Radiation Heat Transfer
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Lecture - 27
Radiation heat transfer during flow over flat plate

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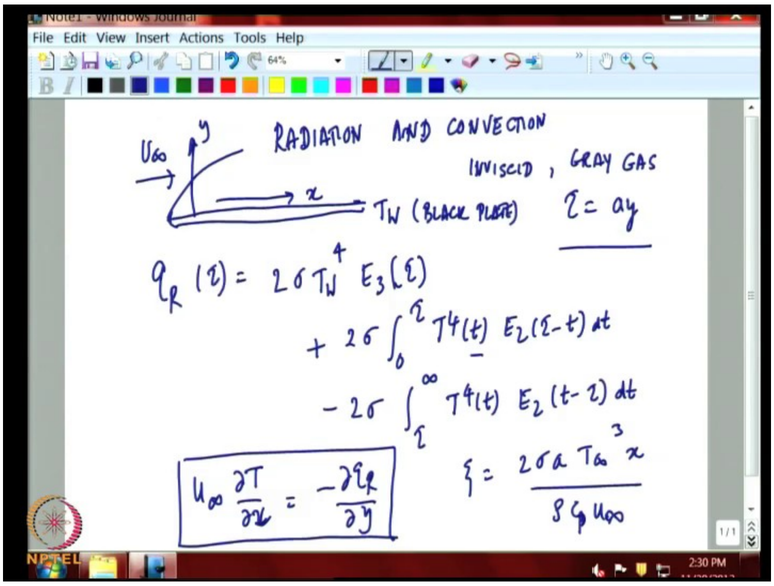

In the last class, we are looking at interaction between radiation and convection. We considered a very simple problem of a flow over a flat plate. We looked at a very elementary example, where the flow was inviscid that is effect of viscosity were not important and the gas was gray.

This of course a very ideal example, not related to a real world, but it illustrates clearly the role that radiation plays in convective heat transfer. The radiative flux is varying in the y direction and the flow is going in the x direction. The radiative flux variation in the y direction was written in terms of tau, the optical thickness. This is nothing but a into y; where a is the absorption coefficient. If T wall is the wall temperature, we consider a black plate, then the radiative flux, will be two sigma T to the power of four E 3 of tau E 3 of the exponential integration function represented by the angle of integration plus the radiation from the gas below a given layer and radiative flux from above the given layer, we go to infinitely thick region there.

This is the radiative transfer equation. The corresponding energy balance equation that we are trying to solve, of which the q R expression is right here. But in order to work in a non-dimensional frame we transform both the x coordinate, the y coordinate of non dimensional form y coordinate is already non dimensionalize by the absorption coefficient. The x coordinate, we non dimensionalize it as follows: two sigma a T infinity cube x by rho C p infinity, where we have linearized equation because we remember that convection depends linearly on temperature. Well, radiation depends on a temperature to the power of four. If we want to get a simple analytical solution, we need to linearize the rate transfer equation.

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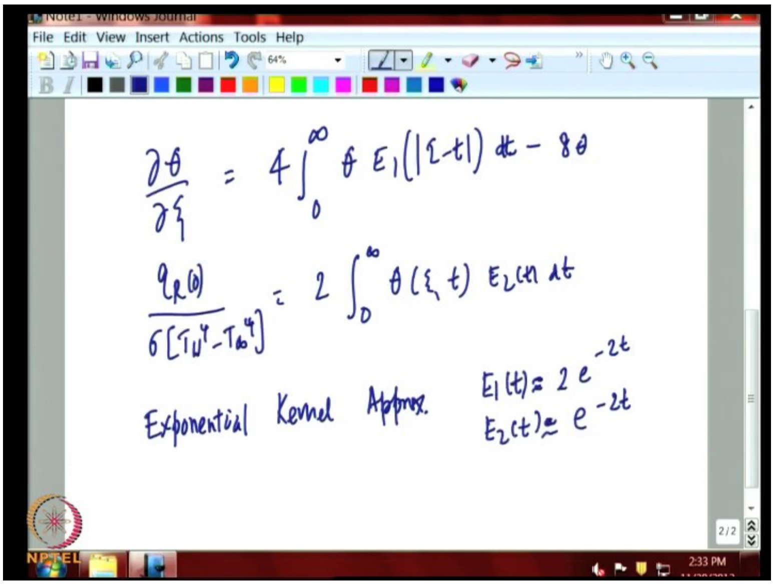

Once we linearize it, we get the following expression for the energy balance. This expression was obtained by linearizing the radiative transfer equation and differentiating it, and if you look at the expression, we will find that the important parameters that comes into play here is the relative role played by radiation versus convection.

We will now solve the equation now, now solve the equation. Before that, we will also mention the wall flux. The wall flux is a very important expression that we need to obtain. That can be written as the ratio of the wall flux without radiation, without convection. Once we have solved for theta, we substitute here to get the wall flux.

Now, the standard way to solve this problem is what we had already discussed earlier; is the “Exponential Kernel approximation”. In this case we will take a slightly different form than what we used earlier. E one of t is assumed to two e to the power of minus two t and E two of t is e to the minus two t. Now, the way this approximation is done that E two of t is a good approximation at t equal to zero here; because it is one. But E one of t, at t goes to infinity, but only we have taken only value two. Now, this is still a good approximation because this goes into the good interval. When we integrate E 1 to give E two, then we will find that it will not really affect the answer too much.

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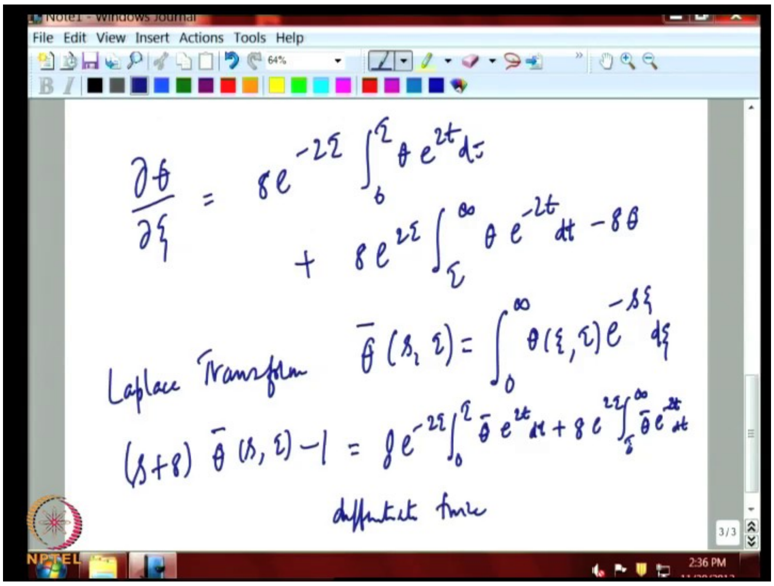

Once we have done this, the expression for temperature becomes the expression that we have for variation of temperature after it is linearized. We have done the kernel approximation. In order to solve the equation, we apply the Laplace transform. Many of us would have used Laplace transform in transient conduction or convection problem.

