Radiation Heat Transfer
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Lecture - 32
Radiation with Internal Heat Source

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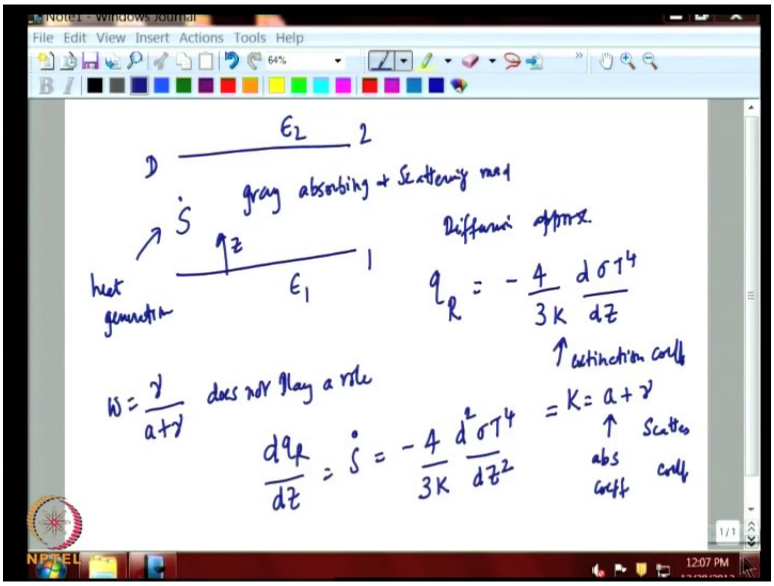

In this lecture, we take an example of solving slightly complicated problem of heat radiation transfer between two plates; surface one and surface two. This is emissivity, epsilon one; this is epsilon two and this is the gray absorbing and scattering medium in between Let us consider a situation where the heat generation is S dot and the depth of the plate is D.

We want to get the full temperature distribution in this medium and to keep the problem simple; we will use a diffusion approximation. We will treat radiation as a diffusion process, which we have discussed earlier in our lectures where, we said that in the thick limit we can treat radiation as the diffusion process and if you recall, we showed that q R equal to minus 4 by 3 K d sigma T to the power of 4 by d Z.

Now this is the extinction coefficient, which is the sum of absorption and scattering. This is absorption coefficient; this is the scattering coefficient. These two are combined to give you the extinction coefficient. Note that in the diffusion limit, the single scattering albedo omega plays no role. So, scattering behaves similar to absorption we have extinction.

The role of scattering really is to remove the photon from the path and, it does not play any other role. But, this we remember in view of the approximation we have made. Now, we also know that the first law of Thermodynamics states that d q R d Z has to be equal to S dot; heat generation per unit volume. From that equation we have here; it is minus 4 by 3 K d square sigma T to the power of four by d Z square. That is your differential equation. Solving this part is easy, because the method is straight forward. The challenge comes in when you want to apply the boundary condition. We integrate the equation; that is, easy integration.

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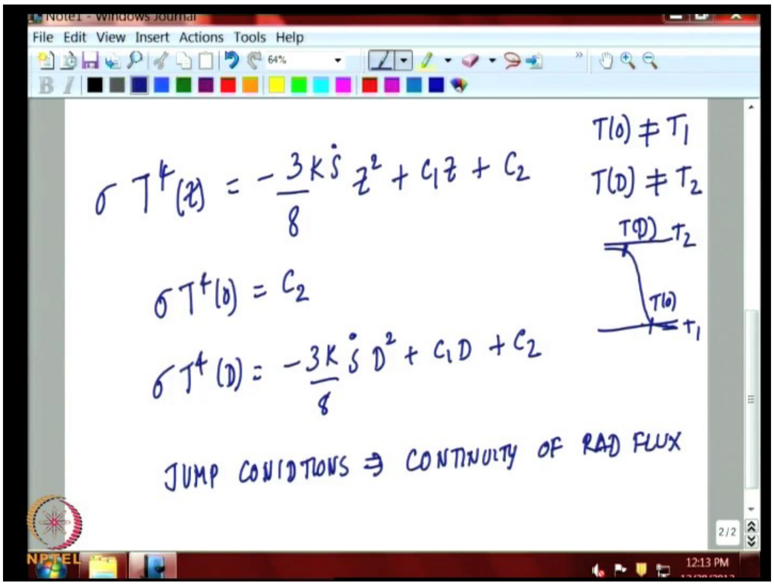

We will get sigma T to the power of four function of Z. The variation of sigma T to the power of four as a function of Z will come out as minus 3 by 8 K into S dot. This is what shown in meter cube; this meter per Watts meter square. This is same as this unit into Z square because you have done two integrations; and into C one Z plus C 2. This is a complete solution to the problem. But, the key as part of the problem is to apply the boundary condition correctly we recall that in the diffusion problem, it is very important to recognize that there is a jump in temperature at the wall.

