Radiation Heat Transfer
Prof. J. Srinivasan
Centre for Atmospheric and Oceanic Sciences
Indian Institute of Science, Bangalore

Lecture - 3
Properties of real surfaces

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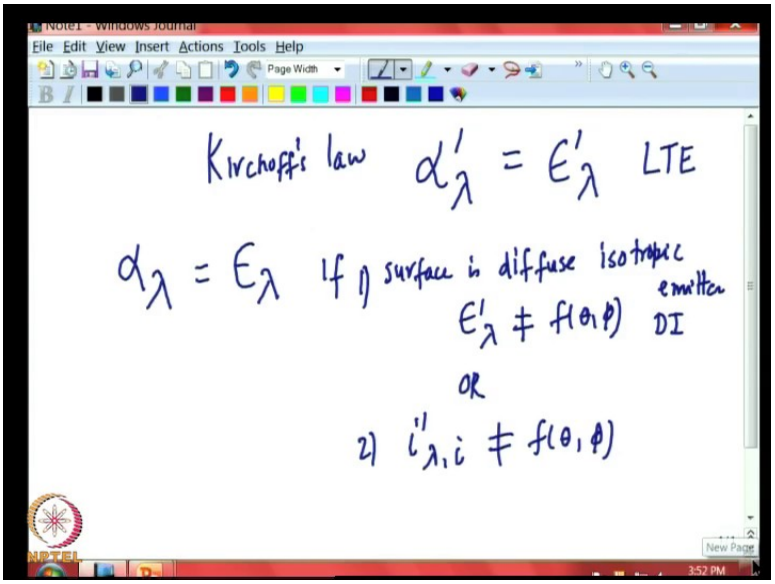

In the last class, we were talking about Kirchhoff's law and Kirchhoff's law relates the ability of a surface to absorb radiation, with its ability to emitted radiation. The most fundamental statement of Kirchhoff's law is that, the directional spectral absorptivity is equal to the directional spectral emissivity. And the only condition for this is that there has to be local thermodynamic equilibrium.

Now, we like to know when we can say the hemispherical spectral emissivity is equal to hemispherical spectral absorptivity. Although this is not always true, it is true if the surface is a diffuse isotropic emitter which will mean that the directional spectral emissivity is not a function of θ and φ. So, such a surface we will call as a DI surface; (Diffuse Isotropic).

Here diffuse is a word used to indicate that emissivity is not a function of θ, and φ, is used to indicate that emissivity is not functional azimuthal angle φ. So, this one requirement or the second requirement, that can be imposed that the radiation that is incident on the object; is not a function of θ and φ. We see that in order to make sure that the hemispherical spectral absorptivity is equal to hemispherical spectral emissivity, we have to put a condition either on the surface or on the incoming radiation. That is the surface has to have a property which is independent of angle or the incoming radiation has to be independent of angle.

In real world , the chances that the incoming radiation is independent of angle θ and φ is very rare. It can occur in a condition outside the laboratory under cloudy conditions, where the radiation of the sun may be roughly equal in all direction. But in most other situation that we encounter, incoming radiation will be a function of θ and φ. The condition, of the surface being diffuse isotropic, although not that common can be a good approximation which we will see later.

When we look at actual properties of surfaces under the assumption that real surface, is really not diffuse isotropic . If we assume diffuse isotropic , we may make an error in the estimation of the hemispherical value and that error can be of the order of 10 percent. Now, this kind of error is usually accepted in this subject because the data on radiative properties of surfaces is not easily available. They are not tabulated extensively as is the case with thermal conductivity or other properties of surfaces. The accuracy of these tabulations is also not that liable and therefore, as an engineer one may be willing to make an approximation, which causes an error which is less than the error in the input data.

So, in any real life problem one has to have a data from some source and if one believes that the data can cause an error of the order of 10 percent; then one should be unhappy about making approximation which is the order of 10 percent. In many engineering problems one would not make a large error when we assume the surface to be a diffuse isotropic emitter, and hence Kirchhoff's law in this form is useful in many real life situations.

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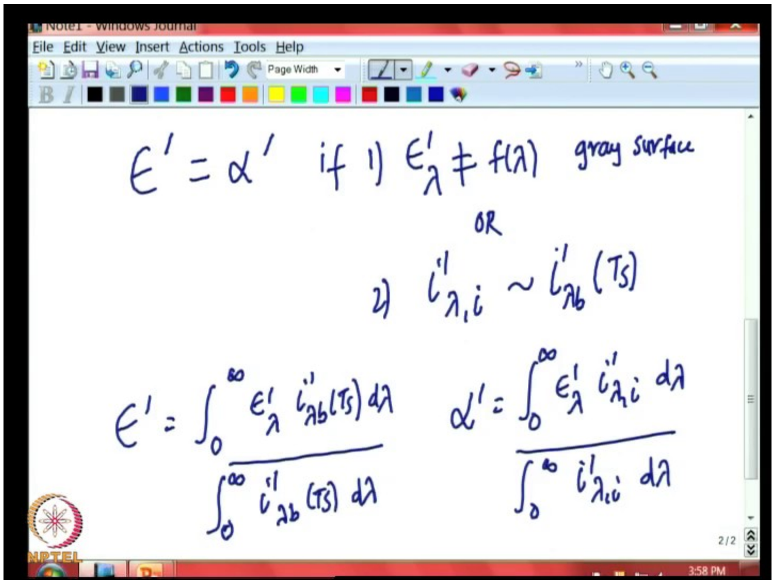