The transient term here really is psi here because it involves psi is related to x and infinity and it is essentially a virtual time coordinate. Now, Laplace transforms of theta; it becomes theta bar in a function of s and tau. By the definition of Laplace transform by zero to infinity theta of psi; the time like coordinate. This is how we transform theta of psi and tau to theta bar of s n tau. Then, we apply the standard rules of Laplace transform for differentials and so on. We get this expression. This is the one definition, which gives you one at a time equal to zero.

The right hand side becomes theta eight e to the power of minus two tau zero to tau theta bar zero to two t plus eight e to the power of two tau to infinity theta bar e to the power of minus two t d t. If we now apply the Kernel approximation; that is differentiate twice and eliminate the integral terms.

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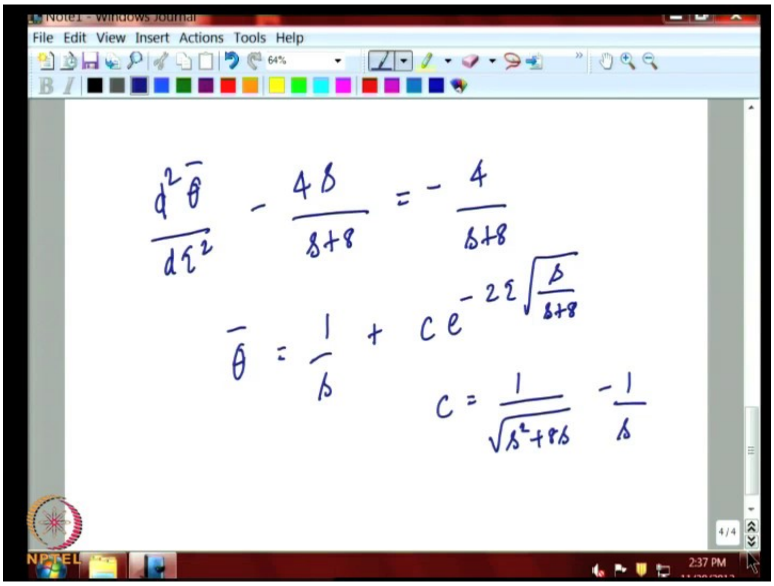

We will get the following differential equation for theta. This is the final expression we get after Laplace transform and Kernel approximation and eliminating the integral terms. One can show that easily that the solution to this equation o d really; ordinary differential is one over s plus C e to the power of minus two tau rou of s by s plus eight. Where, the constant C can be shown to be one over s; this simple o d and so this is the non emergence term, where theta is taken as one over s, not a function of tau. This term drops out. We get this equal to that and, this is the solution of the homogenous equation. We need to transform this result in to the psi coordinate. We apply inverse transform.

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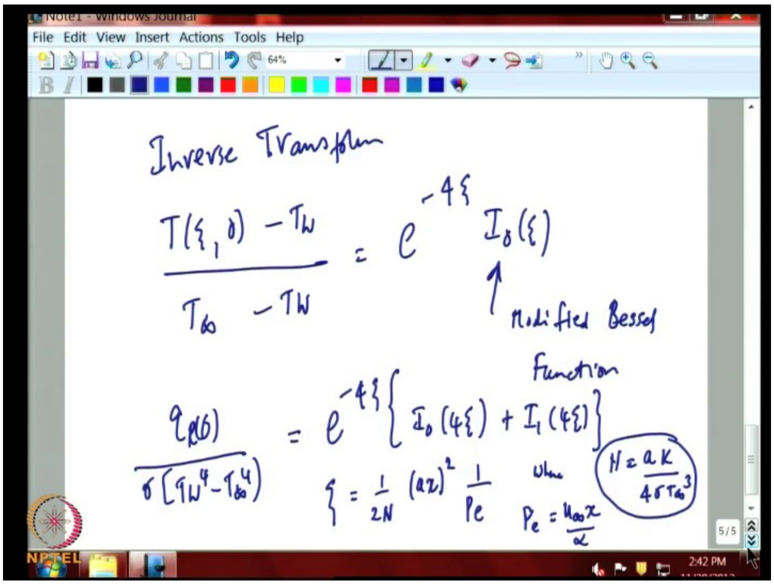

After applying the inverse transform of the Laplace expression, which is available in many text books, you got the following result for theta. This is the Bessel function or more correctly a modified Bessel function. This is well known and available in both text books.

Once we got that, we substitute it back in the equation for flux we get the flux at the wall, which is what we want in terms of two modified Bessel functions. We can see that we are able to obtain an analytical solution for the radiative flux at the wall and the temperature distribution in terms of exponentials and the Bessel function. Now, if you want to understand what this psi; psi can be rewritten in terms of the conduction radiation parameter, which we discussed earlier the length scale and what is known as Peclet number in heat transfer.

It is an expression where N we already defined as a into k by four sigma T infinity cubed, which came up in the conduction radiation problem; T as the conductivity. .The number is nothing but u infinity x by alpha. So, finally the functional dependence of temperature and flux depends upon the conduction radiation parameter here; in terms of absorption coefficient and in terms of the conductivity of the gas constant and temperature of the gas in the infinity the Peclet number.

As we can see in the limit of a very large Peclet number, this constant is going to be very small. We can think of simple results for the convection transfer. We will be able to recover some other solutions we obtained last time for radiation and convection based on series approximation.

