Radiation Heat Transfer

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Lecture - 35
Non-Isotropic Scattering

In the last lecture we were looking at the use of the two stream equations to derive the reflectivity of layer which is scattering. This could be a layer which is a layer of particles in insulation or it could be a cloud layer. The aim was to derive the overall reflection from this layer; we had used the following equations.

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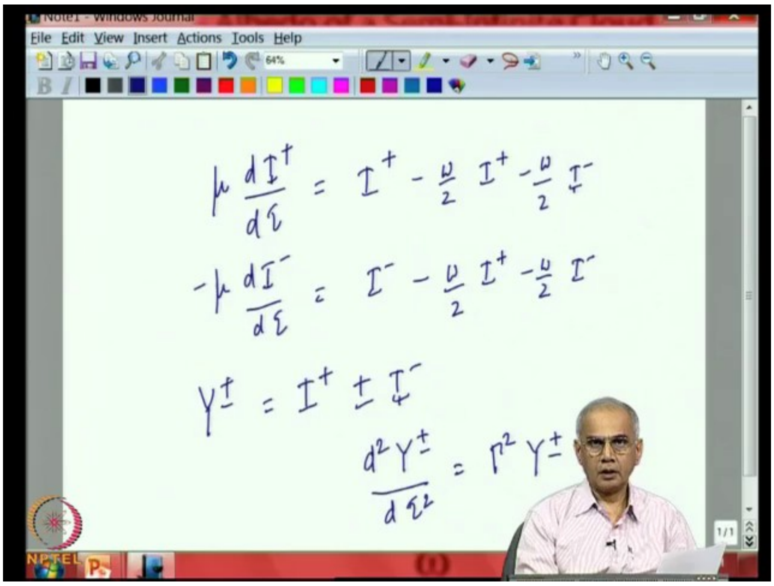

We have looked at the general equation for the upward intensities and a similar expression for downward intensities. So, here we have two equations in two unknowns We can solve it, The solution to the equation can obtained in a very simple way in terms of new variables $Y^{+/-}$. We can define Y plus or minus equal to I plus or minus I minus into, second order equation which says d square y plus or minus by d tau square is equal to tau squared y plus or minus.

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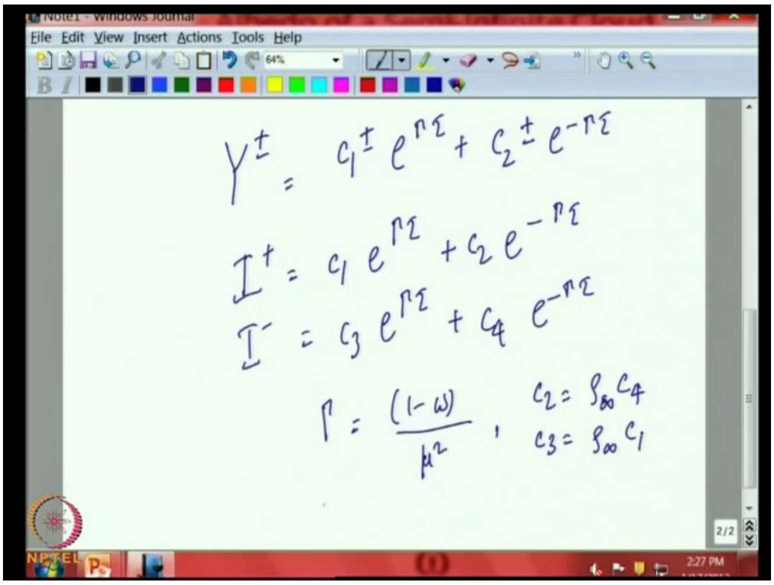

Now, this can be solved very easily. The solution that you get will indicate that Y plus or minus equal to c 1 plus or minus e to the power of gamma tau plus c 2 plus or minus e to the power minus gamma tau. This is a simple solution but , now the real challenge is to find c 1 and c 2 and related to the equal, so we can right in term of I plus or minus with an algorithm of the form is c 1 e to power of gamma tau c 2 e to power of minus gamma tau. We can read in as c 3 tau where gamma then the 1 minus in the scattering out albedo by h square.

We see that the intensity reflected depends on this albedo is 1, there is no damping and as a single scattering albedo goes below one there is more absorption, so that is more damping. It all depends on angle, if the angle is very large there is more path length. There is more damping, so we can relate c 2 to c 4 and c 3 to this one. We will get c 2 is rho infinity*c 4 and c 3 is to infinity*c 1 and where rho infinity as we will see later is related to this overall reflectivity of the medium.

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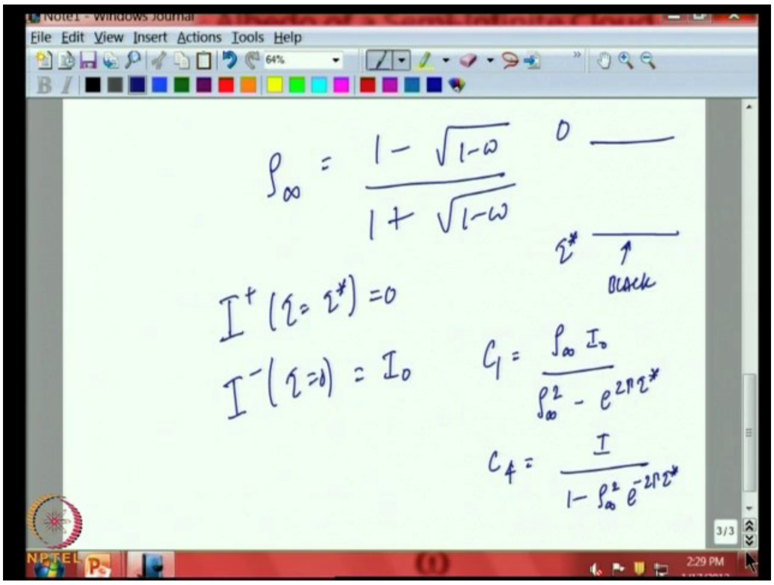

The boundary condition, we need to use in order to solve for c 1 and c 4, now the two unknowns is that the upper flux at tau equals tau star is 0. That is lower boundary this tau is star upper boundary 0, there assuming a black boundary, so there is no reflection. And at the top the incoming radiation is equal to I_o.

Once this is done, we can solve for c 1 and c 4 which are the two unknowns equation, we will get c 1 as rho infinity into I 0 by rho infinity squared minus e to power of 2 gamma tau star. And c 4 as I by 1 minus 0 infinity squared e to the power of minus 2 gamma tau star. This is the complete total solution, but the final interest in this problem is to get and expression for flux out of this layer, which is scattering and the transition.

