Radiation Heat Transfer
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Lecture - 5
Shape factor

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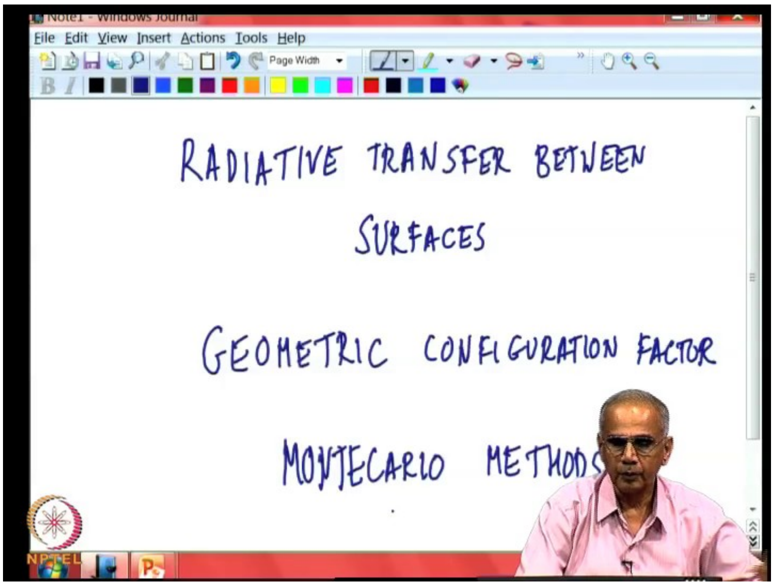

Today we start a new part of the course, which is radiative transfer between surfaces. So far, we have looked at emission by blackbodies and the radiative properties of real surfaces. Now, we are ready to ask this question, what fraction of radiation leaving a surface actually reaches another surface?

Now, this is a very important issue in engineering application. We would like to know of the radiation leaving a flame, what fraction reaches to the certain wall of a boiler or we would like know of the radiation leaving a certain wall of an enclosure, what fraction of it will reach another wall. This problem is a three dimensional geometry, because as we know radiation leaves the surface in all directions. We would like to know what fraction actually reaches another surface in order to calculate the radiation impinging on the surface. This is purely a geometry problem.

But if the radiation leaving the surface depends strongly on an angle, then not only geometry is the issue, we also have to worry about the directional properties of the radiation emitted or reflected by the surface. There are many examples in real life, where one can assume that these surfaces we are dealing with are the diffuse-isotropic emitters and reflectors. In such a case, we will show that the geometry part of the problem gets separated from the radiation part of the problem. In today's class we will define what is known as geometric configuration factor. This factor is a very important quantity. It tells us what is the fraction of radiation that leaves one surface and reaches another surface. And under certain conditions, this is purely dependent on geometric configuration of the two surfaces, and nothing else. It makes the analysis of radiation problem somewhat simpler because we can separate the radiation part of the problem and the geometry part of the problem easily. We will see some examples, where this makes our computation somewhat easier.

If the surfaces we are dealing with are not diffuse isotropic, then the geometry and the surface property gets entangled and we have to do a very complicated kind of analysis. If we encounter such condition where the properties of surfaces are very complex function of angle, then we use what is known as Monte Carlo method. We will discuss this Monte Carlo method near the end of the course. These methods actually follow every photon that is emitted or reflected by a surface and follow the photon till they are absorbed. If it is reflected, then it will follow the photon after reflection and ultimately until the photon is absorbed in another surface.

Today because of the availability of high speed computers, it is possible for us to follow millions of photons from their birth and their subsequent reflection and absorption. One can do the life history of all the photons and compute the total radiative flux. This method is very computer intensive, but has an advantage that it can deal with any complex geometry, in any complex situation. We use the geometric configuration factor. Now, let us now look at the example here.

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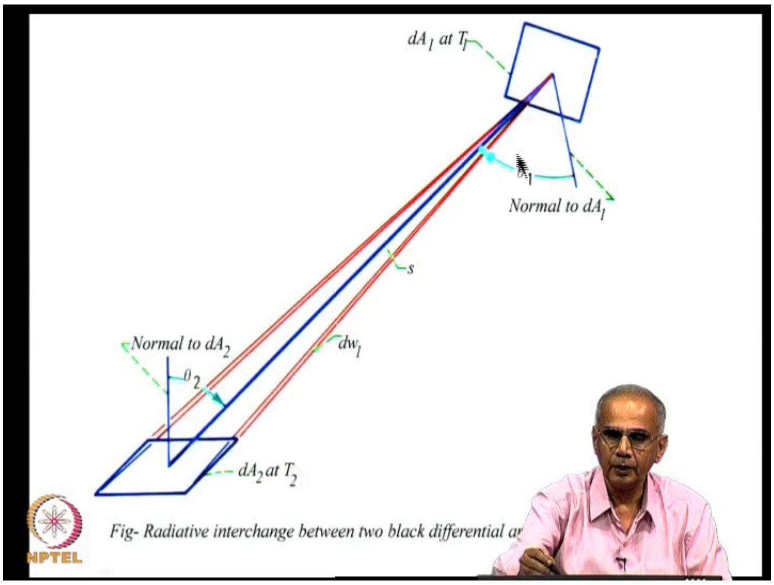

So, we want to ask if the radiation leaving the surface dA_1, elemental surface, what fraction reaches the surface two, dA_2. Radiation leaving the surface two and a certain fraction is intercepted by surface dA_2, and the fraction of the radiation leaving surface one is intercepted by dA_1. The surface two depends upon the orientation of the surface two with respect to surface one. It depends on the distances, that is, the distance between the center of the surface one and surface two and also the angle θ_2 and θ_1. These are the angles between the line joining these two elemental surfaces and the local normal. These are local normal.

