Radiation Heat Transfer
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Lecture - 37
Approximate Methods in Scattering: 2

Today, we will look at some of the results that was obtained by solving those equations.

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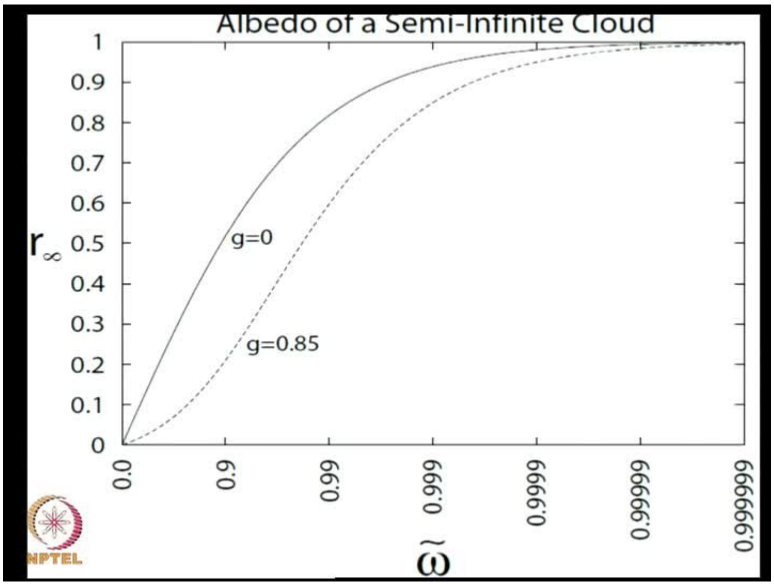

We can when the based on the equation derived in the last class, We can estimate the reflectivity of a layer of particles depending upon the single scattering Albedo, which is the x axis here, and the Albedo of the cloud of particles is the y axis. This result is shown for two different climate parameters. One it is isotropic scattering, which we had derived earlier. In this case scattering is in uniform in all directions. We find that the reflectivity of the particles increase a linearly initially, for small values of single scaring Albedo, that is when there is lot of absorption less scattering.

As scattering increases and albedo approaches the value of 1, which is to be accepted but notice that the x axis is not linear. It is logarithmic. The approach of the Albedo of the cloud of particles to unity is very slow. We can see that as the single scattering albedo goes from 0.9 to 0.99, Albedo increases from around 0.4 to 0.8 and from that you reach one takes lot more increase in the single scattering Albedo. Next, we see the case where there is more forward scattering. If there is more forward scattering, which is more than isotropic than you must realize that there is less backward scattering. So, Albedo is nothing but backward scattering. If there is less backward scattering the Albedo of cloud of particles will less than for isotropic scattering, all the way from 0 value of single scattering Albedo, all the way to all value of 1.

But the basic shape of the curve is similar to the isotropic scattering except that the amount is somewhat reduced, because the scattering is not isotropic and more of the photons are being scattered in the forward direction. Hence, less photon are available for reflection for forward scattering. This gives an idea of the kinds of errors one makes when assumes isotropic scattering. We can see clearly that if the single scattering Albedo is quite low or less, then 0,9 or even less than 0.99 there is the least difference between the results for isotropic scattering and non isotropic scattering. Indicating that by that we have to very careful about assumption of isotropic scattering, we must be a aware that when you assume isotropic scattering We can over estimate the Albedo of the cloud of particles by as much as 400 percent.

If the single scattering Albedo is 0.9 but if single scattering Albedo is very, very large 0.999 is close to 1, then it does not matter what assumption you made about the nature scattering. What this really means is the particles are predominantly scattering, than We can assume that the Albedo will approach 1 irrespective of whether the nature of scattering is isotropic or non isotropic. If the single scattering Albedo is large, ultimately the photons will be back scattered after large number of scattering within the medium. Large number of events of scattering will ultimately take it back to leave the slab of particles.

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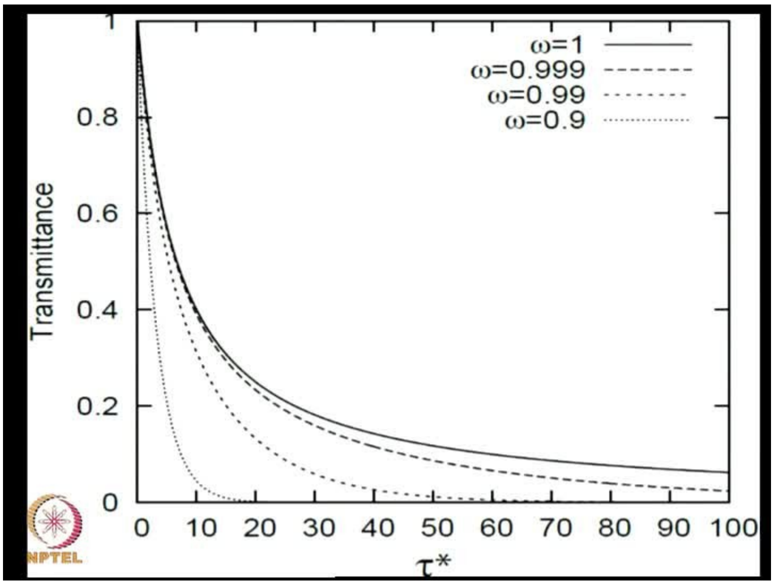

It does not matter whether this isotropic or non isotropic as long as single scattering Albedo is very large. Ultimately it will all be scattered out of the system. By the same contain let look at the effect of the total optical depth of the slab of particles and the transmittance. Now you see that as the single scattering Albedo increases, the transmission is also increasing. That is the understandable because as a single scattering Albedo increases from 0.9 to 1 the absorption is decreasing. If the absorption is decreasing transmission should increase.

