Radiation Heat Transfer
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Lecture - 4
Spectral and Directional Variations

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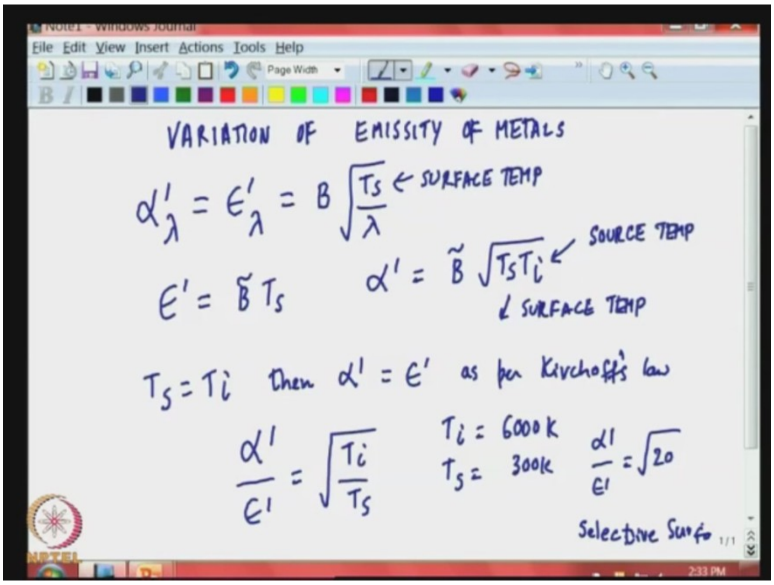

VARIATION OF EMISSITY OF METALS

$\alpha'_\lambda = \epsilon'_\lambda = B\sqrt{\frac{T_s}{\lambda}}$ ← SURFACE TEMP

$\epsilon' = \tilde{B} T_s$ $\quad$ $\alpha' = \tilde{B}\sqrt{T_s T_i}$ ↙ SOURCE TEMP

↑ SURFACE TEMP

$T_s = T_i$ then $\alpha' = \epsilon'$ as per Kirchoff's law

$\frac{\alpha'}{\epsilon'} = \sqrt{\frac{T_i}{T_s}}$ $\quad$ $T_i = 6000\,k$ $\quad$ $T_s = 300\,k$ $\quad$ $\frac{\alpha'}{\epsilon'} = \sqrt{20}$

Selective Surfa

In the last lecture, we looked at the variation of emissivity of metals. We saw that if we assume the directional spectral absorptivity is equal to the directional spectral emissivity is assumed to vary as square root of the temperature and inversely proportional to wave length. Then, we saw that the directional total emissivity varied linearly as the temperature and the directional total absorptivity varied as root of T s and T i, where T i being the source temperature. We clearly saw that the emissivity of metal depends only on its surface temperature, whereas the absorptivity depends both on the surface temperature and the source temperature. This distinction was brought clearly, that if by chance T s was equal to T i then alpha prime was equal to epsilon prime as per Kirchhoff's law.

In general we saw that the ratio of alpha prime to epsilon prime goes as root of T i by T s. Clearly showing that in many cases for example, if T i is close to 6000 Kelvin, the sun's temperature and T s is 300 Kelvin then alpha prime by epsilon prime is root of 20 which is around 4.5. In the case of metals we see that the solar absorptivity is about 4 to 5 times larger than the infrared emissivity. This could be called as selective surface, a surface whose property depends on the wave length and which enables one to draw it graphically.

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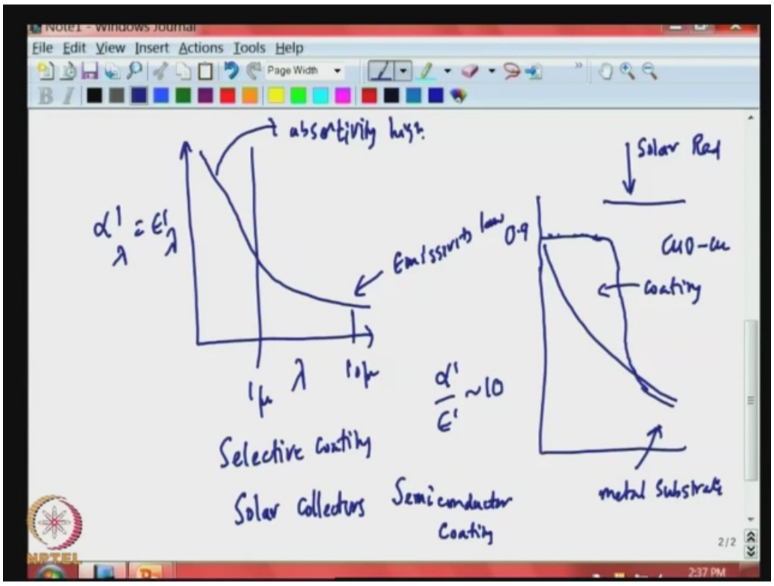

We can write, alpha prime lambda equal to epsilon prime lambda and it varies as 1 over square root of lambda. We can really see that in the wave length below 1 micron absorptivity is high while at higher wave length, let us say around 10 micron emissivity is low. When such a material is put out in the sun with the solar radiation impinging on it, it will reach temperature higher than a surface whose properties do not vary with wave length. We saw just now that the absorptivity of metal is about 4 times higher in the solar region compared to the infrared region. We take a metal substrate and put a semiconductor coating to improve it further. For example, it can be copper oxide on copper. Then this copper oxide will absorb strongly in the solar region and then it will be transparent in the , far infrared region and will come down rapidly in the infrared region.

So, in the infrared region we will see, the effect of the substrate. This semiconductor coating gives the surface a higher absorptivity in solar region and then merges with the substrate. This additional absorptivity is given by the coating. By coating typically a micron or less thick, improves the absorptivity in the solar region, while keeping the emissivity in the infrared region and unchanged. By that the ratio of alpha prime to epsilon prime can be as high as 10 , that is almost twice that of pure metal. These selective coatings are used in solar collectors to achieve higher temperatures and we will show some examples little later in this lecture. Now, let us look at some other examples of materials and see how the emissivity of materials vary with temperature.

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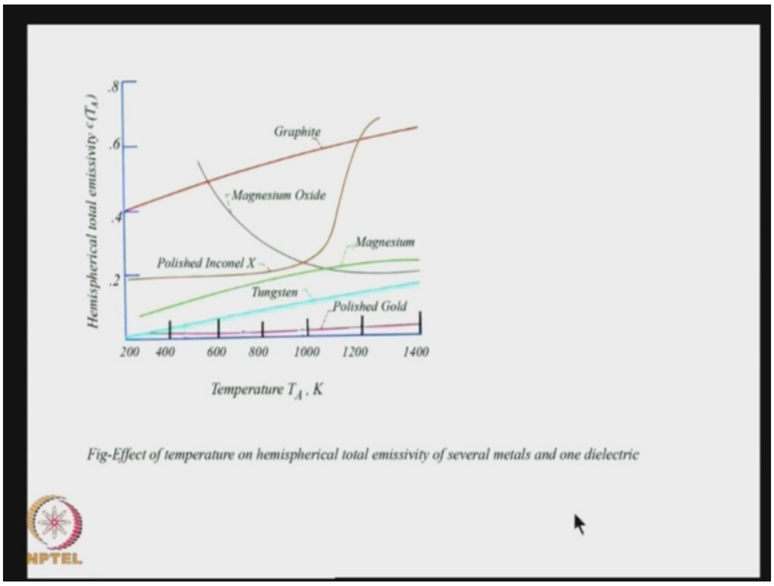

Fig-Effect of temperature on hemispherical total emissivity of several metals and one dielectric

In this figure, you see that the emissivity of graphite varies roughly linearly with temperature although it is not a metal but a nonmetal. We see that tungsten, which is a metal, has a linear variation with emissivity. Polished gold has roughly linear variation of emissivity with temperature. Sodium and magnesium shows somewhat non-linear behavior. While in oxide layers such as magnesium oxide, the emissivity decreases with temperature. Generally metals show an increase in emissivity with temperature.