The series approximation is valid, when size is very small. When the size is very small, that is, the N is very large or spectrum is very large, then this becomes very small. Then, we can expand this i zero, i one and e to the power of minus four psi in terms of linear terms. Then, we will get the result, which we had obtained earlier using this series approximation.

This is more generalized result valid for psi. We can look upon the result, which we got in the last class are the special case of the result., which is a joint solution which combines Laplace transform and the Kernel approximation to get the solution valid for all psi. Only because of this solution if you look at it, is the gray gas approximation which will limit its use for any practical application; but, still if we want to get a quick idea about how strongly the convection influences the radiation, this is the starting point to decide whether we want to do a more accurate or a more elaborate calculation of interaction between the radiation and convection.

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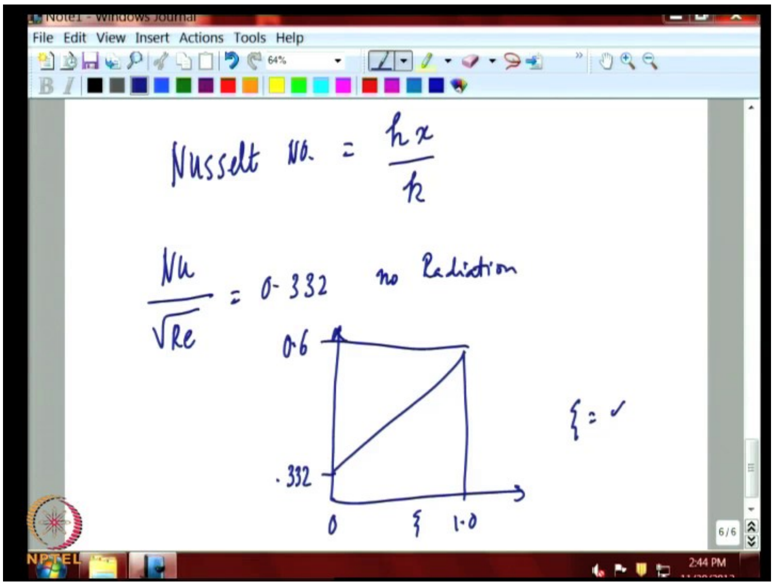

Now, just to give a flavor of the kind of effect this result has, we will look at Nusselt number. All of us have heard of the Nusselt number; which is a non dimensional measure of the heat transfer coefficient. In this case, the Nusselt number will be defined by h x by k; where h is the transfer coefficient and x is the horizontal scale, and k is the conductivity of the gas. And, many of you would be aware that for the normal flow over flat plate this typically comes out as 0.332; when there is no radiation.

If there is radiation, this is going to be modified. If you plug those numbers in to the expression we are given just now, you will see that when size is very small, we start at 0.332 because that is in the limit as radiation being unimportant. As we go to larger and larger psi, it goes on increasing and when you reach psi around one, we can reach value around 0.6. So, what the radiation does is it modifies the temperature distribution such that, the gradient of the temperature near the wall increases for almost a factor of two, when the psi increases from a very low value to one.

This is a good way to a get a rough idea about impact radiation on convection. In a given problem if you want to decide whether you have to account for radiation, we should be able to estimate the value of psi for the given situation. And, if the psi value is indeed of the order of one, then we should realize that the Nusselt number or the heat transfer coefficient by convection increases to almost factor of two in the presence of radiation transfer. This can be simple analysis which is useful in dealing with problems in which there is some interaction between radiation and convection.

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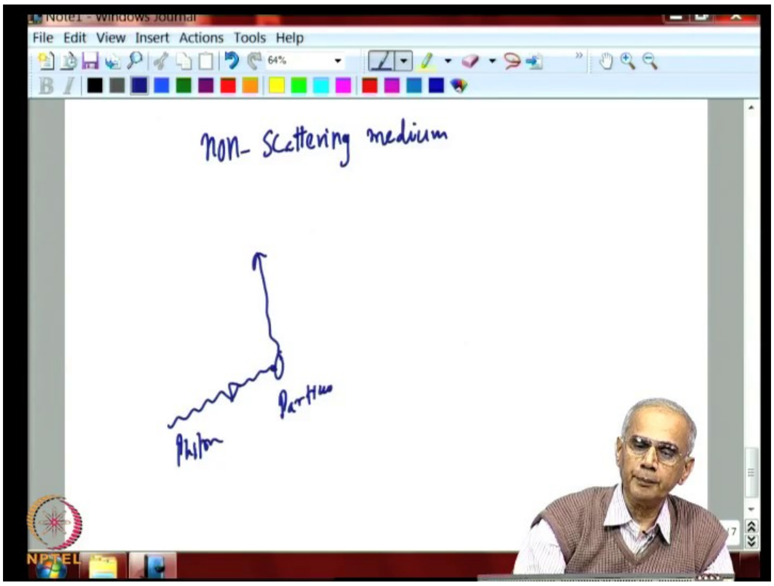

Now, in all the problems you have done so far, we have neglected scattering. We have always considered only a non scattering medium. That is a serious limitation because there are many examples both in Engineering and in atmosphere where the presence of particles, liquid or solid tend to scatter the photon. That is, photons change the direction. That was non incorporated in our discussion so far. We have to assume that the gas was a pure absorber and not the scatterer. The reason was scattering is treated at the end of this lectures because it makes life very complicated. So, photon which is traveling in one direction encounters a particle, and that particle now will change the direction in which photon is going.

The presence of particles in a medium fundamentally alters the direction of movement of the photons. That makes this geometry problem much more complicated. Remember that, so far we have deal with the non scattering medium and we could generally treat direction effects through some kind of Kernel approximation through the mean direction and calculated it. We have got very good answers as we saw just now for the convection problem. But, when the scattering is very important, then we cannot treat directional variations very casually. We will now look at scattering.