Again one notices that when the total optical depth is of order of 10, there is a large impact of the single scattering Albedo on the transmittance. We can see that when single scattering goes from 0.9 to 1, that transmittance can go from 0 to 0.4. On the other hand here at very large optical depth, it does not really matter, what the single scattering Albedo is, ultimately the transmission is very, very small, because again the optical will be large the photon gets many, many opportunities to be absorbed within the medium. As theoretical increases you are going to find that the transmission is going to go down irrespective of the value of the single scattering.

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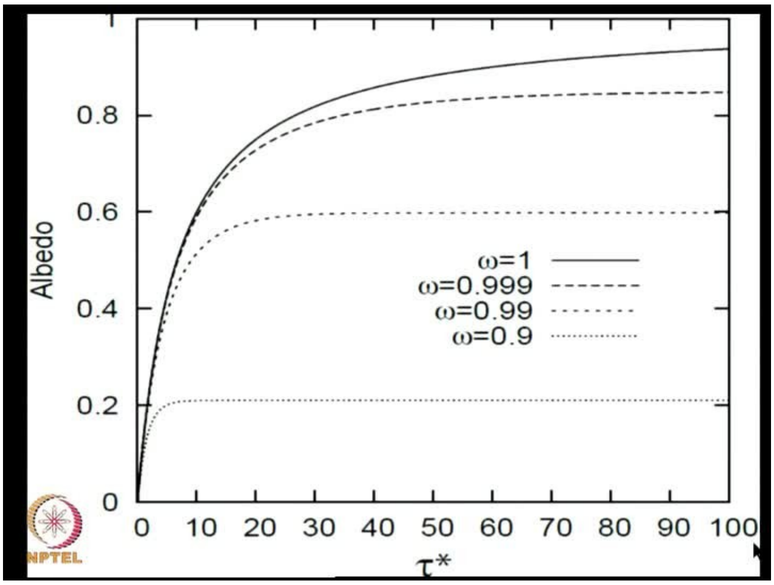

These are very useful lessons one can learn from a simple model because it gives insight into the behavior of the system. Now, in the next graph, we see the again the connection between Albedo which is back scattering and optical depth for various values of single scattering albedo. Since, Albedo again we will see that is the optical is very small, Albedo will be very small, irrespective of the Albedo the cloud will be small irrespectively of the value of the single scattering Albedo, because the optical depth is very small. But when the optical depth is large, the photon has many, many events within the system, then the single scattering Albedo matters. We can see that as the optical becomes of the order of 10 going from 0.9 single scattering Albedo to 1 increase the cloud Albedo by 300 percent.

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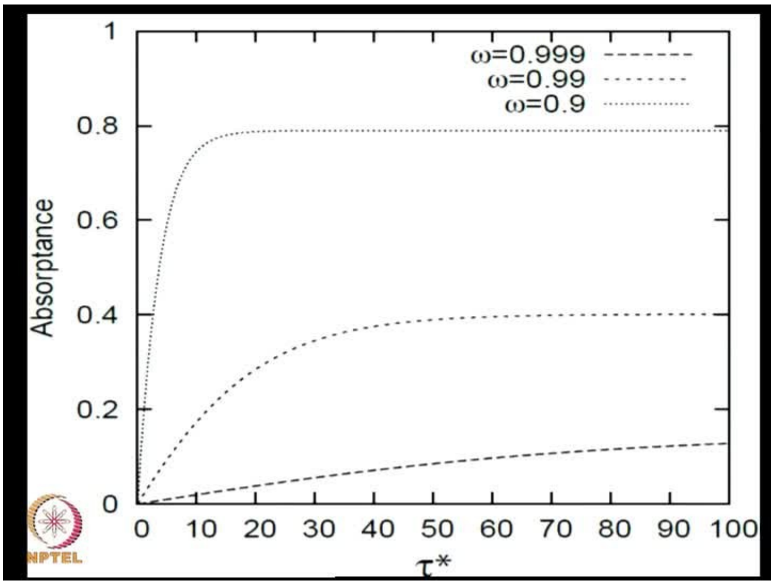

If the optical depth is the order of 100, then going from 0.2, the single scattering Albedo a point Albedo 0.2 while 1 causes Albedo 0.95. So, again there is a large increase in the cloud Albedo in response changes in the scattering Albedo, when the optical depth is large. Now, the next quantity is the opposite of 1 minus Albedo, when the body is absorbed in this system and again you see clearly, that if the Albedo is small absorption is small, does not matter what the single scattering Albedo is, while at the high optical depth the amount absorbed depends very much on the single scattering Albedo. When the Albedo is 0.9 the absorption is 0.8. So, at every scattering encounter there is only ten percent chance of being absorbed.