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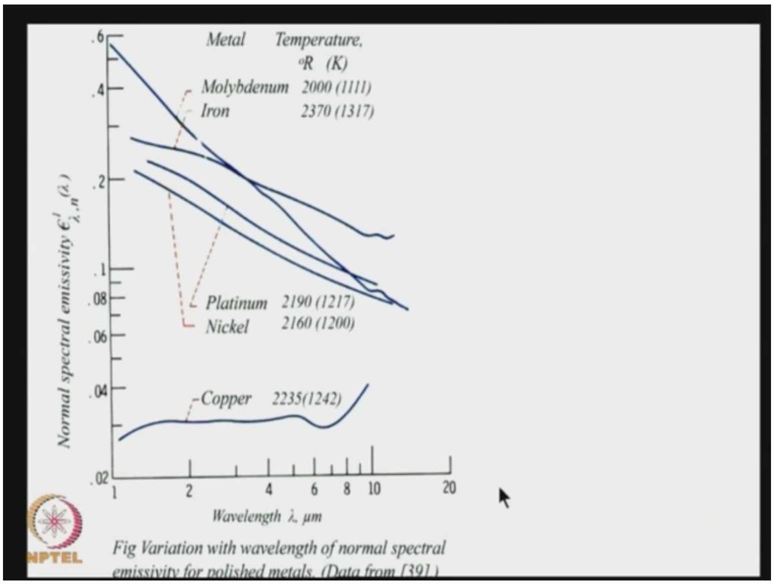

Fig Variation with wavelength of normal spectral emissivity for polished metals. (Data from [39].)

In the above figure we see the dependence of the directional spectral emissivity of the material. The data is shown in forms of squares and circles, while the line in blue or green shows the theoretical relation that we had assumed earlier as proportional to square root of temperature and inversely proportional to square root of lambda. This behavior is called Hagen-Rubens relation. When the metal is oxidized, that is titanium is oxidized, the emissivity increases. They are called the oxide layer. We can see that although the data does not agree precisely with the simple theory, it shows similar trend overall or at decrease in emissivity as the wave length decreases. This is the important reason why the emissivity of metals increases with temperature.

Now, there are few more results here showing a variation in spectral emissivity with wave length. For example, we have molybdenum, platinum and nickel. They all show a decrease in emissivity with wave length. Although, the slope is somewhat different for different materials, but interestingly copper does not show that effect. Copper seems to have an emissivity which does not vary that much with wave length. The reason, why some metals do not show the predictions of simple theory is because, there could be a thin oxide layer that is seen at high temperatures.

Unless we keep them in an inert atmosphere and oxide layer forms on a metal, the emissivity will be then controlled by the oxide layer and not metal. So, we try to look for data from various sources here to be show whether the metal, whose emissivity is given, is really a pure metal with no impurities or not. That is whether it is really free of oxide layers as the oxide layers have a huge impact on the emissivity of materials, which we will see further in the subsequent slides.

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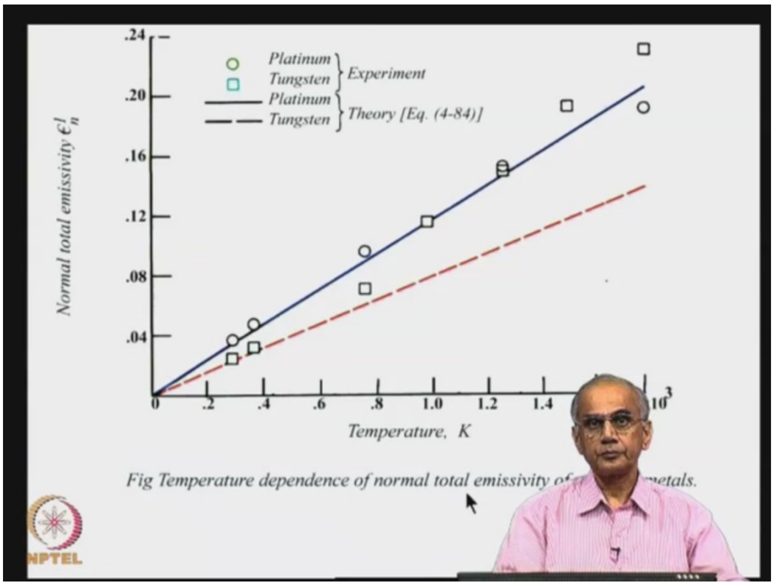

Fig Temperature dependence of normal total emissivity of metals.

Now, here again we show the normal total emissivity as a function of temperature in 1000s of degrees Kelvin. We see clearly that, in the case of tungsten the theory is not in very good agreement. Tungsten is the red line here. The blue line shows the value of platinum, and we can see clearly that the theory for platinum agrees quite well with the data of platinum. This indicates that for platinum the theory agrees while in the case of tungsten they do not agree. This could be on account of a small oxide layer on the tungsten surface, which is not there in the case of platinum. So, these issues must be always be looked into when trying to use data on emissivity of metals. Because oxide layers are usually there, they are not easy to avoid unless the metal is heated in an inert environment like Argon. Otherwise oxide layers will form and one has to account for the oxide layer effects.

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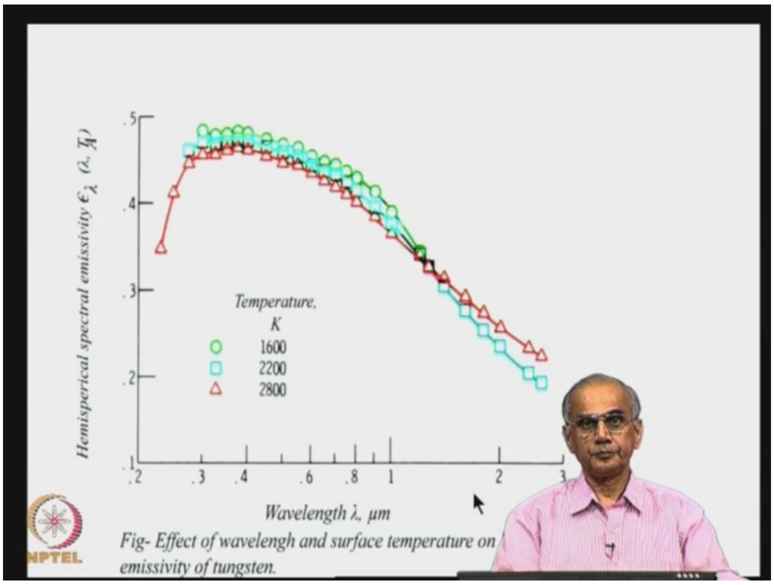

Fig- Effect of wavelengh and surface temperature on emissivity of tungsten.