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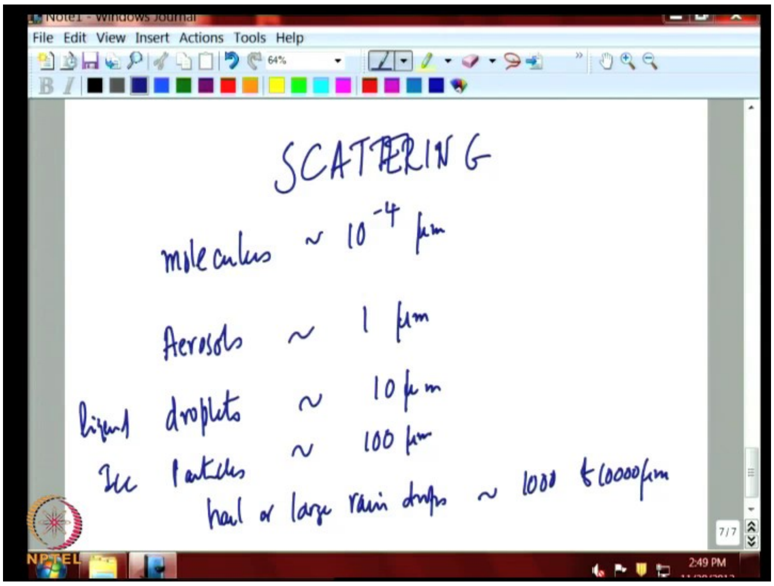

Now before getting in to the basic scattering, let us now first discuss what the scatterers that are present are. The smallest scatterers are molecules, obviously. The molecules have dimensions of the order of ten to the power of four micron. That effect is not really very large, except at certain wavelength. On the other hand there are things called aerosols, which are liquid and solid particles in the atmosphere or in the combustion chamber; soot is one example. Their dimensions are typically around one micron. Since the dimension of the aerosol is about ten thousand times larger than the dimension of molecule, it has higher scattering effect.

Even larger types of scatterers are droplets, which can easily go to the size of ten micron, if it is a liquid droplet; Rain droplet, for example. Then, you come to ice particles. They can be even larger dimensions than the ice droplets. Finally we have hail, a large rain drop. This can easily reach of the order of thousand to ten thousand microns.

We find is that, the particles that are there in the gaseous medium that we encounter either in Engineering or in the atmosphere or in the earth's atmosphere can range from ten to the power of minus four micron to ten to the power of plus four micron. This is range of particle sizes. As we will realize, their effect on the radiation are different.

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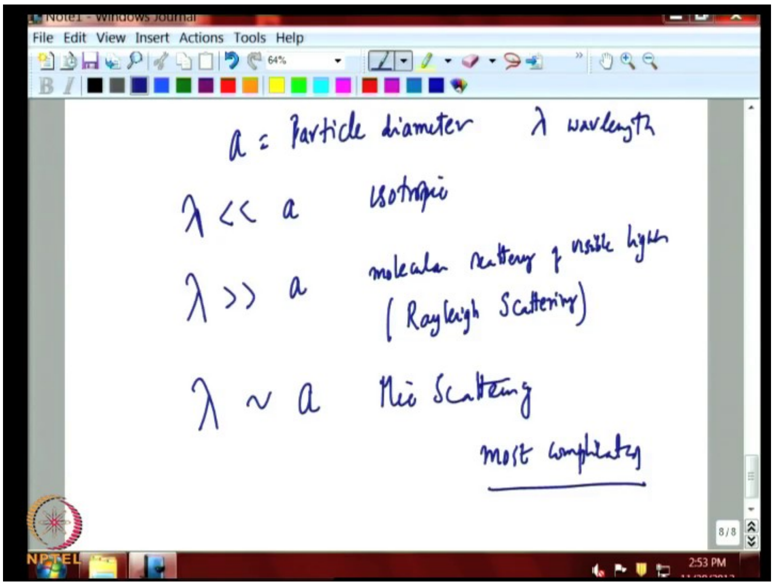

The typical example that we will look at now are if we consider 'a' as the particle diameter and lambda is the wavelength of light which is being scattered. Then, if the wavelength of light is much smaller than the particle diameter, the scattering is isotropic in independent direction.

This is the easiest case to deal with because we do not have to then worry about directional effects. On the other hand, if the wavelength of the light is much much larger than particle dimension, this happens in the case of molecular scattering of visible light. Then, what we get is what you must have heard of in your Physics classes is called

Rayleigh scattering. This is something which we all learnt in our earlier courses. Finally, that wavelength of photon is comparable to dimensions of the object concerned; we get what is known as Mie scattering.

We want to appreciate the fact that, if the wavelength of light is very small compared to the dimension of the object, then life is very easy. Scattering is pretty much isotropic. We will show that the treatment of isotropic scattering makes the problem very very simple. On the other hand, if the wavelength of the photon is much larger than the dimension of the particles which are scattering like the molecules, so we will have visible light and which is of the order of 0.7 micron and molecule of the size obtained by minus four micron. Obviously, lambda is much greater than a and we get the standard Rayleigh scattering, which we will discuss little later. But, many many cases that we encounter in both and Engineering and our atmosphere is where the wavelength of the radiation is comparable to the size of particle. In which case, a Mie scattering which is the most complicated. This is most complicated case this is more difficult to calculate and until the advent of the computer, this problem is not easily handled at all.

Today because of the available computers, we can handle Mie scattering quite easily and involves a lot of calculations. But, these calculations are fairly straight forward. Now, let us illustrate how we handle scattering in a personal simple way before we go on to more accurate treatment of scattering.

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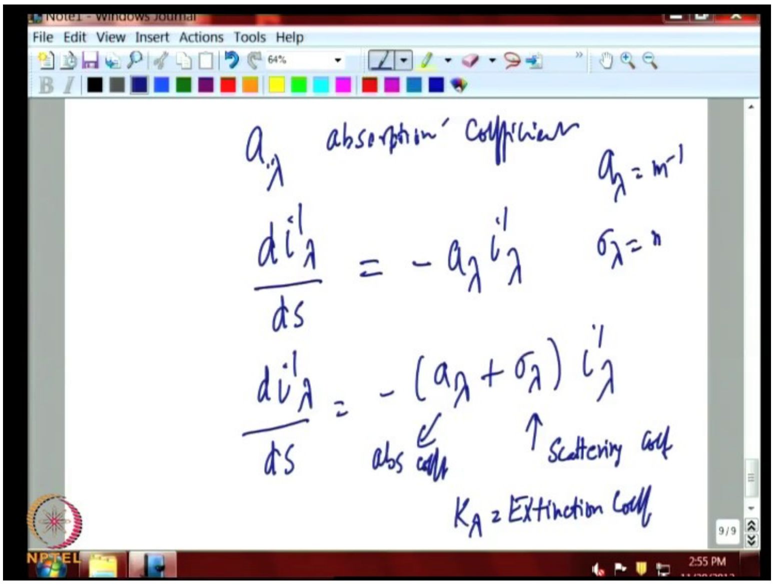

Now, we have defined the absorption coefficient. An absorption coefficient was, if we recall related to the fact that the intensity along a certain path for the minus a lambda i prime lambda. This is how we define the absorption coefficient. Now, when the absorption and scattering occur together, then the attenuation is both due to absorption and scattering. We need one more term in the attenuation term. This is scattering coefficient now. This is absorption coefficient which we have discussed earlier. The combined thing is called the extinction coefficient.