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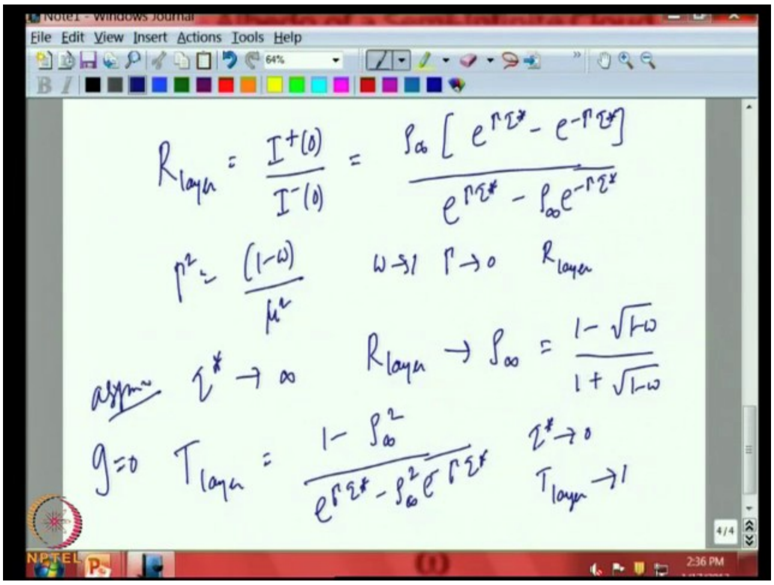

In terms of the ratio of the upper the downward flux that is all it is, so, reflectivity of the layer nothing but I plus of 0 the upper flux divided by radiation that is coming down, that is reflectivity. This we can write as rho infinity e gamma tau star minus gamma plus or minus tau star divided by e gamma tau star minus rho infinity square e minus gamma tau star. So, this is an expression where reflectivity. We can look at various limits of this expression, if you recall gamma was related to the single scattering albedo.

The single scattering albedo is 1, so gamma square if you recall was 1 minus omega by u square, so if omega tends to 1, then gamma tends to 0. So, gamma 0 These things of course the r layer would tend to 0, but one needs be careful here as we have 1 minus rho infinity a body l 0 in the top, so the layer will not reflective.

Now let us look at the other limit when the gamma of the layer is very large, of infinite region then this term will drop out. If tau star tends to infinity, you see r layer tends to rho infinity which is not surprising, but that is why the term 1 mm is we scarred was related to the gamma We define rho infinity, we recall in terms of e single scalp scattering albedo. So rho infinity which is nothing but 1 minus rho to mega by 1 plus 1 omega, and single scattering albedo is 1, hence rho infinity tends to 1.

In the optically thick limit of this layer single scattering albedo is the only parameter that in flow is the result. The optical depth itself is not that that critical. We look at the transparent of the layer will come out as 1 minus rho infinity square by e to power of gamma tau star minus rho infinity square minus gamma tau star. Again we see here in the limit as the transfer very large of course the tau, tau star becomes very large then the transfer is go to 0 that is not surprising, but when the tau star tends to 0 here.

Then we have an interesting issue, which is that the layer transmittance will tend to 1 that is because this term goes to 1, this term goes to 1. Y plus one is going to infinity squared which, so in the thin optical thin limit of course transmittance has to be one that will not be too surprising. Let us look at the other case where tau star tends to 0 on the top we saw that the r layer tends to 0 which is a bit surprising. But, what we can do is we can look at this second order term here 1 plus x normal says we get 2 gamma tau star and at the bottom you will get 1 minus rho infinity.

The 2 gamma tau star term will play some role which is not completely negligible term. Now, one weakness of the derivation that we are not just done is that we have assumed g to be 0, that is asymmetric parameter was assume to be 0. That can be a serious limitation and cases where the scattering is not symmetric, so now, let us look at how we get the result modified, when you include the effect of asymmetric scattering. We extend is result we just not derived to asymmetric scattering and what asymmetric scattering does is only a minor modification this to this equation.

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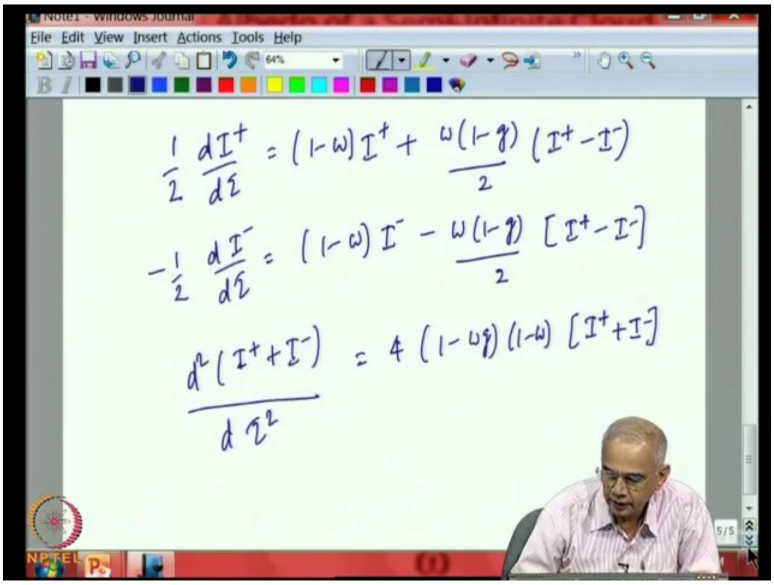

The basic equation, now assuming mu to be half 1 minus omega I plus the big difference comes here. We have g factor which was missing in our previous derivation. It was notified that g was 0 in the original analysis, so it looked different, here g is not 0. The second expression involves the asymmetric parameter also, so that there are two equations in two unknown, but they are coupled because I minus appears I plus equation and I plus I may, so you want eliminate they are, we can combined them like last time.

We get a expression for I plus I minus and it comes in just adding this equations and will get it as 1 minus omega g into 1 minus omega and I plus I minus. We are getting now a single equation for I plus I minus we can get similar equation for the I plus minus I minus, so we won't write it down to the similar.