Now let us go to the other approximations, when we apply Kirchhoff's law to see if we can apply Kirchhoff's law of the form directional total emissivity is equal to directional total absorptivity. Now, we saw in the last class that, this can be assumed, if the directional spectral emissivity of the surface was not a function of wavelength (λ); this is called a gray surface. Gray surface is one whose radiative properties are independent of wave length. This is not very common because most surfaces as we will see later; show strong variation of emissivity with wave length. So, this approximation can be done in a few cases, but may not be good approximation in most situations.

In many cases, we cannot assume the surface to be gray. Then the only choice we have is to assume that the incoming radiation is proportional to radiation from a black body at temperature of the surface This is useful, because if we recall in the last lecture, the expression for emissivity was an integral of spectral emissivity weighted by the black body emissivity at the temperature of the incoming radiation and divided by sigma $\sum T_s^4$/ЈJ.

Now we will do the integral for absorptivity. We will rewrite this as an integral of 0 to ∞ $i_{\lambda b}$ T_s dλ. The primary difference between absorptivity and emissivity calculation while integrating over wave length is, in the case of emissivity we are using the spectral radiation black body intensity at the surface temperature as a weighting factor while in the case of absorptivity it is the spectral radiation of the incoming radiation.

These two can be equal only if either the emissivity is independent of the wave length and comes out as the integral, in which case this will cancel out and we get the first condition. Secondly, if by chance the intensity of the incoming radiation happens to be proportional to the black body intensity at the temperature surface, it can be equal. In a real world where the radiation impinges on an object, it happens to be similar to a black body intensity at the surface temperature of the object.

In a laboratory, for example, if we are doing an experiment; we might be able to arrange a source which is proportional to the intensity of the black body radiation at the surface temperature. But, in all other real life situations, outside the laboratory, this condition will not be met. Therefore, the chances that we will be able to assume that the directional total emissivity is equal to directional total absorptivity will be very rare. There are very few surfaces in the real world which are gray that is the property is independent of wavelength. There are few situations in the real world, where the radiation that impinges on an object, happens to be proportional to the black body intensity at the surface temperature. This is an approximation one has to make with great care, but we will see many textbooks where they will state this condition as the Kirchhoff's law.

We emphasize that Kirchhoff's law states that directional spectral emissivity equals directional spectral absorptivity. Kirchhoff's law does not say about directional total emissivity equal directional total absorptivity. These two become equal only if one of these conditions is satisfied. There are not too many occasions that we will get to be able to make this kind of assumption. One has to use this kind of approximation with great care. Sometimes in real life engineering problem one is forced to make certain approximations, but one should know the consequences and the error that may be introduced by these approximations.

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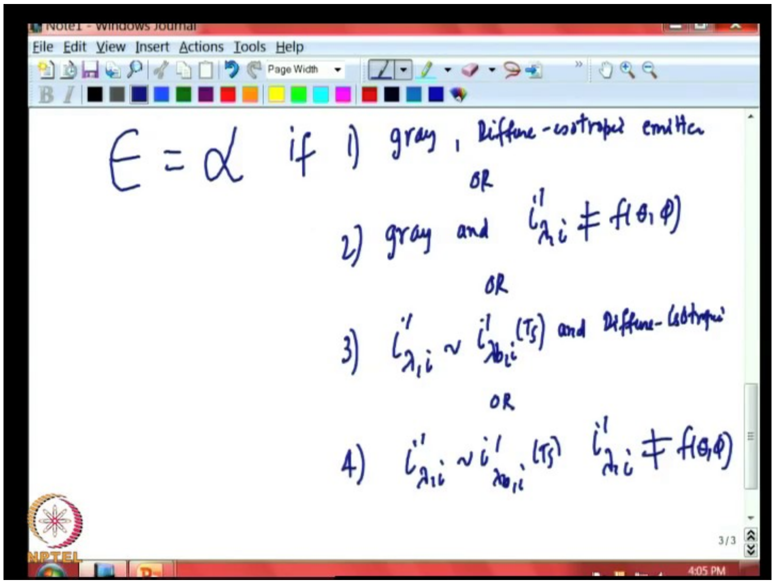

In a more general case one would like to assume that hemispherical total emissivity equals hemispherical total absorptivity. There are books which state this as Kirchhoff's law, but this is not Kirchhoff's law. This is a condition where, we are starting with Kirchhoff's law and we have to perform two integrations; one over wavelength and the other over angle to get at this point. In order for these two to be equal we have to impose conditions on the directional property of the surface or the incoming radiation and the spectral properties of the surface or the incoming radiation. Let us stipulate what the conditions are. This follows directly from what was already done through wavelength or angle integration. When these integrations are done together, then we need to impose two conditions in each case. For example, one can say that the surface is gray and diffuse isotropic emitter.

