Radiation Heat Transfer
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Lecture - 26
Interaction between radiation and other modes of heat transfer

In the lectures we have discussed so far, the role of Radiation Heat Transfer in gases. But we have treated radiation in isolation, but in real problems radiation occurs along with conduction and convection. We need to look at problems where more than one mode of heat of transfer occurs together.

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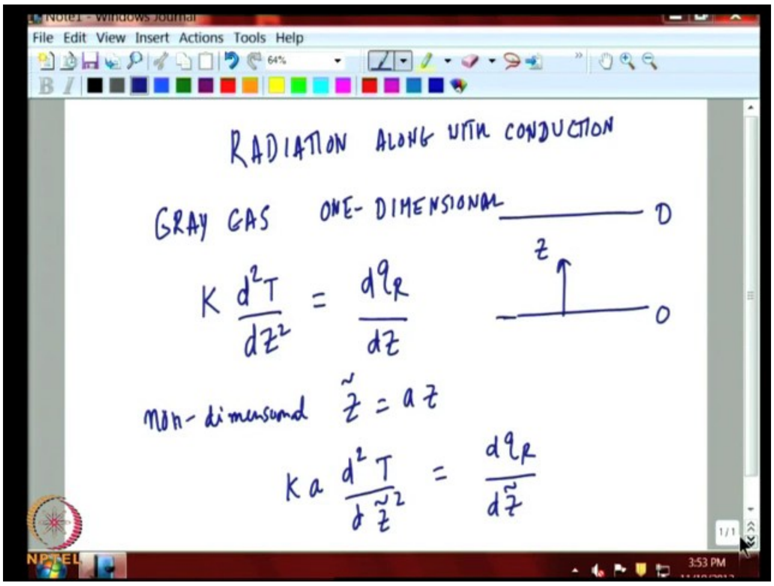

Today we look at radiation along with conduction heat transfer. This is the simplest extension of the pure radiation problem we had encountered, so far. The simplest problem, we can do is assume a gray gas, one dimensional to illustrate the manner in which radiation interacts with conduction. So, the simplest balance is conduction along with radiation, where Z is the vertical coordinate. We have two parallel plates distance D apart with the gas here, which has both radiation and conduction heat transfer.

It would be convenient, to redefine of our z coordinate, non dimensionalize z, as a into z. That becomes z tilde which is non-dimensional. If do that the above equation becomes k a d square T by d z tilde squared is equal to d q R d z tilde. If you want to further proceed, we need to now write down the dimensions of radiative flux, in terms of the integral equation that we had, derived earlier. Now, if we expand this term and write this down in terms of all the terms in the equation the first term of course, is the conduction term.

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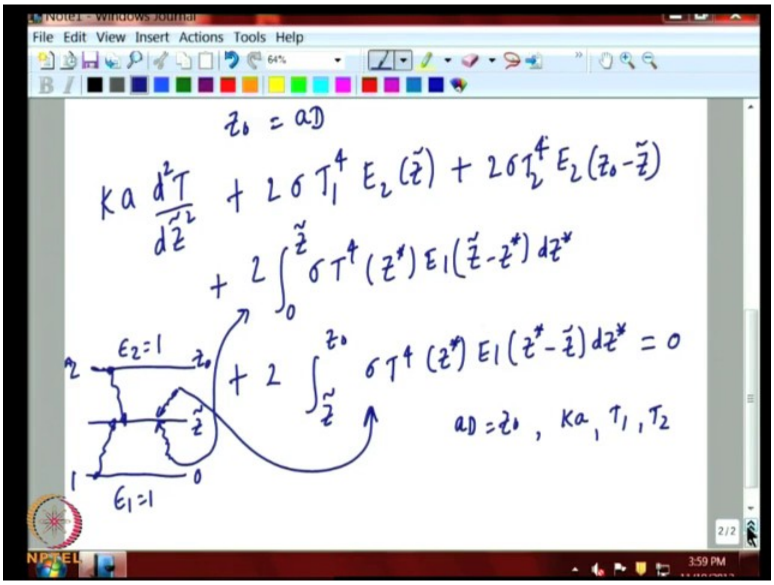

It is already there, then we have, the surface emission from the lower surface absorbed by the gas. Then we have the emission from the upper surface. We will define the z 0 now, z 0 is nothing but a multiplied by D that is you might call it as the total optical depth, measure of the highest non dimensional optical depth in your example. Then we have twice 0 to z tilde sigma T to the power of 4 z star E 1 of z tilde minus z star d z star the next term, twice z tilde to z 0.

