Radiation Heat Transfer
Prof. J. Srinivasan
Centre for Atmospheric and Oceanic Sciences
Indian Institute of Science, Bangalore

Lecture - 33
Particulate Scattering

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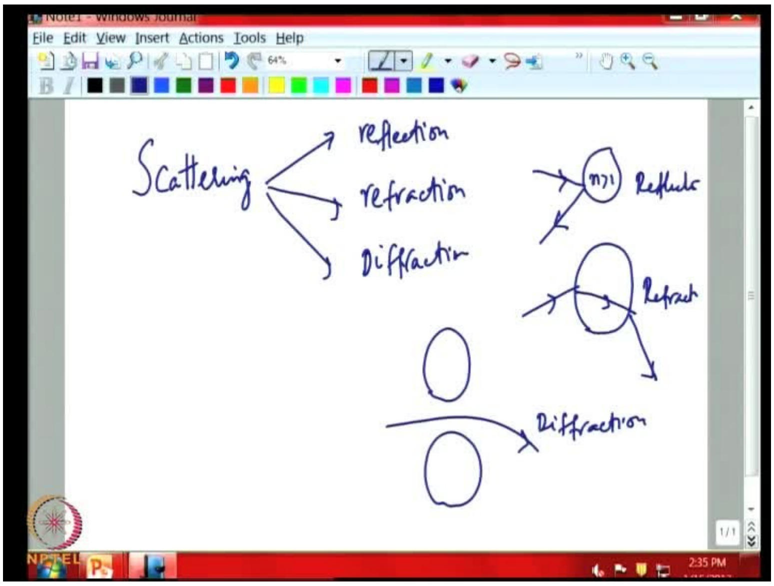

In this lecture, we continue our discussion on scattering. Scattering his a broad term, which includes three process; one is reflection, the second is refraction, third is diffraction. So, all of them essentially bend the ray of light in reflection, as all of us know a ray striking, let us say liquid drop will definitely if n is greater than 1, it will reflect the radiation. We also know that this same droplet, if it is transparent can also refract. This is reflection, this is refraction. In addition to reflection and refraction, there is also diffraction. In the sense that there are the droplets here, a ray of light might just bend, on account of the influence of the droplet. In the first two cases that is reflection and refraction, the ray actually touches the particle and either reflects of it or ray goes through the particle and gets refracted.

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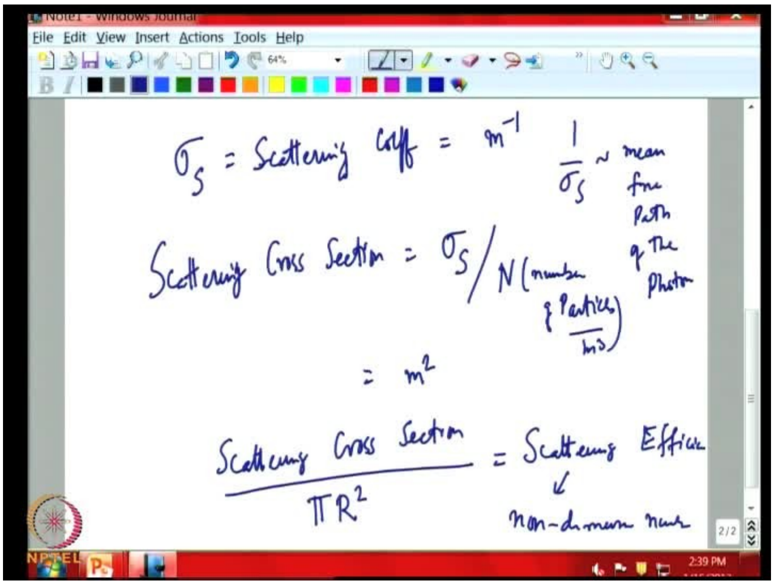

But in the case of diffraction the ray may not come near the particle, it can just go past it. Some area near particle the influence of the particle electrical field will cause the ray to bend. The reason why you have to understand, that all the three play a role will become clear later when we discuss little bit in detail, how refraction plays a role. Now, so far we had defined the scattering problem in terms of the scattering coefficient, which was in units of meter minus 1. This was our standard way of defining scattering we normally prefer this because 1 over sigma is a measure of mean free path of the photon.

That is the mean distance travelled by the photon before it is scattered. There are other definitions to measure the scattering the one that is commonly used is the scattering cross section, which is nothing but sigma S is multiplied by divided by number of particles per meter cube. These are units of meter square. A lot of people like the term scattering cross section because it gives an idea of the approximate surface area, which is influenced by scattering.

If we take the scattering cross section and divide by the cross-sectional area of the particle, you get the scattering efficiency. This is also preferred by many people because this is a non dimensional number. We prefer the scattering coefficient because one over scattering coefficient is the measure of the mean free part of the photon. Others like scattering cross section, which gives an idea what area is influenced by scattering.

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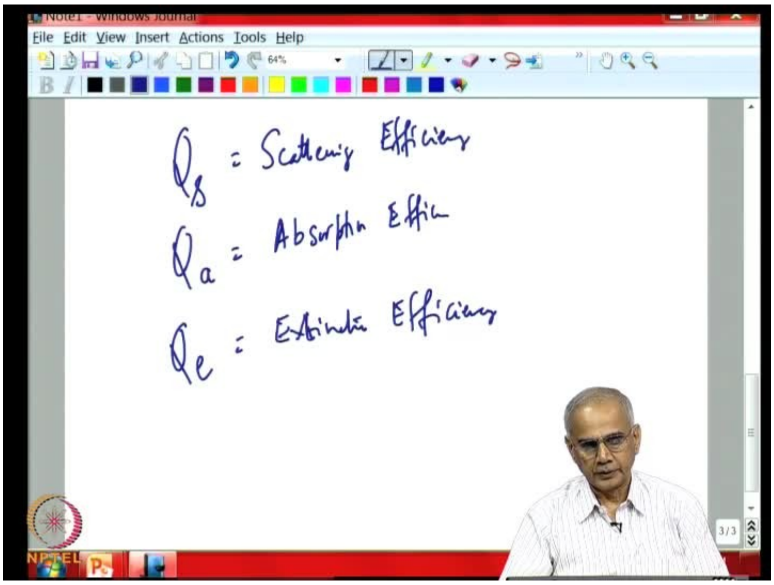

It is nothing but scattering coefficient divided by the number of particles per meter cube. Finally, we have scattering cross section divided by pi R square, which gives scattering efficiency; this is a non dimensional number. This gives an idea in relation to the area of the particle, what is the scattering effect. That is generally called Q e; similarly we can have absorption efficiency.

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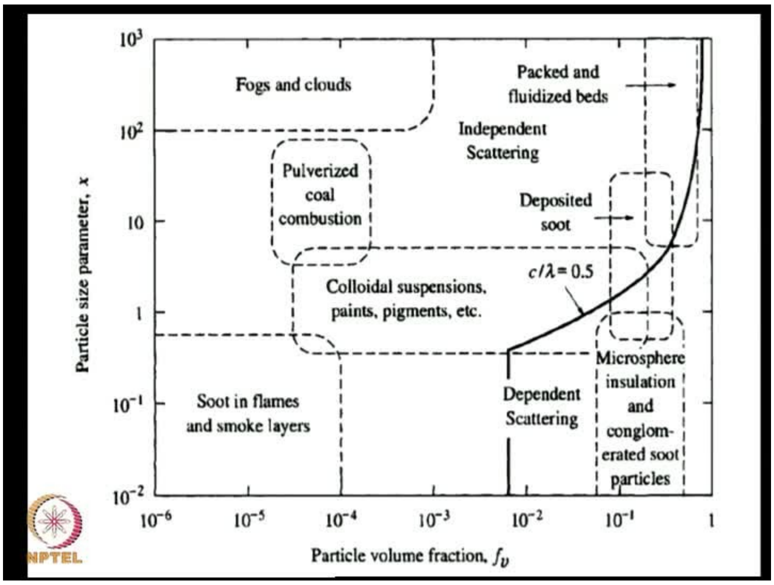

We can also talk about extinction efficiency. So, all these three are non dimensional numbers, which give an idea of the efficiency of the scattering, absorption or extinction, in relation to the cross circle area of the object. Now, let us see how these things look like for a typical particle. Before we do that, let us get an idea of the, how scattering influenced by presence of other particles. So, far we have discussed primarily scattering by single particle The question is whether this assumption is valid, when there are large number of particles, because then scattering from other particles will influence scattering by the particle under consideration.