The extinction coefficient is sum of absorption and scattering. The photon can disappear from the given direction, either because it is absorbed, it is gone or it has been scattered and hence does not appear along the given direction. Extinction coefficient is both due to absorption or scattering. Now, the units of absorption coefficient all of you know are meter minus one. That is the same unit for the scattering coefficient.

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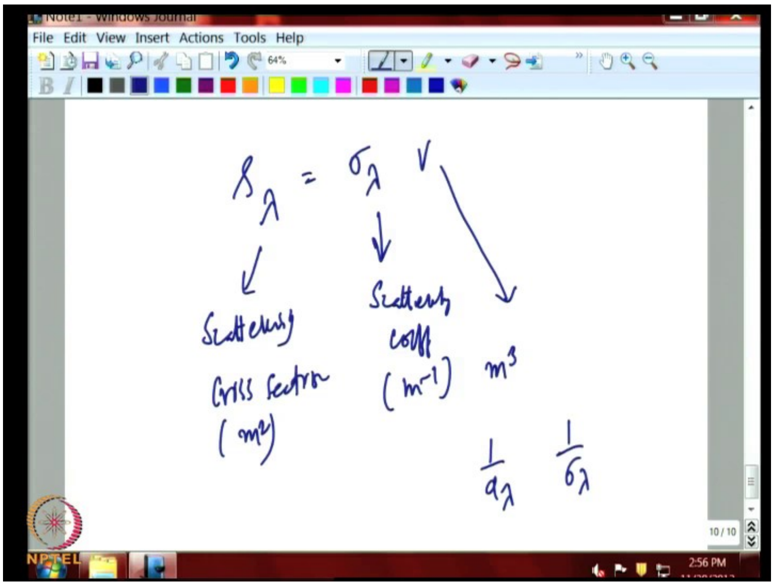

But, the scattering coefficient also has got other standard way as refining, in terms of cross section. This is scattering cross section defined in terms of scattering coefficient, which has units of meter minus one and volume which has units of meter cubed. This gives you meter squared. The cross section is a very popular term used both in Nuclear Engineering as well as in radiation. It has a physical meaning. It is the effective area that the object offers to the incoming radiation of a T obstruction in some sense.

So, many people prefer to use cross section, but we prefer to use absorption coefficient and scattering coefficient; because both absorption coefficient and scattering coefficient, when you take the inverse ratio you get the typical photon mean free path. That is convenient way to understand the role of photon.

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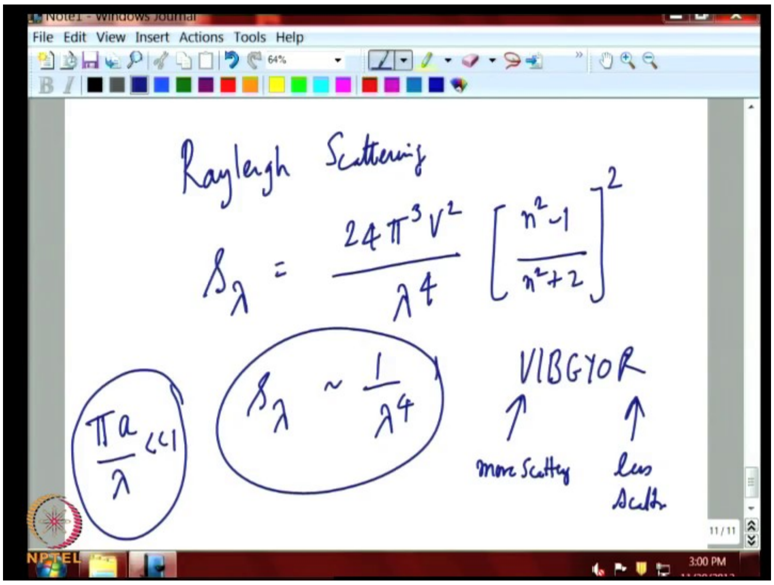

Now, we will give a simple example of the Rayleigh scattering, which all of you have heard about in your Physics courses. Rayleigh scattering occurs when visible light is scattered by molecules. The scattering coefficient there and can be written in terms of a volume of the scattered and the refractive index of the particle.

This is a fairly well known expression for the Rayleigh scattering, showing that the Rayleigh scattering goes as one over lambda to the power of four. Many of you have heard that this is what makes the sky blue in color because the molecules of atmosphere which scatter the sun light which is in visible photon points of micron tend to scatter. If you look at VIBGYOR of sun light, there is more scattering of violet light than red light. More from the white light, lot of violet is removed and what is remaining is red.

That partly will explain, during sun set why you see a red sun because during sun set the amount of particle, molecules between you and the sun is much larger and many of the photons in the Violet, Indigo, Blue, up to Green are removed the sun tends to look more orange or red in color. So, Rayleigh scattering is a very simple example. But, remember this Rayleigh scattering works only if the diameter of the object is much smaller than the wavelength of the light.

It only applies to scattering of light by molecules. But, if we look at all of the particles we discussed like the scattering by soot particles in a furnace, scattering by liquid droplets in the atmosphere, scattering by ice particles, there this equation will not be valid. This is because we cannot satisfy this condition that the radius of the particle is much smaller than the wavelength of the light. In such a case we have to deal with Mie scattering.