Let us take an example take the temperature of the wall. This will not be equal to the temperature of the gas adjacent to the wall; note that T of zero is not equal to T one and T of D, at the upper wall, is not equal to T two that is the key issue. We must appreciate that in radiation heat transfer problems, continuity of temperature is not guaranteed. It may happen in a real problem, if conduction plays an important role. In this problem, we have neglected conduction.

But if conduction is there, it will ensure that the discontinuity near the wall is removed by the conduction process. But, in the absence of conduction you have to allow for the jump to occur. When we look at the value of this, as that equal to zero, this is equal C two, but not equal to T one to the power of four. Then, we ask what the value of sigma T to the power of four at the top surface is; spacing was D, if you recall. This will come out as minus 3 K by 8 S dot D square plus C 1 D plus C 2. We have got two results now. We know what C 2 is; we need to find C one. Now C 2 can be substituted here as sigma; so, here in fact C one also.

But, in order to relate this finally, remember that these two quantities are unknown quantities. These are not known. Just to remind you that this is T two; this is T one. And, so there is a temperature jump here; temperature goes like this and there is a temperature jump here. We must recognize it is of two jumps here; one at the top wall and one at the bottom wall. This temperature is T of D and that is not equal to T two. Here we have T of 0; that is not equal to T 1. We have to relate the T of 0 and T of 2.

We have to use what we have derived earlier called the jump condition; that is essentially applying the concept of flux continuity. This is always guaranteed in any problem. So, although the temperature is not continuous, the flux has to be continuous. From the continuity of the flux we have two conditions which we have derived earlier.

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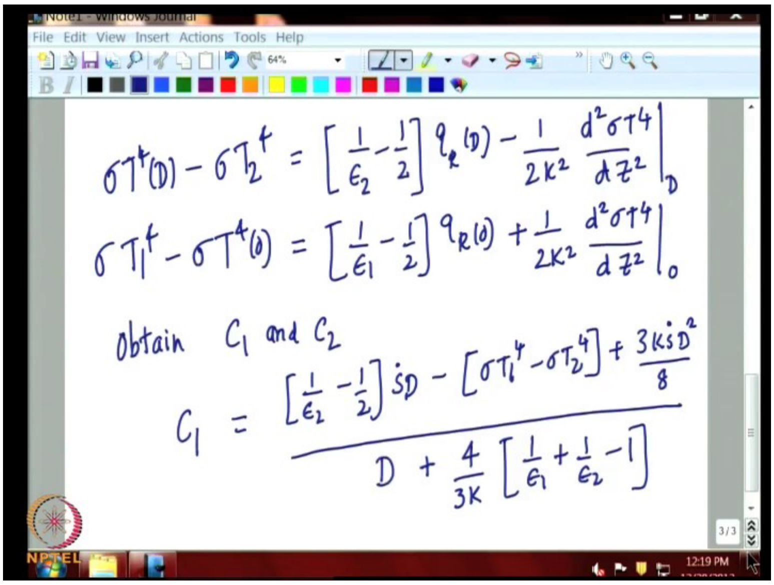

If we recall, which is that sigma T to the power of four at the top surface minus sigma, wall temperature, has to be equal to one minus the emissivity of that top wall minus half and q R at the top wall minus one by two K square d square sigma T to the power of four d Z square. This we recalled was derived by expanding series and retaining the leading terms till valid at the top surface.

Similarly at the bottom surface, the wall temperature minus the gas temperature at the wall, they are not equal; recall; This is equal to now one by epsilon 1 minus half q R at 0 plus 1 by 2 K square d square T to the power of 4 d Z square at Z equal to 0. We have to use this flux condition in conjunction with the solution to the equation that we have just obtained to eliminate these two unknowns. That is, the temperature of the gas near the bottom wall and temperature of the gas near the top wall are unknown quantities.

There we replaced by these quantities. When we do that, we will get the following. We have to solve for solve for C; obtain constant C 1 and C 2 in terms of the wall temperatures. Everything has to be related to the wall temperature. C 2 was the quantity because C 2 was equal to this. C two will be related to C one also. First we solve for C 1, and the solution we will get is one by epsilon one, one by epsilon two minus half into S dot into D minus sigma T one to the power of four minus sigma T to the power of four plus three K S dot D square divided by eight.

This has come from here. This is obtained from that derivative divided by D plus 4 by 3 K into one by epsilon one plus one by epsilon two minus one. So, although this is somewhat tedious in long derivation, it is important to understand the implications here. Let us make sure that in the process of derivation, we did not make any errors. The best way to check errors in this complex algebra is to look at limiting cases. Suppose there was no heat generation, this problem had been solved earlier in one of the lectures. We did for black walls. But, it can also it is not black walls. We can see that internal heat generation; this term drops out, this term drops out and you are left with these terms and suppose you took epsilon one as one epsilon two as one, so even this becomes simple; so, D plus 4 by 3 K.

This can be related to the problem we did earlier for the case where there was no internal heat generation. That is the radiative equilibrium problem. This is the extension of that result to the case, where radiative equilibrium does not exist because there is internal heat generation.