That can depend on particle volume refraction. In a given volume, let us say 1 meter cube and ask yourself, what fraction of the volume is occupied by particles? Then if the volume is very small like 1 millionth or 1000^{th}, we can be quite sure that, we are in the independent scattering domain that is, which one here that is the domain where each particle acts as though other particles do not exist.

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absorption efficiency factor: $$Q_{abs} = \frac{C_{abs}}{\pi a^2},$$

scattering efficiency factor: $$Q_{sca} = \frac{C_{sca}}{\pi a^2},$$

extinction efficiency factor: $$Q_{ext} = \frac{C_{ext}}{\pi a^2},$$

$$Q_{ext} = Q_{abs} + Q_{sca}.$$

$$g = \overline{\cos\Theta} = \frac{1}{4\pi}\int_{4\pi} \Phi(\Theta)\cos\Theta\, d\Omega.$$

We can see that typically in the case of fog the clouds, pulverized coal combustion or soot in flames or colloidal suspensions and liquids, this is generally satisfied   as long as the particle volume fraction is less than 1 by 100. When the particle volume fraction approaches 10 percent or more, we can see that there are interactions between particles. This is mainly encountered in understanding radiation through insulation, where the solid particles are very close together. We expect that adjacent particles will relevant to rays on a given particle, but we will discuss mostly independent scattering, because that is the problem which is easy to tackle to begin with. Let us remember about the three efficiency factors; the absorption efficiency, which is the absorption cross section by pi R square by pi a square scattering efficiency scattering cross section by pi a square and extinction efficiency which is sum of these two is the third term.

There is also another important factor called asymmetry parameter because when there is scattering, we would like to know how asymmetric the scattering is with respect to forward or backward scattering. This is very important because that determines where the photons will travel. The mean value were you have cos theta is defined as 1 over 4 pi included all 4 pi solid angle the probability of scattering in a given angle, then call the omega.

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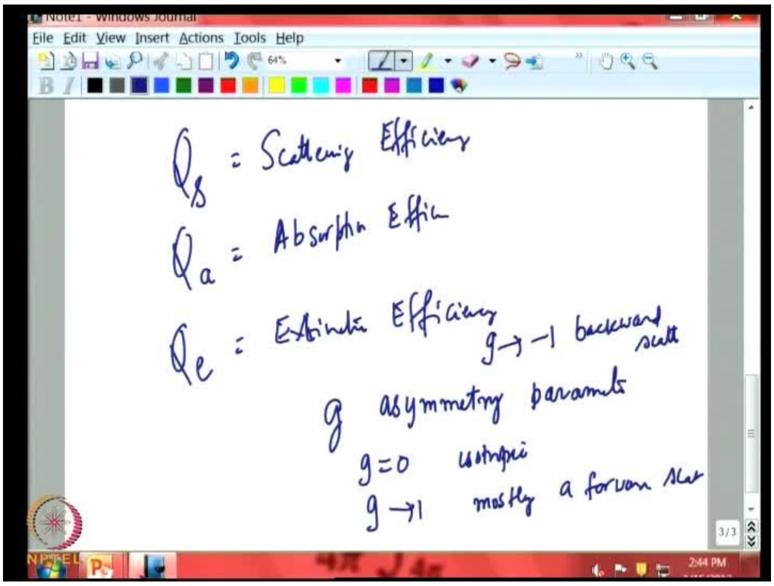

Now, this thing can be understood as follows: g is the asymmetry parameter if g equals to 0 of course, the scattering symmetric. Essentially all isotropic scattering will show g equals to 0. If g tends towards 1, it implies mostly a forward scattering situation, while g tend into minus 1 will indicate mostly backward scattering. So, knowing the value of g you get some idea about, how asymmetric the scattering process is.

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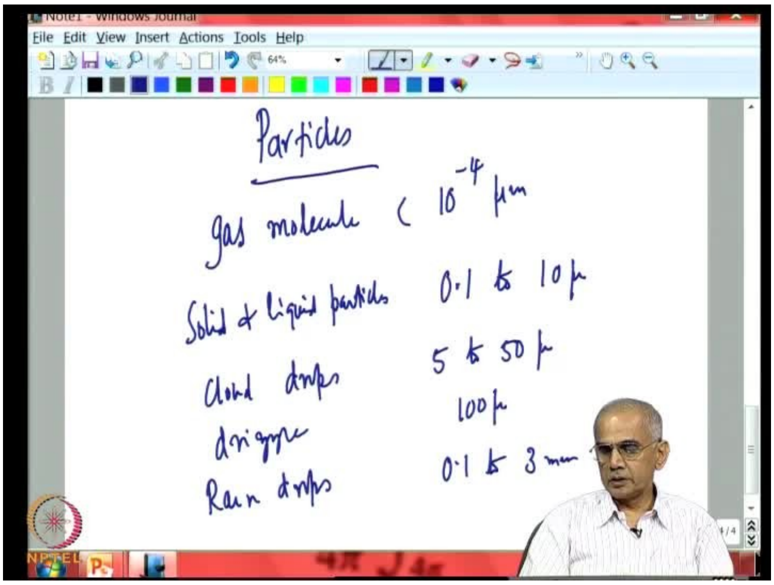

The negative value of g more backward scattering, a positive value of g indicates more process scattering. Now, let us look at typical sizes of particles that we encounter in various situations. The kind of particles encountered is of course, gas molecules are always there, which scatters radiation, but their size is of less than 10 to the power of minus 4 micro meters. These are very small particles when we encounter them.

Then typical solid and liquid particles we encounter droplets in combustion or cloud liquid particles, they are all in the same size range point 1 to 10 micron. Now, this as we can imagine is much larger than, what you see in the case of gas molecule then we have cloud drops, which are typically in the size range of 5 to 50 micron. Then you have drizzle, which are larger droplets. They go up to 100 micron and when you have rain drops which are very large particles, which actually fall to the ground.

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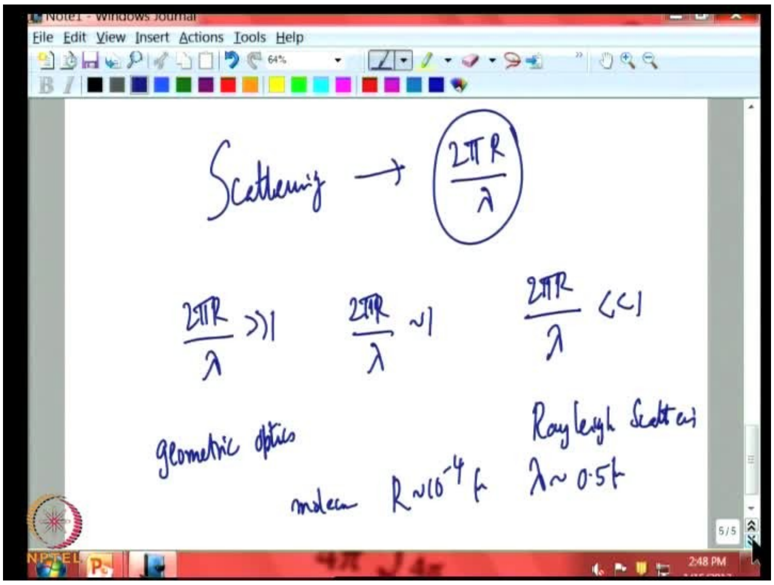

We are talking about 0.1 to 3 millimeter. We can see that the range of particles that we encounter in radiation spans a range of almost 100,000 the smallest, may be a million actually. Take very small gas molecules to very large rain drops, the size range is really a million. Now, this is worth remembering because finally, scattering is governed by a size parameter, which is normally defined as 2 pi R by lambda. The radius of the particle is large compared to the wavelength that is one limit.