The condition is imposed only on the surface. This is very rare, and there are not too many real life situations where you will find surface that is both gray and emits in a diffuse isotropic fashion. But if that happens then these two are equal, ; or the second possibility is we keep the surface gray and it so happens that the incoming radiation is not depending on angle. The incoming radiation is equal in all directions and the surface is gray. This also is of course very rare. The third is that the incoming radiation, is proportional to the black body intensity at the surface temperature and the surface is diffuse isotropic.

We have to assume that the surface has known angular dependence of emissivity and that the incoming radiation happens to come from a black body at the surface temperature. Again these two are very rare conditions to be met in real world. Finally one can assume that the incoming radiation, is again proportional to black body intensity at the surface temperature and also this radiation is not a function of angle. It is incident uniformly on the surface. We can see there are four conditions under which hemispherical total emissivity can be equal to hemispherical total absorptivity but these four different conditions are rarely satisfied.

In general it is important to recognize that one can rarely use this assumption, that hemispherical total emissivity equals hemispherical total absorptivity in real world situations. There are occasions where one may be forced to be make such approximations, but when we do, we must carefully examine the results we get, by making such approximations. Because, it may lead to serious errors in estimation of radiative fluxes or some other property and if the errors are large then we need to go back and question whether this assumption is valid or not.

The above equation is not Kirchhoff's law. Kirchhoff's laws states that directional spectral emissivity is equal to directional spectral absorptivity. That is always valid in engineering situation, but if we want to make assumption that the hemispherical spectral emissivity equals hemispherical spectral absorptivity that is hemispherical total emissivity is equal hemispherical total absorptivity, then we have to make one of these four assumptions, which are rarely true in real world situations.. Later in this course we will do problems in which we may make such assumptions.

Even though that it is not a good approximation, it is done in the class room in a way to solve some simple problems to illustrate some feature of radiation transfer knowing well that in the real world this is not valid. It is more of a teaching tool rather than solving a real problem. We have spent a lot of time discussing Kirchhoff's law because in many text books in heat transfer , Kirchhoff's law is stated rather casually and in rather cavalier fashion. There are books which state that alpha equals epsilon is Kirchhoff's law, there are books which state alpha lambda equals to epsilon lambda is Kirchhoff's law, and there are other book which state alpha prime equals epsilon prime as Kirchhoff's law. All these are not true. The only true statement is alpha prime lambda equals epsilon prime lambda, which is the basic statement of Kirchhoff's law.

The other assumptions that people use are valid under certain stringent assumptions and it is important to ask oneself, whether those assumptions are invalid. There are real world situations where one will be forced to make some assumption, but we must do that consciously and be ready to examine the consequence of making such assumptions on the estimate of either radiative fluxes or temperature and be ready to evaluate the impact of assumption on the errors in the estimates of some values.

As long as one is fully aware of the assumption, that has been made and its consequence, one can make some assumptions to see what happens. In this course we will make some of these assumptions because that simplifies the problem substantially. But ultimately when we compare the results obtained with real world, we find there are large differences and we must be willing to go back and question some of the assumptions we have made. Now that completes the discussion on Kirchhoff's law which is a very important law in radiation heat transfer and law which has been misquoted and misused in many situations.

Now so far, we have looked at absorptivity and emissivity, the two important properties of a surface, but these properties are not directly measured. For example, if we want to measure emissivity of the surface, we should compare the emission of that surface with that of a black body at the same temperature. That is not easy to perform in the laboratory, by taking a ratio of the real emission to that of a blackbody at the same temperature. Similarly, in order to measure absorptivity of a surface, you must measure the amount of radiation absorbed and divided by the radiant incident to the surface. These are quantities are not easy to measure directly.

So, the traditional method of estimating emissivity and absorptivity is not through direct measurement of this quantity, but by indirect inference by measuring reflectivity. Reflectivity is a property which is somewhat easy to measure. We send a ray impinging on an object and measure how much is reflected. We will see immediately that once we measure the reflectivity of an object, we can infer from that, what is the absorptivity by applying the first law of thermo dynamics.