This is the conduction heat transfer; this is the radiation which is absorbed from the emission by the bottom surface. Next the radiation absorbed from the emission of the top surface. Third term is radiation emitted by the layers of gas between 0 and z tilde and let us define this in our z 0, this is z tilde, and this is z 0. This is surface 3 and surface 2. Both of surfaces have unit emissivity. So, they are black surface that is what makes this problem somewhat simpler.

This term is radiation emitted by this layer and this is the radiation emitted by this layer, both these are to be understood. There is first term coming from here, absorbed here. When term starting from here, absorbed here, the third one is this one and the fourth one is this one that is what we are trying to balance at the level z tilde. Now, to proceed further it will be good to non dimensionalize the equations because, it enables us to extract important parameter because, ultimately this equation has to be solved on the computer.

We going to solve on the computer, we would like to keep the number of independent parameters that we have to vary to have minimum. right now for example, one parameter we have course is A D non dimensional parameter, then you are seeing k a the parameter coming in here, we have T 1 we have T 2. If we have solve this problem numerically, will have to solve it by various values of A D which is our z 0. That varies z 0 would have range, then you have to vary k a you have to vary T 1 we have to vary T 2.

So, let say we are interested in solving for 10 different z 0s, 10 values of k a, 10 values T 1, T 2 then we can see that, you have to solve the problem about 10,000 times. In spite of the large computing power you might have, this is a lot of exercise and not only will it take time to solve, so many cases then you have the challenge of interpreting the information in suitable way. It is smart to look upon ways to non-dimensionalize equations right here. We need to reduce the number of parameters you need to vary; now we have 4 parameters let us see whether we can reduce it to 1 or 2.

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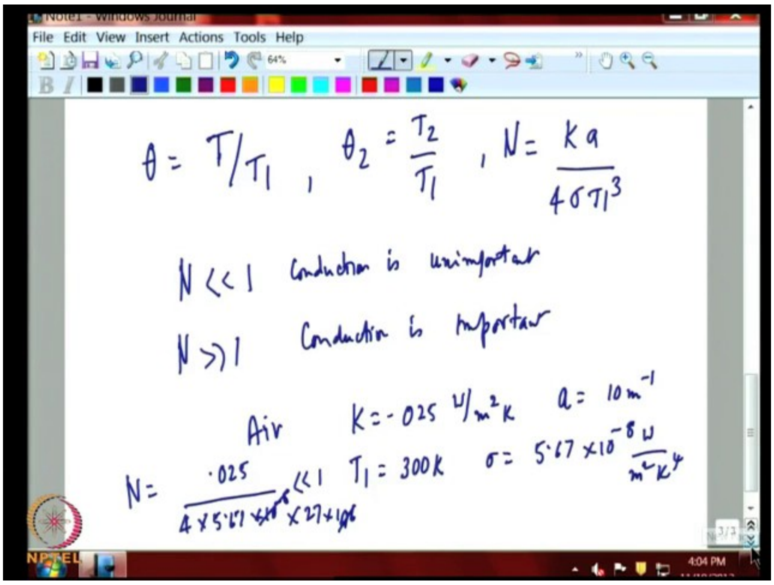

If we look carefully we can see that if you non dimensionalize temperature as T by T 1 we can do it in various ways but, this is 1 of the ways, then you will have 2 parameters coming out this analysis, 1 is the ratio of T 2 to T 1 as called ratio 2 and conduction radiation parameter this for is our convenience, this not really needed, but Now, this parameter is very important, this is the rested ratio temperature, this parameter is very important because, if N is much, much less than 1 conduction is unimportant condition.