This is really one limit or this radius the particle is small compared to the other limit. The third limit is when is in the order of one. When the fund is very large you are governed by geometric optics, some of which you are already studied it in your high school. The problem becomes very simple; it is almost a flat surface. We know the larger optics, geometric optics. If this is very small, we have the limit known as the Rayleigh scattering and as we can imagine if you are dealing with molecules of the size 10 to the power of minus 4 micron, and let us say you are dealing with visible light, let us say point 5 micron, so we can easily see that the quantity 2 pi R lambda will be much less than 1.

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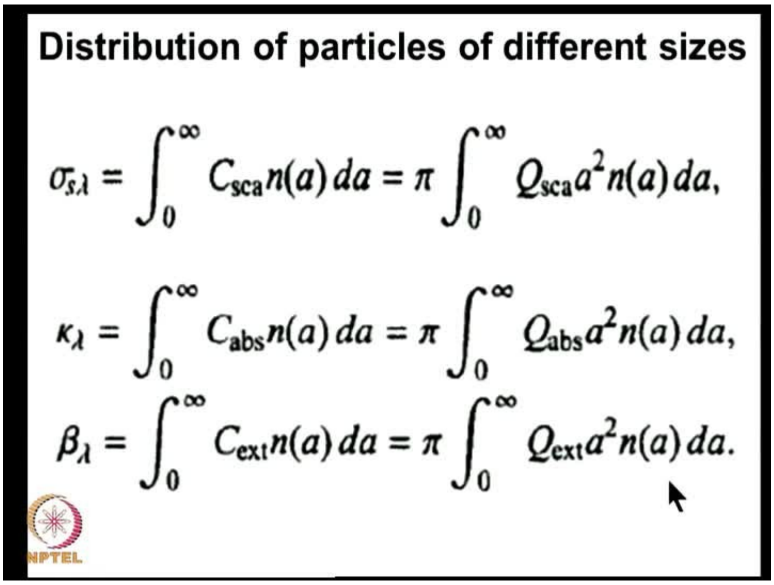

Hence, Rayleigh scattering is the right approximation to make when sunlight is scattered by gas molecules. This phenomenon is very important and we do see examples of that in every day experience of rainbows and other phenomenon of scattering, which we will discuss somewhat later. Now, the fact that this parameter 2 pi R by lambda plays an very important role can easily be seen by looking at calculations of these scattering parameters.

But before we do this, let us now look at; we define the relation between scattering efficiencies and scattering coefficient for single particles. Suppose, you have distributed particles in various sizes, then when we convert cross section to scattering coefficient we have to multiply with the distribution particles as given there. This can be written also in the terms of scattering efficiency and pi R square pi a square and so on. This is the most general definition of the scattering coefficient, absorption coefficient and extinction coefficient, based on the distribution of particles in the system. We can extend whatever we discuss for our particle of homogenous size of same radii to particles covering the whole range of radii, then we can use this equation too.

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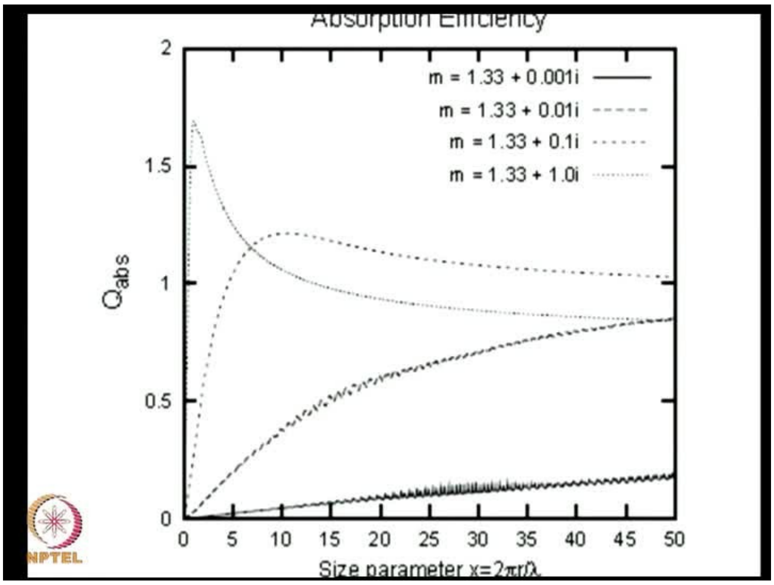

Now, let us take a typical value of absorption efficiency that is absorption cross square divided by pi R square. As a function of these size parameter 2 pi R by lambda, yes. Let us now look at how the absorption efficiency depends upon the size parameter, notice that absorption efficiency is always not always less than 1, it does go above 1, so we can see that as we vary the imaginary component of the refractive index of water.

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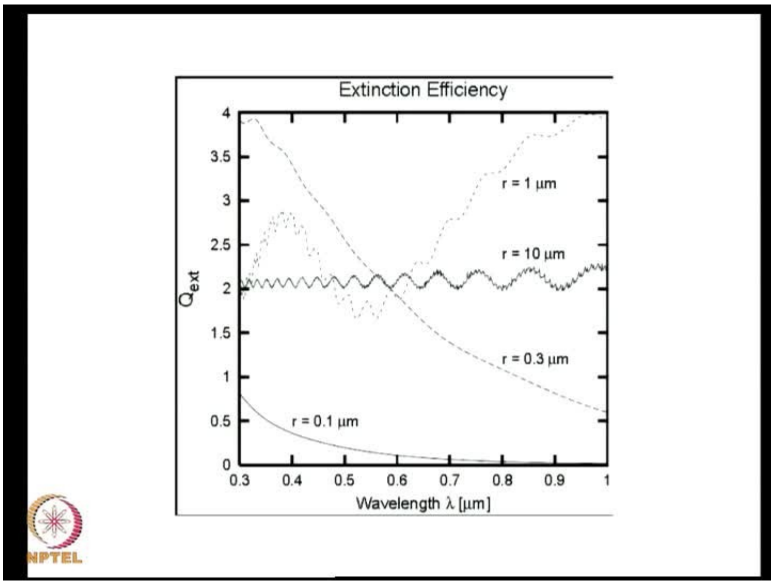

Water has a real component of refractive index of 1.33, we take the imaginary component and vary it, and we can see that the highly absorbing liquid droplet here this is highly absorbing.

As initially absorption efficiency is rising with the size, but finally it asymptotes to a value around 1.2. But in other cases, in most cases it is increasing slowly with the Size parameter, but in the case of highest absorption case, the absorption peaks. Then absorption goes down. Now, let us look at extinction efficiency which includes both absorption and scattering and see how it varies with wavelength the size of particle.

What we observe here is that, when the size of particle is comparable to wavelength, then the exchange of it starts from a value of around 2 to above 2 then it goes back and generally it remains above 2 for a particle size of 1 micron. But the particle is smaller can say absorption is strong only at low wavelength. As the wavelength decreases, the absorb resistance goes down quite substantially, for particle of small dimension and this for example, would partly explain how small particles found in combustion chambers or found in the atmosphere.

They particles in the visible region are important, but in the region beyond the infrared beyond its effect goes down. These particles especially aerosol particles, which have such a strong influence on the solar radiation, they do not influence much the infra radiation from the earth, so they are selective. They reflect the suns radiation, but they may not reflect the radiation emitted by the earth. But as a particles size becomes large of the order of 10 micron, you see that the scattering efficiency hovers around the value of 2. As we recall earlier this two should not be surprising, because scattering includes reflection, refraction and diffraction.