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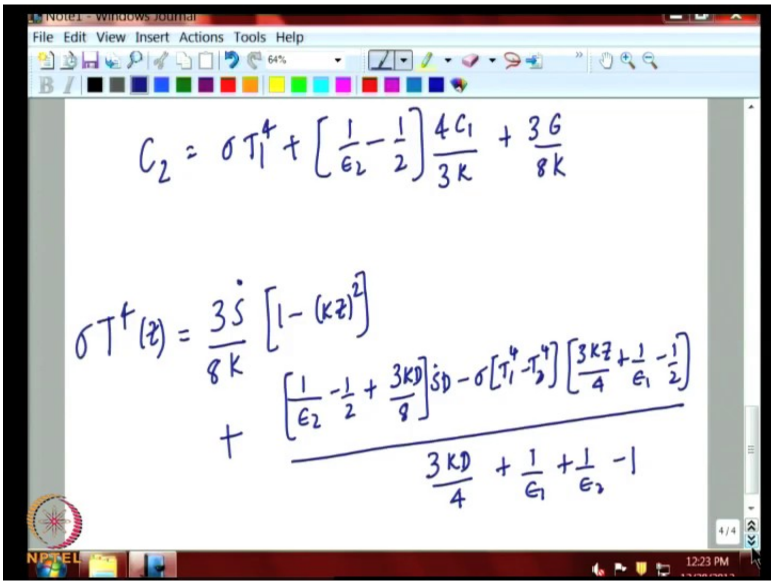

The C two will be related to C one and C two will come out as sigma T one to the power of four plus one by epsilon two minus one, , minus half, that is the boundary condition requirement, into 4 C one by 3 K plus 3 G by 8 K; where K is the extinct coefficient, not conductivity. We put all these into the final result, so that we get a feel for the final result. That will come out as sigma T to the power of four, a function of Z; this is equal to 3 S dot by 8 K one minus K Z the whole square plus one by epsilon two minus half plus 3 K D by 8 into S dot D minus sigma T one to the power of four minus sigma T two to the power of four. This is not easily completed here.

We write it below, so that we can easily see the result; so, plus one by epsilon two minus half plus 3 K D by 8 into S dot D minus sigma T one to the power of four minus T two to the power of four into 3 K Z by four one by epsilon one minus half, the entire thing is divided by three K D by 4 plus 1 by 1 by epsilon one plus one by epsilon two minus one.

This is a fairly complicated and long derivation. It is important to check whether we have done all the things correctly. It is always good to look at limiting cases. We can show that we get the correct limiting solution. One thing which we know well from our previous result is limiting cases.

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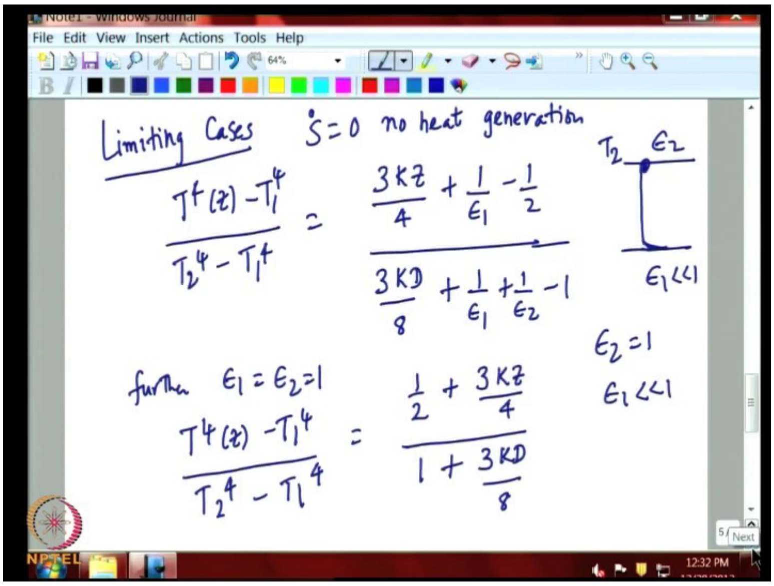

We will see limiting cases here, so to verify that result. One limiting case is of course, no heat generation. This is the case we had really done earlier in our lectures when looking at how to use the diffusion approximation. Let us see how simple the following result is. We can easily verify that this standard non-emission temperature. Problems come over that 3 K Z by 4 one by epsilon one minus half divided by 3 K D by eight by one by epsilon one plus one by epsilon two minus one. This is important, if there is heat generation. This is your standard radiative equilibrium solution for the case of the optically thick limit, which have been derived earlier.

We can simplify this further. Suppose, further we assume that the two walls are black; that case of how we looked at. This becomes half. We have; this is your half. We have half plus 3 K Z by four divided by one zero one plus 3 K D by 8. This must look familiar to because we had solved similar problem earlier. Wherein we had shown that; for example, if I take Z as zero, we have shown that the radiation slip at the bottom wall was equal to half by one plus 3 by 8 kappa naught. We have used it; we called it as the kappa naught; K into D is non dimensional optical depth.