Now, we will derive that expression. It tells us what fraction radiation leaving the surface one reaches the surface two. After that, we will see under what condition this quantity is a purely a geometric factor. We will show that if the radiation leaving the surface is diffuse-isotropic. (independent angle θ_1 and φ_1), then this fraction is a geometric factor.

In the next few lectures we will discuss the techniques of calculating this factor purely from the geometry of the problem. There are several interesting techniques available to calculate this factor. We will go through those techniques.

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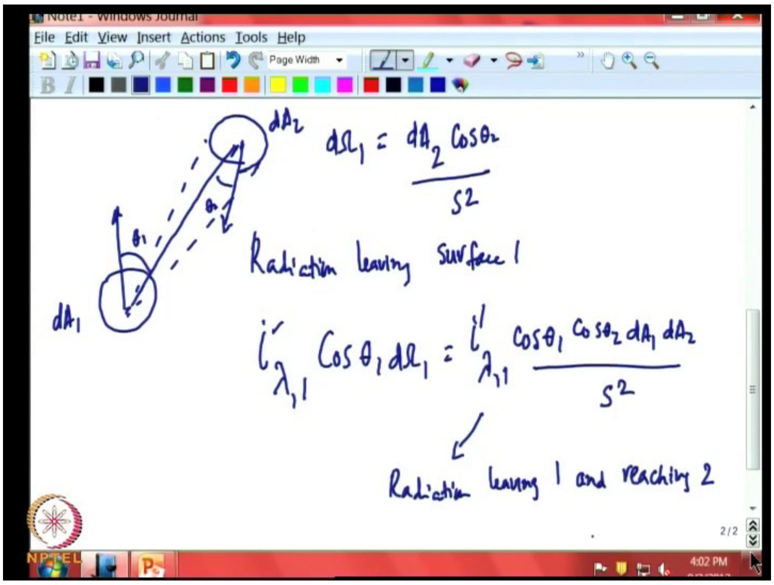

Now, let us derive this expression for geometric configuration factor. We will draw a simple figure. The surface dA_1 and dA_2 has a normal. The angle between the line joining these two surfaces and the normal to these surfaces are θ_1 and θ_2 respectively. And, we asked what is the solid angle subtended by surface two and surface one. By definition of the solid angle, d omega is nothing but area projected of surface two divided by the distance S squared. So, this is the solid angle subtended by surface two in surface one. Now, let us ask what is the radiation leaving surface one.

This, from the definition of intensity $i'_{\lambda 1}1 \, \text{Cos}\theta_1 \, d\Omega \, d\lambda$. This is the radiation leaving surface one in the direction towards surface two and radiation that leaves one, which arrives in surface two times the solid angle subtended with surface two, which is $dA2\cos\theta_2/S^2$. This is the solid angle that subtended by two in one. We have to define what is $d\Omega_1$. We rewrite this as $d\Omega_1$. This can be further expanded to $i'_{\lambda 1} \, \text{Cos}\theta_1 \, d\Omega_1 \cos\theta_2 \, dA_2/S^2$. Now this is the radiation that is leaving one and reaching two. We have to find the total radiation leaving surface 1 in all directions.

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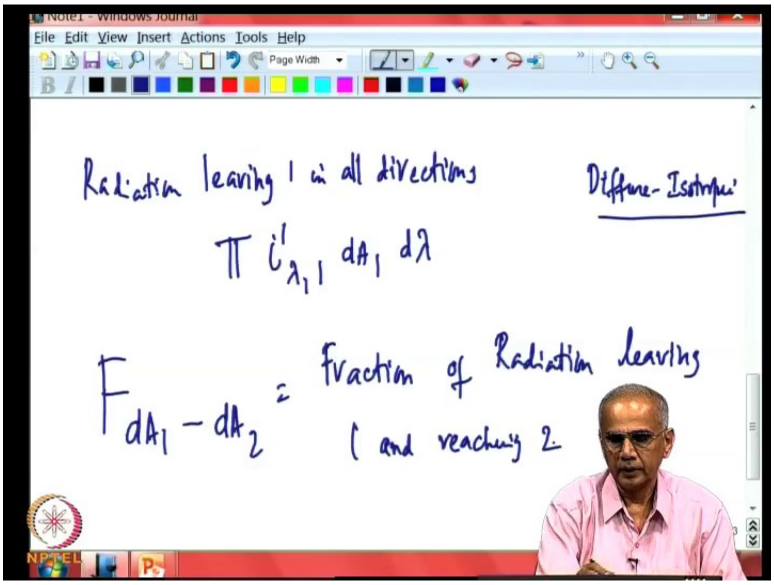

If we take all directions and integrate to all directions, then radiation leaving surface one in all directions is nothing but $\pi\ i'_{\lambda 1}1\ dA_1\ d\lambda$. While doing this integration, we are assuming that radiation leaving surface one is diffuse-isotropic. Only then one can calculate this quantity. Without assumption of diffuse-isotropic, this quantity is a function of θ and φ. If given the values for θ and φ we can integrate and get the answer. Now, we say what is the fraction of radiation leaving surface one and reaching surface two. That is the definition of $F_{dA1\text{-}dA2}$.