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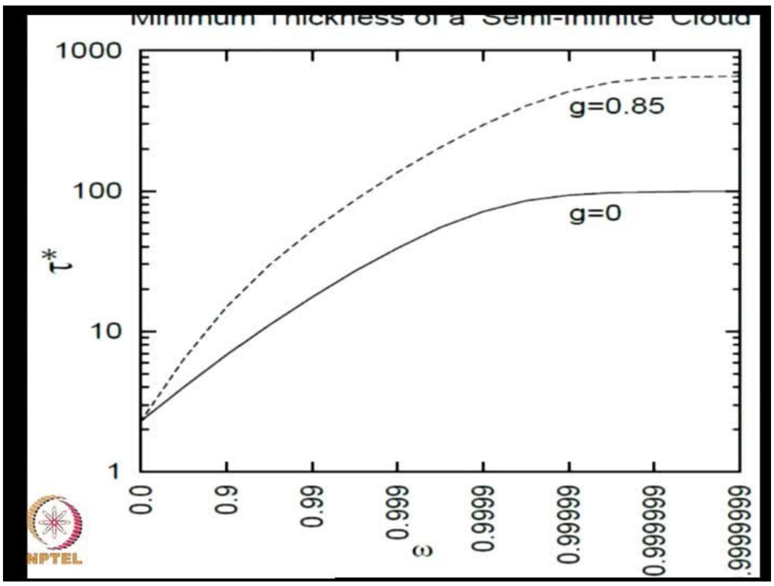

But since the optical depth so large, there are many, many opportunities for the photon to absorb ultimately 80 percent photons are absorbed. While the single scattering Albedo is 0.999 then the changes being observed will be only 10 percent. So, again you see very interesting non-linear relationship between single scattering Albedo and the total photons absorbed by this slab of particles. Here you see the importance of minimum thickness. Very often there are simple results for semi infinite medium available and one need to ask whether the assumption is a valid approximation in a given situation.

This result here in this graph will help you to decide up to what optical depth considered as semi infinite and that depends upon the different single scattering Albedo. As the single scattering Albedo increases, the amount scattering events increase. We need a must thicker layer of the medium for what you do fitted it as semi infinite. It also depends upon whether you have more forward scattering.

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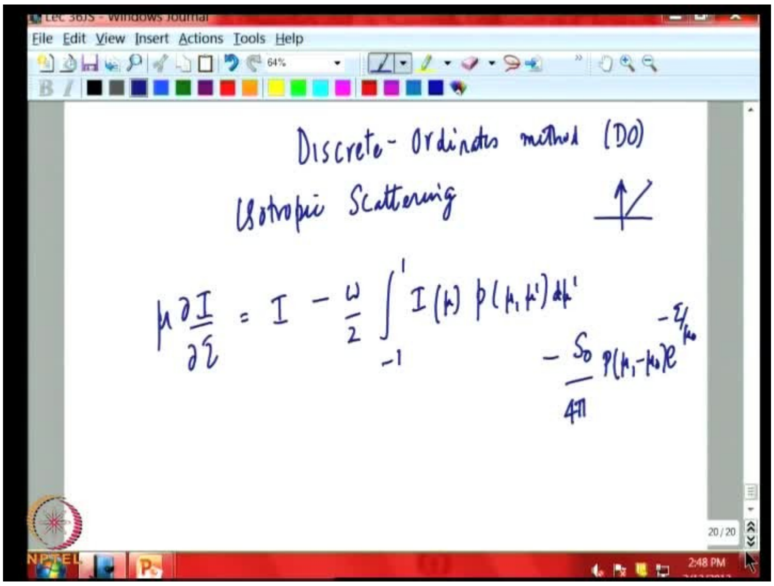

We have isotropic scattering, so with this we conclude the discussion on the use of simple scattering models to estimate the Albedo of a cloud, a particles as a function of the single scattering Albedo and assumption about asymmetry parameter. Now, we move on to another technique, involving the scattering this technique is called the discrete ordinates method also called DOM

So, let us assume that we are dealing with system of particles which are scattering and the scattering is isotropic. This is just to make the analysis simple. This can be extended to non-isotropic scattering also, but the method is best illustrated by isotropic scattering. Now, for radiation which is assumed to be azimuthally averaged, once you do that your equation for intensity. In the absence of emission, reads like this is the phrase function we can also add direct absorption from solar energy radiation coming directly on the medium. This diffused radiation that is different from the direct radiation can be written in terms of the incoming radiation as 0 and phase function of u

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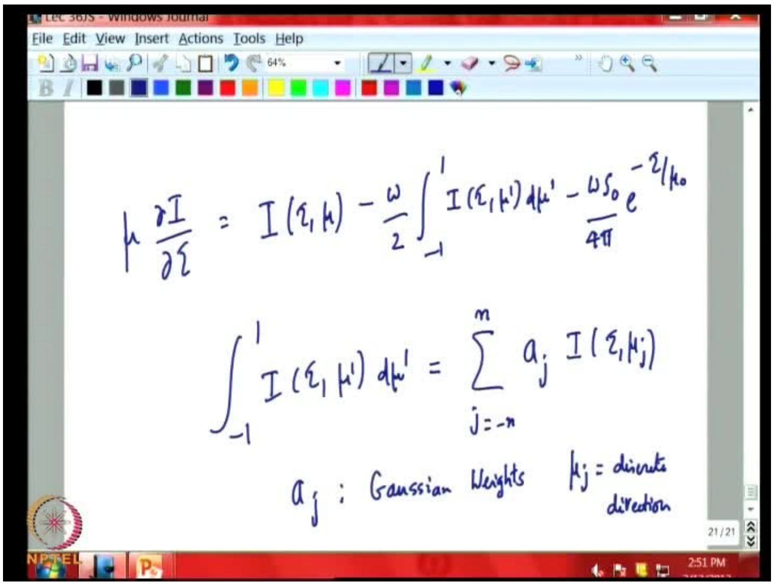