Now, here we are showing the variation of the spectral emissivity of tungsten with wave length. We see that beyond 1 micron this decreasing effect of emissivity with wave length is seen clearly and follows the theory which we discussed. But at low wave length around the visible region, we see that emissivity is almost constant. This shows that the simple assumption we made that the emissivity is a function of wave length and varies as one over square wave length is valid only beyond 1 micron, below 1 micron other effects coming to play. The simple theory that we have proposed may not be valid always.

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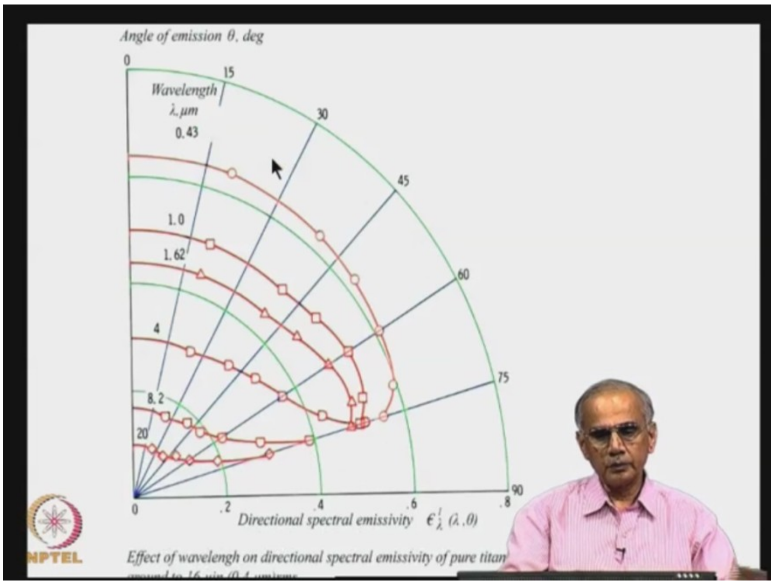

Effect of wavelength on directional spectral emissivity of pure titan

Now, here we show the directional variation emissivity at various wavelengths. This is pure titanium with a certain surface finish. We see that in the visible wave length the emissivity is shown as the function of the angle normal to the surface of the plate. We see emissivity is almost constant, from 0 degrees to almost 60 and then starts decreasing. This is the typical behavior of many materials. Their emissivity is roughly constant up to around 60 degrees and then they start decreasing. This tendency is seen at also 1 micron and 1.62 micron.

But beyond that at 4, 8 and 20 micron, we see that emissivity is continuously increasing up to about 75 degrees and then it starts decreasing. So, the behavior of emissivity with respect to angle, is wavelength dependent. At low wave length, it has one behavior and at high wavelength, it has a different behavior. When we are modeling the variation emissivity of metals, the angular dependence of emissivity as a function of wave length must be accounted for. We can see that variation is very different between low and high wavelengths.

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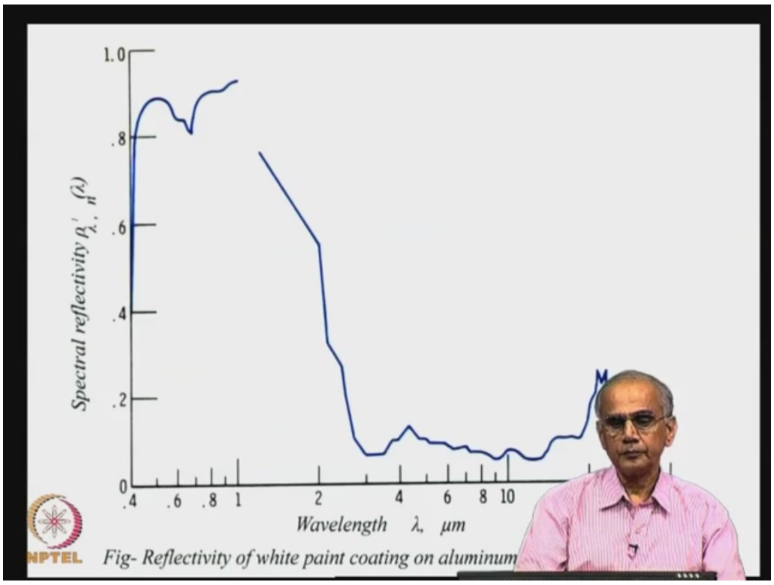

Fig- Reflectivity of white paint coating on aluminum

Now, here is an interesting example of the effects of coating on metals. In the above figure what we have shown is the reflectivity of white paint that is coated on aluminum. We see that white paint is a very good reflector in the visible. It has a value around 0.9. We expect white paint to be nice and reflective in the visible. But when we go to the infrared beyond about 3 micron we see that the reflectivity is very, very low. Reflectivity is around 0.1, that means, it absorbs 90 percent of the radiation. So it is almost black in this region. This is an interesting factor. . When we look at the surface we know it is white and hence we know it is a reflective surface. But we cannot conclude anything about the behavior of the surface in the infrared. In the infrared, it is not a good reflector.

The white paint is a selective surface, whose behavior below 1 micron and above 1 micron is very different. This factor must be accounted for, when we are trying to model the emissivity of white paint on aluminum. We have to account for the fact that the reflectivity of the material is very different is around 0.9 in the visible and around 0.1 in the infrared. When white paint is coated on aluminum, it is still a good emitter in the in the infrared. That is why, it is a very good paint, used to paint outdoor containers, which contain let us say petrol or other flammable material and we want to keep the temperature low. When coated with white paint, the white paint is all reflective in the region where solar radiation is impinging on the tank. Hence it will not absorb solar radiation.

At the same time, whatever radiation is absorbed, it is able to emit very efficiently in the infrared. Hence, it will keep the tank temperature sufficiently low, so that it can be safe. . This is the good example where selective coating like white paint keeps the temperature of the surface low. When we coat copper oxide and copper, it makes the surface a very strong absorber in the visible and a very poor emitter in the infrared. In the case of copper oxide and copper the selective coating actually enables it to reach very high temperature.

One can use a selective coating that is a semiconductor on metals like copper oxide. Copper helps one to get high temperature when subject to solar radiation. But if we coat a metal with white paint it enables one to keep the surface at a low temperature. We are able to achieve both, by suitable selective coating, like copper oxide and copper or a cool coating, like white paint on aluminum. In this way temperatures are controlled in satellites or in solar energy applications or other situations in engineering, where we want to control the surface temperature, on surfaces subjected to radiation.