We have shown that there is a temperature slip at both walls at both Z equal to zero, Z equal to d, this numbers exactly match what we had obtained for the case of radiative transfer between two black walls with the heat generation. Now, here we had covered the case of heat generation, no heat generation and the role of wall emissivity.

We can see that if the wall emissivities are very low, then these terms become very important; Let us now look at how we looked at the limiting case. At the bottom wall, we saw it is half by 3 K D by 8 and at the top wall it is half plus 3 by K D by 4 by one plus 3 K D by 8. In the both cases the sometimes exist.

Now, in the presence of wall which is not black, we have two additional terms coming in. They actually can increase the jumps. Now, let us look at the interesting case where the top wall is black, but the bottom wall is highly reflective. If we look at the case where the top wall is black, this term will drop out and the bottom wall is reflective, this term will dominate. This term will dominate here.

Indeed epsilon two is equal to one and epsilon one is much much less than one, highly reflective bottom wall. We will find is that this term drops out; this term dominates. If indeed it dominates over the term having the extinction coefficient, then the temperature, if this is very very small epsilon one. This term dominates. This quantity will approach one, which means the temperature everywhere will be equal to T two and the bottom wall will play just the role because if the bottom wall becomes highly reflective and the top wall is black, so the gas is truly radiatively coupled only to the top wall. The temperature of the gas will approach T two. This T two; temperature of the gas will approach T 2 here. There will be a strong decoupling at the bottom. This is not surprising in this example; because if the heat generation is zero, the gas can only get heat from the two walls. If the emissivity of the bottom wall is very low, no heat transfer occurs to the bottom wall of the gas. Gas is very radiatively coupled only to the top wall and the temperature there will approach T two. There will be a huge temperature slip at the bottom.

We find this interesting situation that is prevailing here; where, in the absence of heat generation and when the top wall is black and bottom wall is highly reflective, the temperature of the gas will approach the top temperature. We will essentially be completely decoupled from the bottom condition.

Now, let us go back to the previous thing we derived. Having understood that case, let us look at the full problem here; which involves heat generation and it involves heat generation as well as in the first, let us take the case of black walls in which this becomes one plus 3 K D by 4. This will introduce a half here. Here we will see that now we have to see the relative importance of the heat generation verses the wall induced heat transfer in the medium.

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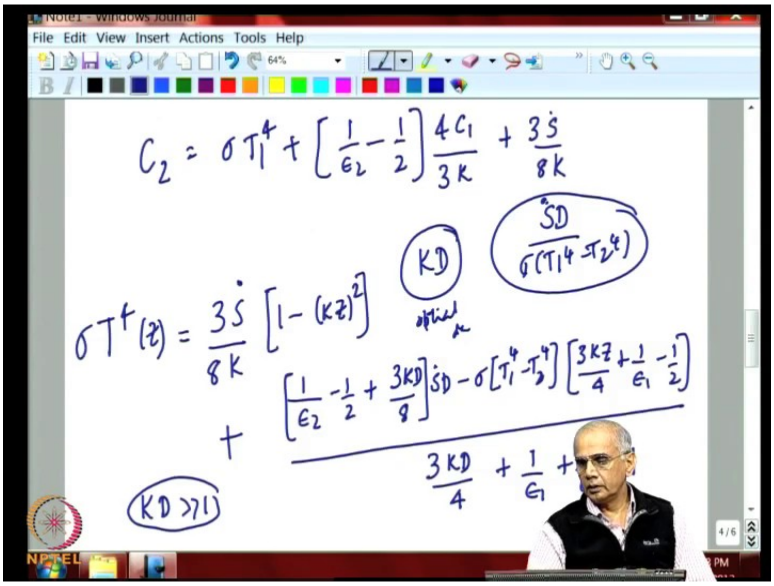

We can see clearly that, now there is a new parameter coming in; there is a heat generation into D by sigma. This will determine the importance of this term. This quantity is large; this term will dominate. We can essentially ignore this term. In that limit if the emissivity of the two walls is small, this will become large this term will drop out and it will be very simple quadratic expression here. But, on the other hand if the heat generation is small compared to the flux difference within two walls, then this term drops out, then we are left only with dealing with this expression here.

In this more complicated problem where heat is added, volumetrically requires as well as the heat is coming from the two walls, the relative importance of the internal heat generation and surface heat transfer non-dimensional parameter. For example, if we are dealing with a problem of conduction heat transfer, problem of combustion heat transfer in a furnace and you have an idea about rate heat generation due to combustion, then by calculating this parameter a priory, we will quickly get a feel for how important the heat generation term is, which influences the wall fluxes.

If we take that limit where this term is small compared to this term, then once more you see that. Then, we have to worry about the contribution of this term, which is a constant contribution here. Remember that, this term is small; this portion not varying with space. So, merely adds a constant value to the variable component here. Then, the importance depends upon one more parameter, which is not obvious here. The optical depth is KD. If this is very large, then this term becomes small because this becomes very large. This term becomes more important than this term.