The radiation which in a scattering medium is coming in all directions; if you assume that this scattering is isotropic scattering, this is 1, we can integrate over all of the new that simple expression in which the new integration the p r minus 1. Finally, we have an expression which says the rate of change of intensity with optical depth is equal to the value of that at that tau and mu minus the single scattering Albedo. How it varies with mu here minus omega direct radiation by 4π, e to power of tau by mu

This is the simple expression for isotropic scattering. Now we would like to solve this equation for intensity and in order to do that our main complication is this integral. This integral what is going to cause a big problem for us, because the unknown function is integral. We adopt the following strategy, we replace this integral (minus 1 to plus 1 I dove u prime d prime) by a simple quadrature. Now, today all of you are computer savvy we know that there are many methods available to replace integral by summation. So, here aj are the Gaussian weights and b_j are the various directions, discreet.

So, essentially what we are doing is replacing the continuous variation in this integral by discreet directions. We have to continuously vary mu from 0 to 1 or for minus 1 to plus 1. Now, we are going to decide to replace this by may be 2, 4, 6, 8 or 16 directions and then each of the direction we will find an appropriate function. Now, once we substitute this back here your system let us to make sure we understand, what is going on here, once we substitute that gets the following equation.

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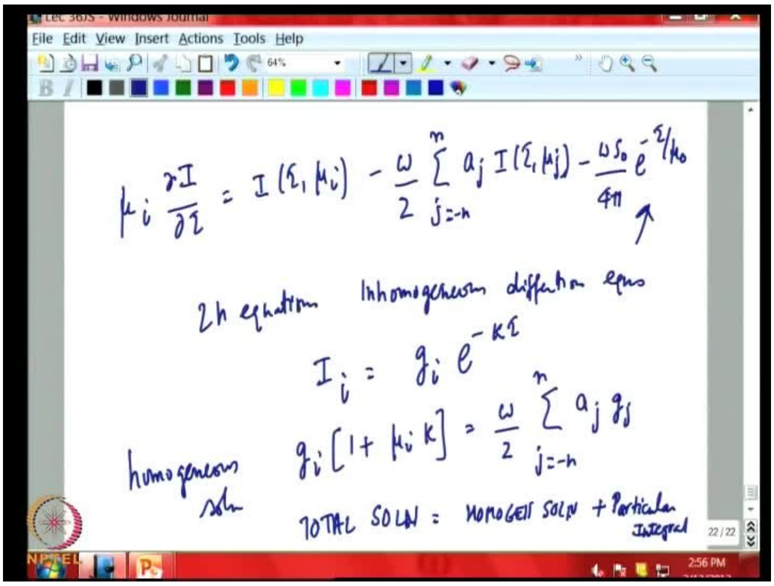

This is the integral that is now replaced by summation; this already coming in specific angle is 0. We have now, but to an equation for 2, different values mu. That is we have and it is inhomogeneous because of this term, differential equation. There are well known techniques available to solve a set of simultaneous different equations that are linear. We have learnt to solve the equation. So, let us say that we assume that in Ith directions in the Ith direction the solution of the kind some gi e to the power the minus k times tau.

We insert this form of the solution in to the equation and collect all the terms with the same mu(direction). Then you will get g i into 1 plus mu i k is equal to omega by 2 sigma(j equals to minus n to plus n) a j. This is the homogeneous solution; we are counting through the second part, the homogenous solution. We are going to take care of the non-homogeneous solution because the total solution will say is equal to homogeneous solution plus particular integral. All of us have done this kind of exercise in which and non homogeneous equation, first ignore the non-homogeneous parts of the equations how much is part how much we get the solution and then add the particular integral, which will ensure that the non homogeneous term is taken into account.

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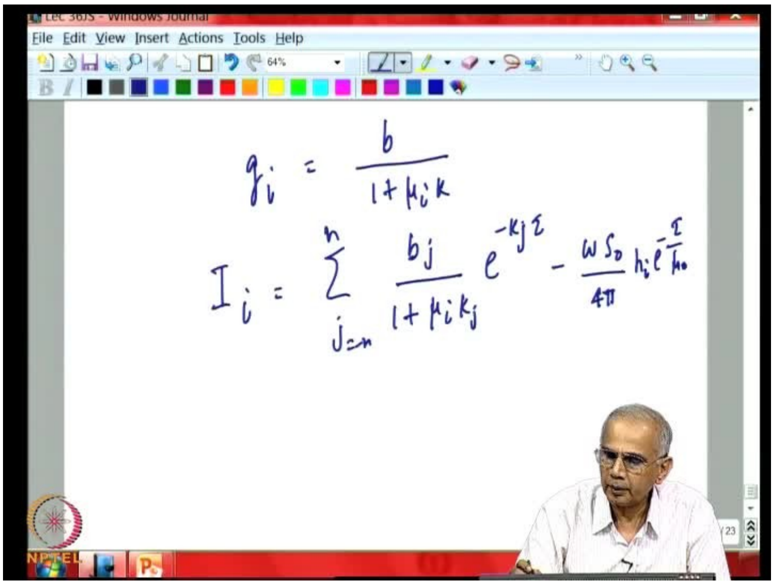

So, g i which is the unknown concern in this problem, will be of the form some let it be i, from b 1 plus mui into k. So, general solution now we can write down for the i th direction will be sigma over j b j plus 1 plus mu i k j e minus k j into Tau. This is a form of the solution for the homogenous term. Then you have minus omega single scattering Albedo as 0 by 4 point into h i another concert e to the power of minus tau of mu 0. So, still there is a task of finding the b j and the hi and so that depends on the body condition We will not get into the details of how we get, the we have a set of 2n equations are available.