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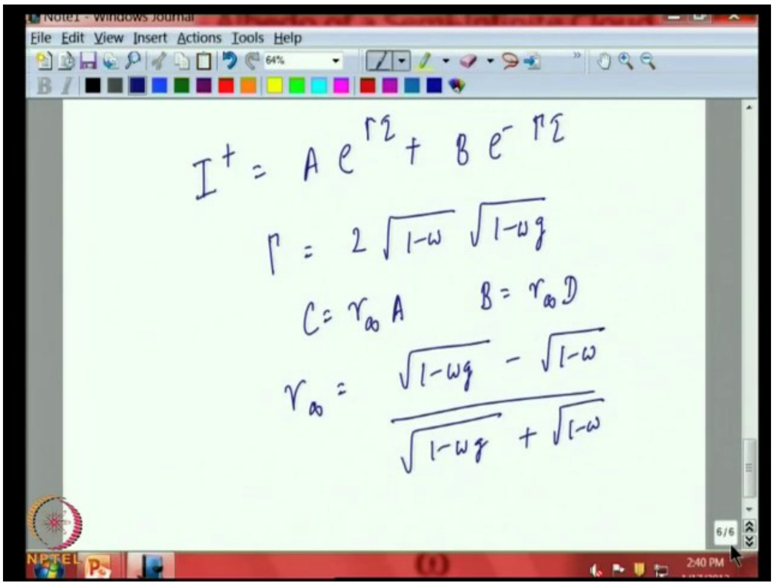

Hence the solution to the equation is I plus will come out to the A to the power of gamma tau B e to the power of minus gamma tau, now gamma is little bit different from the previous we been have 2 square root of omega 1 minus omega g. So the asymmetry parameter is coming in here, which was missing in the last derivation, then as before the relate c to A and b to D those are essentially quite common.

We notice the difference between the previous derivations, which involved symmetric scattering to here which is asymmetric scattering; here g is not equal to 0. We get the expression where if we put g equals to 0, you recover back what we did in the last derivation. Now, the next step as in the previous exercise is to use the one designation and derive expressions, for reflectivity and transmitivity that the same logic as before.

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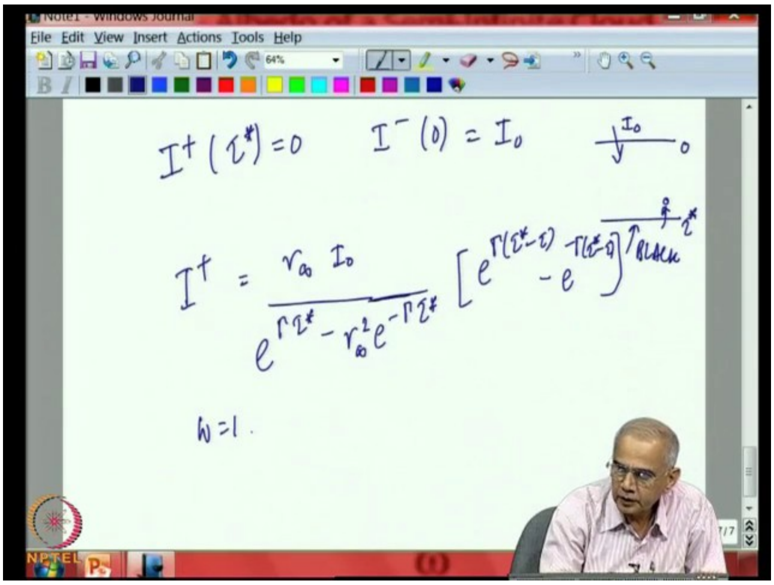

Let us now assume and use the boundary condition which is that I plus at the bottom of the is equal to 0 black surface, and I minus at the top is equal Io. Essentially, what we saying is we have two plates, we have Io coming in at 0 and at tau star there is no reflected radiation, because this is black that is assumption we are making. Once, done that expression for I plus comes out as R infinity Io pi to the power gamma 1 for tau star minus r into infinity square e to the power of minus gamma tau star into e to the power of minus gamma tau star minus tau minus gamma tau star minus gamma.

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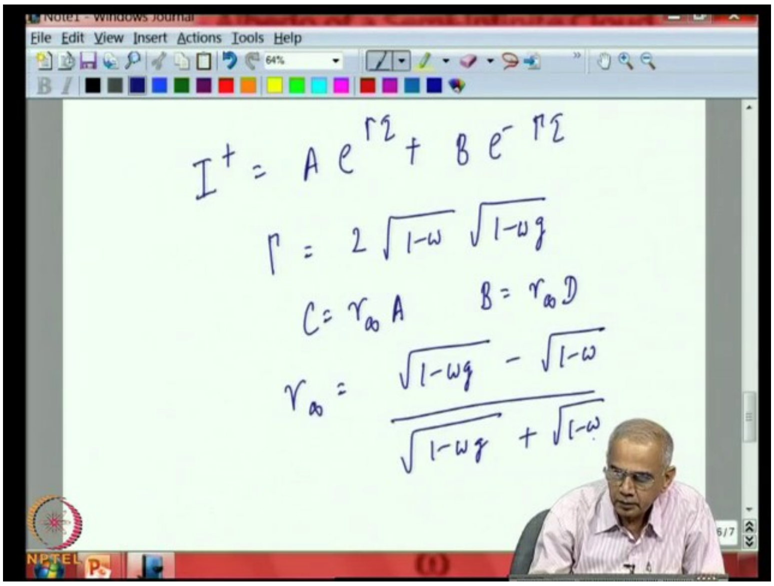

The I plus now notice that when omega equals 1 in the previous expression I got omega equals 1 this term drops out, this term becomes 1, so omega equals 1 R finite equal to 1 irrespective of the value of g.

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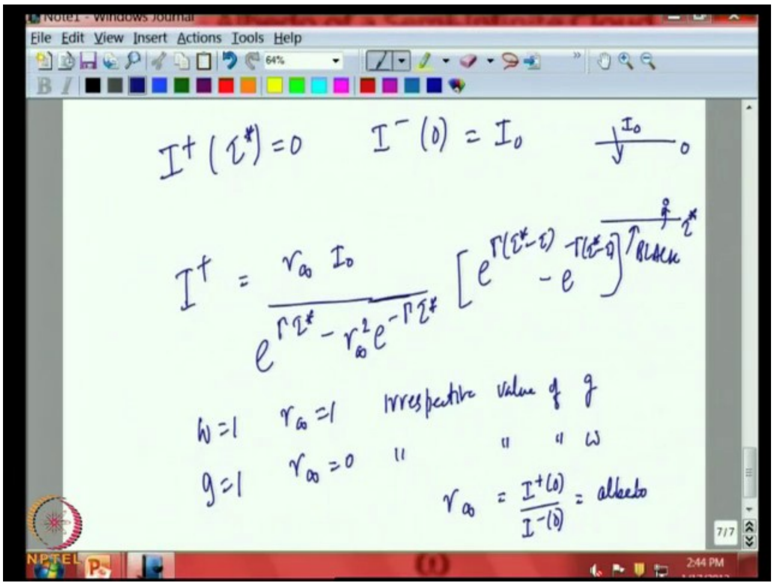

That is a new insight which is that is the single scattering albedo tends to 1; that means, medium is not absorbing then the R infinity value, which is measure of reflectivity of the medium, in the limit of very thick medium does not depend upon the asymmetric scattering. So, this result is surprising, but what this implies is that in the limit of non absorbing medium, it does not matter how the scattering occurs, it is symmetrically or asymmetrically, ultimately all the photons have to emerge at the top.