This becomes; this is small compared to this term. Again we have got an interesting situation, where the temperature distribution is not that the gas temperature is essentially uniform in the bulk except for the sharp drop near the two walls. In particular consequence is happening in the optically thick limit and when the internal heat generation dominates over the wall flux, and then we find in the optically thick limit, the high heat generation inside the enclosure makes the temperature distribution almost uniform. The effect of the wall is only seen very close to the two boundaries as jump condition.

This result gives us a lot of insight about the two important non-dimensional parameters in this case. One is the optical depth; that is, the extinct coefficient times the spacing between the two walls. That is one and as we recall because you are looking at the diffusion approximation. Thus, it can be useful only in this limit, ok; when K D is much greater than one. But, the other parameter, the relative importance of the internal heat generation to the wall heat transfer is determined by second parameter. If that parameter is important related to this parameter, then the linear term will play a role. While, if this term is small, then our quadratic term created by volumetric heat generation will play a role.

We see that this kind of interpretation of the solution would be very difficult, if this problem was solved numerically. For example, suppose we wanted to solve the full problem without making the diffusion approximation, which limits it to the high optical depth, then we have to solve the full integral differential equation, involving essentially four parameters.

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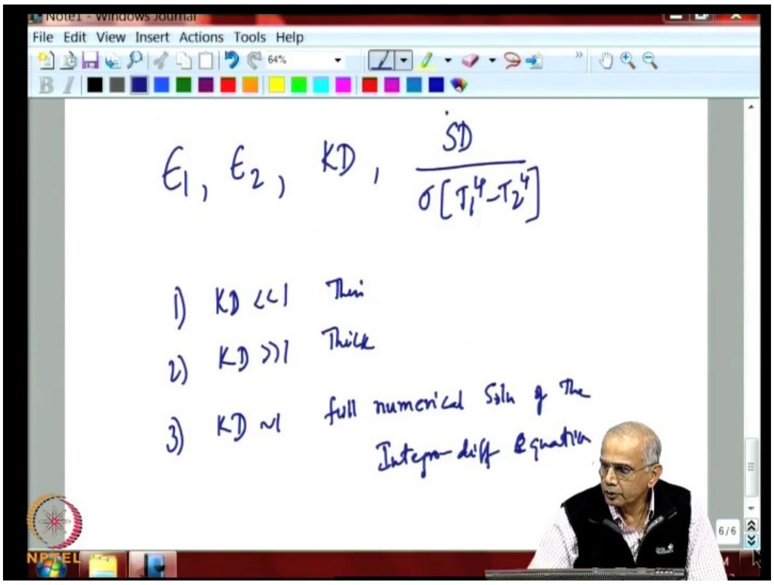

Let us highlight that. The four parameters are emissivity of wall one, emissivity of wall two, the optical depth and the relative importance of internal heat generation. The problem would be quite tedious solving for all this four parameters; not about parameters, but remember that finally to interpret the solution which are appearing would be much more difficult than that is possible in this simple case. That is why, when solving such complex problems, you do look at the two limits first. Then, when we call two intermediate limit, then we go to full numerical solution of the interpreted limit; this at that point. But, this solution can will better interpreted if we understood the limiting case because of the complex interaction between the relative importance of the internal heat source upon the wall flux, the optical depth of the medium and the wall emissivities. This has to be kept in mind when looking at such problems.

We want to indicate to you the kind of solution that we have got earlier for pure radiative equilibrium, in which case the solution was not given this form. But, somewhat modified at form. There we had; in the case radiative equilibrium, the problem was somewhat simpler because of wall flux was a constant so that, it did not pose any special condition.

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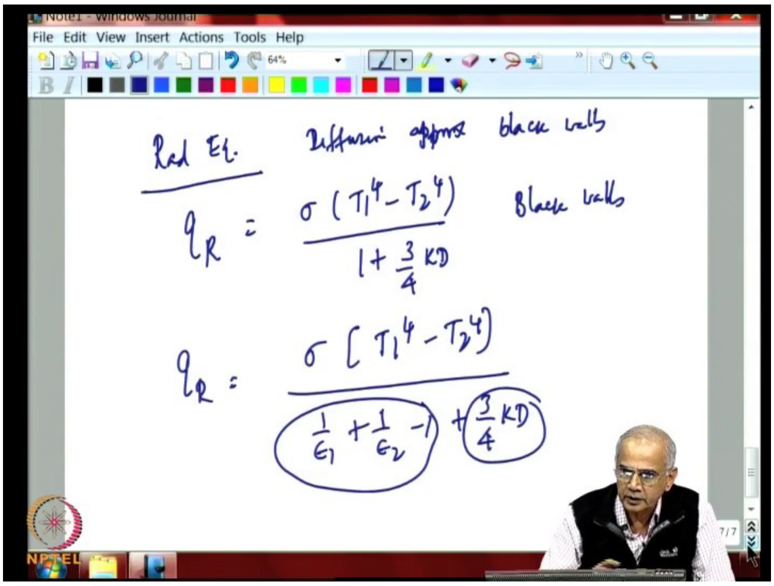

If we have radiative equilibrium and diffusion approximation and black walls, which is what we covered in the earlier lectures the result we got was q R. This is result we recall and which we had. Then, we also solved the problem where the wall was not black; this is for black walls. For non-black walls, we extend this result using the wall condition.