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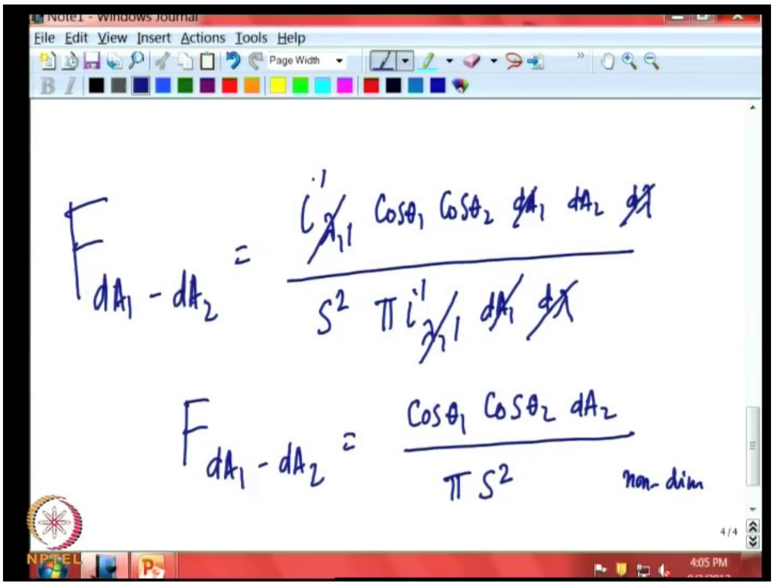

$F_{dA1-dA2}$ equals the fraction of radiation leaving one and reaching two. The radiation which leaves one reaches two divided by radiation leaving one in all direction.

We can see that $i'_{\lambda 1}$ and $d_{\lambda 1}$ will cancel out here. Now there is a simple relation for differential geometric configuration factor. It is $\cos\theta_1 \cos\theta_2/\pi S^2$. That is the dA_2.

We should notice that this is a non-dimensional number because it is a fraction. This is non-dimensional because it represents the fraction and it depends only on the angle θ_1 and θ_2 , area of surface two and the distance. So, there will be some relation.

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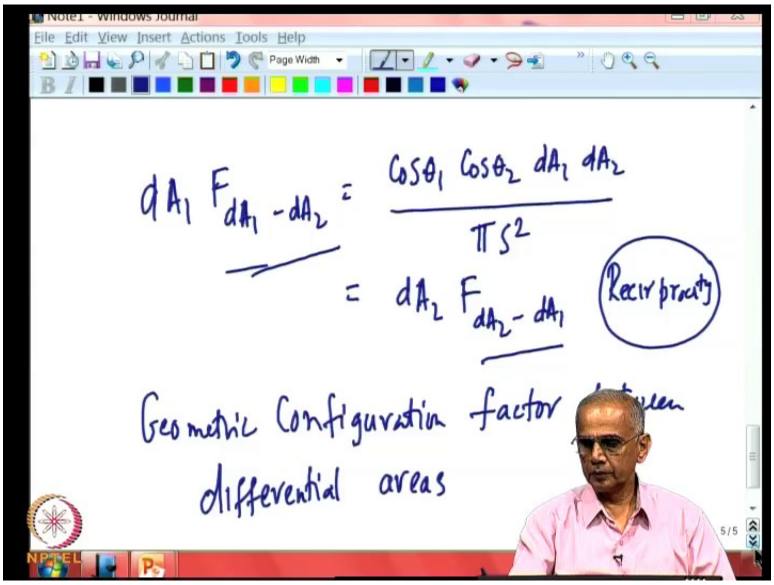

Now, this relation has some interesting features. Notice that dA_1 $F_{dA1-dA2}$ will be equal to $\cos\theta_1 \cos\theta_2 dA_1 dA_2/\pi S^2$. This is a symmetric function. We can interchange θ_1 and θ_2. Therefore by symmetry, it is also equal to dA_2 $F_{dA2-dA1}$. This is known as reciprocity. The reciprocity is a very important property telling us that, the total radiation leaving one and arrives at two and the same radiation leaving at two and arriving at one. This will be very useful because if we are able to calculate this quantity, then we also know this quantity; because this depends only on the area ratio.