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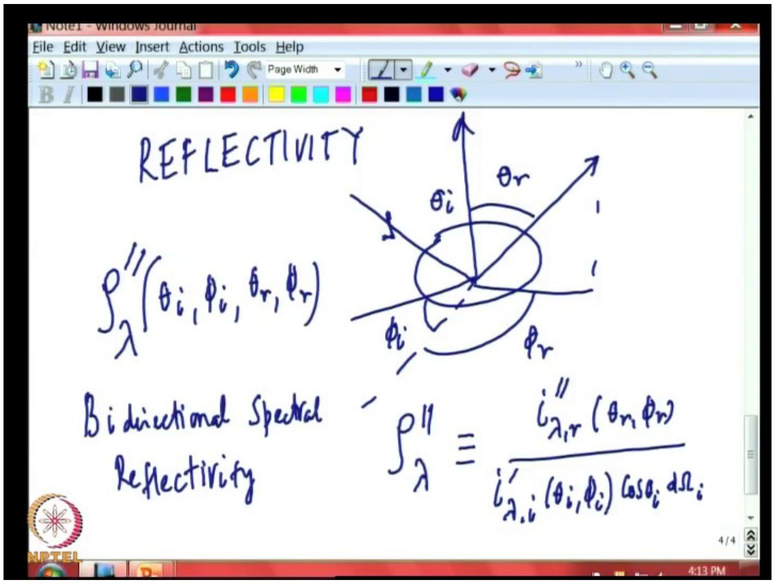

Reflectivity is a quantity which is somewhat complicated. It is because reflectivity depends on two angles. Suppose a surface, has a normal and it has radiation coming in at a certain angle θ. This is called incident angle θ_i, and with respect to some reference it has azimuthal angle φ_i and it is reflected in some other angle. So, that angle is θ_r and the azimuthal angle is φ_r. We see that problem is more complicated because the reflectivity rho 'ρ' depends on two angles and on wave length also.

The reflectivity is a function of θ_i, φ_i, θ_r, φ_r and wavelength λ. The basic quantity in reflectivity is called the bi directional spectral. This is the bi directional spectral reflectivity.

It is defined as the intensity of the reflected radiation at angle θ_r and φ_r divided by radiation incident at a given angle θ_i φ_i $\cos\theta_i$ and the incident solid angle. This definition is more complicated than that for emissivity and absorptivity. Comparing the intensity of the reflected radiation to the total intensity of the incoming radiation. We would like to know how much radiation is reflected in all directions from the surface, from the energy incident on this surface. In other words, of the total radiation impinging on an object, we would like to know how much radiation is absorbed and how much is reflected. We do not really care in what directions it is reflected. We would like to integrate over all out going angles.

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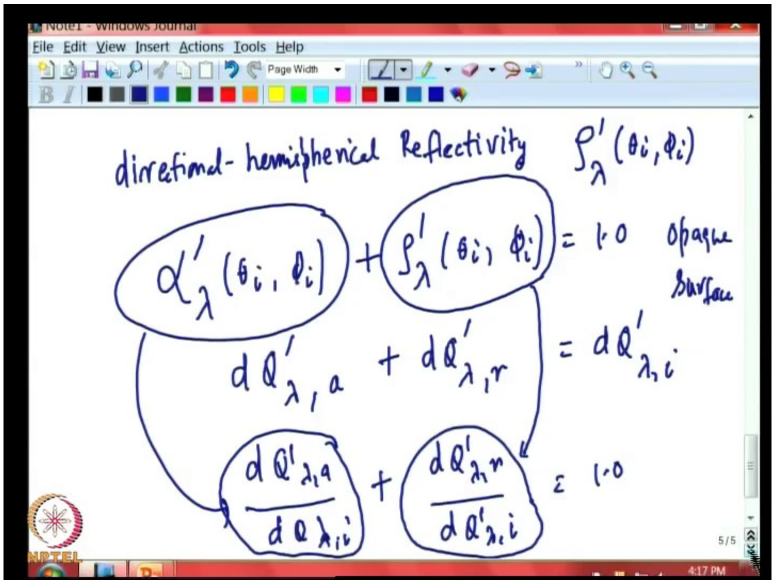

We will define now, the quantity called directional hemispherical reflectivity. The symbols for this is one prime , it will be a function of only of incoming radiation, because we are integrating over all out going angles. We are not concerned about where the radiation is reflected. We want to know what is the total radiant reflected over all angles or an entire hemisphere about the surface. The reason why this quantity is very important is that, this follows from first law of thermo dynamics. Suppose we have a direct spectral absorptivity for radiation coming in at angles θ_i, φ_i. This plus energy reflected in all directions, is equal to 1. So, this follows from the first law of thermo dynamics of total radiation impinging on an object. Let us call that as $dQ'\lambda_i$., Certain radiation is absorbed and certain radiation is reflected in all directions.

We can see that this quantity $\acute{\alpha}_\lambda$ is nothing but ratio of the energy absorbed to the energy incident. The directional spectral reflectivity or directional hemispherical reflectivity is nothing but ratio of the energy reflected in all directions to that incident energy. This follows from the first law of thermo dynamics for an opaque surface. If the surface is not opaque, then we will extend this. Now let suppose surface is not opaque, let us imagine here a glass. A glass can absorb radiation , reflect radiation, and can transmit radiation.

