving surface 1, and reaching surface 2 must cross the both the cross strings, and that any ray leaving 1 and not reaching 2 must go through the parallel strings.

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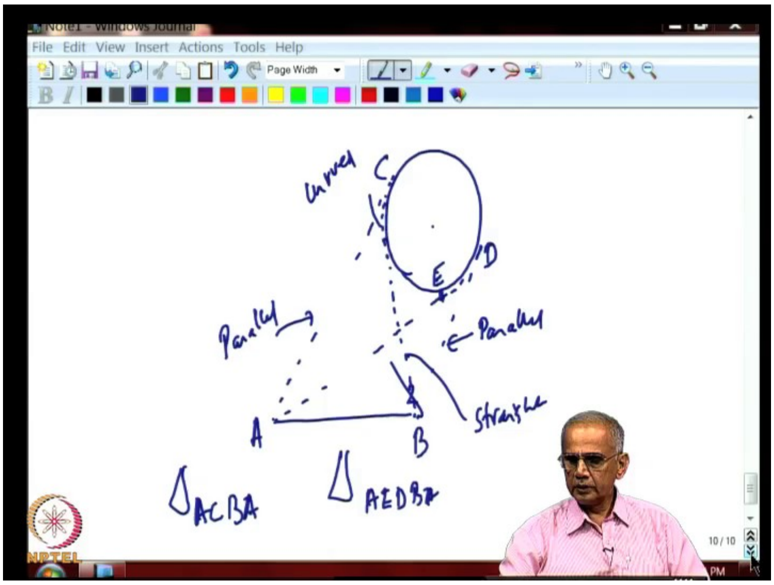

Now, let us take one more example where the surface is a circle. So, we have this surface 1 and surface 2. First we draw a tangent and then we draw one right along this and then were can draw tangent to this. This should be actually be in dotted lines. Similarly, we draw this until it touches the tangent and then go like these. We can see that the two parallel strings are the straight lines but the cross strings involve a curve portion and a straight portion.

What we have done really is that we again constructed two triangles. Let us call this A, B, C and D. Here one triangle is A, C, B, A. This triangle consists of non concave surfaces. We can apply the triangular rule. We will call this tangent E, A, E, D, B, A. The other triangle is A, E, D, B, A with non concave surfaces with the common base A, B. We can apply the same rule that we have applied to derived the Hottel's cross strings method, and we will get the sum of the two cross strings minus some other two parallel strings by twice the length of A, B. Now, there are situations where one may not even have to draw this strings directly.

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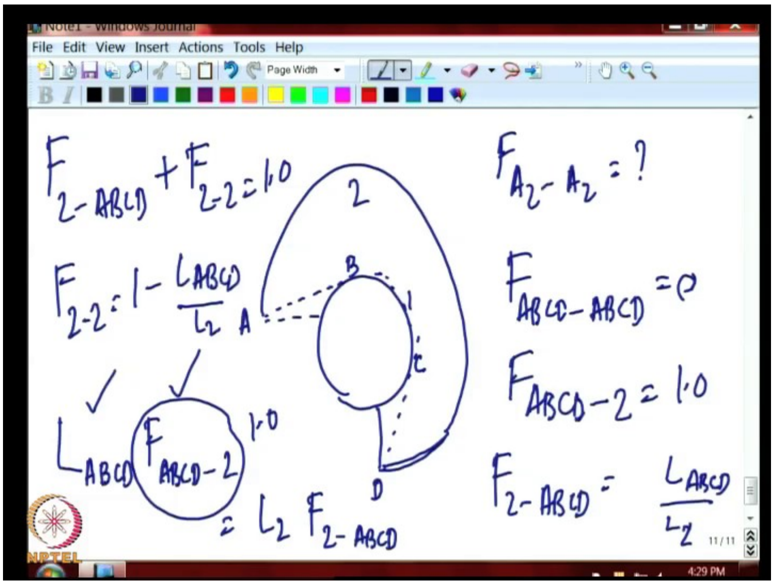

For example, suppose there are two circular tubes, a hollow tube with surface 2 and a solid tube with surface 1. We want to find F_{A2-A2}. The best way to do this problem is to construct an imaginary surface. We notice that this surface A is tangent to B on the curved surface and the curved surface C is tangent to D. A, B, C, D is a convex surface. Therefore $F_{ABCD-ABCD}$ is 0 and it cannot see itself because it is convex.

The only surface A, B, C, D can see is surface 2. Therefore, $F_{ABCD-2} = 1$ because radiation leaving A, B, C, D cannot see self and it does not see any other surface except 2. Hence all the radiation leaving A, B, C, D has to reach 2. So, this is constructed for a convex surface. By applying reciprocity $L_{ABCD} F_{ABCD-2} = L_2 F_{2-ABCD}$.

We know that F_{ABCD-2} is equal to 1. So, we can obtained the fact that $F_{2-ABCD} = L_{ABCD} / L_2$. , But we want F 2 to F A 2 A 2 and that comes out easily because we know that $F_{2-ABCD} + F_{2-2} = 1$. By applying the law concentrate of energy, any radiation leaving has to come back to itself or go to A, B, C, D.