Now, this geometric configuration factor between differential areas requires only one condition; that is, we are dealing with diffuse-isotropic emitters and reflectors. So, once we have made that assumption, this geometric factor comes out naturally. So, as long as the radiation emitted and reflected by a surface is diffuse-isotropic, then we are guaranteed that the relation between radiation leaving surface d A 1 and arriving surface d A 2 is purely a geometric factor. But as we can realize, this equation although very useful, is not very practical; because in application in Engineering one doesn't need $F_{dA2-dA1}$, but we need F_{A1-A2}. That is the geometric configuration factor between finite areas.

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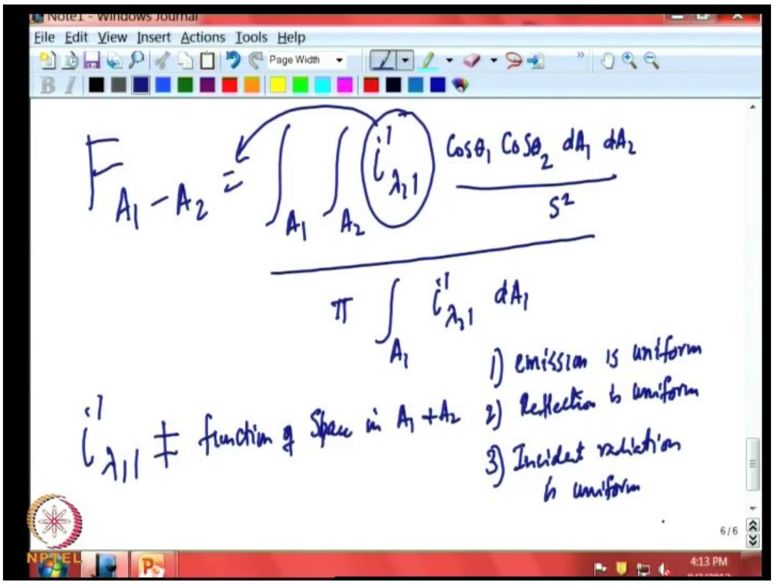

This is a quantity that is really of great interest to us. Now the question is, we have just shown that the geometric configuration factor between differential areas is completely a geometric factor, if the surfaces emit and reflect in a diffuse-isotropic fashion. But is that conditions sufficient for calculating this factor between finite areas? The answer is, no, one has to apply some more conditions.

If we want to calculate within finite area, then we have to integrate over the two surfaces A 1 and A 2. This intensity leaving surface one $\cos\theta_1 \cos\theta_2 dA_1 dA_2$. This is the quantity in numerator. In the denominator, we are looking at π times what is the total radiation leaving surface one. Now, if we look at this general expression for the geometric configuration factor between finite areas, we will find that this will be a purely geometric factor, only if i'_λ is not a function of space in A 1 and A 2.

If the intensity leaving surface one is varying with space then $i'_{\lambda 1}$ in this integration cannot be taken out. If you want to take out this term $i'_{\lambda 1}$ out of the integration, then it should not be a function of space in one and two. This will happen if emission and reflection is uniform, and does not vary with space and finally incident radiation is uniform. Only then we can guarantee that this quantity is independent of angle, and independent of space, and we take it out of the integration. We then have a simple expression. Now, this is not commonly met in real situations. Especially, the condition that the incident radiation should be uniform is rarely met because most surfaces we are dealing with, gets radiation from all directions and the radiation incoming from different surfaces impinging on a given surface will not be uniform.

So, if the incoming radiation coming at surface is not spatially uniform, then there is a chance that the radiation leaving the surface will not be uniform in space. Even though the properties of surface like reflectivity are uniform, the reflecting radiation will be non-uniform.

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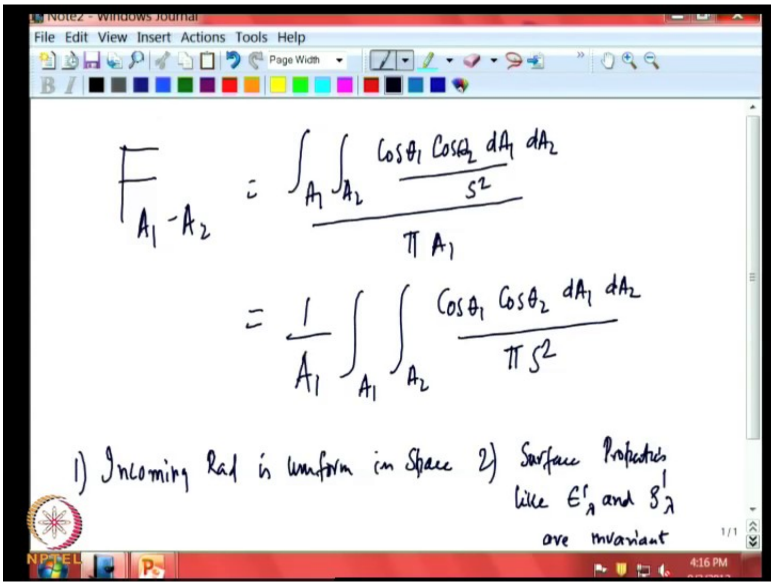

So, one must remember that in this condition, finally we get for F_{A1-A2}, as an integral of A_1 and A_2. This is written as $1/A_1 \; {}_{A1}\int^{A2} \cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2/\pi S^2$. This is now a truly geometric factor. It depends on the relative orientation of θ_1 and θ_2 of the two surfaces. This is a very useful quantity to have only if incoming radiation is uniform in space.