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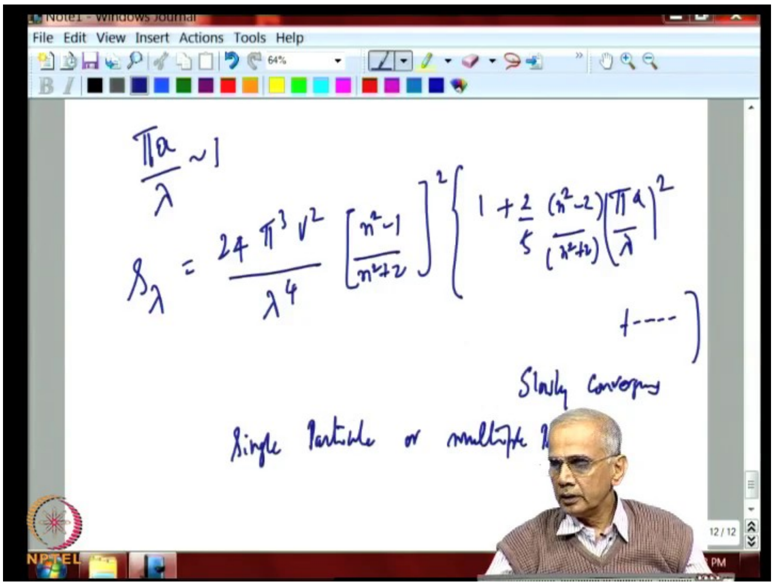

In Mie scattering we expand; the problem is that the radius of the object is comparable to the wavelength of light. Our Rayleigh scattering value is a leading term and expansion. Then we have many more terms which we have to deal with. The series expansion terms, many many terms and higher the value of pi lambda, we have to take more terms in to account. This is the fairly difficult expression to calculate slowly converging infinite series. So this calculation normally is very time consuming. We will not find simple results for a Mie scattering in any text book because it is much more complicated equation that we have to deal with.

Now, another problem that comes in scattering which we do not normally consider is, whether your scattering is by a single particle or multiple particles. As we can imagine this carrying a single particle, it is very easily to a take care of; only one particle and the photon are influenced by that particle. If there are many many particles in a given volume and if they are very close to each other, then there can be interaction between the particles, scattering with the other. The expression we give here for both Rayleigh scattering and Mie scattering is for a single particle, completely isolated. Then this expression is easy. But, if there are many many particles, then the photon scattering from one particle will come and interact with another particle. The problem is much more complicated. We will not have an occasion to look at such complicated cases, but we can mostly show that, today because of the availability of the computer we can actually tackle this problem in computational or proper software that is used for computation of these complicated scattering case. Now, let us look at how the basic equation of transfer is fundamentally altered by the presence of radiation.

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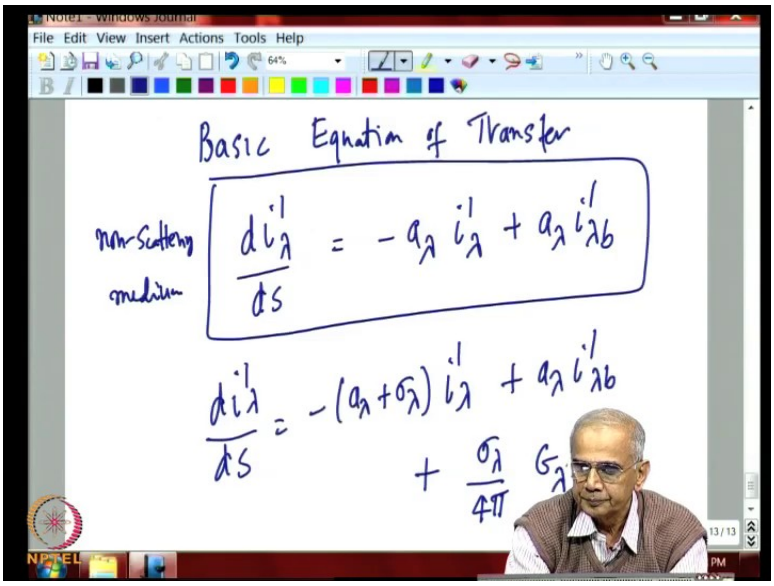

We started with our basic equation of transfer for radiation. If we recall, this expression was as follows for a non scattering medium to begin with. This is the expression we use for a non scattering medium. If we have scattering medium, this expression is substantially changed. First thing we already had indicated that, the attenuation term is enhanced by scattering function. This emission term will not change. There is one much term what is called scattering in that is, the photons from other direction are scattered in to a given direction of interest to you because of the presence of scattering this is written as sigma lambda by four pi times G lambda.

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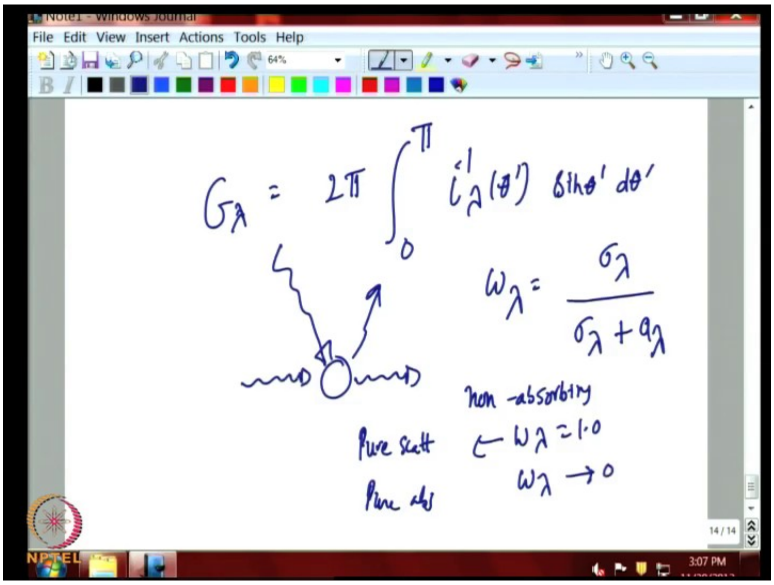

Where, G lambda is defined as two pi zero to pi. They neglect the changes in pi direction, so only the theta direction is concerned with. Especially, this term is going to add photons. What is happening is, we had a photon which can be scattered off by a particle; that is, the first expression. The photon from another direction can be scattered in this direction also. This is the term which adds to your intensity because photon travelling in some other direction has been now started in to the given direction of interest to you. The expression we wrote can be rewritten in terms of what is known as single scattering albedo; which is ratio of this scattering to the scattering plus absorption.