Now, similarly when you take symmetric parameter equal to 1 means primarily forward scattering, now R infinity is 0, irrespective of the value of omega. Again this is interesting It does not matter what the single scattering albedo is as long as most of the scattering is in the forward direction, then there is not much chance of back scattering or reflection back, so that R infinity tends to 0.

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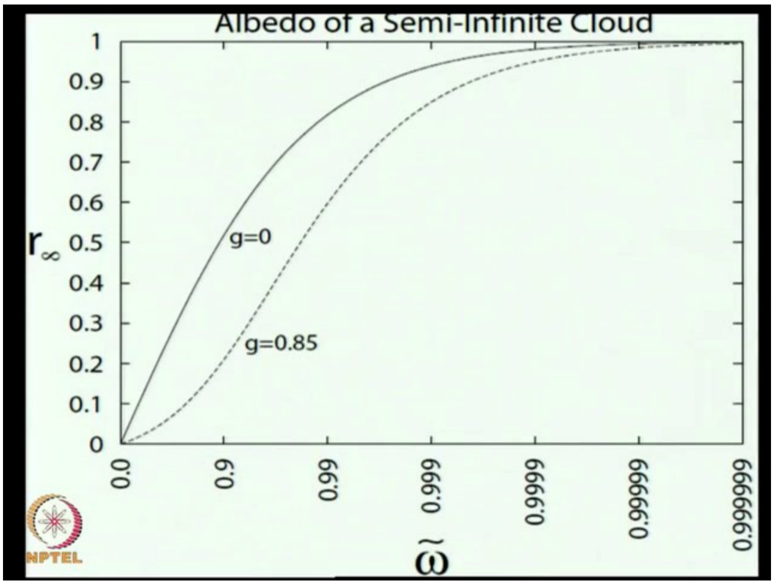

Now some of the results we obtained here we can also look at in terms of figures, now here is the albedo, reflectivity of a semi infinite cloud, when tau star is very large. As a function of the single scattering albedo on the x axis on a log scale, is the log scale here and R infinity is this parameter.

Now, what we notice clearly is that would be just right expression is what, so r infinity here is very important because this is nothing but I plus of 0 by I minus 0, that is the albedo of the scattering layer that is why we are looking at that expression.

This quantity the albedo of the cloud of particles as a function of the single scattering albedo for two different asymmetry parameters, one is 0, which is nothing but symmetric scattering, the other is g equals to 0.85, which is primarily forward scattering. It states that when the single scattering albedo is below 0.9 of actually, 0.99 the albedo of the cloud is very sensitive to the value of g that is the asymmetry parameter.

When we go from symmetric to asymmetric scattering the albedo can go down 0.5 to 0.1 a huge difference, but as you go to very high single scattering albedo like 0.99 or 0.49, We can see that it does not matter where that the scattering is symmetric or asymmetric ultimately your reaching the value of 1. It is quite clear whether the scattering or symmetric, asymmetric is usually crucial for those clouds of particles, where the single scattering albedo is less than 0.9, then we can see there is some preference.

If we have medium with a very single scattering albedo, then we do not have to worry about the scattering symmetric or asymmetric. But, in more real situations where there is some absorption in the cloud, then we are going to be somewhere in this regime. We can see here that there is a large difference between the albedo which is 0.05, for a symmetric scattering to around 0.35. There is a decrease in the albedo the cloud by factor of 7 as we go from g of the 0 to 0.85, so this must keep up in mind at single scattering albedo is very important parameter in this situation.

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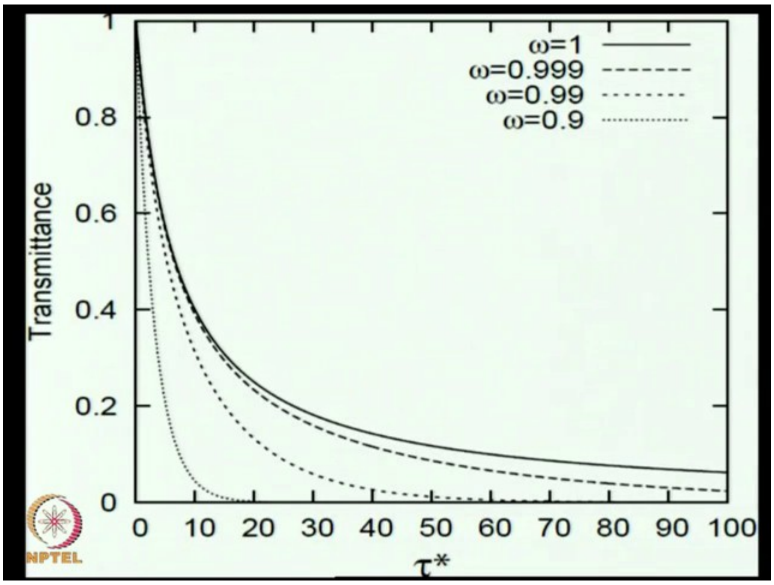

Now, let us look at transmittance, the transmittance you do expect it to go down as the optical depth of the cloud increases, so this goes down as optical depth increases, but the way it goes down is sensitive to single scattering albedo. When, the single scattering albedo is equal to 1, it goes down in a certain way, but if it is 0.9 it falls much more sharply. This is not surprising because at 0.9, the lot of absorption is going on, so naturally the transmittance will drop to 0, but you get out of 10. While, in the case of the single scattering albedo been 0.99 or 1 the absorption is not playing much of a role