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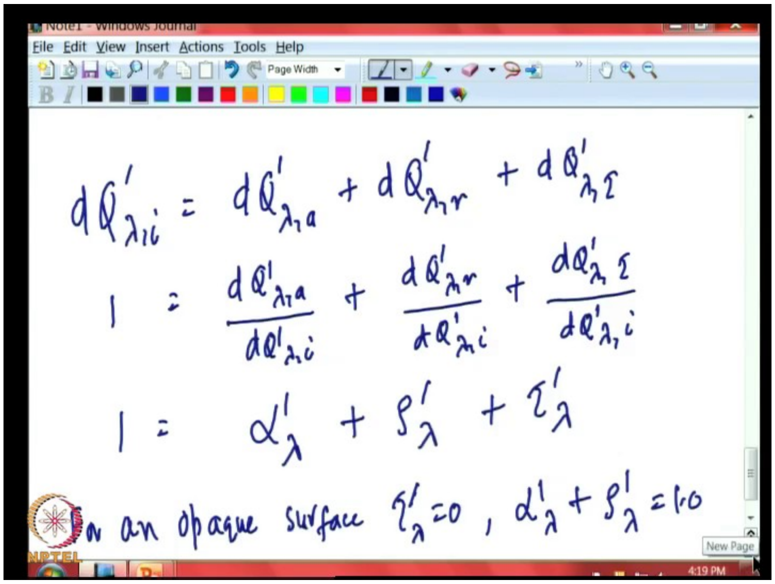

In such a case we must extend this to a total radiation incident, that can either be absorbed or reflected or transmitted. Now we divide by this quantity we get 1. By definition this is $\acute{\alpha}_{\lambda}$, ρ'_{λ}, and this by definition is τ'_{λ}. We are saying that the fraction of radiation absorbed, the fraction of radiation reflected in all directions and the fraction of radiation transmitted has to be equal to one. For an opaque surface, we have $\tau'_{\lambda} = 0$. Therefore, $\acute{\alpha}_{\lambda} + \rho'_{\lambda} = 1$.

This is a very useful equation because it is difficult to measure absorptivity directly by measuring these two quantities. It is easier to measure reflectivity. So, by a measure reflectivity, one can infer what is the absorptivity from this equation.

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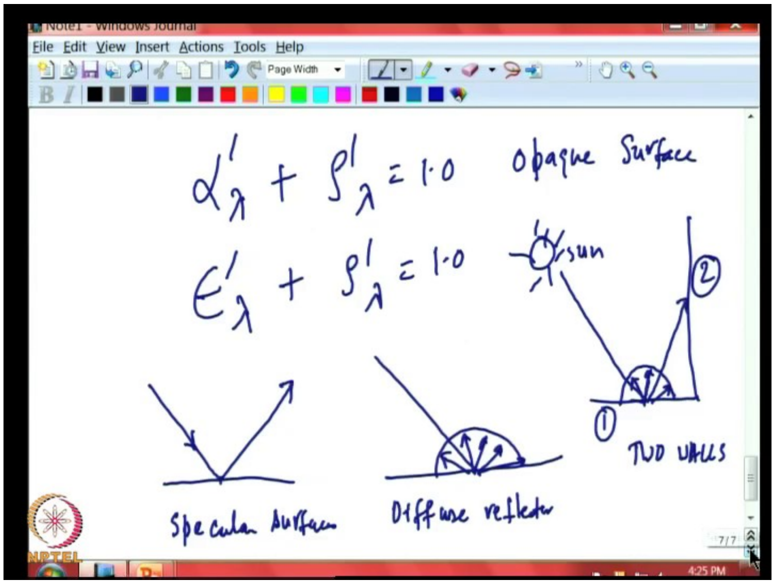

Now we apply Kirchhoff's law to the above equation for opaque surface. We get $\epsilon'_{\lambda} = \acute{\alpha}_{\lambda}$. We can see that, we can infer the emissivity of a surface or the directional spectral emissivity, by measuring the directional hemispherical reflectivity. This equation is used frequently to infer the absorptivity and emissivity of surfaces without directly measuring them, but by indirectly inferring them by measuring reflectivity This is quite common because instrument for measuring reflectivity are available and we measure those values and from there one can infer the emissivity and absorptivity of the surfaces. But reflectivity which is a complex quantity, but here we are dealing only with directional hemispherical quantity, which is similar to absorptivity and emissivity.

But the real surfaces for example, can reflect like a mirror. These are called specular surfaces. They follow the Snell's law which we have learnt in our schools. They are surfaces which reflect equally in all directions called diffuse reflector. But all real surfaces are in between that is they are neither purely specular, nor purely diffuse. In a real surface part of the reflection will be similar to a mirror. The remaining part will be varying with direction.

The real challenge is to characterize such a surface. We find the average over all outgoing angles and get the directional hemispherical value. Later in this course we will see, that there are some situations wherein it becomes important to know the direction in which radiation is reflected. For example, suppose we have two walls adjacent to each other and radiation coming from the sun impinges on this wall. We want to know how much radiation hits this wall.. If radiation coming from the sun hits wall1 then we would like to know how much is reflected inside the house.