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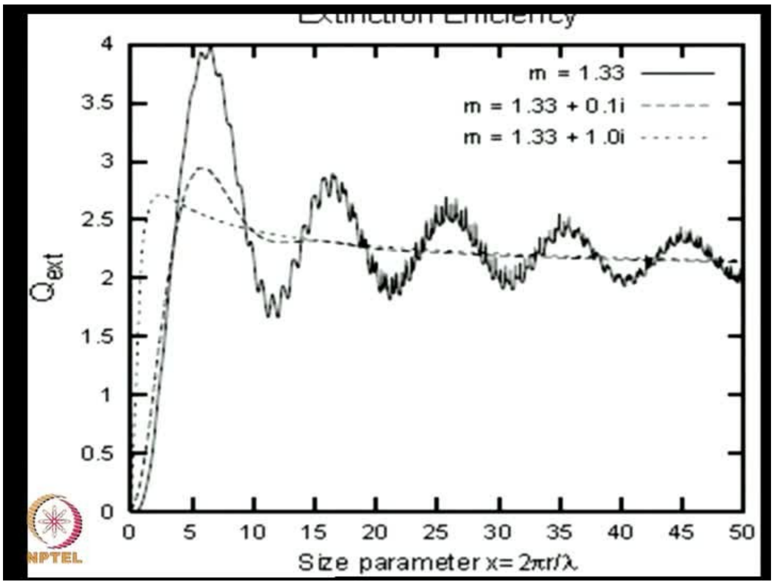

So, when all the three process occur together, the effective cross sectional area can exceed pi R square because cross section of the particle This is happening at 10 micron particle. But of course, smaller particles like 0.3 micron, that influences especially infra red somewhat less. Now, there is one more way of showing the same result now, in terms of size parameter 2 pi R by lambda, what do you see is that, when the absorption is low as in the case of pure scattering, there is a varied behavior, which is absent when the absorptions very strong.

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$$Q_{sca} = \frac{8}{3}\left|\frac{m^2 - 1}{m^2 + 2}\right|^2 x^4$$

$$Q_{sca} \propto \frac{1}{\lambda^4} \propto \nu^4$$

$$\Phi(\Theta) = \tfrac{3}{4}(1 + \cos^2 \Theta),$$

This is very strong like here 1.01, the various effects are cancelled out the effect increases initially the extinction increases, then it will decreases. This is the behavior of extinction efficiency, as a function of such parameter for various values of refractive index. Now, this quantity is all calculated based on Mie theory, which is established for this purpose. You see that for the value limit where the size of the particle is small compared to the wavelength, then we can develop an analytical solution.

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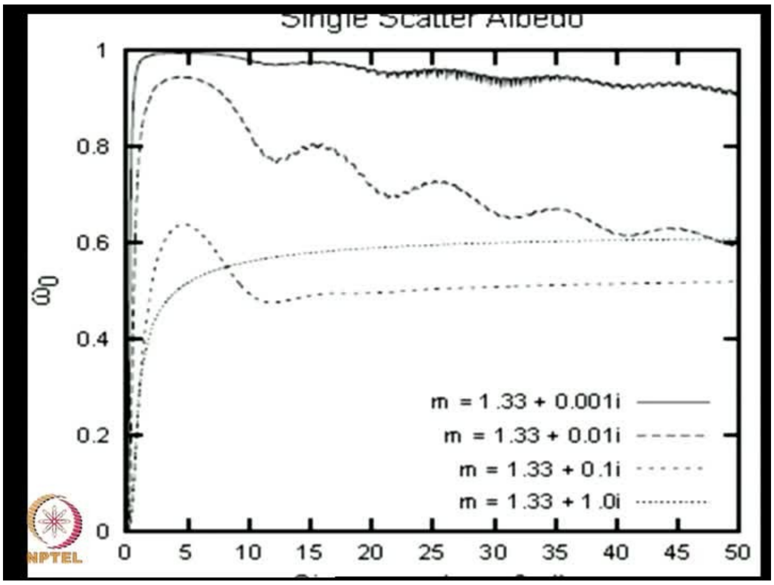

One of them on the top is the scattering efficiency. We can see that it goes has 1 over lambda to the power of 4, which is well known Rayleigh scattering formula. Here what is interesting is that, in this limit the scattering efficiency goes 1 over lambda to the power of 4, which many you of heard about as the way the scattering of solar radiation by these particles will go as 1 over lambda to the power of 4. The scattering phase fraction pi is of course; somewhat different from one is used to isotropic scattering.

Now, we have talked about absorption efficiency, scattering efficiency, but not the important parameter called the single scattering Albedo. There is a ratio, of scattering cross section to absorption plus scattering cross section. The number has to be less than 1. What we notice here is that, when the imaginary component of the refractive index is large, it does have an influence on the single scattering Albedo, otherwise when the absorption is essentiality close to 1 for many cases.

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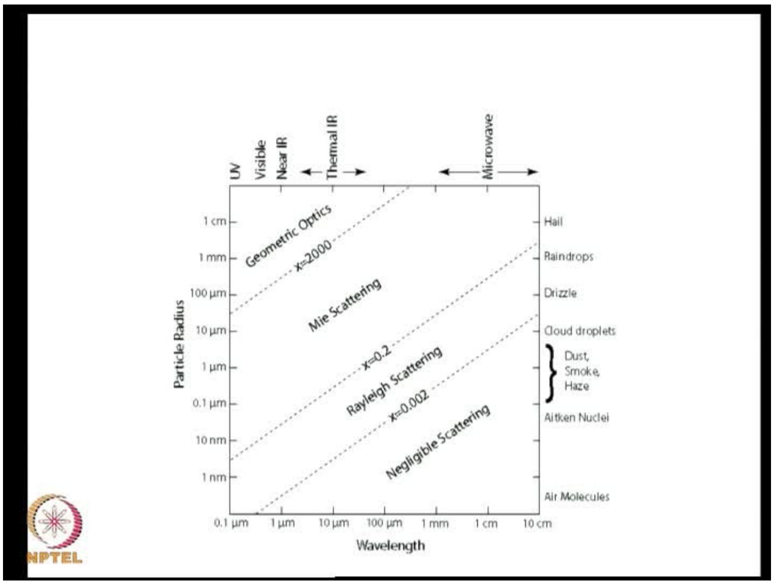

This is important number. The difference between single scattering Albedo calculations, if the particles are strongly absorbing we can have stronger influence, because it is in the denominator. Now, in this graph there is a clear relation between wavelength radiation particles, radius the kind of regimes that you are in. If the particle radius is in the order of 1 micron, then all wavelengths from 0.1 to 100 will be governed by this scattering.

But, if we are willing to make an approximation relevant to the work in which the particle is large about one micron, but we are not sure how effective this scattering process is. We can use the chart to see, which of this scattering is important. This scattering is the most general calculation, there is a solution to the Mie equation, and the key point is that, if the size of the particle is 2000 times the size of the particle we see the geometric optics applies. We can see that this will happen only around 1 micron there are rain drops and hail influencing your answer.

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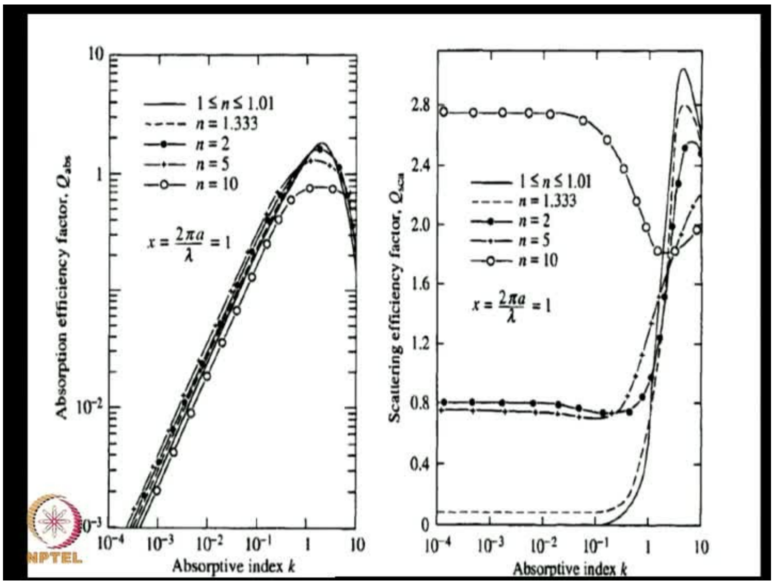

But, in most situation you should be able to deal with Mie scattering and point that are not revealed and would take some effort to develop now. Cases where the size scale is quite small and mist case we can solve as a special case. Now, this figure it out the absorption efficiency and scattering efficiency as show as a refraction absorptive index k, that is the real part the imaginary part of the refractive index of that material. We can see here that the highest absorption emissivity occurs, when the absorption index of the order 1 and right in the heart of the scattering process, so this occurs simultaneously. But they do keep a tab when the calculation is done.