Secondly, surface properties like emissivity and reflectivity are invariant in space. We want radiation to come uniformly on that surface. And the surface properties like emissivity and reflectivity should also not vary in space. This condition is not met commonly in many practical applications. We have to keep that in mind that, the use of geometric configuration factor between finite areas is not always valid, unless we have a new condition that the incoming radiation is uniform spatially, and this surface properties like emissivity and reflectivity do not vary in space.

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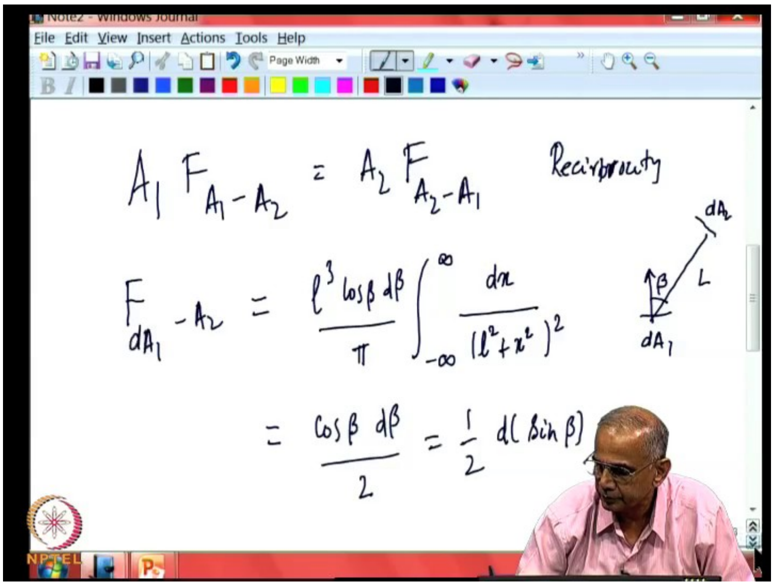

We can also see from this symmetry of this particular equation. We can also see the following conditions are satisfied A_1 $F_{A1\text{-}A2} = A2$ $F_{A2\text{-}A1}$. This is called reciprocity. This is a very useful property. If you happened to get hold of $F_{A1\text{-}A2}$ from some source, we also know $F_{A2\text{-}A1}$ immediately. It helps us to generate new shape factors or geometric factors based on available values. This is a basic parameter which we will be now studying in the next few lectures. We should try to understand under what condition these can be applied and what are the real life examples, where this can be implemented.

Now in today's computer world, in principle, this integration which we talked about, can be done on the computer. But, one must recall that although this can be done on the computer fairly easily, it is important for one to actually tabulate correctly the limits of the integration because that the computer will not know. We have to indicate what are limits of the integration that is A_1 and A_2. Once we have given those limits, there are many software packages which will do the integration for us. But for simple geometries, these configurations have already been done and given in various tables in books. We will take a few examples to illustrate the values of geometric factors. The very common geometry which we will encounter quite often in the real world are, quantities for which integrations have already been conducted and the relationship is already available to us.

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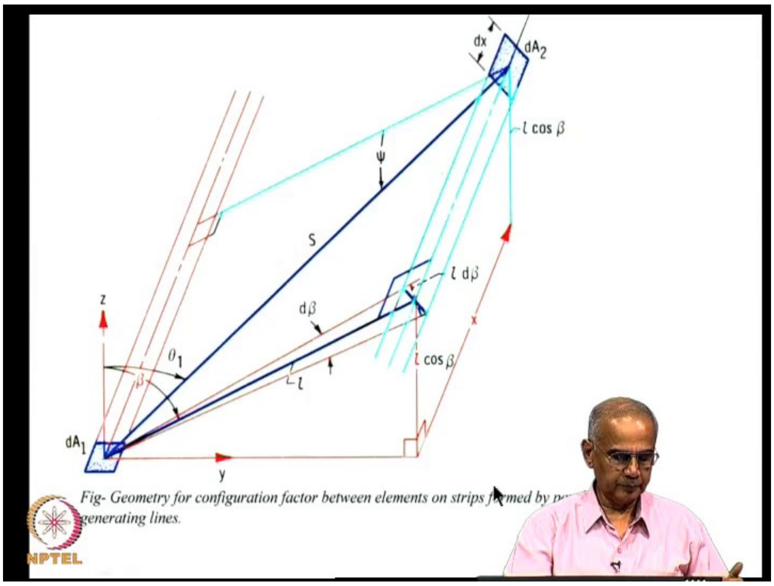

One example that we will now look at is that between a small area dA_1 and the long element. So, $F_{dA1\text{-}dA2}$ is parallel well defined quantity from the definition given there. Now, we need to integrate over this length to get the configuration between element dA_1 and a strip A_2. In this case, the basic dA_1 dA_2 we know from the definition, but we need to integrate over one of the elements. That is the element dA_2 to get a relation between $F_{dA1\text{-}dA2}$ to strip one.