Now, these constants are to be from the boundary conditions and so we are going to assume that these boundary conditions can be discovered by appropriate conditions at the two boundaries. This is an example of a method which is really an extension of the method we developed for two stream approximation.

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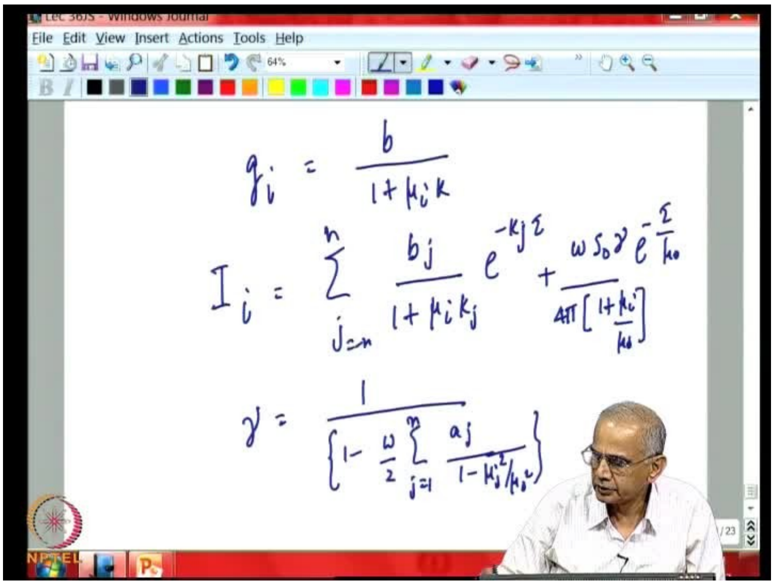

So, two stream approximations can also be thought of as a discreet ordinate method expect that the number of streams is 2(up and down here). We have gone to a more elaborate technique here we have that something more elaborate. Final form solution we can write down in terms of particular integral, then we may write down little more carefully particularly integral here. We will write the homogenous solution The real task of finding this constant will be gamma is nothing but 1 by 1 minus omega by 2 ,sigma is equal 1 of a j minus 1 minus 2 j square by mu 0 squares. So, normally in all the particular obtain in the similar by the way and the task of satisfying the boundary condition is left to the b j equation is in a homogeneous terms and they are so adjusted that the intensity satisfying either top, at the bottom at all the interfaces. The simultaneous can be done value easily, but it occurs in some other technique. Lot of algebra is involved but today, this algebra is not as tedious as one may think.

That was the illustration of the discrete ordinate method you must appreciate that there is nothing but very systematic extension of the two stream approximations, which we took one steam up one steam down this format can be solved coordinates 4 steam or 8 or 16 and today with the available computers and the very attractive prospects in parallel computing, running this code on a parallel computer using large number of coordinates will not take much time.

So, such routine calculation which were considered very challenging 50 years ago and people like Nobel Laureate S. Chandrashekar developed very elegant and complex methods to solve the problem with least effort that now is not needed. Now, we can use the brute force of the computer to solve any tedious calculation without worrying about time or the computational effort.

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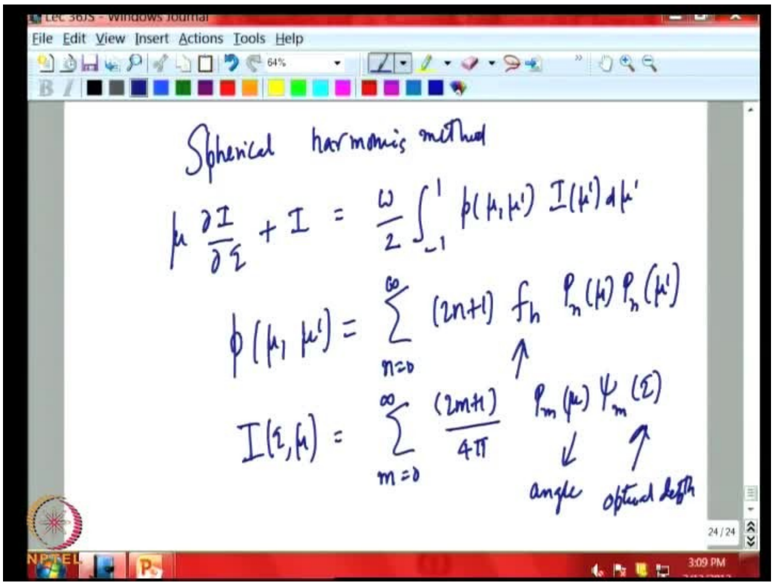

That is why discrete ordinate method has now become popular. There are standard codes available, which will solve the full scattering code, very easily. Also this is very popular method today and there are software available which easily take care of the solution of large number of simultaneous equations, using the discrete ordinate method. Now, it is also important to highlight a few other techniques which have been developed in this area and the technique which is also been quite popular is the spherical harmonic method.