Now, now on the right hand we see that the scattering coefficient is plotted here, have to answer absorption efficiency, scattering efficiency, extinction efficiency shows a more complex dependence. Both of them are plotted for size parameter above 1. We can see that in the case of absorption efficiency the real part refractive index plays no role, whether it is 10 or 1 in the graphs are pretty much going together. But if you come to the scattering efficiency factor, you see that refractive plays very important role, when refractive index is close to 1.

Here, we see that mainly scattering unless the absorption index is very large, but as the refractive index increase from 1 to 1.33 to 2 to 10, we see a large increase in scattering efficiency is a factor. Showing that scattering is much more influenced by real refractive index, while the absorption coefficient is more influenced by the imaginary part of the refractive index, scattering shows some more complex behavior, when the absorption is very large or very small, there is a simple linear dependence on the value of n, but when you go much larger absorption there is much more complicated behavior.

Again notice that when n is very large the scattering efficiency factor can exceed 2. Now, this does surprise many students because they cannot imagine how a particle with a cross section of pi R square can have a scattering cross section, which is larger than pi R square. But one should remember that scattering involves reflection, refraction and diffraction, and it is diffraction which causes the ultimate the scattering efficiency to be about two in such situations.

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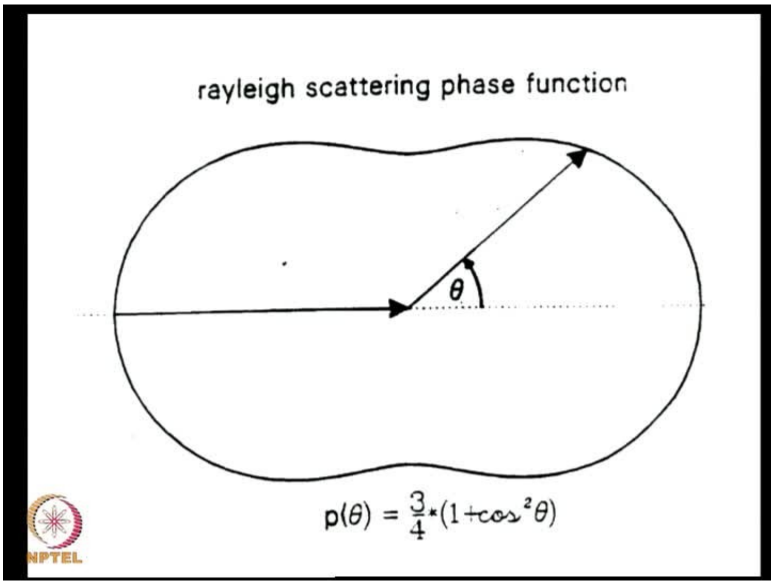

Now, typical phase function for Rayleigh scattering is shown here what we say is for the fore-aft symmetry, the forward and backward scattering are symmetrical, but the scattering on the side is small. This is the Rayleigh scattering feature. Now, indicates the Rayleigh scattering, we already saw that the scattering efficiency goes 1 over lambda to the power of 4, which is well known as the Rayleigh scattering effect, who's impact is on rainbows and hallows and so on. We will discuss these, in one of the subsequent lectures.

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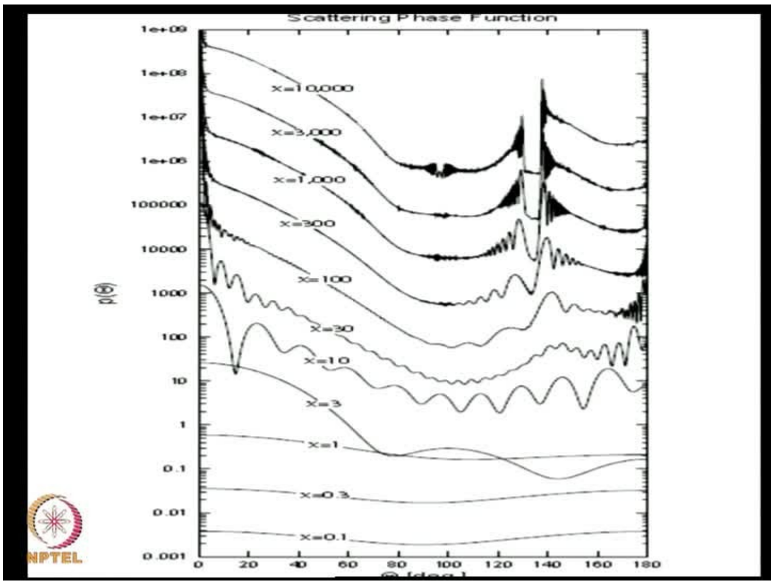

Now, here we see the true complexity of scattering phase function as a function of the angle theta where theta is defined by this term here. We can see that depending on the size parameter 2 pi R by lambda, this scattering function can display pretty complex behavior. What you see is that, when the scattering when the size parameters of order of 1, or small really, the behavior of the curve is rather uniform. But when it go to size parameter range of 100 to 1000 to 10,000, there are very interesting resonance kind of phenomenon where the resonance applies in the, in the values scattering at a given angle.

This kind of feature here this kind of phenomenon where there are sharp changes in the scattering efficiency at certain angles, these are very important from the point of view of explaining number of interesting optical phenomenon in the atmosphere This will be discussed in the substituent lecture. Because of a large increase scattering at certain angle, these phenomena create very interesting optical effects.

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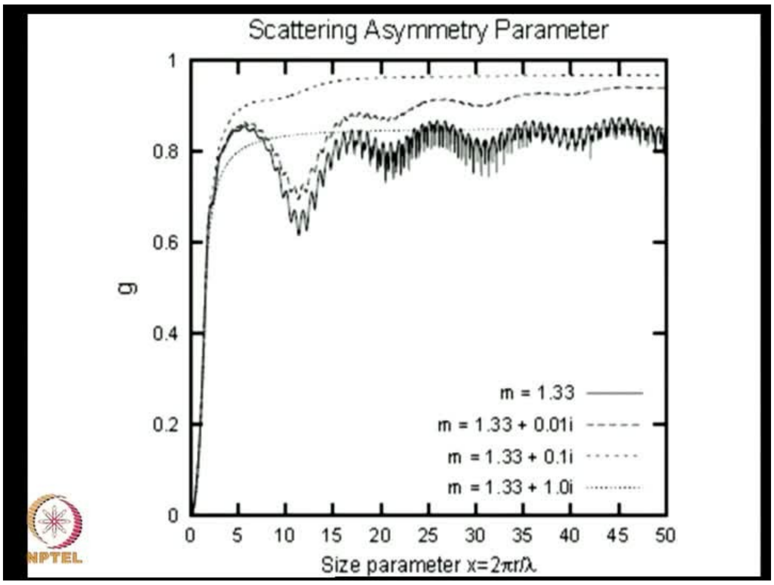

Now, here is the plot of the asymmetry parameter, as a function of the size parameter as well as the different values of the imaginary part of the refractive index. Then what we see is that if there is no absorption, that is the imaginary part of the origin index is 0, then you see complicated behavior, these are large increase in g from 0, which is symmetric scattering to the full high value. Then there are interesting oscillations with very special scales. This is what we expect in the v scattering domain. Lots of interesting features we saw in the phase function also.

As the amount of absorption increases that is as the imaginary part refractive index starts increasing, you see that it becomes much more smooth behavior because of absorption. The g part increase with 0 to around point n remains constant around 0.8 0.9, what remains is that you primarily have forward scattering because g is greater than, g is positive. It is primarily forward scattering and this behavior now will be seen graphically in some example.