For that we must have a detailed account of how the energy is reflected by the pavement in front of the house. Depending on that directional reflectivity, one can estimate how much of the reflected radiation enters the house. For such a situation one needs a detailed account of how much energy is reflected in one direction by such a surface.

On the other hand we are only interested in understanding the temperature of wall one. The temperature of wall one does not depend on how much energy is reflected in which direction. It mainly depends on how much radiation is absorbed from the sun and how much radiation is reflected in all directions. For estimating the temperature of wall one, we do not need detailed information about in which direction radiance is reflected. But to estimate the temperature of wall two, we need to know what fraction of radiance of the sun is reflected in which direction.

We can clearly see that depending on the problem and situation certain information is very necessary or in some other situation, it may be not necessary at all. In certain situations we do not need details about where radiation is reflected. But in certain other situations, we have to worry about where the radiation is reflected. Now before we get into these applications, we need to now have an understanding about emissivity and absorptivity. For that we take an example which illustrates the difference between absorptivity and emissivity.

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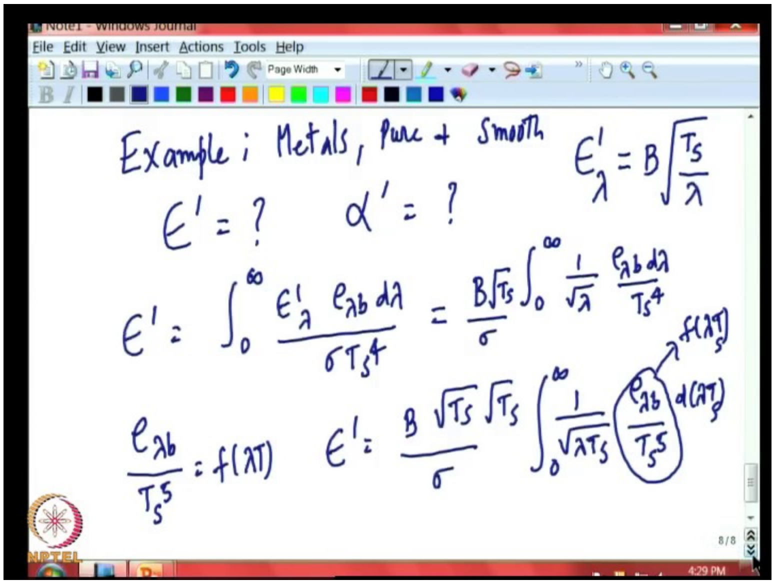

We take as an example such as a metal. Metals which are both pure and has a smooth finishing is taken. One can show that, for such metals the directional spectral emissivity can be written as some constant 'B' which is the property of the metal, times the square root of temperature T_s of the surface and inversely proportional to the square root of the wavelength λ. It is an approximation that is not always valid. Let us assume that this is valid for a surface. One wants to know if this is given what is the value of the directional total emissivity and what is the value of directional total absorptivity.

If we recall, the directional total emissivity can be written as an integral from 0 to ∞ $\epsilon'_\lambda e_{\lambda b} d_\lambda / \sigma T_s^4$. This is the basic definition of directional total emissivity. We can take the B out as B is the constant. We can rewrite this as T_s is not a function of wavelength. We write this as an integral of 0 to ∞ $1 / \sqrt{\lambda} (e_{\lambda b} d_\lambda) / T_s^4$. We also take the σ out.

If we recall from the previous lectures, one of the interesting properties of $e_{\lambda b}/ T_s^5$ is a function only of λT. This was the useful result you can infer from the basic Planck's black body formula . Using that we will rewrite down ϵ' as $B \sqrt{T_s} / \sigma$. Now we want this to be a function of lambda T only. So, we multiply numerator and denominator by $\sqrt{T_s}$. We get integral 0 to ∞ $1 / \sqrt{\lambda} T_s (e_{\lambda b}/ T_s^5) d (\lambda T_s)$.

We notice that this thing which is circled here is the function of λT only. This quantity will be some number.

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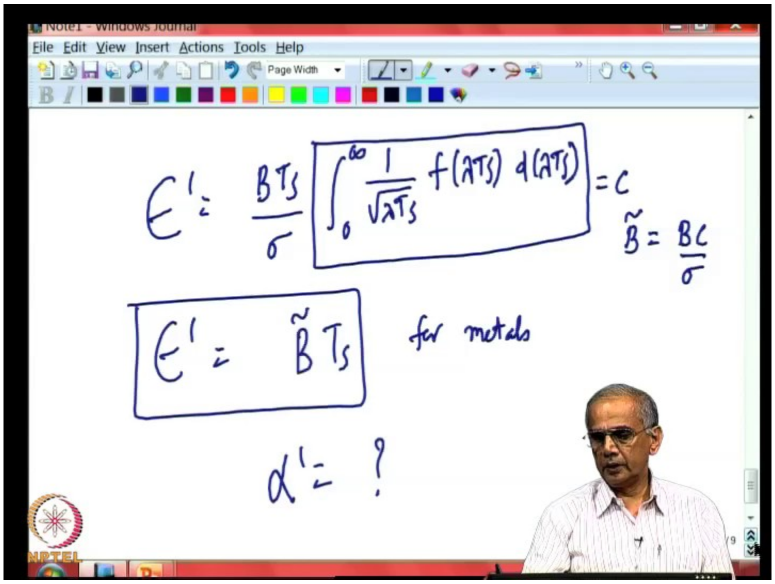