We see that it gives us some similarity between the solution where we got today with the earlier solution. The key point to remember is that the term three-fourth K D or three-fourth kappa naught; as we had called it earlier was the term which was the contribution of the gas, which are the contribution of this surface. We have similar result today; identical result today; as far as the denominator of the temperature distribution is concerned. We can actually also compare the heat fluxes; the heat flux if we recall and it is only that the difference between this solution; the one we got today is that in the radiative equilibrium problem, this is the constant. We can easily write out.

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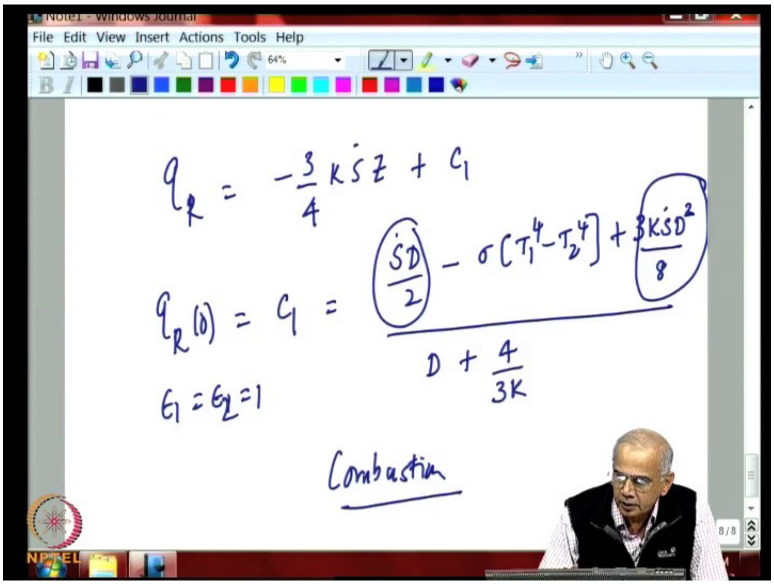

In the case of the present problem, radiative flux is not a constant. We can write this radiative flux as minus 3 by 4 K S dot Z plus C one, which we already highlighted. In this case because of the internal heat generation, heat flux is not a constant; it is varying linearly with Z. It is not so easy to compare the previous result. But, for example, suppose if we take q R Z is equal to zero; which could be a simple example. That is equal to C one and that we have readily available.

Let us put that down for comparison and let us take also black walls for time being. If we do that, we will get the following result which could be compared with what we got earlier. This is the result we will get for our black walls in the present case. If we compare with what we wrote down in the last slide, you will see that again we will get one by three-fourth; like that we will get. But, we see that in the numerator much more important role played by heat source system with the flux system.

In the previous problem, the flux term did not play; the heat source term was not there. The source was not clearly highlighted. But, in this problem we can see clearly the internal heat source term here can become quite important. We completely alter the even the sign of the flux is; it is so happens. This is a complex problem highlight that in problem involving combustion, the heat flux at the wall may be controlled more by the internal heat generation term, instead of the term involving wall temperature difference. That has to be kept in mind when you solve such problems. So, with this we complete;

complete this discussion about the radiative role of internal heat generation and fluxes at the wall.

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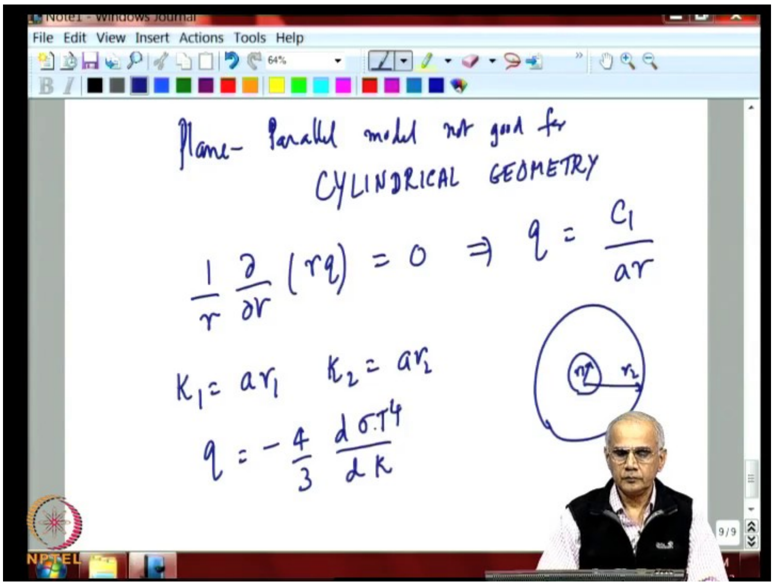

Now, in the remaining time we need to to solve another simple problem. For all the problems which we have done in radiation heat transfer, involved plane-parallel model. But, all of us should realize that this is not the only Geometry that we deal with in the real world. In the real world, we also deal with the Geometry that concerns Cylindrical Geometry. The Cylindrical Geometry is encountered frequently in Engineering; especially, when you deal with internal combustion engines, where a huge amount heat is released and the lot of radiation is transferred. If we want deal with such situations, you have to now extend the approach you have taken in the radiative transfer formulation to look at the Radial Geometry.