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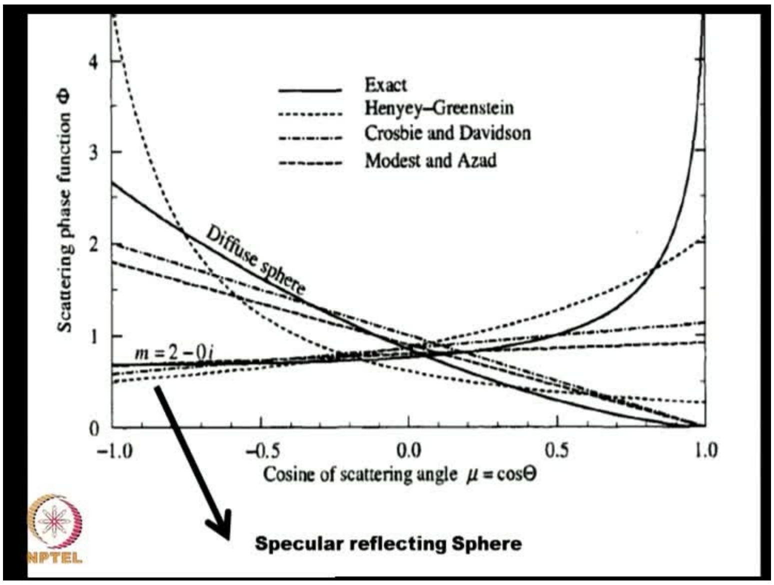

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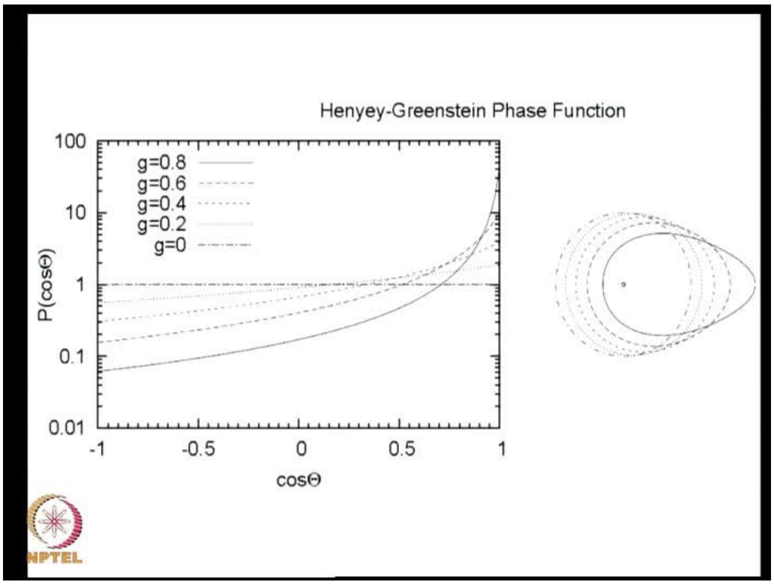

Here is the example of well known Henyey-Greenstein phase function, which shows that the scattering is peaked in the forward direction The scattering becomes more symmetric and when of course, when g approaches 0 and that is this one. Henyey- Greenstein phase function is a quite a popular function used to take care of the asymmetry in the scattering phenomenon, but is not the accurate depiction of the Mie scattering formulae because of the approximation.

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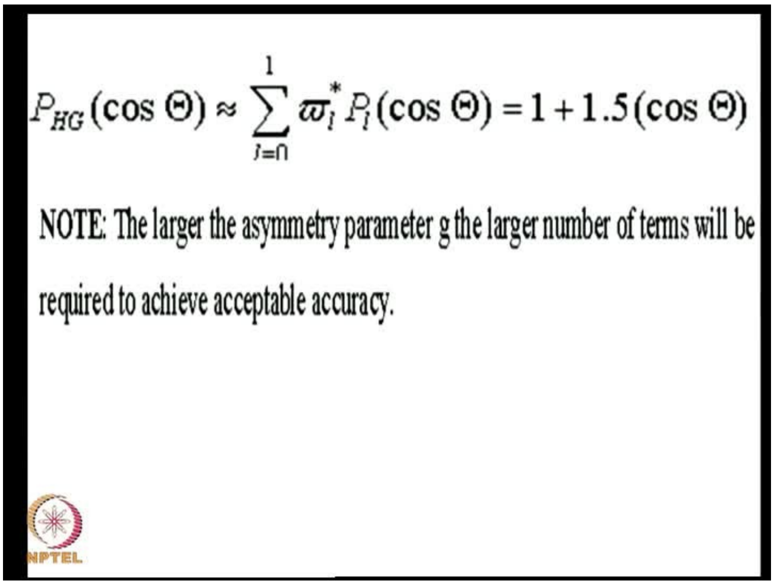

This is one kind of approximation, which called the Henyey-Greenstein, which takes into account the asymmetry in the scattering.

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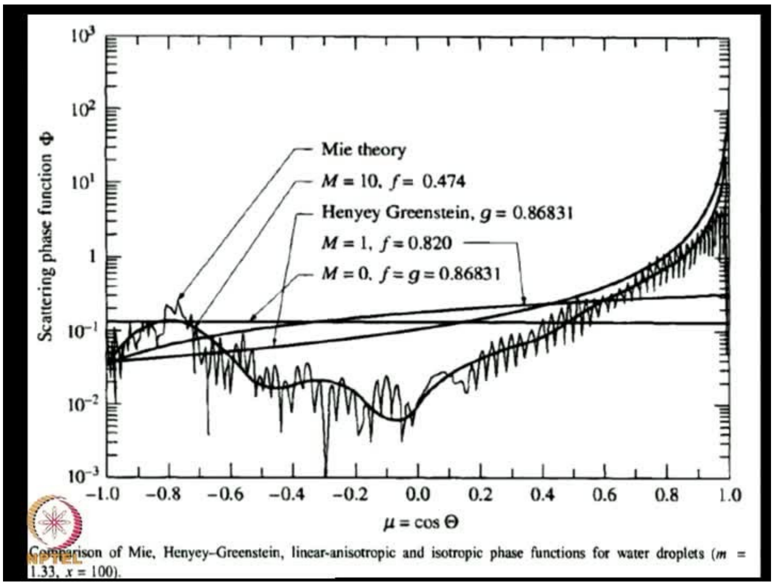

In the comparison of the Henyey-Greenstein phase function, which is this one, which is simple formula with actual Mie scattering expression here. We can see quite clearly that the actual fraction is very complicated, which can be simplified by suitable. approximation of course, but, the Henyey-Greenstein function captures the basic shape of the phase function quite well from mu in the positive domain, but mu and mu negative one can say that the Henyey-Greenstein approximation shows very large error in negative mu cases.

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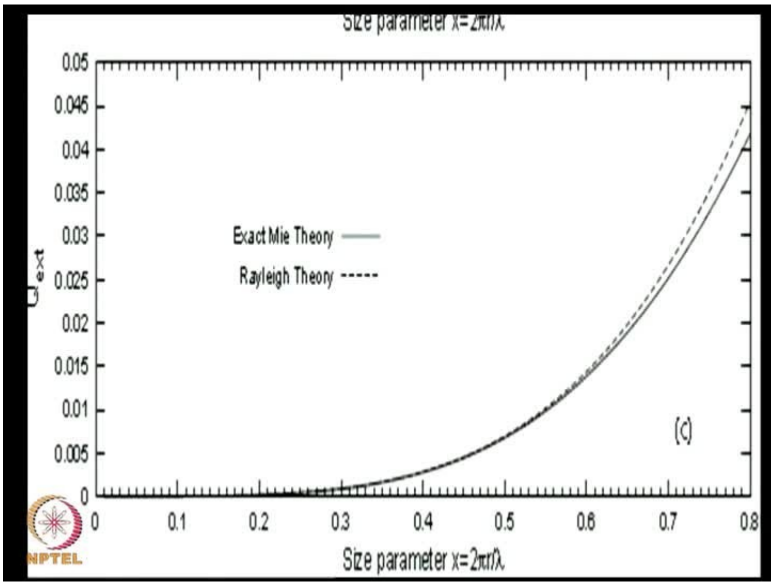

Now, here is the comparison between the extinct coefficient for the exact Mie calculation which is the approximate Rayleigh calculation and this video clearly shows that as long as the size parameter is less than around 0.6 or so. If less than 0.6, there is hardly any difference between Rayleigh and Mie scattering. But as it approaches 1, the differences start emerging and exact Mie theory predicts less scattering efficiency than the Rayleigh theory.