Conductivity is so, small that if N is much, much greater than 1 conduction is important. So, even before solving the actual problem, you will get a rough and ready idea or whether conduction is going to play very important role in this problem. So, this is very good measure, for example, let us take a gas like air, whose conductivity is of the order of 0.025 Watts per meter square Kelvin. Let's assume absorption coefficient for air containing let say, carbon oxide, water vapor, and methane, so on let us say an average value as 10 meter minus 1. This as an example and let us say, we are looking at a phenomena at room temperature, around temperature, 300 K this we know is 5.61 by 8 watt meter square Kelvin to power of 4. This will give you N value, of the order of 0.25 4 in to 5.67 into the power of minus 8 into T 1 cubed this will be 27, 10 to the power of 6. Now, this will roughly come out to be 10 to the power of minus 2 here, goes up 2.5 about 20. It is 10th this is fairly small.

What we see very interestingly here, that even at room temperature for gas like air the chances that, conduction will be important is not very large. but; however, if we take an example of gas, at much lower temperature, lets say T 1 we are looking at cryogenic application. For example, T 1 could be on the order of 100 degree Kelvin, then conduction becomes very important, but at room temperature for gases, unless it is highly absorbing gas, like pure carbon oxide or pure water vapor in which case this quantity can much larger, this simple analysis shows that, conduction may not be very important, compared to radiation.

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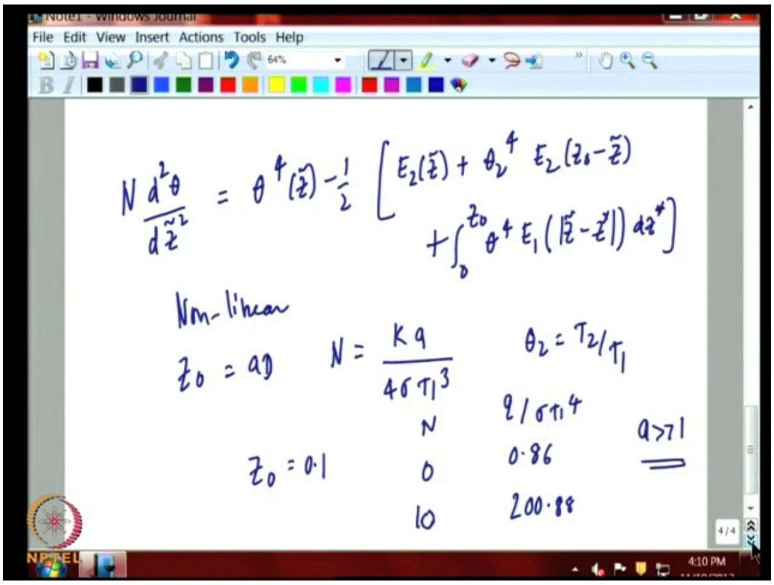

Now, let us write down complete of non-dimensional equation. This whole equation look like N d square theta d z tilde square is equal to theta to the power of 4 function of z tilde minus half E 2. We see this equation now is much more easy to use because, of the non dimensionalization that has been done.

We are writing this in a compact form of the integral term, which contains both the terms that, it was drawn in the previous expression, this is the modulus here. We must remember that, in actually solving equation numerically it expands this from 0 to z tilde z tilde to z 0, but here we have written for convenience in a similar expression, this is matter of compactness for use.

Now, this problem has to be solved on the computer because, as soon as the conduction, radiation interaction, radiation goes as T to the power of 4 conduction goes linearly in T when you combine the 2 we are going have problem of non linearity. The problem is non linear. Hence, the only way we can solve numerically and we will give some results from the numerical solution, to give a flavor of how conduction influences radiation.

There are two main parameters z 0 and N, which is the ratio of conduction to radiation. We also have third parameter, which is theta 2 this T 2 by T 1. We will not worry about this, so much. Now, let’s look at typical example of how, the radiation heat transfer from plate 1 to plate 2 place is influenced by variation of N. So, let take z 0 of 0.1 this is fairly non absorbing gas and let us look at 2 values of N, 0 and 10; 0 is case where conduction is not there at all.

What is the value here q by sigma T 1 to the power of 4. If there is no conduction this value will come to be around 0.86, where when this conduction dominating in a big way, this can be 200.88. What we see clearly here, that if value of N in this example was of the order of 10 there is conduction which is of the order 10 times larger than radiation, then the heat transfer rate from the surface of the bottom plate, almost goes up more than 200 times.