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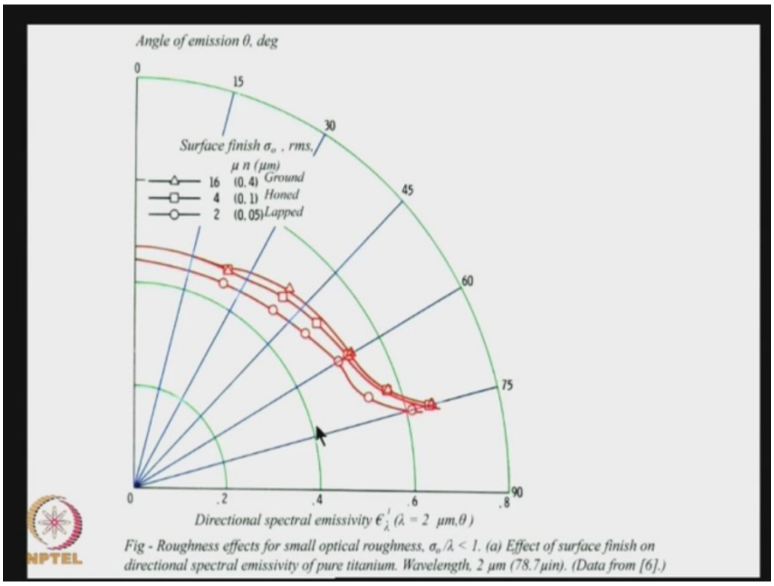

Fig - Roughness effects for small optical roughness, $\sigma_o/\lambda < 1$. (a) Effect of surface finish on directional spectral emissivity of pure titanium. Wavelength, 2 μm (78.7μin). (Data from [6].)

Now, here is an example of the effect of surface finish on the directional variation of emissivity. Here, we see an example of 3 different conditions, ground surface, honed surface and lapped surface. When we use ground, honed or lapped surface for getting a smoother surface, the roughness and height goes down from around 0.4 micron to point 0.05 micron. The smoother the surface is, higher is the reflectivity and hence lower is the emissivity. The emissivity is lower for the lapped surface and higher for ground surface. But in spite of the honing and grinding or lapping, the angular variation of emissivity has not changed much, it remains almost similar.

The state of the surface has not altered its angular dependence , but it has altered the value of emissivity. Hence it is not dependent on angle. For pure titanium we can have a single function which will account for the angular variation of emissivity, with data, without worrying about the type of surface finish the object has. We notice here that these measurements have been made for a wave length of 2 micron. The reason why this grinding honing or lapping has not had any large impact on the directional variation here, is because the wavelength of the radiation we are dealing with is larger than the roughness side that is depicted here.

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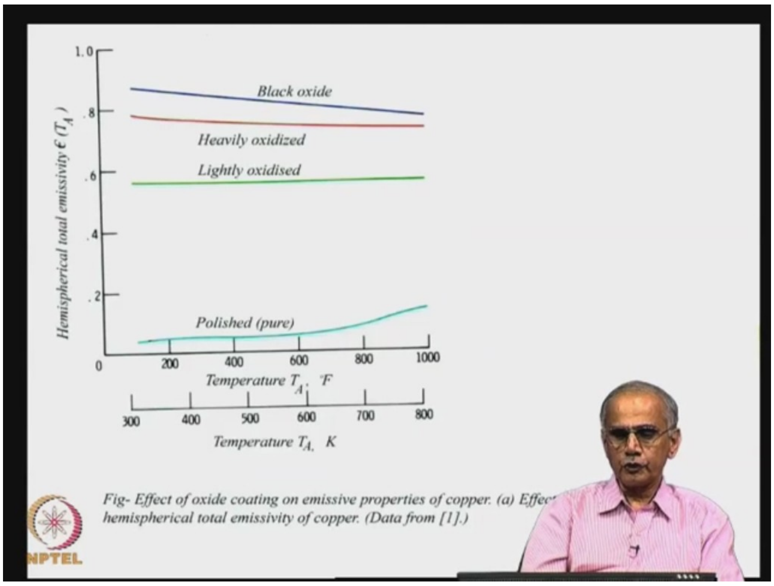

Fig- Effect of oxide coating on emissive properties of copper. (a) Effec
hemispherical total emissivity of copper. (Data from [1].)

The above figure shows how the hemispherical total emissivity when integrated over both angle and wave length, varies with temperature for copper. When we have pure polished copper with no oxide layer, then the emissivity is very low at around point 0.05. But this situation will not prevail for long time in the real world. Very soon a thin oxide layer will form, and we see that the emissivity suddenly jumps from 0.05 to around 0.55.

The emissivity goes up 11 times on a account of the oxide layer. If the oxide layer, increase in thickness it can go to up to 0.8 and when we make the oxide layer very thick, we can even get a black oxide. This is the standard coating used on solar collectors to increase the temperature of the collector. We can see that the oxide layer plays a very important role and can change the emissivity from point 0.05 to 0.9. So, oxide layer should always be recorded while estimating the emissivity of metals.

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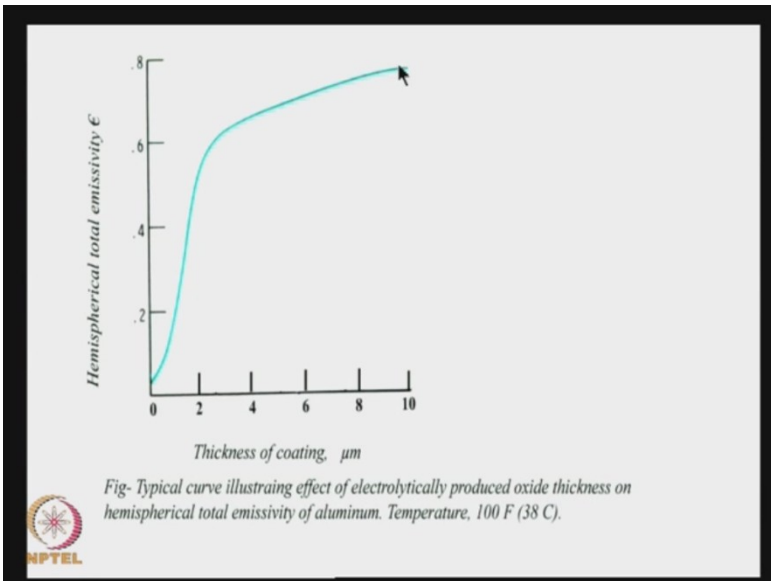

Fig- Typical curve illustraing effect of electrolytically produced oxide thickness on hemispherical total emissivity of aluminum. Temperature, 100 F (38 C).

This is depicted in a somewhat different way in the above figure to show the hemispherical total emissivity as a function of coating thickness for aluminum. And you can see that initially, the emissivity increase rapidly from point 0.05 to around 0.5. When the oxide coating is about 2 micron, beyond 2 micron, it increases, but somewhat slowly goes from around 0.5 to around 0.8 micron. This gives a rough idea as to the impact of the thickness of the oxide layer on the emissivity of metals. Hence it is important to have some information about oxide layer thickness, before we assume the emissivity of metals, because in engineering practice pure metals with no oxide are very rare. One should be looking out for a certain amount of oxide layer thickness, which has a substantial impact on the emissivity of the metal.