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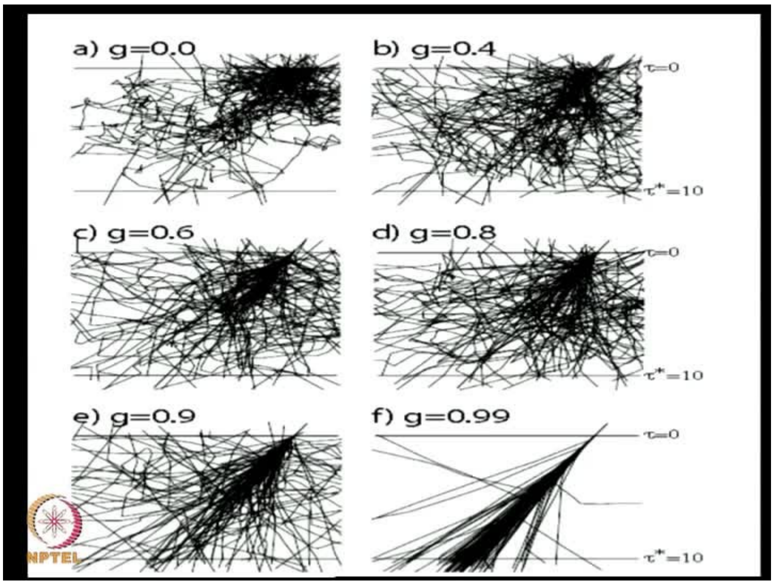

Now, the best way to understand the role of asymmetry parameter is to do Monte Carlo ray tracing. In Monte Carlo simulations, many photons are seeded. Their path is followed till the photon is absorbed. Now, here is the stimulation for g equals equal to 1, these are forward scattering the particles are been sent from this side here. We can see that most of the particles are scattering only in the forward direction and hundreds of particles shown here most of them are doing primarily forward scattering, a few of them are in this side.

Now, the same problem done by Monte Carlo scattering for g equals to 0 is very, very different, because this scattering has symmetrical characteristic, the photon paths show a very random behavior. Here the behavior is somewhat controlled by forward scattering. Here it is almost random and so the photons are taking all kinds of paths, which one would not have expected. But, we can see that even if g equals to 0.9, the photon trajectory show a very complicated structure. So, quite clearly when g equals to 0 or 0.9 should not matter too much because of the way the Monte Carlo formation examines the effect of asymmetry parameter. But quite clearly shows that the asymmetry parameter has a quite close to 1.

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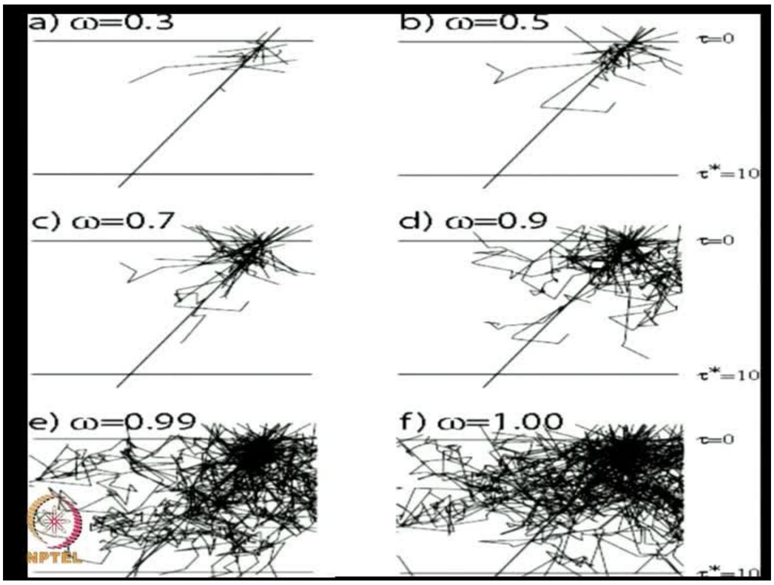

But we have to go further to get this simple forward looking scattering phase function, the same result has the previous one. Now, here Monte Carlo shows the impact of single scattering albedo, when the single scattering albedo is close to 1, you have lot of scattering going on. Hence, this is the case we saw earlier for g small, this kind of behavior not seen for g equal to 1. But when the secondary scattering albedo is very, very small that means the particle is primarily absorbing, and then you see clearly that the preferential mode is forward scattering. Since here albedo is very important parameter in this respect because it shows a related role played by absorption and scattering. When the absorption dominates as it does in this case, we see that the forward scattering is preferred more. Otherwise, it was all reflecting particles, and then you had more distribution like this.

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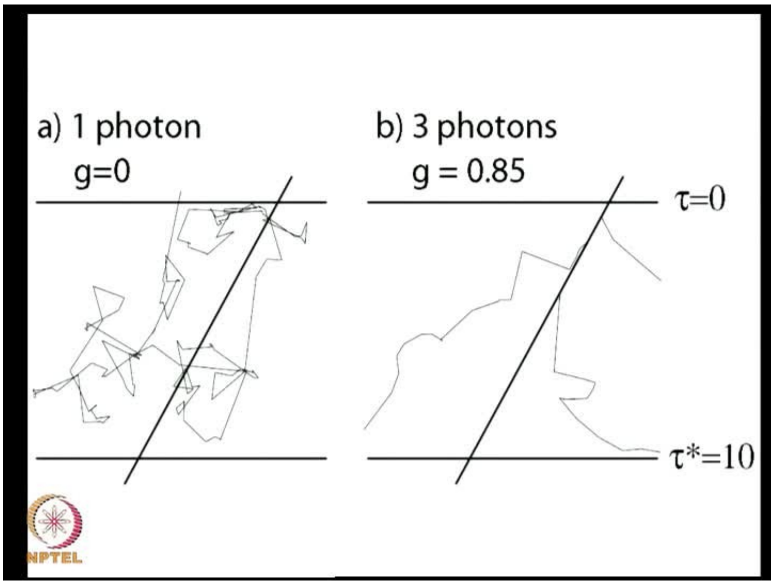

Here is the example of number of photons, the single photon with symmetric scattering, we can see moves all around, while here g equals to 0.85 predominantly forward scattering. We can see that the primary rays are going down and have no preferential direction on the side.

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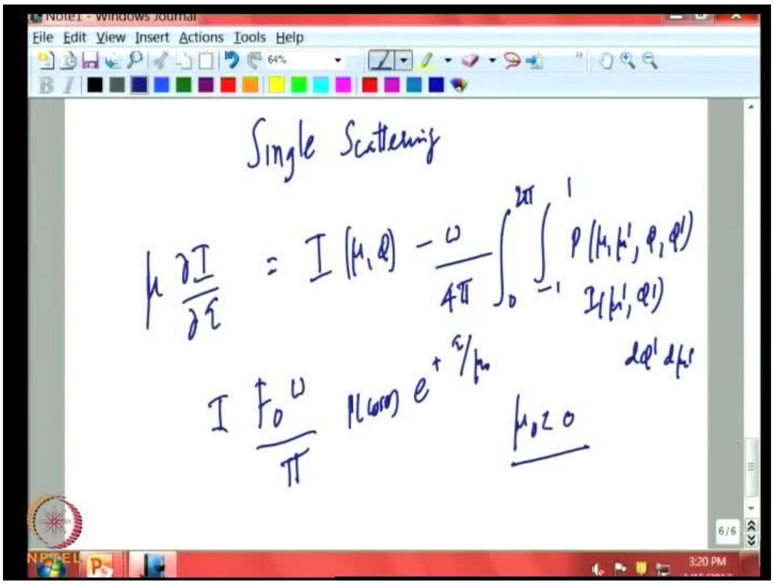

Now, we take an example. Suppose we take our case of single scattering, that problem is rarely well posed. Now, typical example could be sunlight coming in from the top of the atmosphere and being scattered by particles only once and by now you know that the basic formulation is a standard ray tracing equation. We see here that the second term here depends very much upon the direction i u prime. Suppose, we assume that the emission gas is small and has no relevance, then the i u prime can only be the solar input.