Now, this integration can be carried out fairly easily. This is purely a geometric problem. We will now write down a few steps indicating how this integration is done. We will look at this integration from area$F_{dA1\text{-}dA2}$. A_2 is the strip that we are looking at. We can look at the basic geometry of the problem and wherein the distance, the normal distance between the two elements. One element is dA_1. The other element is shown as above in the figure as dA_2. The projected distance between the normal to the plane is 'L'.

One can do all the simple geometry that this calls for and one can easily show that this angle between the normal to the surface is β. θ_2 is the three dimensional angle. In the above figure we can see θ_1 is the angle obtained when we draw the vector S joining

dA_1 and dA_2, while β is the projection of this on the plane. Once we know that β and this distance is l, $\cos\beta$ is the vertical distance here.

We will now use the geometry and will stick to the notation in the figure. We will draw a small cube $\cos\beta$. This is purely a problem in geometry, by π. We then integrate in the x direction from minus infinity to plus infinity, the normal to the plane of this board. It will come out as $dx/(l^2+x^2)^2$.

Now, we can do the integration that is fairly straight forward. We will get that answer now as $\cos\beta\ d\beta/2$. So, this is a very simple result indicating that the geometric factor between the element dA_1 and a finite area A_2 which is the strip, is nothing but $\cos\beta\ d\beta/2$. It can also be written as $1/2\ d(\sin\beta)$. We will examine the result when we integrate with A_1 and A_2 when we look at the geometric factor between two surfaces, which are infinite in one direction. That is, they are very long in one direction. Such examples occur in practical situations, where one dimension may be in furnace or some other situation or a cloud; where one dimension is very large compared to the other dimension.

Then, the problem is essentially a two dimensional. But, remember that in radiation the basic phenomena in three dimensional, although one dimensional is long compared to the other dimension, we still have to take into account the three dimensional nature of the phenomena, which cannot be ignored. We are going to addresses this issue by calculating geometric configuration factor in those situations, wherein one dimension is extremely long compared to the other two dimensions. We want to know whether some simplification we have arrived at will enable us to calculate things without actually doing the integration. This method is quite useful and powerful and has a simple geometry and simple interpretation in terms of energy.

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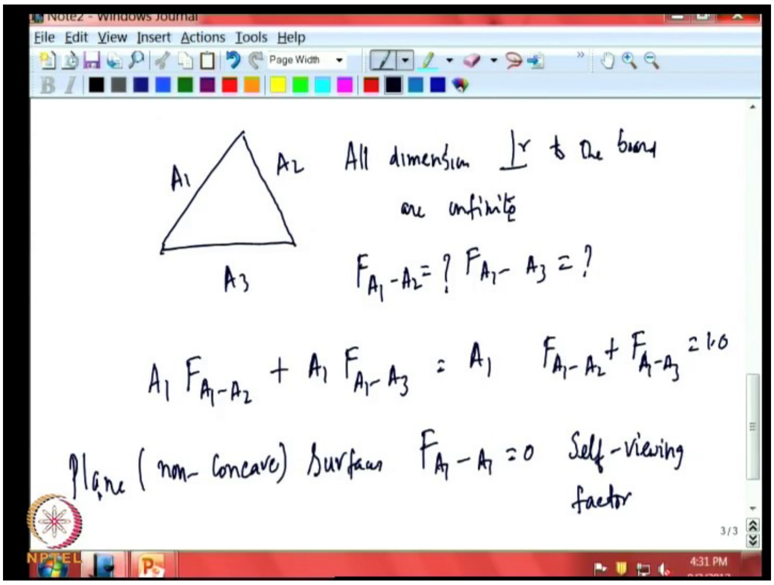

We will look at an example now. We will take a simple example of a triangular enclosure triangular system. Imagine, we have a triangular enclosure A_1 A_2 A_3. All dimensions that are perpendicular to the board are very long and are infinite. We are looking at the problem where this dimension is extremely long compared to what is there in this plane. The question is, does such a configuration simplify the problem. Now, we will appeal to reciprocity. We know F_{A1A2} and F_{A1A3}. We want to calculate these quantities without actually performing integration. This is possible because we can write down energy balance that is the radiation leaving surface one and reaching surface two as well as radiation leaving surface one and reaching surface three, as equal to radiation in the surface one.

This shows that $F_{A1\text{-}A2} + F_{A1\text{-}A3} = 1$. This is because these are plane surfaces. A plane surface cannot see itself. For a plane surface, that is, which are not concave, F_{A1A1} is 0. This is called as self-viewing factor. The self-viewing factor is the fraction of radiation leaving a surface which comes back to itself. Now, this can happen only when the surfaces are plane or they are convex. The self-viewing factor is 0. We are considering a case, where all the three surfaces A_1, A_2, A_3 are completely non- self viewing.