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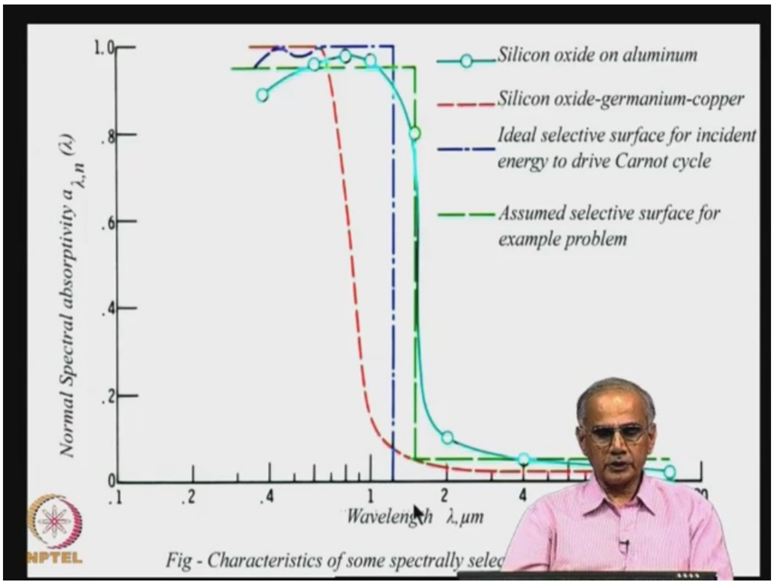

Fig - Characteristics of some spectrally selec

The above figure is an illustration of the impact of surface finish and oxide layer on Inconel X. For example, we see that newly polished Inconel X has an emissivity close to 0.2. But if it is not polished, it is 0.4 micron. But once the oxide layer forms on it, the emissivity jumps to around 0.75 micron. So, here again, we see the important role played by the oxide layer and the importance of the impact of polished material after it is received. The polished material's emissivity goes down. Wherever an oxide layer forms on the surface its emissivity jumps back to 0.8. This again is an important issue in making estimate of emissivity in metals. We go back to our discussion on selective surfaces. A selective surface is one whose absorptivity or emissivity is a strong question of wave length. Here is picture of the observed normal spectral absorptivity of silicon oxide coating on aluminum. We can see that it falls around 0.9 in the region below 1 micron and it rapidly decreases to less than 0.1 beyond 2 micron.

This is the kind of material we will need for solar collectors to absorb the sun's radiation strongly, and at the same time not to emit radiation that is coming in the infrared from the surface. Now, real surfaces , have complex behavior from modeling prospective, which we can replace it by a surface with absorptivity of 0.9 or 0.95 in the region below 1.5 micron. We assume it to be rapidly changing to an emissivity around point 0.05 beyond 1.5 micron. This is simplification one can make or if one wants to be a little more realistic, one can make an approximation, as one can assume 1 in the solar region and then rapidly considering to almost 0, in the region beyond 1.1 micron. So, these kinds of approximations are made to calculate. .

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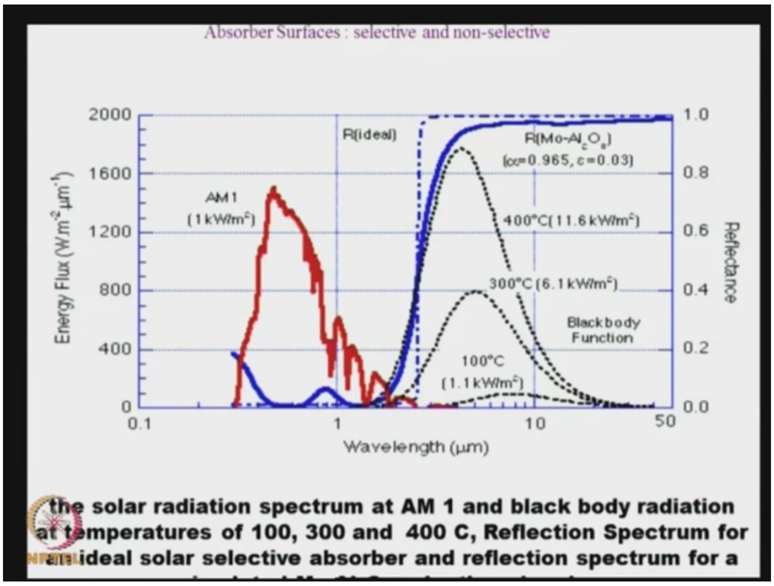

Absorber Surfaces : selective and non-selective

the solar radiation spectrum at AM 1 and black body radiation at temperatures of 100, 300 and 400 C, Reflection Spectrum for an ideal solar selective absorber and reflection spectrum for a

Here is the realistic depiction of the incoming solar radiation, which is around 1000 watts square meter. We can see that it peaks around 0.5 to 0.6 micron. In the same figure we have also shown, the emissions of black body at 100^0C, 300^0C and 400^0C. We find that up to 400^0C most of the emission from the surface is confined to the region where the reflectivity in the material is quite high or alternatively emissivity is very low. After 400^0C, the emissivity which is low in this region is advantages in not losing heat. At the same time, when the same material is confined to the solar wavelength region that is around 0.5 to around 1.1 micron, the reflective is very low.

This is molybdenum aluminum oxide, which is already a good absorber. We can see that it is very useful to get the surface to reach temperatures of the order of 400^0C without losing too much radiation in the region where the solar radiation exist. This kind of coating is now being used routinely in solar collectors all over the world. The basic principle here, is to have this coating thin enough not to alter the emissivity of the base material , that is already quite low. At same time we need to ensure that the surface absorbs all the radiation that is coming from the sun.

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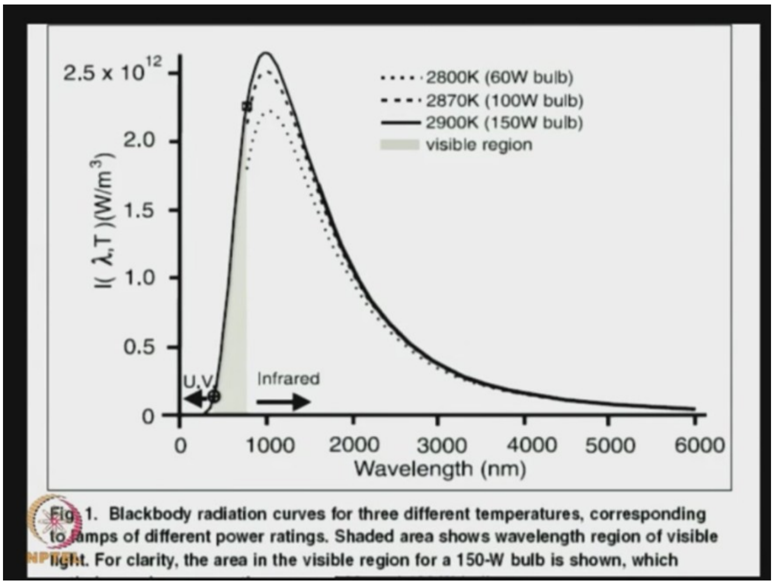

Fig. 1. Blackbody radiation curves for three different temperatures, corresponding to lamps of different power ratings. Shaded area shows wavelength region of visible light. For clarity, the area in the visible region for a 150-W bulb is shown, which

Another example that we will discuss in this lecture is the temperature variation from a tungsten filament lamp. We use a 65 watt bulb, 100 watt bulb and 450 watt bulb. As we go to higher and higher voltage of the bulb, the temperature or the filament is increasing slightly from 2800^0K to 2900^0K. If we plot the black body emissivity power at those temperatures, we see that the tungsten filament lamp peaks at a wave length of around 1 micron. The tungsten filament lamp is not peaking in the visible.