We will take a simple case just to illustrate the point. In Radial Geometry first requirement from first law of Thermodynamics is that this equals to zero. We are looking at radiative equilibrium problem. In under radiative equilibrium what you have decided for is not q equals to constant. That is not going to happen here. From here all of us realize that q will be equal to C one by absorbing coefficient that you stay here. We can assume what we did earlier; so, K into r. We define kappa one as a times r one and kappa times r two. Two optical depth; that is, we have a cylinder here and another cylinder here; this is r one this is r two.

These are the two optical depths of interest to you. Then, again we will appeal to the diffusion model and then we write q as minus four by third d of sigma T to the power by d kappa optical depth.

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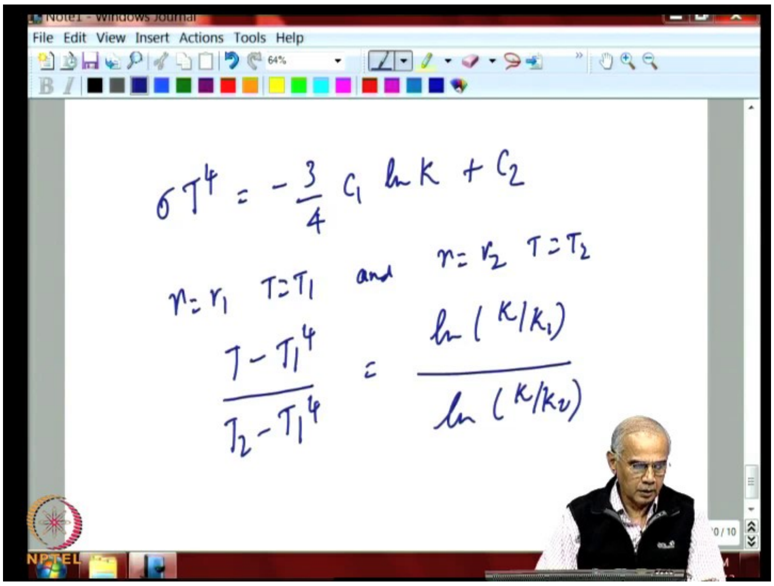

If we do that, we recall that the solution will be plus C two. This is a quite well known. If we want to get the temperature distribution, given this one, we can appeal to in that limit we can appeal to the continuity of the temperature and we can say that at r equals to r one, T equals to T one and r equals to r two, T equals to T two, then your result will be very simple. We will get at temperature distribution. But, if we want to get the heat flux in this case, you need to now appeal to the flux boundary condition that we have discussed in the earlier lectures.

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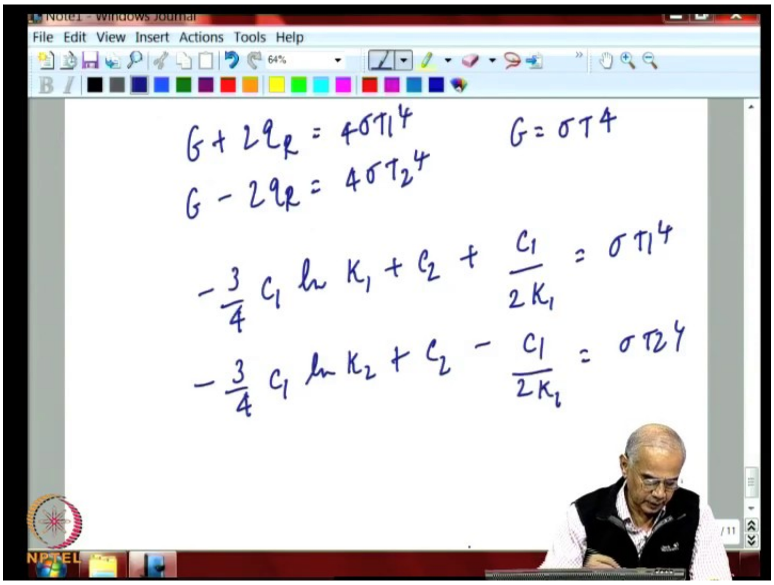

If we recall, the mean intensity G plus two q R was equal to this at the bottom and the mean intensity minus two q R was equal to the top condition. Under radiative equilibrium you know that G is any way equal to sigma T to the power of four; that also we recalled. Hence substitution of all this into the three equations will give you three-fourth C one log kappa one plus C two, and C one by two kappa one will be equal to sigma T one to the power of four. In heat flux, this is the intensity. Now, in the other wall will get minus C one by two kappa one sigma T two to the power of four. Then, we can solve for C one and C two and calculate the radiative heat flux.