Since we know the value of F_{2-ABCD} from this equation, by substitution finally we get $F_{2-ABCD} = L_{ABCD} / L_2$ only.

Here we are not directly applying the Hottel's cross strings method, but this spirit has been used which is a convex surface cannot see itself, and if a concave surface is bounded completely by another surface then all the radiation leaving the concave surface has to reach itself. We give a few more exampled to highlight this part, by a very simple example.

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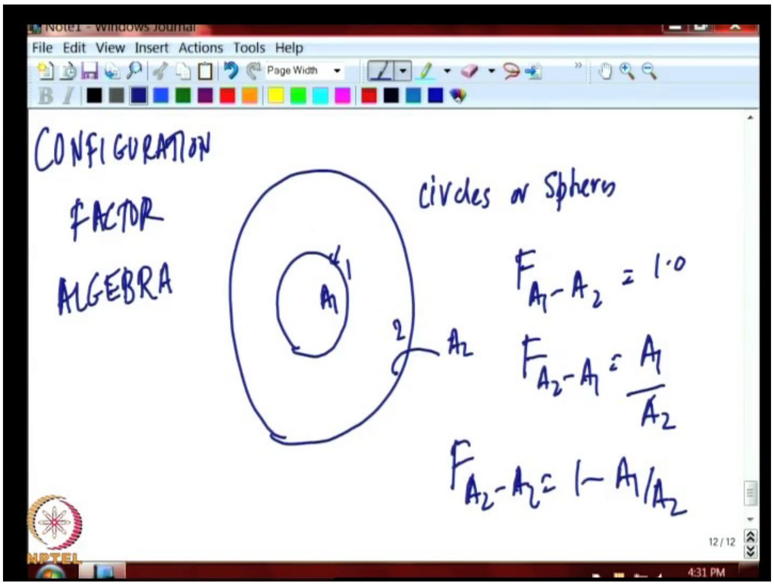

Suppose, there are two concentric circles or spheres which has surface 1 with area A_1 and surface 2 with area A_2 then $F_{A1-A2} = 1$ because 1 cannot see any other surface. From reciprocity $F_{A2-A1} = 1 - A_1/A_2$ and therefore, F_{A2-A2} is 1 minus A 1 by A 2. So, these kinds of simple calculation involving reciprocity and common sense are called configuration factor algebra.

This is essentially using common sense, arguments of symmetry, reciprocity and the first law of thermodynamics. These all can very useful for it will generate expression for shape factors without actually, doing the integration of the equations. Now, let us take few more example of this kind of situation where without integration we use the symmetry of the problem to get the answer,

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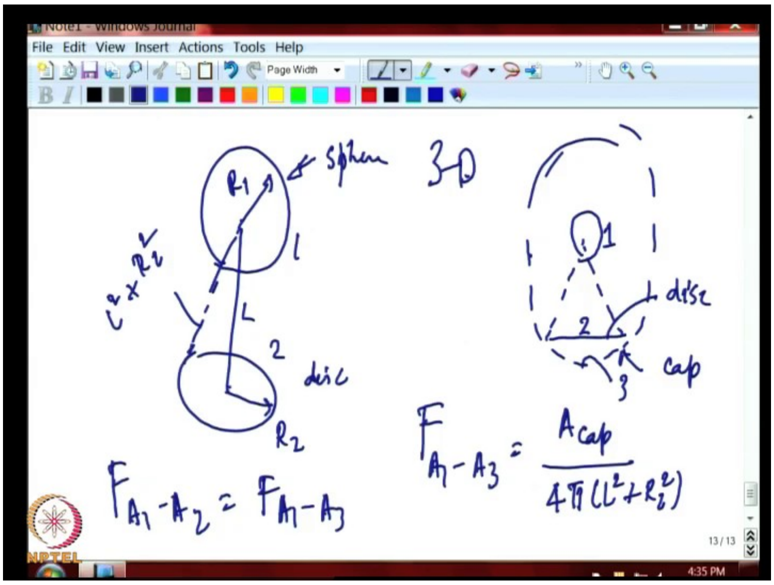

Suppose, we want to calculate from a sphere with surface 1 and radius R_1 to a disk with surface 2 and radius R_2, given the length L. Now, this problem is a 3 dimensional and hence, we cannot use the ideas of the Hottel's cross string method because this is a 3 D problem. But we can appeal to symmetry. If we take this distance which is nothing but $L^2 + R_2^2$ and draw an imaginary sphere, we can see by the symmetry of the problem that radius leaving surface 1 and reaching surface 2, is written as F_{A1-2}.

If we draw an imaginary sphere then $F_{A1-A3} = A_{cap} / 4\pi (L^2+R_2^2)$. The area of the cap can be obtained, from geometry books. What we have done here is we drew an imaginary sphere around the sphere, which encloses this disk.