There is no other possibility and in the case solar input we know that the scattering will depend upon the slant path length. If the F_o is flux coming in, we can write, we can clearly say that where as v 0 is negative, so that this term will d k and one can clearly say that the scattered radiation depends linearly on the single scattering albedo, but non- linearly on the optical depth.

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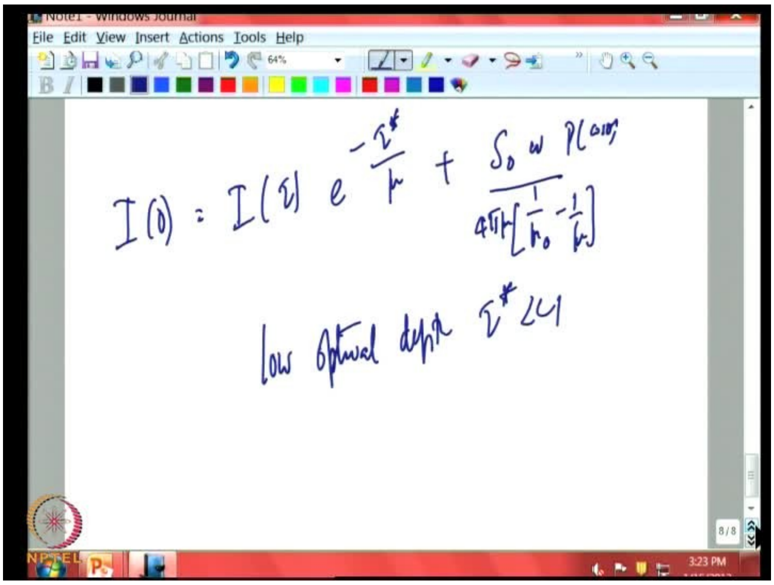

Now, what about the solar radiation? Actually we can solve these equation very easily. We can see that If F 0 the downward flux is influenced. So, I is equals to I tow plus incoming solar radiation. We can see the role of radiation now is coming out quite explicitly. This result is useful to understand the role of collimated beam radiation and how is it scattered in a single scattering mode. That is we are really neglecting the role of other particle in the scattering already into this particle. One can question this by generally this is valid for low optical depth. In that limit can this be easily be usefully discussed as a parameter of some interest.

Now, if we look at some of expression that we have discussed, as long as scattering is elastic, then the expression for the Mie scattering are somewhat easier, but when you include non elastic efforts like absorption, you do have problem with the, way the expression is derived. We saw that when lambda is much is less than the radius of the particle, then this scattering is geometric and does not care what the shape of the object is. But if the particle is much smaller than the size of the object, you have Rayleigh or molecule scattering. There are many application where the wave length of light. The particle size are comparable, in which case you have to deal with Mie scattering.

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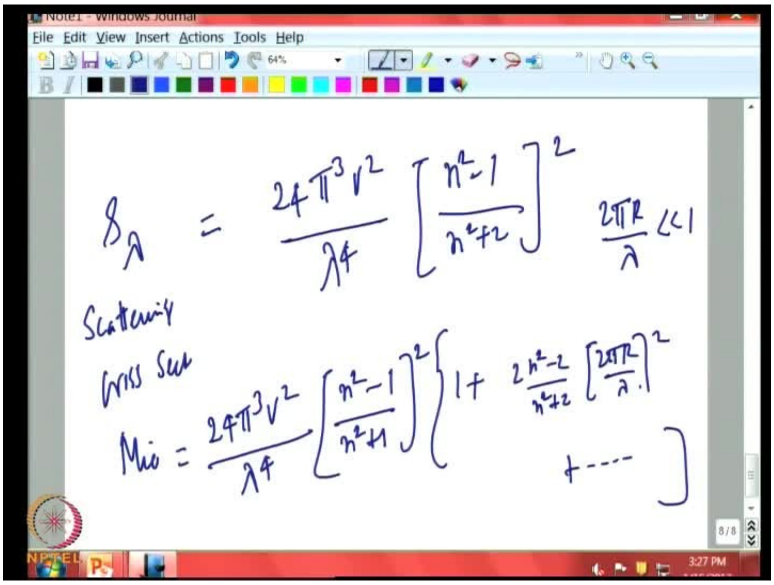

Now, the expression for Mie scattering, we will write down expression for the scattering cross section. Let me call it as lambda for the Rayleigh scattering the expression can be written terms of volume particles. This is especially useful if the particle is not really circular and so this is valid only for very small particles like the molecular scattering. While the Mie scattering formula is much more complicated, the leading terms are same because they have to be same because Rayleigh scattering is a special case of Mie scattering.

The first two terms here are absolutely same, and then you have an expression of series. The way to remember the scattering problem is that the general expression for scattering for any size parameter is that a long and tedious and involves many, many terms in the various scattering equation. But if we are lucky and 2 pi lambda the surface much less than 1, then we get the leading term, which is the Rayleigh scattering. Now, very natural phenomena that we absorb in the sky can be explained in terms of the dependence that coming out here in this equation here. So in the next lecture, we will give the examples of rainbow, halos and glory, which can be explained by the simple expression that we have derived here. But remember all these are singe scattering models. So, one can ask, when is the single scattering approach valid?

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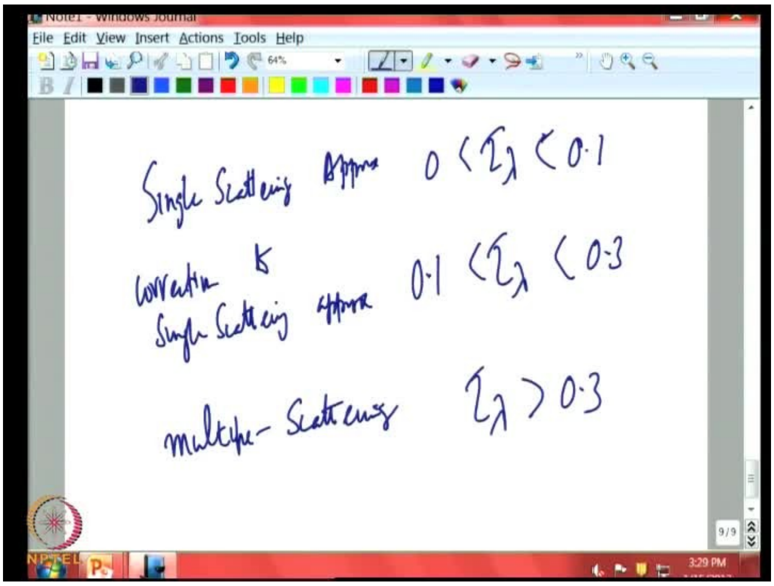

The validity is generally, when the optical depth at that wave length is between zero and 0.1, then one can quite safely say that the single scattering approximation is valid. But if Tau lambda is between 0.1 and 0.3, we can do a correction to a single scattering approximation we can manage to study scattering up to the optical depth of 0.3. But once we go beyond 0.3, then unfortunately the multiple scattering is an unavoidable. If we go into high optical depth multiple scattering phenomena is very much in the picture We need to do that.

As we can all imagine the simple problem is done, so far involving single scattering and an isotropic scattering are all amenable to simple an analytical are almost analytical expressions. But once we go to multiple scattering, this problem is going to get more complicated and that is when we cannot avoid a scheme like Monte Carlo. Monte Carlo scheme is ideally suited to problems of multiple scattering, non isotropic scattering We saw example of the ray racing through Monte Carlo which clearly illustrate the complexity of the problem when the scattering is not isotropic. But there are many problems in the real world where we have to deal with non isotropic scattering and multiple scattering. There we have to take re course to Monte Carlo. The only problem in Monte Carlo is that is not so easy to interpret those results. But in the next class, we will look at real life phenomena, which involves scattering.