ϵ' in this case is $BT_s / \sigma \left[\int_0^\infty 1 / \sqrt{\lambda T_s}\, f(\lambda T_s)\, d(\lambda T_s) \right]$. This quantity is some number, let us say is a constant 'C'. We can integrate this function as we know this function this is a blackbody function. Let us assume that this C into B is some $\tilde{B}$. So, we define $\tilde{B}$ as B into C for convenience. This quantity becomes $\tilde{B} T_s$. BC / σ is $\tilde{B}$. Finally, we will get a simple result that for metals under certain approximation, the directional total emissivity is linearly proportional to this surface temperature. We will see later that this is shown to be reasonably accurate based on experimental observation. When you plot the directional total emissivity of a metal with temperature in many cases, it falls in a linear assumption which is quite valid.

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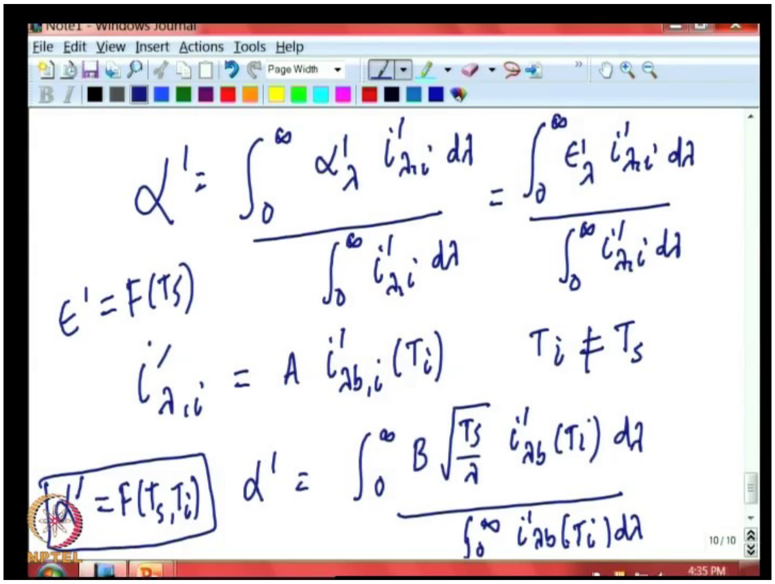

The $\acute{\alpha}$ is little more complicated than ϵ' because as you recall the definition $\acute{\alpha}$ is integral 0 to ∞ $\acute{\alpha}_\lambda$ $i'_{\lambda i}$ d_λ divided by both entire total radiation coming on the entire wavelength. We cannot perform this integration on this equation, unless someone gives us this spectral variation of the incoming radiation. In real life this can be quite complicated, depending upon the environment in which this surface is present and the incoming radiation, which can vary with wavelength in a variety of ways.

Let us take a simple case, where the incoming radiation happens to be let us say some constant A times a black body radiation intensity at some temperature T_i . In general T_i is not equal to surface temperature. So, the equation for $\acute{\alpha}$ is equal to for the metals we are considering; then we write ${}_0\int^\infty B \sqrt{T_s/\lambda}\, i'_{\lambda b} (T_i) d_\lambda / {}_0\int^\infty i'_{\lambda b} (T_i') d_\lambda$. .

What we have here now is the expression, which is the directional total absorptivity and is a function of both, surface temperature and the temperature of the incoming radiation. If it is a black body radiation, this is different from directional total emissivity, which is only a function of surface temperature.

Emissivity is a property of a surface, once we know the type of surface that is metal, non-metal or something else and that the nature surface smooth, rough or other information and its temperature. We can estimate emissivity, irrespective of what radiation is impinging on it. For example, we have a surface of temperature let us say 100^0C inside the room subjective to radiation from the lamps, the emissivity from that surface would be same when taken out to sun light. By keeping the surface temperature same, emissivity will be same whether you are inside the room or outside.

On the other hand for the same surface, the absorptivity of the surface will be different inside the room and outside. Inside the room the absorptivity will depend on the nature of the incoming radiation, the radiation of the lamp or from the wall. While on the outside that surface is subjected to solar radiation and hence its absorptivity will be different. Hence absorptivity is very much dependent upon the environment in which that surface is placed. While emissivity depends only on the surface temperature, the nature's surface metal or non-metal and the surface finish. So, this is a point to remember and to make this clear we have taken this example, but one may wonder as to why absorptivity and emissivity are so fundamentally different; while one depends only on the surface condition and temperature where the other depends on incoming radiation.