The argument is that $F_{A1-A2} = F_{A1-A3}$ any radiation which leaves 1 and crosses 2 has to also reach 3. We are able to derive an expression for F_{A1-A2} without actually integrating any equation by just appealing to symmetry.

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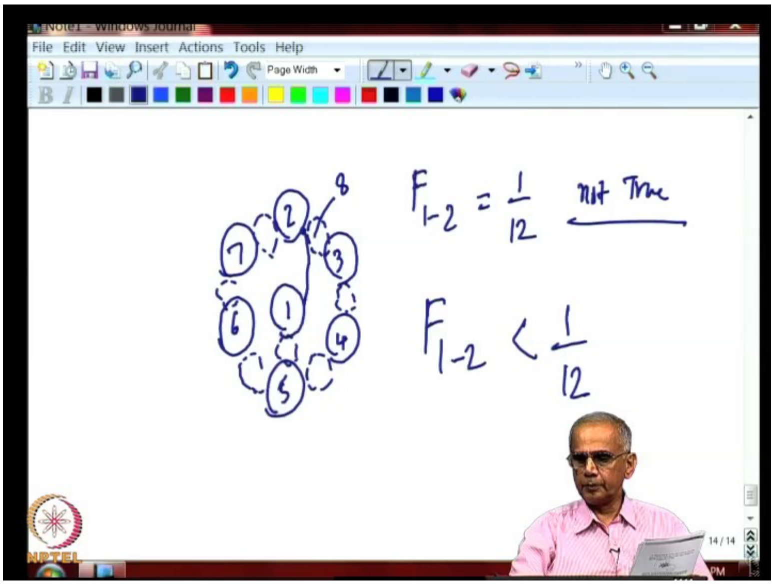

We take an example of a cylinder and think of various other cylinders around it. We want to calculate $F_{1\text{-}2}$. In the above figure, there are 6 cylinders placed symmetrically around the centre. One may be tempted to conclude that $F_{1\text{-}2}$ is equal to 1 by 12, but this is not true because one might mentally construct imaginary cylinders 6 of them.

One may think that by polynomial symmetric, there are 12 cylinders and a cylinder should get one-twelfth of the radiation. We actually construct this and ensure that there is no chance that some other ray, which is going from 1 to 2 is obstructed by a cylinder 8. If that happens than $F_{1\text{-}2}$ will be less than 1 by 12.

One has to be careful with symmetry. They are very powerful concept in this subject, but sometimes you may get a mis-leading impression of a symmetric situation, but in reality because of the intrusion of other objects, the symmetry may not be valid. But as long as we are allowed to the various rays, which are travelling in different directions and one ensures that the possible obstruction is present by some other objects in between, then one can solve the problem. Let me just complete this lecture by one more example just ensure that you are…

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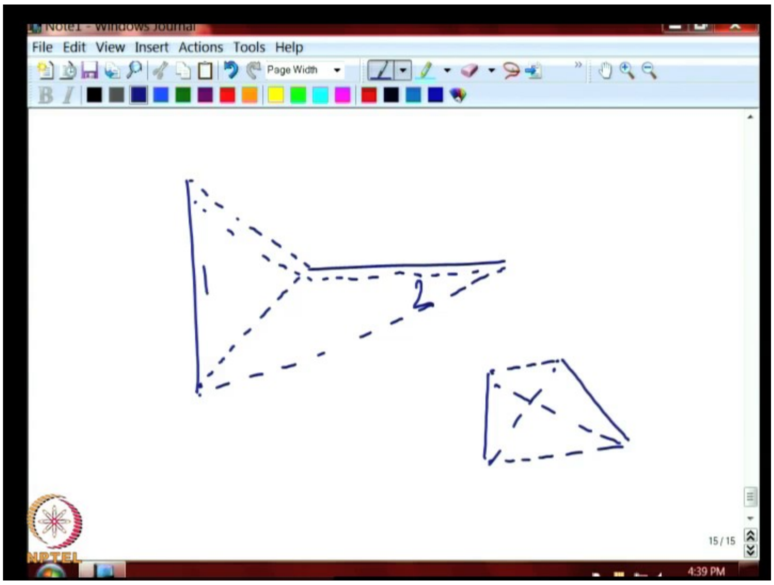

Let us take an example, of surface 1 and surface 2 and everything infinity to the page as shown above. We can easily draw one parallel string and one cross string. One cross string is very easy and for the other cross string we have to apply and draw carefully as shown above.

We can convince ourselves that radiation leaving 1 not reaching 2 , has to go through this parallel string. On the other hand any radiation which leaves 1 and each has to cross the cross strings at one of these points. As we saw in the earlier example of two plates, we mentally rotate the surface to get the result.

We saw in this class that a combination of triangular enclosure results and symmetry. We saw that first law of reciprocity helps us, to obtain shape factors for various simple geometries, but in case it is not possible then we have to resort to actual integration. But today the lots of data are available in many books on radiation heat transfer. One can also search the internet for data on shape factor.