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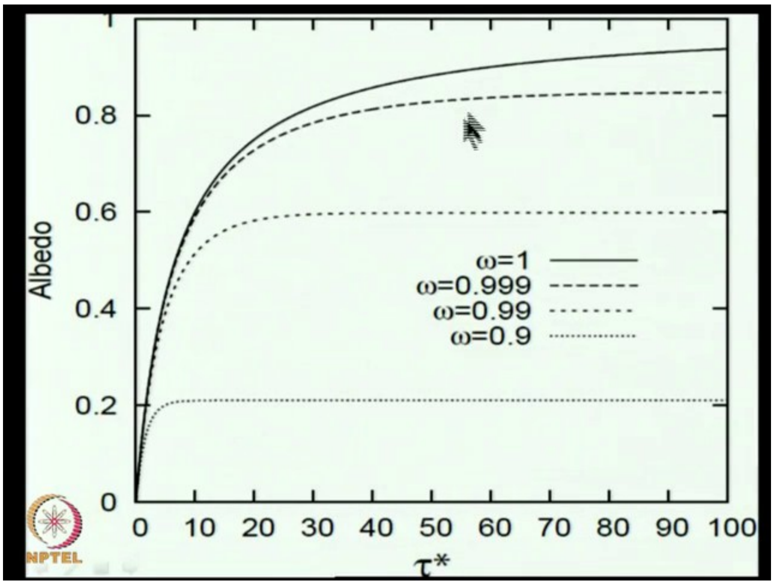

Now, next we look at albedo of this cloud we see that the albedo crowd is dependent on the crowd optical depth not surprising, the single scattering albedo. We see that when singles of course is one, then it requires only optical depth to be of the order of 40 to 50, that you have also almost reached the symmetric value. While, for more absorbing cloud layers with let us say omega equal to 0.9, we see that the you see that the albedo reaches very rapidly a value of around 0.2 at optical depth of around 5.

In absorbing cloud the assumption of semi infinite cloud in valid in for optical depth around 5 because most of the photons are very quickly absorbed and does not require much thickness of the cloud for it behave like a semi infinite cloud. While, when very no absorption in this case you require a thickness cloud of the order 50 or 60, to get the asymptotic value of semi infinite cloud.

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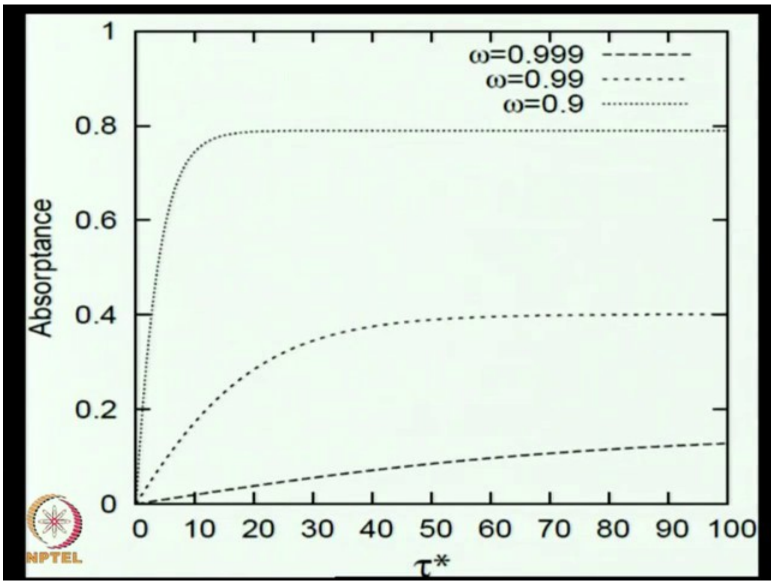

Next we look at the absorptance of the cloud. How much radiation absorbed of course omega is 1 with the absorptance we do not have to worry about it, but even omega is 0.9 here. We can see that the ability of the cloud to absorb radiation reaches a maximum when optical depth reaches 10 of 15, but after that there is no change. So, this again is an indication that clouds which have absorption do not have to be extremely thick before they behave like semi-infinite clouds.

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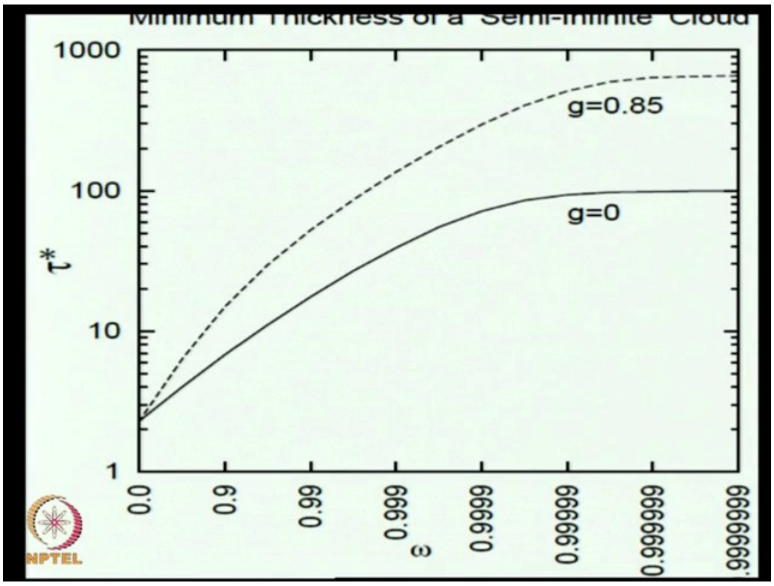

Now, this graph shows the minimum thickness required for the cloud to behave as infinite as a function single scattering albedo the asymmetry parameter. When g= 0 that is the scattering is symmetric, We can see that by the time you have single scattering albedo of 0.999 you have reached thickness of the layer of 100 for clouds to be same infinite. While, if it is only 0.9 which we saw earlier it requires only the thickness of the 4 of 5. We can see that the asymmetric reflection causes the need for higher optical depth for you to declare as semi-infinite.

The asymmetry parameter play a role in determining how thick a cloud should be before it can declare it as semi infinite, now before you go further let us also look at the non absorbing cloud and some depth. The non observing been cloud that we are dealing with here, where omega is equal to equal to 1, that is you have consider it to be have than the reflectivity and transmitivity can be written down in a fairly simple manner.