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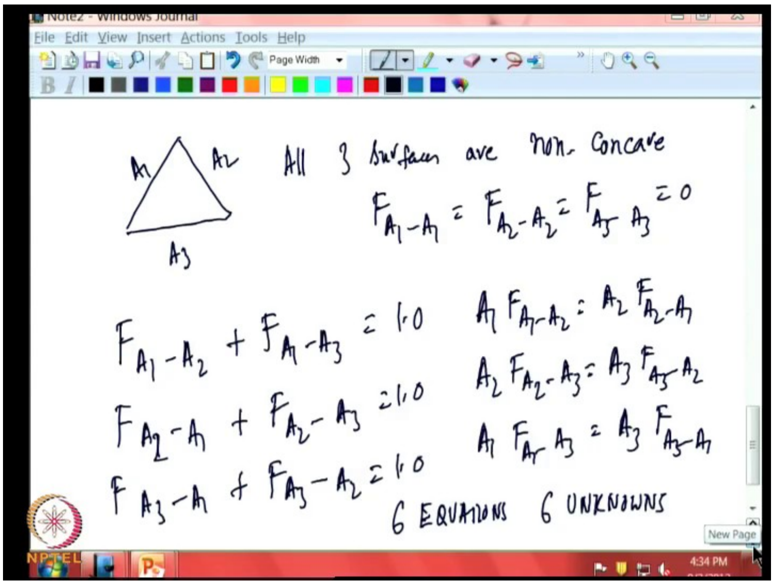

This the surfaces A_1, A_2 and A_3 are all non-concave. Therefore F_{A1-A1} equals F_{A2-A2} equals F_{A3-A3} equals 0. So, they do not see themselves. Now, let us write down the first law of each case. The radiation leaving surface one and surface two are ending in surface three which equals one by first law. This is because whatever is leaving reaches the any of the other two surfaces. This dimension is infinity. Similarly, $F_{A2-A1} + F_{A2-A3} = 1$ and $F_{A3-A1} + F_{A3-A2} = 1$. So, we have three equations here with six unknowns. We can solve for all these values. But we know by reciprocity, which is $A_1 F_{A1-A2} = A_1 F_{A1-A3} = A_2 F_{A2-A1} = A_2 F_{A2-A3} = A_3 F_{A3-A1} = A_3 F_{A3-A2.}$ We now have six equations and six unknowns. We can solve this problem completely without doing any integration. In such a situation the results are very simple; We can just eliminate what we do not want. This simplification implies that this integration is not required.

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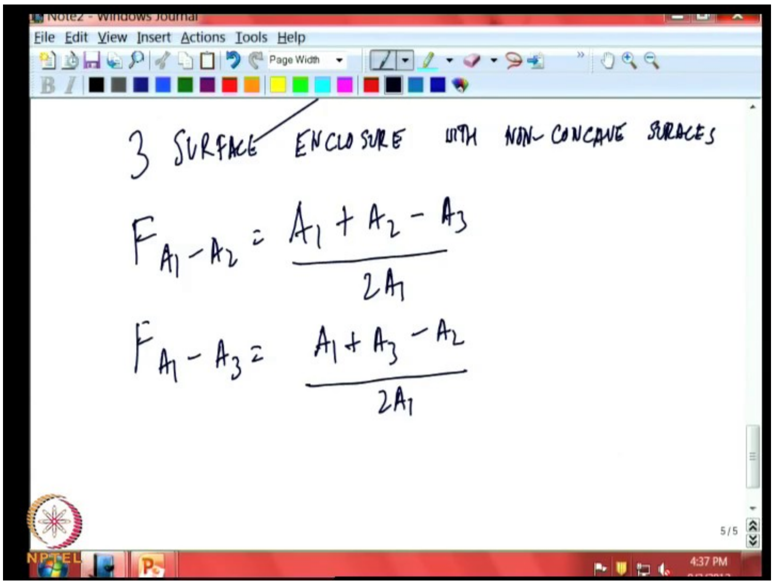

If we encounter a situation with three surfaces which forms enclosure then we can show that $F_{A1\text{-}A2} = A_1+A_2-A_3/2A_1$.

This configuration factor between two finite areas A_1 and A_2, if they are part of an enclosure containing three surfaces and all surfaces are non-concave, is the sum of the area of the two surfaces involved in the interaction minus the area of the third surface divided by twice A_1. Similarly, we can write the other two. $F_{A1\text{-}A3} = A_1+A_3-A_2/2A_1$. This is a very powerful result because we have avoided all integration. We need to know only the surface area of the three surfaces. That is fairly an easy quantity for surfaces of known shape. We have managed to avoid the difficult three dimensional integration that is usually called for.

This can be used in a situation where one dimension is very large compared to the other two dimensions, because the problem becomes essentially a two dimensional problem. But, still the integration that we have to do is three dimensional. We can avoid the integration by appealing to Law of conservation of energy and reciprocity. By combining energy conservation and reciprocity, we get this relationship between the shape factors. This is a very useful result and it has application which we can adopt very easily. There could be situations where we actually do not have three surfaces, but one can always construct a three surface enclosure.