If it is the non absorbing medium, then omega lambda is one; fully scattering medium. If it is a highly absorbing medium this dominates over that, then omega lambda is tending to zero. This is a pure scattering medium; omega lambda is one. This is the pure absorbing medium, where this is much smaller than that.

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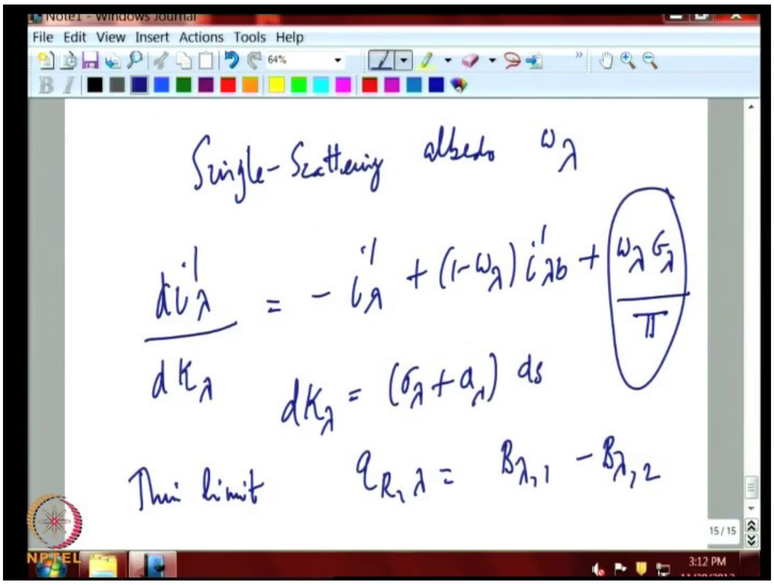

Now, with the definition of this single scattering albedo, we can rewrite our equation for intensity; where, kappa lambda is nothing but sigma lambda plus a lambda into d s. This is the extinction coefficient. Now we see the expression very clearly showing the role of the attenuation absorption. This is the emission term and this is the scattering term.

So we say that, in a non scattering medium omega lambda is zero; this goes away, this goes away. We are back to the original expression. But, in a medium which both scatters and absorbs radiation, we are now linking the intensity in a given direction of wavelengths to not only to the incoming intensity and the intensity of emissive black body, we also let it to another expression called G lambda, which actually connects all the other directions to a given direction.

We solve the equation because in the previous approximation made by the scattering, the photons travelling in the other direction did not affect over result. It will be mainly concerned with photon joining, going in a given direction; whether they were attenuated by the absorption or they were enhanced by emission or by scattering in. But, now the last term is going to bring in the effect of all the other direction on to a given direction.

So, when there is scattering taking place in a given situation that complexity of the problem goes up many many times because now we have to worry about solving the directions pertaining to intensity in all the other directions in order to get the answer in a given direction correctly; because they all are coupled through this term. This term is the most difficult term you have to deal with; because it links the intensity in a given direction to the intensity in every other direction.

When we deal with scattering in more detail, we find that we have to pay much more close attention to the directional nature of the radiation because the intensity in the given direction depends upon the integral of radiations from all other direction. This linkage was missing in the case of pure absorbing medium, in which the scattering was negligible. There, the intensity in a given direction was not linked to the intensity in other direction.

We could treat the angular integration in radiation rather casually, but now we cannot do that. We have to account for it very very carefully. Now, in spite of these complications we can still argue that, in the thin limit when both the scattering and absorption is going to be small, one can still calculate the flux as from surface one minus surface two; so, whether it is scattering or whether it is absorption by both are very small and in optical thin limit we can use that.

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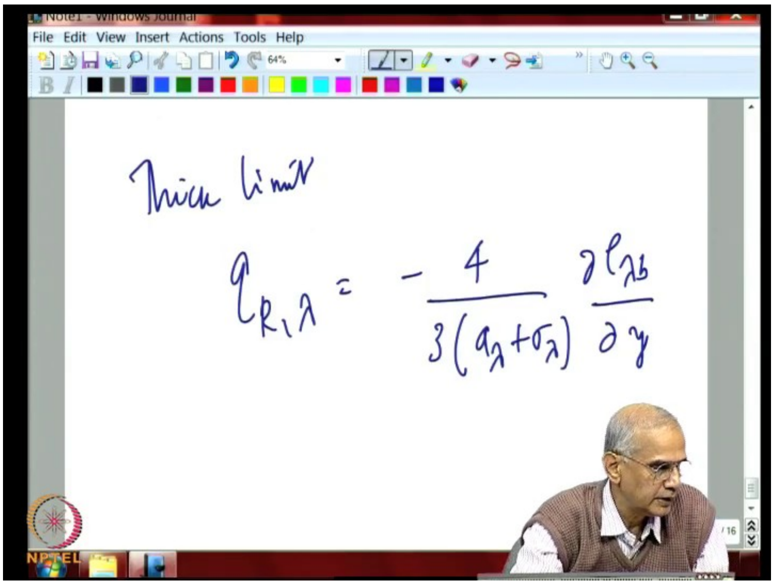

In the thick limit we can use aerosol diffusion model for a scattering medium. We can see that in very simple cases in the thin and thick limit, we are going to easily extend the simple modules for thin, thick limit scattering by nearly modifying the a lambda to a lambda sigma lambda. But, in all other cases we are going to be having a difficulty;

because in the cases were thin and thick limit is not valid, we have to take care of angular nature of radiation extremely carefully.

This will be the part of the discussion in the subsequent lectures as to how we tackle scattering problem, in which the effect of the other angles is felt in the given angle. Now, we compare a more complicate expression to explain what happens in this scattering medium.