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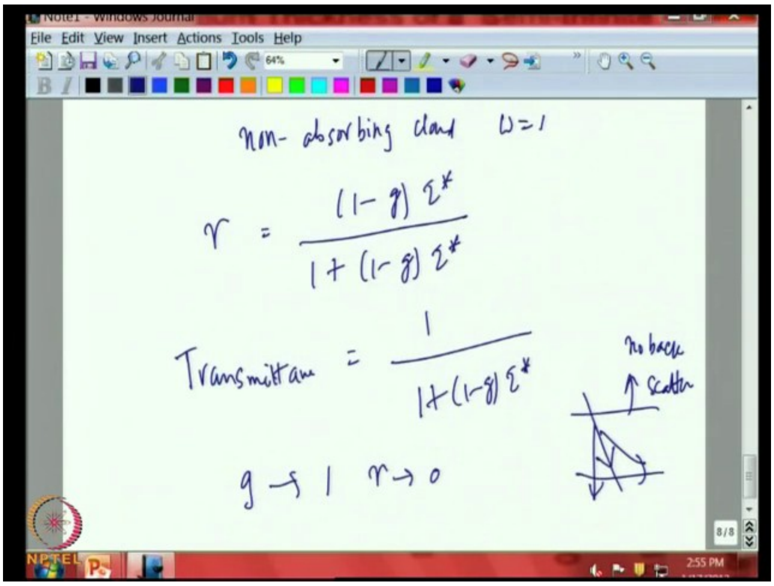

We have a non absorbing cloud, for which omega is equal to 1. Then we can define the reflectivity of this cloud as being equal to 1 minus. Again we can see that unless cloud is extremely thick reflectivity, will not go to 1. The transmittance will be 1 by 1 plus Once more we see the fairly strong impact of non absorbing cloud of the asymmetric parameter. It has huge factor to play, if g is 0 we get a simple result and as g tends to 1 you see that the phenomena is somewhat more complicated.

For example, for R as g tends to 1 and g tends to one to R tends to 0, we recall that as g tends to 1, we only encourage forward scattering. If there is a lot of forward scattering there will be no back scattering, we cannot expect any rays to comeback as most of the scattering is occurring the forward direction. Now, we can seen how we can solve for reflectivity transmissivity, and absorptivity of a layer based on basic properties of the particles in the cloud, and those basic properties are the asymmetry parameter, optical depth and the single scattering albedo.

Now, let us presume that this information is available we want understand the total transmissivity of the layer, so this is done very simply in terms of ray racing, but we will also show that all the ray tracing is basically very meaningful method.

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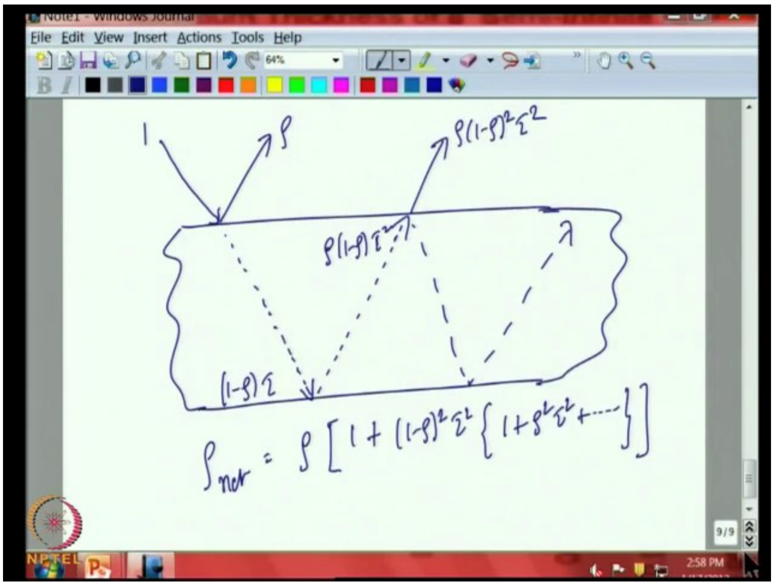

We are let us see earlier of glass those properties are known, radiation comes here at intensity and on the first face it reflect rho back it space. Once it is does that you should recognize that whatever is not reflected will go on transmitted and by the time it comes to the second interface. So, what did not reflect and absorbed is then transmitted then it comes here, once it comes here it gets reflected and so on. Once it comes here it is reflected, now this term has to be rho into 1 minus rho into tau square.

Now, going through this twice and finally, when it comes out here escape reflection here because to the rho into 1 minus. Now, this can be repeated when it has, so we have an infinite series of reflection terms. The expression for net reflectivity of all this comes over that rho first term, the common term here is 1 minus rho square tau square. This is quite understandable because anything has goes backup has to go through this layer twice so that twice square and has to escape reflection of the two interfaces 1 minus rho square, but then this infinite series which can be written as follows.

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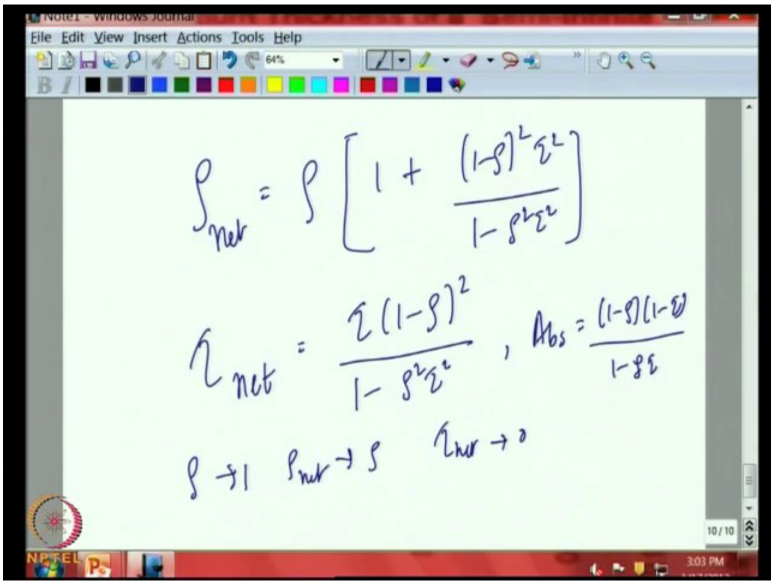

By applying to summation series rule finally, you get the rho infinity as rho, so this expression for the overall reflectivity of the layer I we will call it net reflectivity of the layer, in terms of reflected add each in reparse transmittance. Similarly, one can derive and net transmittance which will come out as tau, now let us examine that result which I obtained by ray racing. Ray racing is physically very satisfying and simple approach, but it is important recall at this stage.

That in ray racing especially, when there are a large number of rays it is easy to neglect or ignore some term, so it cannot be taken for granted that everyone will get this expression. There can be terms that are missed out, so it is not going to be easy to be able to check this result, for example, let us see what happens to the layer if reflectivity tends to 1. These terms drops out and rho net tends to one but on the other hand if the optical depth of the medium because very, very large, because a more complicated expression here this term become very large drops out tau square cancel out the total reflectivity will be a function only of the interface reflectivity.

If you look at the transmission as reflectivity tends to 1 that it is not surprising that transmission is 0, because this particle tend get reflected backwards mostly and ultimately the amount energy transmitted through will go to 0. So, this result is useful to look at the expression for absorption. We can see either rho is 1 or tau is 1. In either case the absorption is zero and it is not surprising if a particle in the medium is reflecting then there is no impact on absorption the transmission tends to 1 then there is of course there is no absorption , if the all photon are transmitted.