. As the name indicates we are going to exploit the spherical harmonic functions. We again start with the same equation which we had earlier, which involves the phase functions. Now, we make some assumptions above the nature of the phase functions, we are going to assume that this phase function, it is depends on mu and mu prime can be expanded in terms of Legendre polynomials. This based on experience that although in the case of Mie scattering, we know that the phase con function a very complex function dependent on mu and mu prime. In spite of that we know that we can explain that complex phase functions in terms of Legendre polynomials, which is in typical optimal function for these applications. u subscribes this here, so advantages in this functions now very clear known.

To go further we need to makes some assumption about I. So, where I intensity we are going to assume that intensity can be expanded in terms of Legendre polynomials and functions which depends only on the optical depth. Essentially you must understand in the spherical harmonic method. The first the harmonic method is replacement of the complex phase function in terms of Legendre polynomials and see these polynomials have been used for long time in many i fields, one can be confident that if you chose specific number of m values in this expansion We can represent any real complex means Mie phase function in terms of this Le gendare polynomials.

If the Mie scattering is very complex, then you need more values of n and then it was for example, for Rayleigh scattering which has a very simple expression. In addition to that in the case intensity we are essentially assuming that we can separate the dependence of the intensity on the angle with that dependence of optical depth. This could be considered as somewhat like the separation of variable technique, we adopt in solving equations. So, here we are separating the dependence of the intensity and optical depth and dependence of intensity on directions.

We are assuming that we can represent the dependence of intensity on both optical depth and angle, both this value can be separated into a product from, one which depends on optical depth multiplied by a function dependent angle. This approximation but based on experience in solving different equations in this separation of variables. We know that this technique will work. Now, we need that will Legendre polynomials on both sides, both for the phase function and for the intensity function. We search it to the back in that equation. You collect powers of like values if mu because we want the equation to be valid for any mu. We collect those functions with like values of mu We are able to get all the unknown function, unknown constant This unknown function of the intensity as function optical depth.

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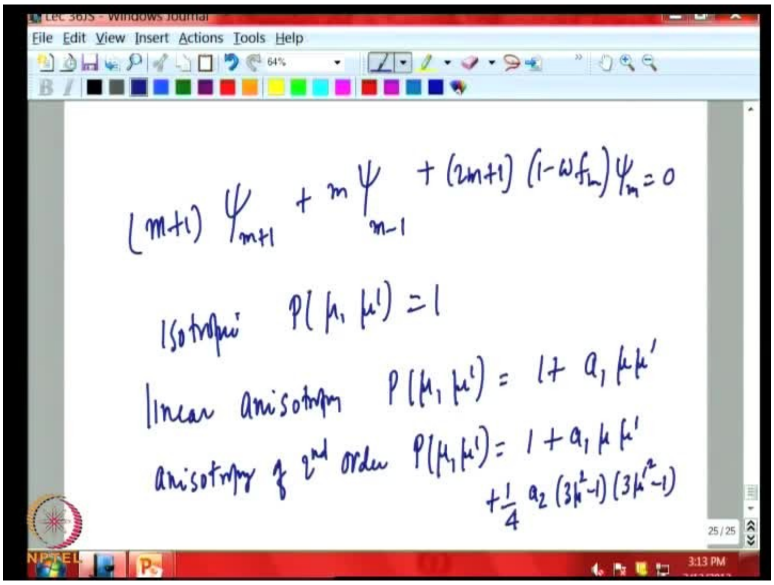

Because subscribe all that we realize that We can have function something like m plus 1 psi m plus 1 plus m psi m minus 1 plus 2 m plus 1 into 1 minus omega f m in the psi m is equal to 0. This kind of equation you will get for the unknown size. Now, this is now a set of simultaneous equations, we can be easily solved and usual standard or technique sounds that is available today and the computer, and so the only difference between this technique and the discrete ordinate technique is that here we are not dividing the dependence of intensity on angle to discrete values. You are allowing for a continuous variation.

This technique in terms angular dependence is bound to be more accurate than what we can do in the cases discrete ordinates. Some of the phase function that people assume of course, we use isotropic, you know that p of mu mu prime is really 1. That is simple form dealt early in this class, but if you go for linear anisotropy, linear and isotropy that is, this is the depending on mu prime, but in very simple way, a simple of function we can assume is of this kind.

It is the call linear anisotropic simple function, which takes into account the departure all the phase function from isotropic. If this is too simple we can go for non linear anisotropic of second order. Non-linear one has typical; the non-linear one had calls dependence on a both mu and mu prime square. This now completes the formulation. So, with assumption one of these phase functions similar form on this.

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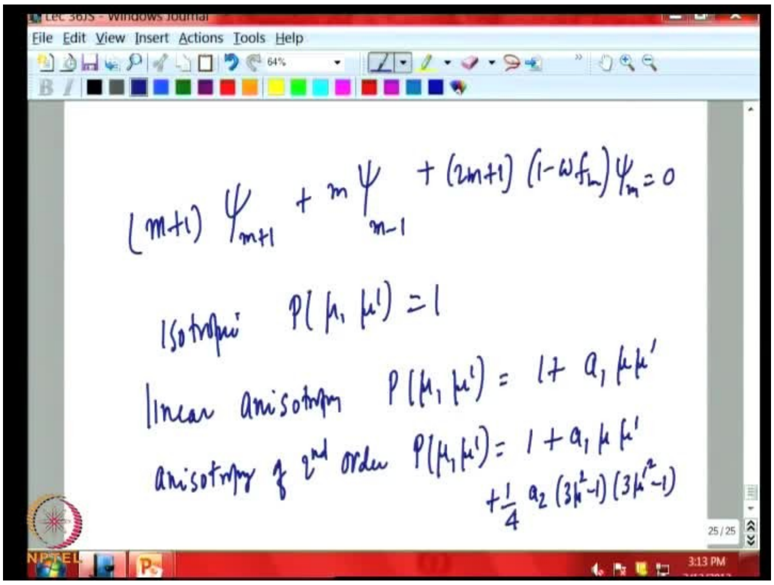