We can see that although we found that, in many cases N is quite small, but if we think of a cryogenic application, where T 1 is very small, then it is possible that this quantity can go about 10. Notice that this quantity can go above 10 also if a is much, much greater than 1, that is the absorption coefficient is much, much larger then also N becomes large. So, either conductivity large or the absorption coefficient of the gas is very large one of the two leads to N being very large. The next important thing which, we want to illustrate is, this is for the small path length.

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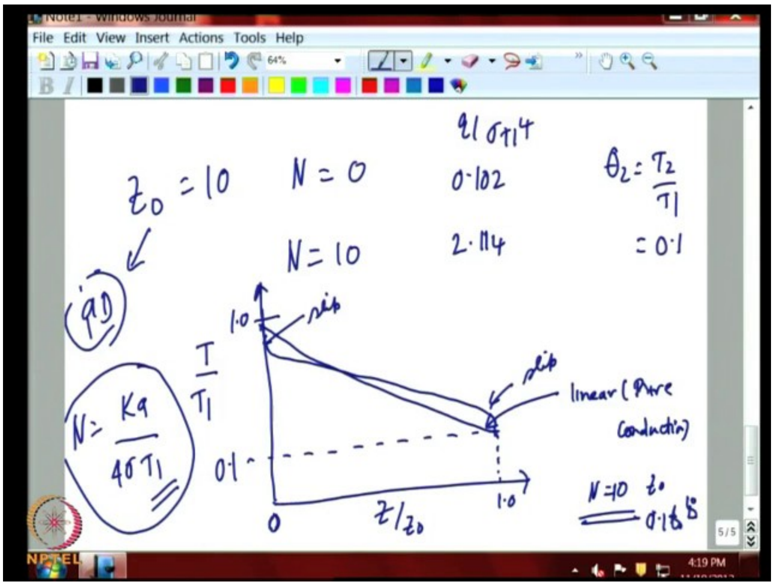

Now, if we go to large path length limit, that is take z 0 of the order of 10 and consider no conduction case, in this case we will get this quantity to be around 0.102 while N equal to 10 will take 2.114 again going from N is equal 0 to 10 the heat flux does increase, but notice that it increases only about 20 times. In the previous case it was 200 times. The overall optical depth in the medium has a role to play. The other thing we must remember is the nature of the temperature profile.

Now, we will plot in terms of T by T 1, T by T 1 maximum value we can reach is 1 and we have taken theta 2 to be T 2 by T 1 of the order of 0.1 at somewhere here. This is z by z 0 which has to go between 0 and 1. If we look at this problem in a pure conduction domain, then you will get a linear profile with pure conduction; however, when we have radiation as we recall from a previous discussion, there is a slip and the temperature profile will look like as shown above.

There will be a sharp change here, which called slip at both walls it represents the fact that when there is a radiation heat transfer, there is no guarantee that the temperature of the gas, near the wall has to be equal to wall temperature, but of course, since we have included conduction here, conduction heat transfer will ensure the continuity of the of the profile near the wall, but that layer where this occurs can be quite thin and it will almost look like an abrupt change.

This simple model illustrates the inter action between conduction and radiation. We found that essentially two parameter problem, one involving the optical depth a D and the other involving the conduction radiation parameter k a by 4 sigma T 1 cube. Now, if you look at this parameter we can see, that the temperature of the surface just plays a role that is, quite understandable because, of the fact that radiation heat transfer goes as fourth power of temperature, higher this value more important radiation compared to conduction.

The second part is the molecular conductivity of gas, that of course, will make conduction more important if case large, but the role of the absorption coefficient is slightly counter intuitive because, you thought that a gas in which the absorption coefficient is large, will be the one where conduction is less important, but you need to be very careful here because, the absorption coefficient, plays two roles in this problem, it comes in this parameter it also comes in this parameter.

When absorption coefficient plays a role through two independent parameter problem, We cannot make any judgment based on the looking at only one. Suppose, you increase a here of course, N goes up a conduction more important, but a is also increasing here. If you look at the results, we have looked at, so far. If you keep N constant, let say N equal to 10 this conduction is important we r z 0 goes from 0.1 to 10 then you saw out heat flux went down, by factor of 100.

All the conduction is dominating according to N value, the role of radiation also comes through of value of z 0 and as you increase value of N. In this case weight interprets them the result, is that if you take a gas in which a fixed, but z 0 is going down because, dimension of the problem has gone up as D as increased. If we have thicker layer of gas, the heat flux goes down and that is absolutely understandable.