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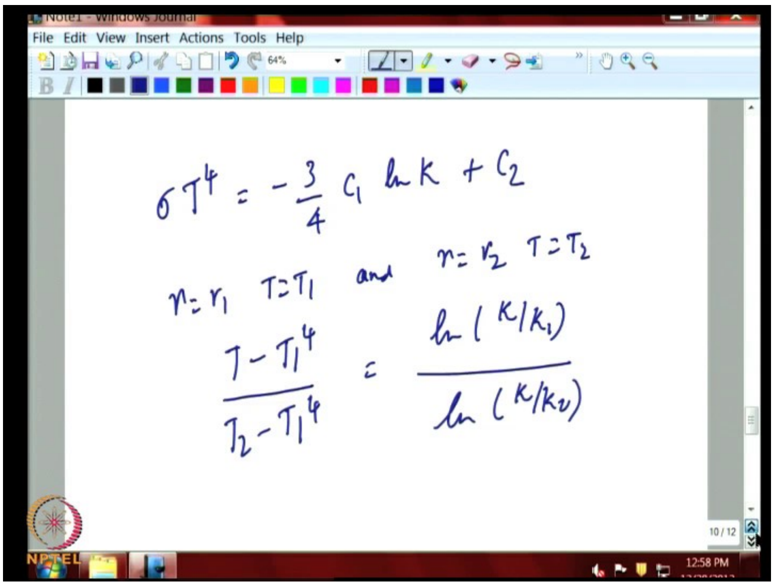

Now, what we see here is that, in the Cylindrical Geometry the problem does get a lot more complex than it does in the plane parallel case. Here only case where we can expect the two results to come close to each other is to look at; when if we recall, this is r one this is r two. We do realize that when there are two approaches to r one and the gap is very small, we do expect the plane parallel model to work. Let us see whether happens. That means kappa two by kappa one approaches one. When we invoke that approximation there, this equation now becomes; this becomes close to one. That is one, plus here what you have to do is you do exactly equal to the problem; of course, this is not there. There is zero, so it is approaches one.

We can write this as K two by K one is equal to one plus delta K; that is, K two minus K one by kappa one. This is a small quantity as kappa equals kappa two. This becomes one plus. It will be three-fourth into K two minus K one; kappa two minus kappa one by kappa one. And, so this problem can now be; this can be the total optical depth. We can see this result now approaches the one result we had; which is one by one plus three-fourth kappa zero. We see that in the Cylindrical Geometry, this is somewhat different result. The heat flux is not constant under radiative equilibrium. We want to reinforce that it is not forgotten.

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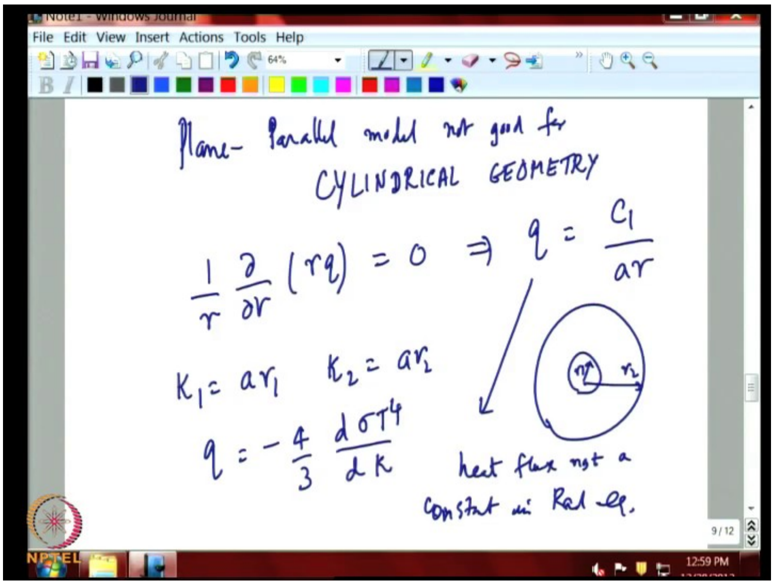

We notice that heat flux is not a constant in radiative equilibrium. This is the new feature of Cylindrical Geometry that even under radiative equilibrium, when there is no competing heat transfer mechanisms, competing energy sources, we cannot say d q R d z is zero and q are the constant because q will indeed decreases as you go from the inner wall to the outer wall. That is reflected by this expression. The invariant, in fact that q is not the invariant, influences the final solution. We get a result somewhat different from what we got for the plane parallel case. But, we are able to show that in the limit as kappa two by kappa one approaches one, then of course we recover that original result. But, it is worth remembering that there are other interesting situations that which we could have exploded. Suppose, the radius two was much larger than the radius one, then this term will drop out; the half plus three-fourth. This effect will dominate as the cylindrical effect.

In those problems like if we want to deal with diffusion combustion chambers, where there is no r one really, similarly r two term is going to zero here, so then this will dominate our result. We took a very simple example of radiative transfer in the thick limit of a Cylindrical Geometry and showed that there are similarities as well as differences. The similarity is a method solution, the differences emerges because of fact that the radiative flux is not a constant at this case, although we are talking about radiative equilibrium.