This is that match the terms on this are all same pair of mu we will get the completion solution required. Now, we can see that this spherical harmonic method this ideally should for computer use, calls for inverting the matrix. This can be done for in the mode more accurate be a having more values of the value of m. We see that the spherical harmonic technique would be superior to discrete ordinate method because it does not discretize side in the direction mu mu prime, it is all continuous variation.

Hence, promises with more accurate with reference to the direction. This technique has been heavily used in the estimation of role of scattering especially in a non isotropic medium. Both of discrete ordinate method and spherical harmonic method have been widely used to solve many problems involving scattering in powders, scattering in clouds, and scattering in aerosols. So, wherever you are dealing with complex scattering forms, one of these techniques is routinely uses and today software is readily available, to calculate the scattering properties of cloud of particles insulation which contents many particles or clouds with water ice and aerosols that scatter sun's radiation.

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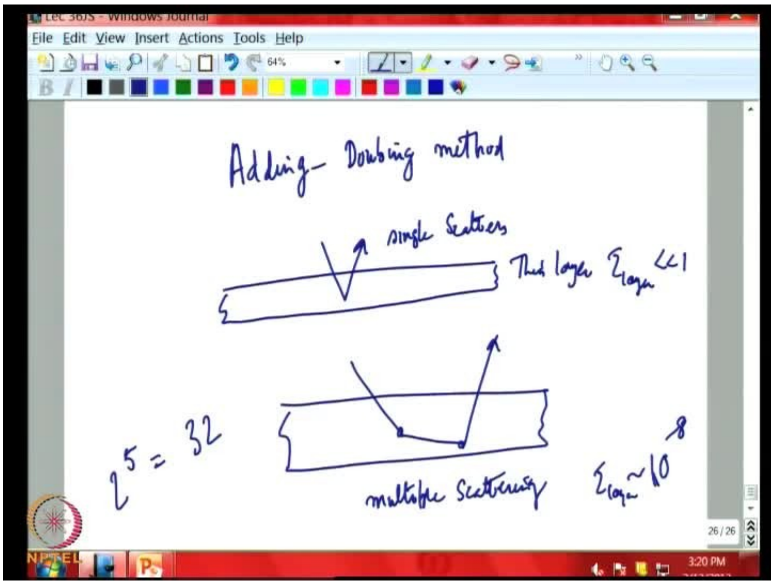

So, what we should understand from all these cases which we have discussed is that with availability of computers, now there are lots of techniques available to solve the problems with various levels of approximation from two stream, four stream to the full spherical harmonic method to resolve the directions very accurately. Now, we will discuss another technique which exploits the simplicity of the equations called the adding-doubling method. This method is valid for a optically thin layer.

Whole optical depth is very small as small as we want. Since, the optical depth is small all we can assume where the only single is scattering. Well if you take a thick layer it involves very complicated multiple scattering. We have derived expressions for reflectivity, transmissivity and absorptivity of layers there scattering. Now, we like to do know whether we can exploit those techniques of solving for thin layers to obtain the solution for very thick layers. Now, this method says that we start with a thin layer and add two thin layers it is the property or doubled layer. We can go and double as many times as we want. So, all of you know that, by time you 5 times there are thirty two times the optical thickness of a single layer.

We start with very thin layer where we are assume that single scattering is valid and doubling the thickness till we reach the desired thickness. This technique is, very elegant solution very difficult solution because solution for a single scattering layer is well established. We recall the expression for reflectivity, transmitivity and absorptivity of a single layer and also two layers. This methodology just continues till your required layer thickness who is optical can with be 30 or 40 or 50 does not matter. We start with a layer with a very low optical thickness.

May be delta Tau of order of may be10 to the power of minus 5 or less to guarantee no multiple scattering. Then go and adding this layers together,. Many people prefer tau to be 10 to the power of this minus 8, not 10 to the power of minus 5 they will be preferred to use as thin as possible. But, they are single; doubling it is sum here other in layer and varies from 32. We can see that this can be useful based can a total layer of thick is all of 1, but doubling it as much as possible.

So, on the computer this technique is very popular because all this demands is that understanding the basics of a thin layer and then go on adding the result of this single layer to get double, 4, 8, 16, times thickness and ultimately you will design your techniques. This is the technique that is also quite popular along with other methods. So, traditionally the method that was popular was discrete ordinates or spherical harmonics techniques.