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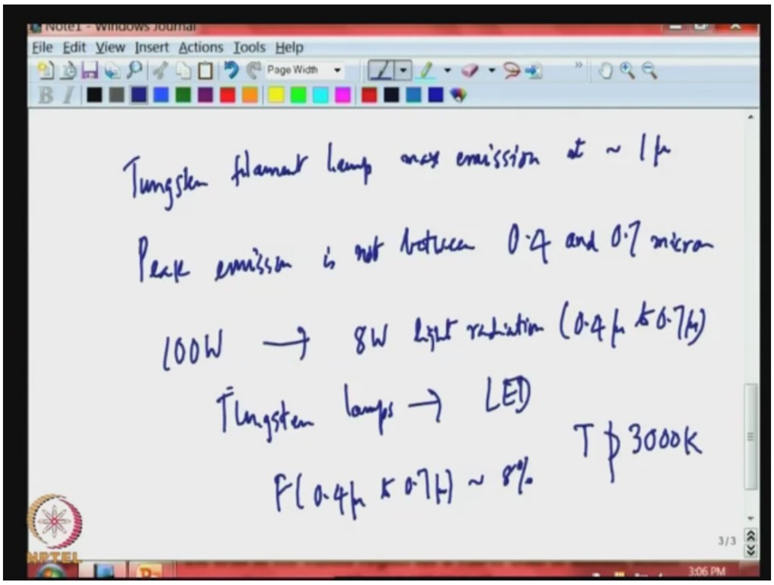

Tungsten filament lamp max emission at ~ 1μ

Peak emission is not between 0.4 and 0.7 micron

100W → 8W light radiation (0.4μ to 0.7μ)

Tungsten lamps → LED

$T \not> 3000K$

F(0.4μ to 0.7μ) ~ 8%

The tungsten filament lamp has maximum emission at around 1 micron. Ideally it should have its peak emission somewhere in the wave length range from 0.4 to 0.7 microns. The peak emission is not between 0.4 and 0.7 micron. This means that tungsten filament lamp is not a very efficient device for producing light. For example, a typical 100 watt lamp will produce only around 8 watts of light radiation that is in the range 0.4 micron to 0.7 micron. Hence the efficiency is around 8%. That is for a 100 watt of electrical energy consumption only 8 watt is coming out of the light.

That is why, in the last 10 years most people, who are energy conscious are switching over from tungsten lamps to light emitting diodes(LED). This is occurring, because efficiency of the LED is much higher than that of tungsten filament lamps. Hence they are more efficient for light energy. The tungsten filament lamp is ideally most suited for infrared heating, because it generates the peak radiation around 1 micron in the infrared. So, if there are application where infrared heating is desirable then one can go for tungsten filament lamp. But for purpose of lighting, tungsten filament lamp is not a good choice, because only about 8 percent of the electrical energy consumed as light that actually comes out as light energy. L E D take a different approach to generate this light source. Note that L E D is not governed by laws of equilibrium thermodynamics.

Tungsten filament lamp is of course, an invention that goes back to the days of Edison. . But it is now not in favor with people who want to improve energy efficiency, because a lamp by its very design cannot really give high efficiency. One should realize by now, that if the fraction of radiation is lying from 0.4 micron to 0.6 micron, energy is around 8%. 92% of the radiation emissivity from a filament lamp comes out as heat. In hot climate, having tungsten lamp actually will add load to the air conditioning , because the tungsten lamp puts out more and more heat and that heat will ultimately heat the room unless there are ways to move the air from the room. . So, the tungsten filament lamp can never achieve high efficiency, because of the fact that it cannot go above 8%. The temperature of the filament lamp cannot exceed 3000^0K. That is the basic limitation of this lamp, as it cannot exceed 3000^0K, because the melting point of tungsten is close to 3000^0K. As we approach the melting point of the material the evaporation rates go up dramatically. Now, we will give an example of the empirical data, we have.

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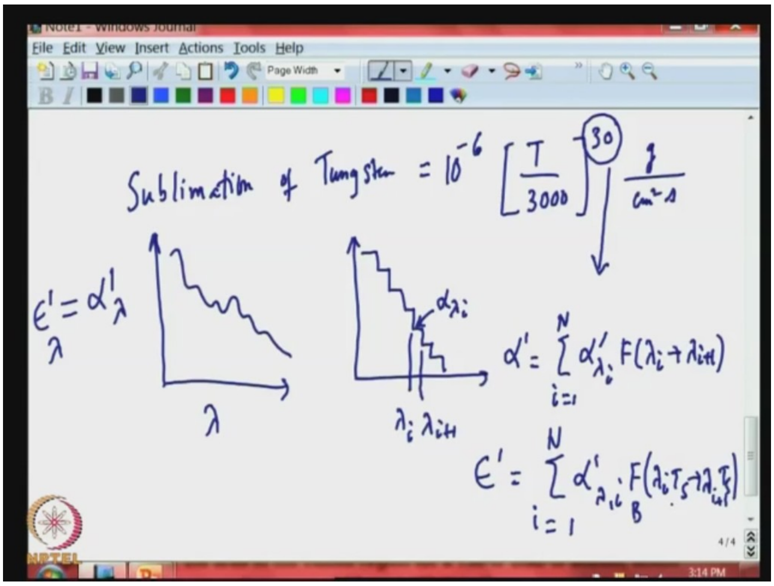

Sublimation of Tungsten $= 10^{-6}\left[\frac{T}{3000}\right]^{\textcircled{30}} \frac{g}{cm^2\text{-}s}$

$\epsilon'_\lambda = \alpha'_\lambda$

$\alpha' = \sum_{i=1}^{N} \alpha'_{\lambda_i} F(\lambda_i \to \lambda_{i+1})$

$\epsilon' = \sum_{i=1}^{N} \alpha'_{\lambda,i} F_B(\lambda_i T_s \to \lambda_{i+1} T_s)$

For sublimation, that is conversion of tungsten to vapor, this value can be shown to be around $10^{-6}[T/3000]^{30}$ gms/cm^2/sec. This is this rate at which tungsten wire sublimates and the key issue here is not the numerical value, but this power. We notice that the sublimation increases almost exponentially with temperature and is going at the rate of T to power 30. So, even a few degree rise in temperature will have a huge impact on the life of the tungsten filament lamp, as most of you realize the tungsten filament is already very thin. As the sublimation occurs and parts of the material become vapor, there will be some point in the filament where the amount of metal is so small that it will break. As a lamp gets older and we switch on the light and the voltage is somewhat higher than normal, then we will see the filament breaking.