Over all the simple model does give as insight about the nature of this process, but it is important to recognize that although ray racing is very powerful tool. It is possible to make some error now and then because one did not account from some streams , so although ray racing is very popular and is very often used , but while actually doing the problem. It is desire to adopt ray is seeing one should be really clear that there is no one or no object which is interfering with the path or which you want to the measure. That question has to be kept in mind before one can adopt this kind of approach, now although we got enough insight because we adopted ray tracing. Its importance is in the fact that there could be alternative way in the deriving the layer mean functions in terms of individual interfaces.

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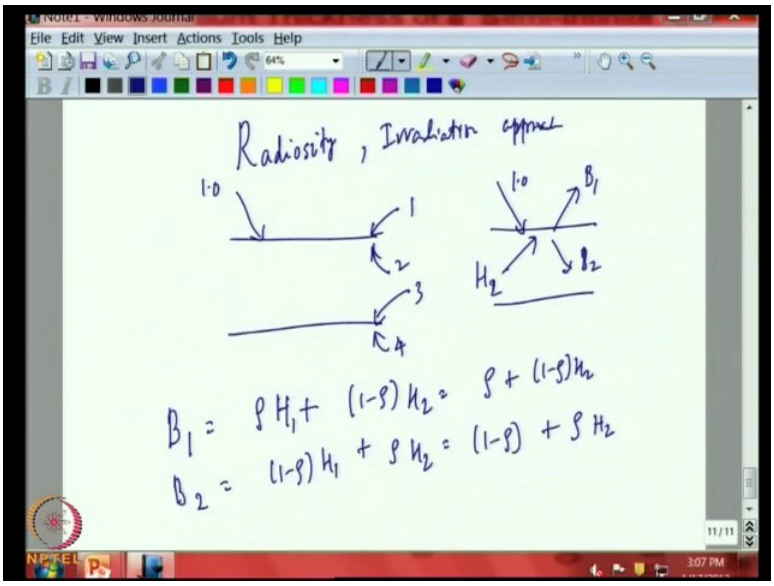

The best way to do it, what we have done a long time ago, is using the radiosity method, so in the Radiosity- irradiation method which we have already adopted when we were dealing with shape factors. We do not want to follow individual rays we only want know the impact of the total number rays in the system, and so we do not do individual counting in which in which case. We can make some mistakes.

We really use this layer as the black box use intensity coming here is 1 This is surface 1, this surface 2, surface is 3, surface 4, The schematic we are going to apply is you going to assume that incoming radiation 1 and outgoing radiation is B 1 and come radiation with 2 is q i 2 and what going on it is B2. Now, I want use q i 2 a notation I want to follow what done we use earlier a value H i or H 1, this is H and other S 1. So, the advantage now that there is radiosity should be 1 here has be equal to what the reflected already times H 1, it is 1 in or case by notation plus.

So, this the second term is equation is 1 minus is rho, so you right is us rho H 1 and H minus is 1 1 minus rho into H 2, this one this become rho plus. Similarly, for B 2 the second term is rho into H 2 and so we write expressions for also B 3 and B 4, so this H 1 is 1 so this comes 1 minus rho plus rho B H 2. We are able right to B 1 in term of H 2 and B 2 also in terms H 2, but we need two more expressions The y are B 3 which rho time H 3 plus 1 minus rho time H 4 incoming radiation.

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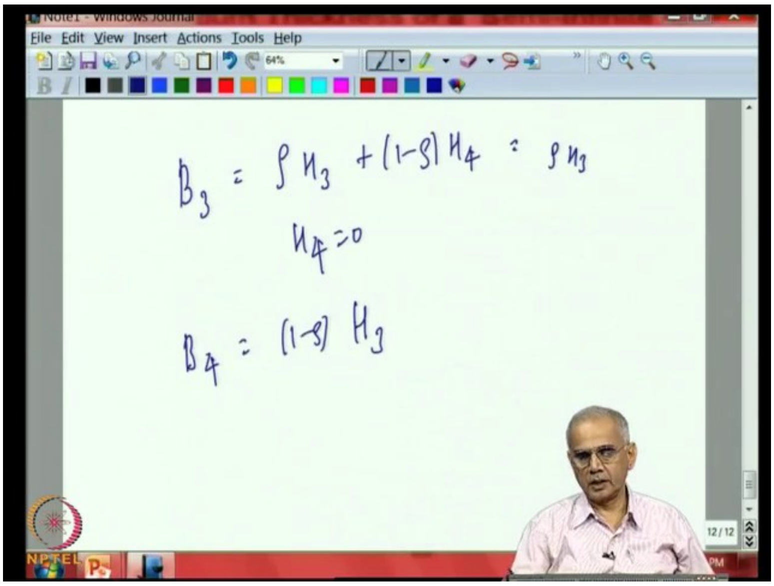

Since there is no radiation below H 4 is 0, similarly we get B 4 this is nothing but 1 minus rho times B 3 extremely, so now we have 4 expressions for the 4 radiation. Now we can calculate the various terms involving net radiation.

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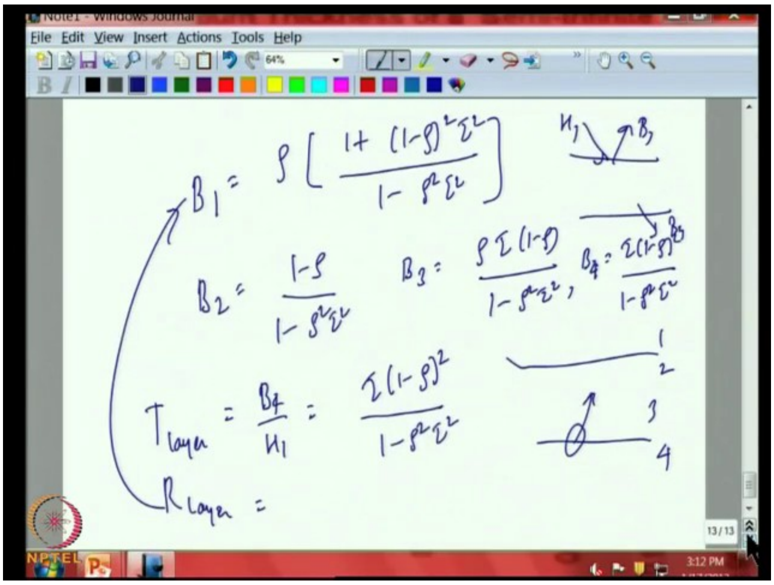

First we solve the radiosity. They will look at how to calculate the other terms, so it is solve for all the 4 with 4 equations, we get B 2 is solved to be B 3 and B 4. We have solved for B 1 B 2 B 3 B 4. The transmittance of the layer is nothing but B 1, because B 1 is the radiation leaving the surface 1 actual really, but we have taken H 1 as 1.