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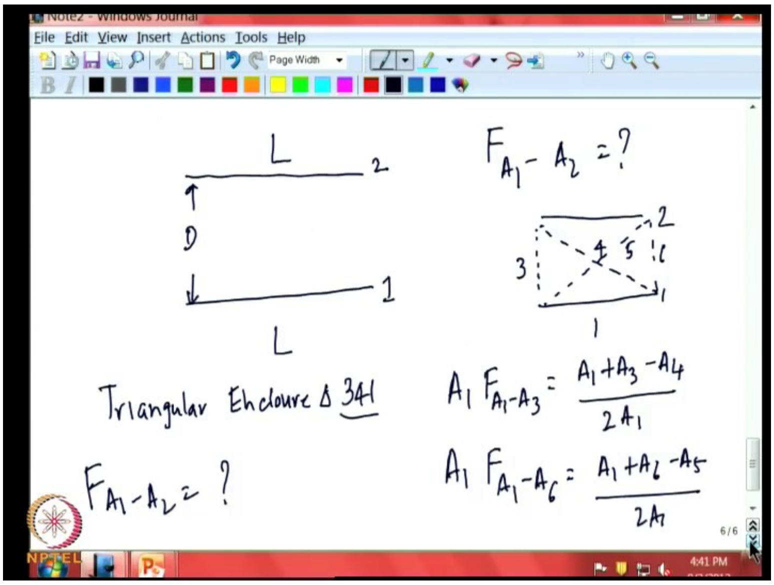

Let us take an example. Suppose there are two parallel plates, may be a furnace for example; We have one plate as surface one, and the other as surface two. The distance between the plates is D. The dimension normal to the board is infinity. We need to see if F_{A1-A2} can be calculated without any integration. Now, the way we will do this is, we will construct an enclosure. We draw two dotted lines connecting the two surfaces. This is an imaginary surface. Let us call this surface three and surface four.

Then, we apply the rule we just derived to this triangular enclosure (refer above figure) ; three, four, one. From our earlier formula we know that A1 F_{A1-A3} is equal to $A_1+A_3-A_4/2A_1$. Similarly, we can construct other triangle here. Let us call this as five and six. We can also say A_1 $F_{A1-A6} = A_1+A_6-A_5/2A_1$. We constructed two triangles on the same base of surface one and applied the triangle rule for non-concave surfaces and arrived at two expressions. Now, what we really want is F_{A1-A2}. We have F_{A1-A3} and F_{A1-A6}. Now, we can apply the first law of Thermodynamics.

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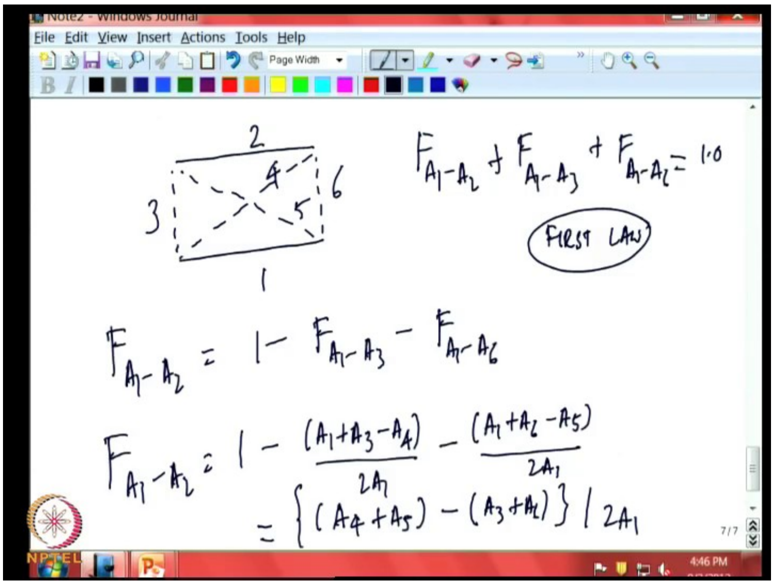

This enclosure, we can say is the radiation leaving surface one, reaching surface two plus radiation leaving surface one and reaching surface three. The radiation leaving surface one reaching surface six has to be equal to one. Therefore, the quantity $F_{A1-A2} = 1 - F_{A1-A3} - F_{A1-A6}$. This is from the first law of thermodynamics. We have already derived the expressions for these two. We can rewrite F_{A1-A2} as $1 - [(A_1+A_3-A_4) / 2A_1] - [(A_1+A_6-A_5) / 2A_1]$.

We have been able to get the configuration factor between surface one and two in terms of just the areas of one, three, four, five and six. These can be easily calculated. Since these dimensions are infinite in this direction, all these are essentially lengths of various surfaces here. We can see there is a minus half here and the one is cancelled out. So, finally we can add four and five. We will get $[(A_4+A_5) - (A_3+A_4)] / 2a_1$. It is a very simple result, which has several physical insights.

In the next lecture we will proceed further in using this method. In this lecture we define a quantity called geometric configuration factor, which is a purely geometric factor when it is applied between finite areas if the radiation leaving that surface is uniform in space. We showed that for certain situations in which one dimension is very long compared to the other, there is a simple way to evaluate this geometric configuration factor without actually doing the integration. In the next lecture will look at how this simplification helps us to solve some interesting problems.