This is because of high voltage the temperature of the filament goes up by 10 to 20 degrees and that is enough to bring about a catastrophic failure. So, that is why people designed it as filament lamp cannot afford to increase temperature beyond 3000^0K. Now, if the temperature of tungsten filament lamp is kept below 3000^0K, then we can see that it can have only 8% of the radiation in the visible. So, the upper limit of the efficiency of the tungsten filament lamp is controlled by the highest temperature that can be achieved there keeping in mind that one needs a lamp to have at least the life of 1000 hours. If we typically keep lights on at night for 2 to 3 hours a day we can expect it to last about an year.

Hence the design of the tungsten filament lamp has to account for the fact that we want a life of at least 1000 hours. In that case we have to keep the temperature somewhat below 3000^0K. . Hence the conversion of the electromagnetic energy to light energy is only 8%. When we put tungsten filament lamp in a room, 92% of the electromagnetic energy that is consumed by the lamp, comes out as heat. In country like India, tungsten filament lamp is heating a room more than lighting. That is why most people are now switching over to light emitting diodes. Although, they are more expensive, their efficiency being higher in the long term they will be cheaper on the basis that the higher the efficiency the lower the consumption of electric energy.

Now, let us take a typical example of any material. Let us define the variation of characteristic spectral wave length of the material. Now, in today's world we will replace that by series of values or steps. Let us say this is 'i' and 'i+1'. So, the directional total absorptivity will be nothing, but summation over all 'i'. There are different regions here from 1 to n. This value let us call it as $\alpha_{\lambda i}$. That is the absorptivity of the material between λ_i and λ_{i+1}. Then we have to know the fraction radiation between λ_i and λ_{i+1}. We have to know how much of the incoming radiation is between these 2 wavelengths to find the absorptivity. Now, the information about the fraction of radiation lying between λ_i and λ_{i+1} can either be provided directly to us by measurement. If we know that the incoming radiation is proportional to a blackbody radiation at a certain temperature, then we can compute this from standard tables available in most heat transfer text books. The fraction radiation lying between 2 wave lengths for a given temperature for blackbody, for the same material, we know that $\acute{\alpha}_\lambda$ is equal to ϵ'_λ by Kirchhoff 's law. For the same material, we can calculate the total emissivity again as ∑ i=1/n. Now $ά_λ$ and $є'_λ$ are the same. We can use $ά_{λi}$, which is a fraction of blackbody radiation; that is weighting function, for emissivity.

Then $λ_iT_s$ to $λ_{i+1}T_s$ is available from tables in all text books. The main difference between how we calculate absorptivity or emissivity is the value 'F' that we use. The value 'F' for absorptivity is based on how the incoming radiation varies with wave length. Whereas in the case of emissivity we go by the temperature of the surface T_s and we look at the blackbody function and compute the fraction of radiation lying between $λ_i$ to $λ_{i+1}$ from the blackbody function formula. Let us take a very simple example to illustrate this.

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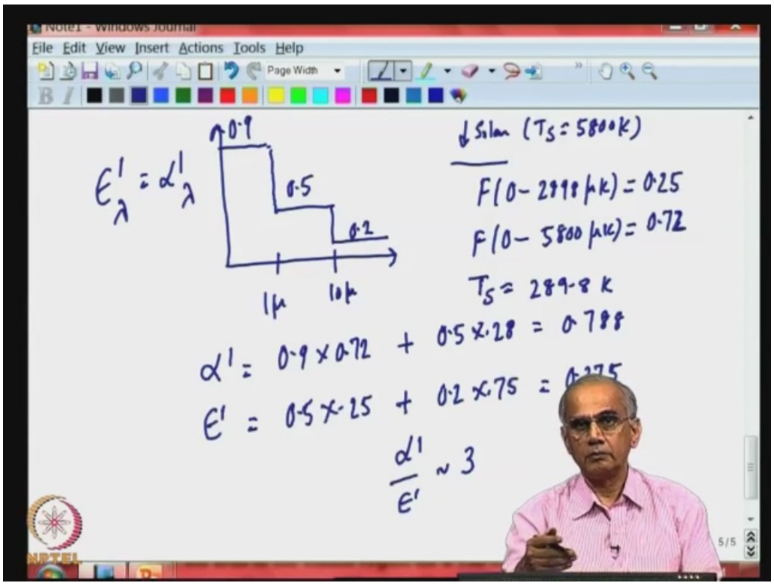

$\epsilon'_\lambda = \alpha'_\lambda$

↓ Solar ($T_S = 5800$ K)

$F(0-2898\ \mu K) = 0.25$

$F(0-5800\ \mu K) = 0.72$

$T_S = 289.8$ K

$\alpha' = 0.9 \times 0.72 + 0.5 \times .28 = 0.788$

$\epsilon' = 0.5 \times .25 + 0.2 \times .75 = 0.275$

$\frac{\alpha'}{\epsilon'} \sim 3$

Imagine a material, whose emissivity varies with its wavelength in a very simple way. Below 1 micron, let us assume it has some value and between 1 and 10 it has is another value and beyond 10, it has a third value. The numbers assumed for this example are 0.9, 0.5 and 0.2. If its variation is more complicated then we can use more number of intervals in between. Let us assume that this material is subjected to solar radiation. We can assume solar radiation to be a blackbody at the temperature of 5800^0K. We know from the blackbody function that from F(0-2898μK), amount of radiation fraction is 0.25. When you go from F(0-5800μK) it is around 0.72. This is available from tables in the back of most books on heat transfer. We can estimate ά as a fraction of radiation lying between 0 to 1 micron there will be F(0-5800 μK) as 0.72. Hence, ά = 0.9x0.72 +0.5x0.28 = 0.788. Here 0.5 into 0.28 radiation beyond 10 micron was the Sun. These two will lead to a value of 0.788 for absorptivity. For same material, we look at directional total emissivity. We have to use an assuming surface temperature material is 289.8^0K assumed for convenience close to room temperature. For this value at 1 micron, there will be no radiation, between 1 and 10 it will be 0.25. So, we take ϵ' = 0.5x25 + 0.2x0.75 = 0.275. We can clearly see that for this material the ratio of ά to ϵ' is close to 3. So, this is one more selective surface like the one we saw in the case of metal. This was just to illustrate how we compute the total properties from spectral properties. If we know piecewise values of these functions, this can be extended to any number of intervals. This can be also automated that is we can add computer program to calculate emissivity and absorptivity at a given temperature of the surface and also a temperature of the incoming radiation if it is a blackbody. Now, let us give another illustration..