So, transmittance comes out as nothing but tau into 1 minus rho square of the transmittance becomes of the transmittance nothing but B 4 H 1, so that will come out as this one. So, this result is not surprising, if reflectivity of layer is 1 of course, there is no transmittance or the transmittance of the layer is 0, then also otherwise things will be simple.

Then what is the absorption of the layer or reflection of layer will come out as equal to B 1, that is the expression. There also can see that if the transmittance of the layer becomes very, very small this term drops out, this one drops out. We get only the first interface reflection that is not surprising. Second interface, so this interface plays no role, if the tau is very, very small that is not much radiation comes to this point to reflect here, so that is the message from that figure.

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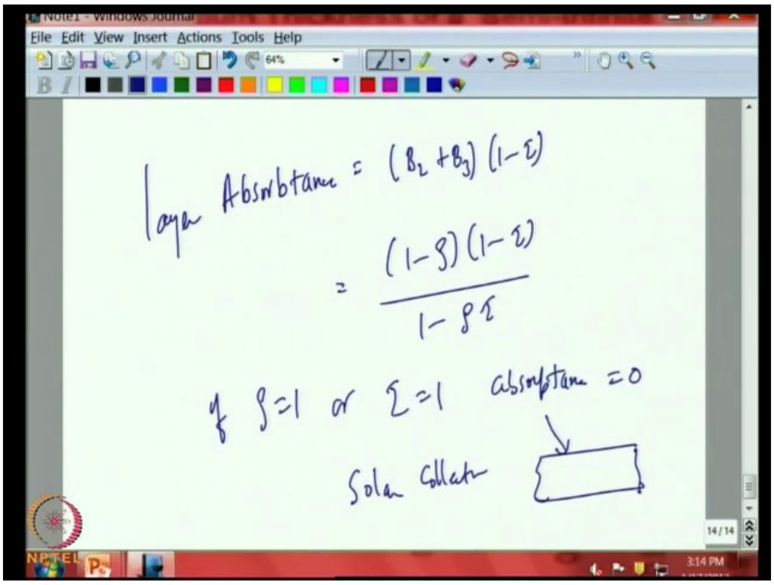

Now finally, we look at absorption, and that will be nothing but whatever radiation is there inside 2 and 3 and or this much is absorbed this is 1 minus rho into 1 minus tau divided by 1 minus rho tau. Again we see that the layer absorptance is 0 reflectivity is 1 or so if rho is 1 or tau is equal to 1, absorptance of the layer is 0, so perfect reflection at the top interface or almost perfect transmittance implies there is no absorption in the medium.

We show that although the same results are obtain from ray racing, and physically they do provide more insight, but the more elegant way of deriving the layer property is from the application of the radiosity-irradiation methodology, we developed earlier in the lecture, because that is very easy to apply, and it involves no book-keeping for photons where you keep track of the photons little harder. So, generally one would recommend for new complex system, we adopt the i radiosity approach rather than actual ray tracing.

Of course, for verification of a result you might verify the answer is correct. Now, these results that we have derived in this lecture are very relevant for solar collectors, solar collectors are typically have two interfaces here. The net absorptance transmittance The reflect layers is pertinent, and if the example the reflectivity of the solar collector is very large, one can add a thin coating to reduce the reflectance in both layers.

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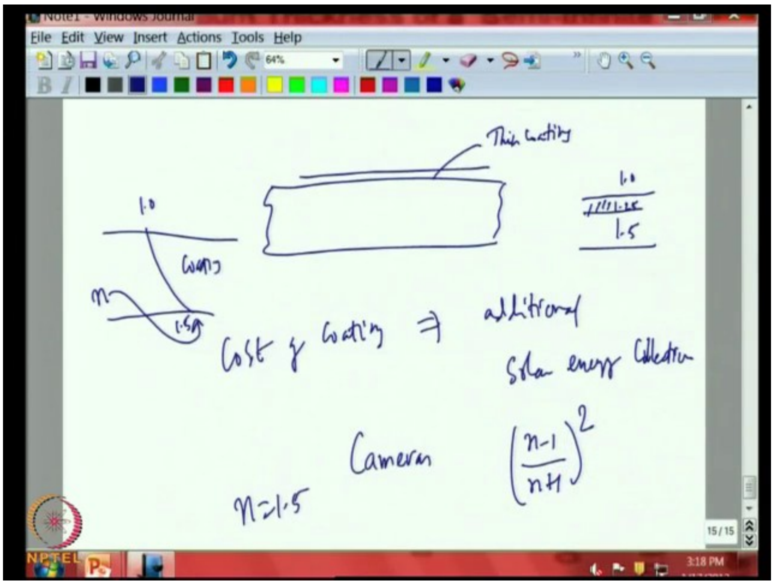

These are now routinely done in many devices, a thin coating does not fundamentally alter the transmission of the medium, but at same time reduce the reflectance, Hence more radiation gets in through the glass plate into the absorber. So, this calculation will help us to calculate to how much additional energy you will get in a solar collector, because doing thin coating and economic analyses to determine whether the cost of coating can be balanced by additional solar energy collection.

We can demonstrate that by using this coating here increasing the solar energy absorbed the collector by 5 or 10 percent, then one can easily estimate what is payback period for that coating that you are using. Now, this a coating idea they are also used in the camera of course to increasing the amount of photons that get into the your camera, into the film or your sensor.

In the case of camera this thin coating is very important, because if you want to do the photography in low light conditions, you will need every photon that is coming from the scene in to your camera to be used or not will lose by reflection or absorption. Most cameras use coating to reduce the reflection the reduction in coating depends on this fact which we had seen earlier. If the refractive index of the glass 1.5, this can be quite large and so if it is replace 1 to 1.5 to one transition, to let say 1.5 to 1.25, 1.25 to 1 thin layer with the depends in between glass and air then we do you reduce the reflectivity of that layer.

This logic can be extended There are coating switch will continuously decrease the n value from 1.5 to 1 in this coating, so this is coating when they can reduce the refractive index, gradually by producing series of deposits of refractive index. So, this is routinely used in many industries like camera and collectors and so on, where loss of photons by reflecting is avoided.

Now, in the next lecture we will proceed to look at Monte Carlo methods, which are the methods we will use if you want to deal with complex system. What we have discussed so far have been very simple systems, wherein simple methods were used now we are going to look at complex systems for which we have go to Monte Carlo methods..