The changes in z 0 and n can be interpreted smart layers long as this same parameter does not appear in both the in both this both these terms. That is issue here, if it appears in only one of them, then we have no problem in interpreting the result, but it appears in both, then we already careful about how you interpret the result. Now, will get some more insight into these interaction problems, by looking at now, radiation It is interaction with convection.

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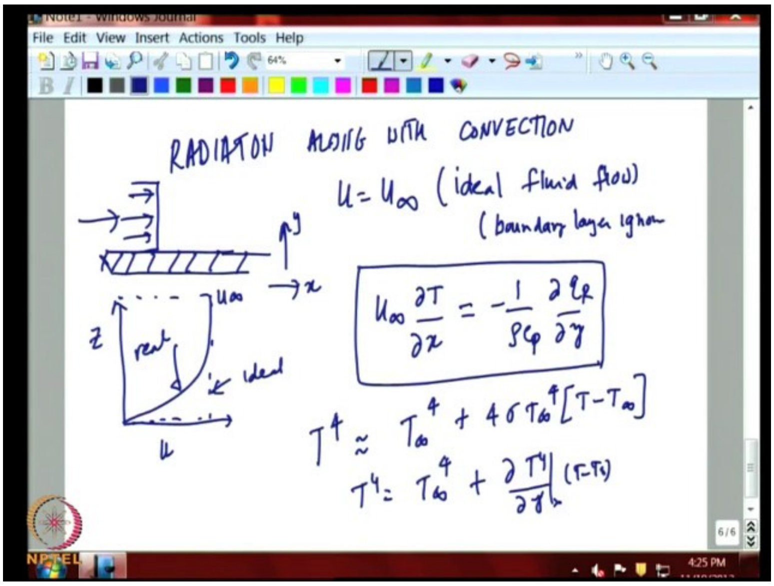

Now, this is the more complicated problem than conduction, you might recognize because, we have been talking about a fluid flow problem recently. We need to look at it little more carefully, than the conduction problem we did, but we can still do the problem in a very simple context, by assuming flow over a plate and since our aim is to primarily understand the role of convection in a qualitative way.

We look at the simplest problem in convection, when we assume that the fluid flowing over the plate at a single velocity U infinity. That is we assume that fluid is ideal that is, neglecting role of the boundary layer. The boundary layer, near the wall has been ignored. Essentially all of you know that you plot U with height above the plate, the normal profile is something like that, will be U infinity. So, what we doing is, we are replacing that by an ideal profile this one.

This is the real profile with ideal profile. In a ideal profile ignores the fact that right close to the wall they velocity has to be 0 but we will neglect it. This is an approximation it is not essential to make this approximation, but it does make it is somewhat easier to interpret the results, if we look at this ideal fluid case first. Now, if you do that, then your advantage that we have let us call this direction x and direction y. Now, we can write down the simple equation as, heat removal by convection by fluid flow is equal to minus 1 over rho c p delta q R delta y.

This is simplest problem we can solve, in the context of convection-radiation interaction we cannot make it simpler than that. And once it provides us some insight, we can proceed to more ambitious models with more terms in the equation. Now, further we will take up a case, where this problem is linearized. Now, we saw that if you do not linearize the radiative transfer equation, the problem becomes highly non-linear and interpretation is also difficult. We like to know that if you write down the equation with linear terms, whether the problem is more tractable.

So, what we do is we replace T to the power of 4 which keeps coming in radiation problem as approximated by the temperature at infinity plus this essential here, simple expansion of T to the power of 4 around T infinity to the power of 4 that is all it is, the simple Taylor's expansion that is saying T to the power of 4 is to the wall and derivative of the T to the power of 4 in terms of sigma t, minus t minus infinity. So, the simple way of expansion is to do the expansion of T to the power of 4.