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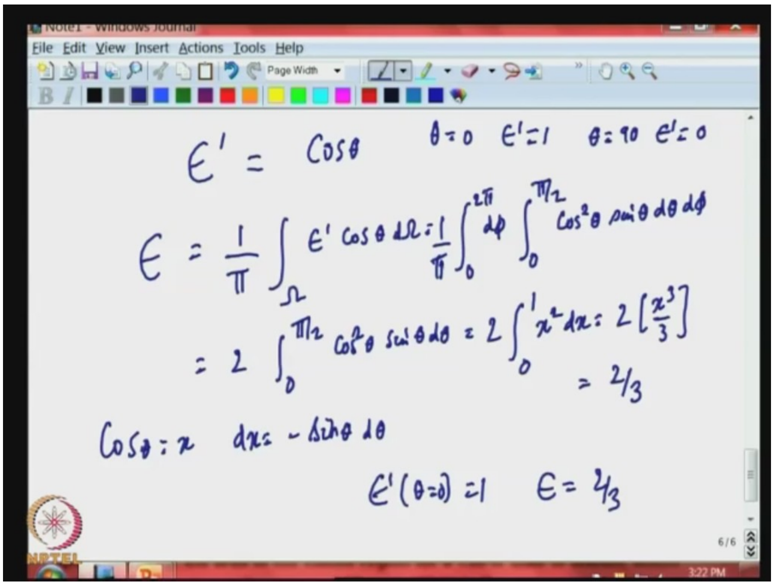

$\epsilon' = \cos\theta \quad \theta = 0 \ \epsilon' = 1 \quad \theta = 90 \ \epsilon' = 0$

$$\epsilon = \frac{1}{\pi}\int_{\Omega} \epsilon' \cos\theta \, d\Omega = \frac{1}{\pi}\int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$= 2\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta = 2\int_0^1 x^2 \, dx = 2\left[\frac{x^3}{3}\right] = 2/3$$

$\cos\theta = x \quad dx = -\sin\theta \, d\theta$

$\epsilon'(\theta = 0) = 1 \quad \epsilon = 2/3$

The directional emissivity of many materials that is values like cosθ, let us assume that θ=0, ϵ' is 1 and at θ=90, ϵ' is 0. There is a very simple variation. The hemispherical emissivity is e' cosθ dΩ. Now, this can be written as $_0\int^{2\pi}$ for the π variation, and $_0\int^{\pi/2}$ e' for cosθ. As there is another cosθ, we get cos^{2}θ. From the definition of dΩ, we get sinθ dθ dφ. i, This becomes 2 $_0\int^{\pi/2}$cos^{2}θ sinθ dθ. We assume cosθ as x. When θ equals to 0, x = 1. As there is a - (minus) sign, this will become 0 to 1 x^2 dx, which will integrate to $2x^3/3$. So, we will get answer of two third. What we find here is that, the directional emissivity varies as cosθ. The hemispherical emissivity is two third of the value of the emissivity at normal (θ equal to zero). So, if directional emissivity at θ equal to 0 is 1. The hemispherical emissivity now comes out as two third.

We can see typically for many materials, the hemispherical emissivity is about two thirds of the directional normal emissivity. This is a useful to have, because one can measure the directional normal emissivity fairly easily. For an opaque material some of the directional normal emissivity and the directional normal reflectivity is equal to 1. If we mention the reflectivity, we get the emissivity. Once we know that if the material has wave similar to what is given here as $\cos\theta$, then your hemispherical value will be two thirds of the normal value. This kind of information can be useful for many applications, wherein we have to estimate the hemisphere value from the directional value.

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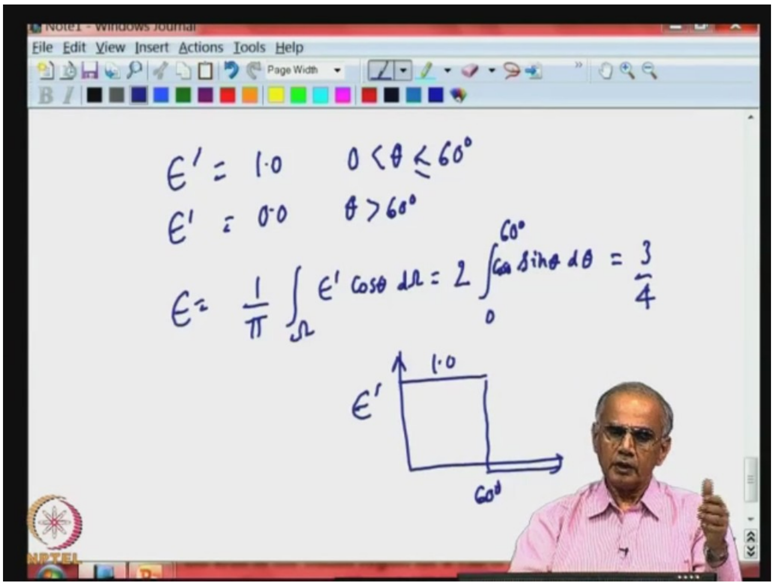

$$\epsilon' = 1.0 \qquad 0 < \theta \leq 60^\circ$$

$$\epsilon' = 0.0 \qquad \theta > 60^\circ$$

$$\epsilon = \frac{1}{\pi}\int_{\Omega} \epsilon' \cos\theta \, d\Omega = 2\int_{0}^{60^\circ} \cos\theta \sin\theta \, d\theta = \frac{3}{4}$$

Emissivity is a constant that is equal to less than 1 for θ between 0 and 60^0 degree. We saw that example in the in the figures discussed earlier, and after that goes rapidly to 0. Let us assume that emissivity goes to 0 for θ greater than 60^0. If it shows this kind of behavior, then ϵ will come out as $1/\pi$. , Once again we integrate over $_{\Omega}\int^{\epsilon'} \cos\theta \, d\Omega$. We again integrate with integer 0 to 60 degree, because that is up to which emissivity is there. We will get $\sin\theta \, d\theta$. The result is a constant 3/4. In this case where the material had emissivity variation which was a constant from 0 to 60 and went to 0 after that, for such material we can see that the hemispherical value is three fourth of the directional normal value. This is higher than what we saw in the previous example, where it was two third. This gives a roughly typical expectation, that the hemispherical value emissivity is somewhere between two thirds to three fourths of the normal emissivity. This information is useful for application where in we may not get the emissivity value at all angles. We have to make do with the value of normal emissivity measured in the laboratory. From there we want to infer the value of the hemispherical emissivity.

What we have done so far is, we have highlighted the fact that for most real materials both emissivity and absorptivity varies strongly with wavelength and somewhat with angle. We have shown as to how Kirchhoff's law can be applied for these materials, carefully not casually as is done in some of the books. We also showed how the spectral variation emissivity of many materials like, metals and several surfaces can be very useful in the design of tanks to keep the tank cool or in designing solar collectors to increase the absorptivity of solar radiation, at the same time keeping the emissivity low.

A good knowledge of the directional and spectral variation emissivity is very useful in the design a many engineering systems. The challenge for the designer very often is to get the right data about these materials. We saw that for metals the radiative property is very sensitive to the oxide layer thickness. When the oxide layer is present, the emissivity is much larger, and when oxide layer is not present emissivity is quite low. Before we can assume the value emissivity for metals we must be quite sure about the amount of oxide that is present in that metal and an oxide layer is inevitable. This is because many metals are easily oxidized and we should expect to have a certain amount of oxide layer unless it is a freshly cut material. With this we complete our discussion on the property of surfaces. We will be going on to the next course to understand, how we estimate the fraction of radiation that is there in one region and how much it reaches another region. So, that will be the topic of the next lecture.