The Kirchhoff's law states that the directional spectral absorptivity equals directional spectral emissivity. Now you must remember Kirchhoff's law is applicable at the most basic level, that is the direction spectral level and it is postulated by Kirchhoff that the two are equal at that level, because at the microscopic level absorption and emission are equally probable events. If there is a surface which has a high absorptivity in a given direction in a given wave length, that surface should also have a high emissivity in the same direction in the same wave length. But when we ask questions related to hemispherical quantity or quantity average of all wavelengths at the total quantity, than that quantity will depend upon the nature of the incoming radiation.

So, one must recognize that the directional spectral quantity is connected directly to the microscopic processes that operate in any surface; while the hemispherical and total quantities depend upon the environment and as to where the surface is placed. So, Kirchhoff's law is rigorously valid only for directional spectral quantity. It is not applicable without approximation to hemispherical quantities or spectral average quantities. Now let us proceed further on this derivation.

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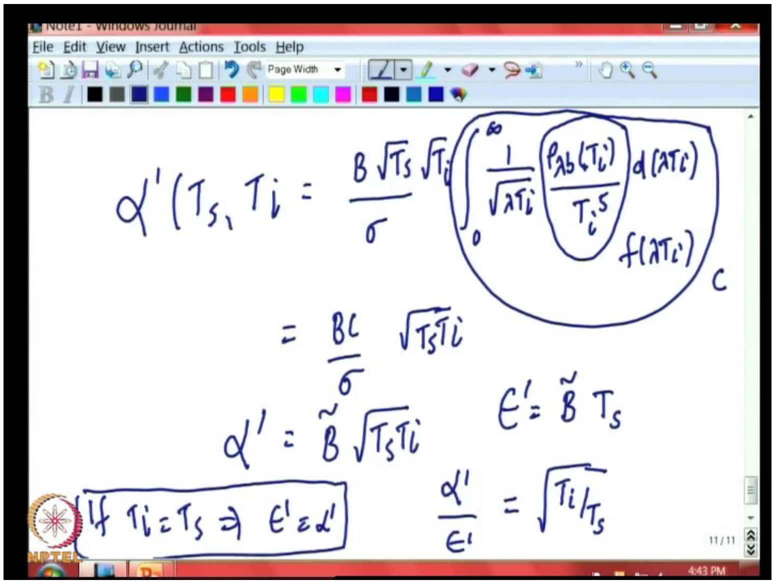

We find that $\acute{\alpha}$, which is now function of T_s and T_i is equal to constant B. We can bring $\sqrt{T_s}$ by σ out. Now we will have to multiply by $\sqrt{T_i}$ which is necessary because the black body function is a function of T_i here. We have $_0\int^\infty 1/\sqrt{\lambda}\ T_i\ e_{\lambda b}\ (T_i)\ /\ T_i^5\ d(\lambda T_i)$. We compare this particular expression with what we use for emissivity, that is T_s. T_i is a function only of $\lambda\ T_i$. So, the entire quantity is a number. This number is same as the number obtained earlier 'C'. Because this integral does not depend on T_i or T_s here because we integrate from 0 to ∞. If we evaluate this quantity numerically, we will get constant 'C' same as before. So, this can be written as (BC / σ) $\sqrt{T_i\ T_s}$.

B's we have already defined as $\tilde{B}$ so we will show now that $\acute{\alpha}$ is $\tilde{B}\sqrt{T_i\ T_s}$, while earlier we showed ϵ' is $\tilde{B}\,T_s$. . So, this is the fundamental difference between directional total emissivity and total absorptivity. Emissivity is linearly propositional to the surface temperature. The directional total absorptivity in this case is d propositional to square root of surface temperature and the temperature of the incoming radiation, if it is a black body. This is a very important result as we can see that the ratio of absorptivity to emissivity of this surface is equal to $\sqrt{T_i / T_s}$. So, this is a very important distinction we must make.

This is a very important result because it is showing that the directional total absorptivity is not equal to the emissivity; and the ratio depends upon the surface temperature and the incident radiation. Also we can see, that if $T_i = T_s$, then ϵ'=ά. But in general they are not equal. This is the reason why the derivation was done, to illustrate and to convince one that in general the total absorptivity of a surface is not equal to total emissivity. It only occurs in rare situation like a gray surface, that is surface whose radiative properties are independent of wave length.. Where the radiation is coming in is such that it is similar to that of a black body at the surface temperature that is $T_i = T_s$. . In this lecture we discussed in detail the Kirchhoff's law and we showed clearly that the equality of absorptivity and emissivity in general can only be obtained under the very special conditions and we set out those conditions clearly. In the next class we will illustrate the importance of the Kirchhoff's law in practical applications like solar collectors. Thank you.