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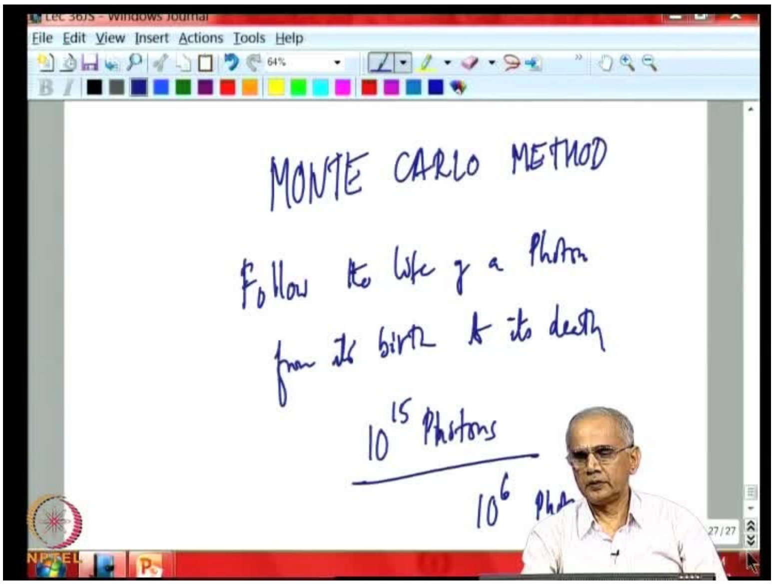

But as computer power increased then the lst approach in this subject has changed dramatically. The present preferred method to solving complex scattering problem is the Monte Carlo method. This named after the famous gambling dens of Monte Carlo where people take a chance to make quick money. In the In Monte Carlo technique we follow the life of every photon. We follow the life of photon from its birth to its death. So, typically the solar radiation incident on earth contains around 10 to the power of 15 photons.

That is a large number, so you would not follow every photon in that encounter because that is even with all power of computer today and the advantages of parallel computing, following every photon that would be very tough. Typically how many photons you need to use is dependent on the problem complexity. Now, one may wonder how a million protons can be used to represent on 10 to the power of 15 photons, which is billion times larger. Now, the hope if the million photons are chosen at random from the larges set of actual 10 to the power of 15 photons. If you chose this truly randomly, then it is a highly probable that all those follow in one in billion photons, the chances are that we will get a result which is not very far from the reality.

This must be thought through carefully. The challenges in Monte Carlo are to prove that we have used sufficient number of photons for our application, so that the final answer has acceptable accuracy. Now, that is on is it the show in expectedly in complex problems where other techniques where the use a pyracarlo method or adding or doubling to show that it given situation, the vertical method will give you result of a bizarre accuracy with over million proton is not easy to show, but lots of new results of come shows that the choices of part of million protons or may be decide them a will give you answers of adequate accuracy, but this has to be actually be shown.

Now, why this method getting popular today? It is getting popular because the computers are getting faster, enables you to follow thousands of photons in parallel because the basic assumptions that each photon does not interfere with the life of another photon. So, all photons have independent life they do not interact each other; if that is the case then of course, you are at liberty it to run this course in parallel.

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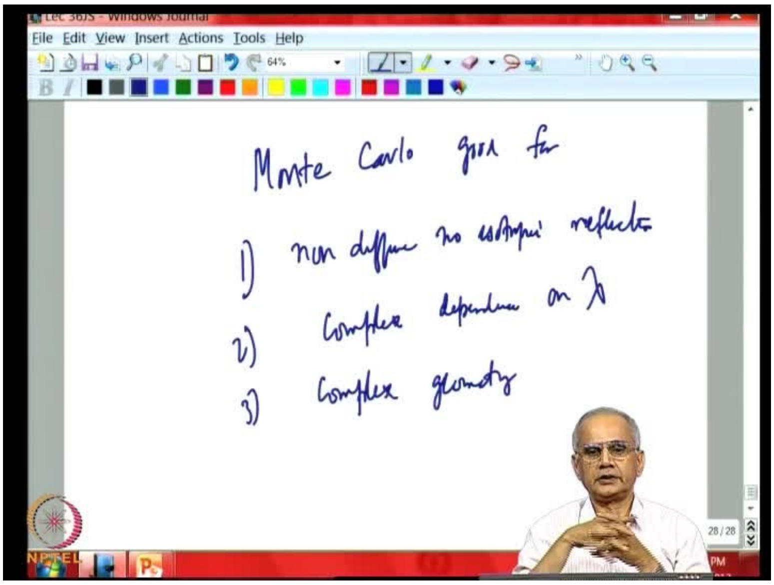

So, parallel computing has given a huge advantage to Monte Carlo and as a computers get faster and people learn to write more efficient course for parallel computing, this technique is going to become the favorite technique for radiation heat transfer. The reason why Monte Carlo will become a popular is because it can handle the problem of any complexity. For example, suppose you deal with the scattering, absorption, emission problem and which there are strong and there is spectral dependence. So, Monte Carlo is good for a non diffuse, non isotropic and reflection functions and functional process.

So, all the tradition techniques that we have discussed so far whether the discrete ordinate method or spherical carbonic method or in the doubling method, all this techniques will become expensive, if the surfaces reflect radiation and non radiation non refuse non as role manner and medium is complex dependence upon on wavelength and the geometry, very complicated tradesmen geometry.

The advantage Monte Carlo is that it is does not really worry about the complex geometry or, complex wavelength dependence or complex directional reflection functions, because once this is specified, this method will take this into account all the complexities and they rate which the computing time increases with complexity, may not be that large. So, Monte Carlo is always being looked at as the technique for solving problems with large complexity, without increasing the computation time by large margins. That is of the spirit of Monte

Carlo, but one can give a generic Monte Carlo, but to discuss a specific thing, little difficult This will be taken up in next lecture.