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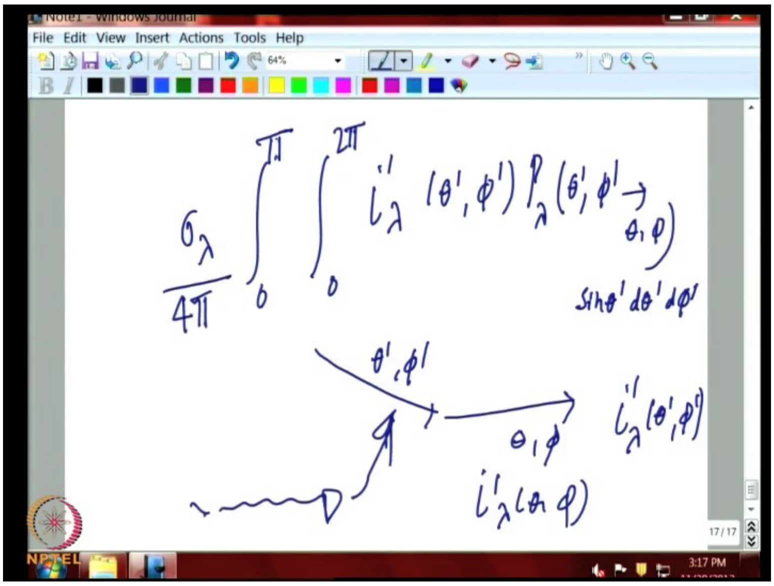

The last term in which the symbol be G lambda can actually be written as sigma lambda. You have to worry about two direction; zero to pi and zero to two pi. That is the azimuthal angle and the other angle. This sigma lambda by four pi, this part we all know. We have to worry about the i prime lambda in the direction different from your direction. The probability of scattering from theta prime pi prime to theta phi; this is the key expression; times sin theta prime d theta prime d phi prime.

So this is the expression, which decides how a photon traveling in a given direction theta prime phi prime lands up in your direction because of scattering. So, scattering is linking the radiation coming in the direction theta prime phi prime to radiation in the theta phi. Now, this linkage is missing in the pure absorption problem. Where, in a pure absorption problem we have merely concerned with photons in a given direction being absorbed by gas molecules or scattered out. There are slow scattering in, in the pure absorbing problem. In the scattering problem, this term which due to scattering change the direction of photon from theta prime phi prime to theta phi. That is what causes a lot of complication; because in out of five, you solve for i prime lambda theta phi. We must not quickly want the intensity in some other direction theta prime phi prime. We have to link the intensity of radiation at a given angle of interest to you to the intensity of radiation in any other direction that is there.

So, all radiation problems that involves scattering demands that you solve the scattering equation very carefully because any simple approximation that you make for the scattering term must be carefully considered; because unless it is isotropic scattering which is a very rare case, in all the other cases the scattering problem is going to make life very hard. Now, we will illustrate that issue of how scattering influences your answer by a simple example here.

So, before we go further in to the example, let us write down the expression for radiative flux in an absorbing, scattering medium. This is essentially the extension of the derivation which we did earlier for pure gas radiation. We will see that in additional to the expression we had earlier; simple expression we had.

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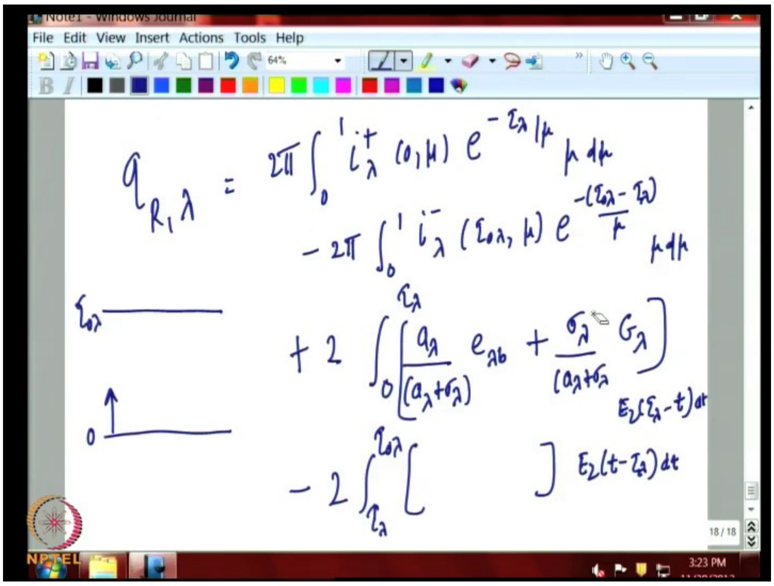

We will have a now much more complicated expression, which relates q R lambda. If we recall the first term that comes from the wall, so we will put a two here and this is from the bottom wall zero. The top wall will be tau zero lambda. First the radiation of the bottom wall has to be integrated over all angles. Taking into account attenuation; this is same as what we did earlier. This part is not anything new. Same way, the second expression for the top term is minus two pi, again zero to one i minus lambda tau zero lambda mu e to the power of tau zero lambda minus tau lambda by mu d mu.

These two are essentially as same as what in last time. Now, the two expression which involve in emission will also involves scattering term. We have two in to zero to tau lambda. We have an expression for in terms of kappa lambda by kappa lambda plus sigma lambda. This is the absorption coefficient by the scattering coefficient. And, so this expression will be now kappa lambda by beta lambda; where beta lambda is kappa lambda plus sigma lambda.

This one involves e lambda b that is the emission term; plus, so we just write it that in a clear way, so that all these expressions what you have seen earlier. Kappa lambda, we will write in terms of a lambda, so that there is no confusion in the notation compared to what we used earlier. The ratio of the absorption the scattering plus ratio of scattering to absorption plus scattering times the G lambda, which is the expression which involves scattering term. The entire thing is of course multiplied by E 2.

Finally, the last term is two tau lambda to tau zero lambda. E 2 of t minus tau lambda dt; so, we want to see how the original expression varies with the flux. It did not have these two terms. Now, we have two additional terms and start the integral now. What makes this very complicated as all of we will appreciate is because there is an additional unknown. There was already an unknown in the problem, which is the unknown temperature distribution between the gas; now, this unknown distribution of intensity in the gas.

We have to solve for both. The problem gets an order of magnitude more complex because the way you treat the angular variation now is much more critical than what it was in the case where there was no scattering. So, basically the pure absorption, emission problems were much simpler to deal with compared to what we have now deal with, when we look at combined absorption and scattering. There will be much more heavy mathematical derivations to take care of that, and so problem becomes lot more complex and much more difficult to interpret the physics of the problem. This we will take up in the next lecture.