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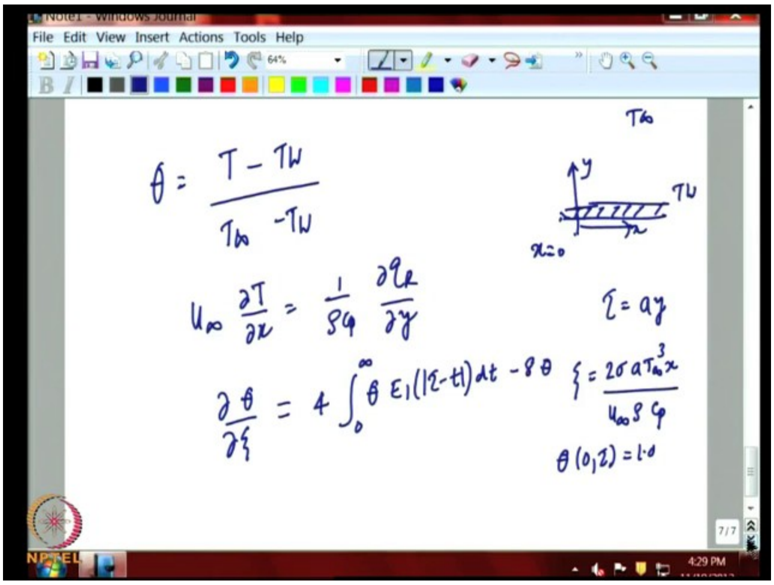

So, once we linearize equation then of course, we again want to non dimensionalize to make it easy, in terms of wall temperature and temperature at infinity. We have the plate, T infinity, temperature wall this is x coordinate, this are y coordinate and that is a non dimensional temperature, all the linearization is done properly, then the equation can be rewritten. So, the old dimensional equation this can be converted to non dimensional equation, as delta theta delta psi.

We will define psi right, now 4 integral 0 to infinity theta into E1 of mod (tau minus t) d t minus 8 theta this comes about because, of the non dimensional terms and linearization that we have done, in this we have two parameter, one is tau which is the optical depth, non dimensional terms of a and y, a is absorption coefficient gas and psi is the non dimensional x scale, there is a non dimensional y coordinate and tau is the non dimensional y coordinate. The only boundary equation we have, is that at x equals to 0 beginning somewhere here, theta=1 at x= 0 and any y.

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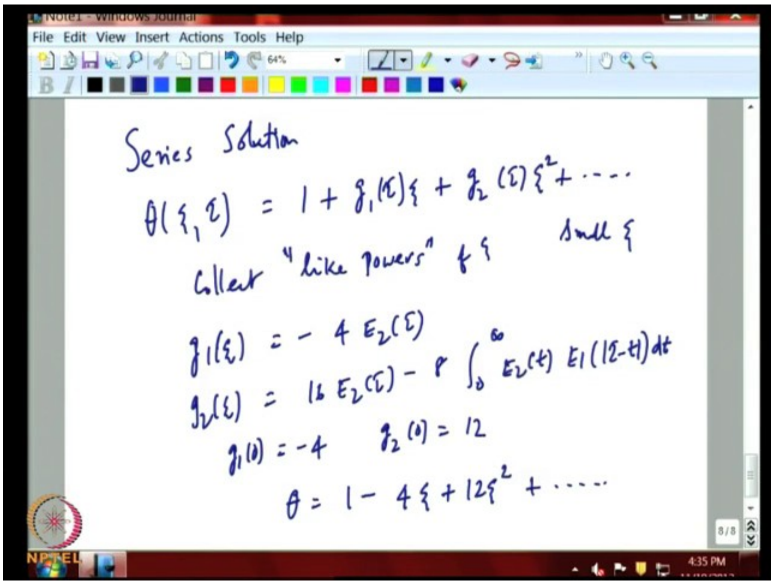

We can solve this problem analytically by various interesting approximation and one of the approximation we can do is to do a series expansion. They will give you that result first, to show you how the result looks like and later we will do more elaborate calculation. So, first example is what is called the series solution. In a series solution, you expand the temperature profile the depends on x co ordinate and the y coordinate, in terms of the values at psi 0 which equal to 1 and hope that we can expand this by the following form.

We are expanding the non dimensional temperature, in terms of the two functions. So, psi and tau and psi is the x coordinate and tau is y coordinate. So, as you probably realize, this is solution which will really work for small psi, got a series expansion we want to converge we will converge only psi is less than 1. Although one might think that this is a very limited approach, we remember that even if we have solved this equation in the computer, you will have some difficulty close to the origin, because of unusual behavior of this function near the origin of this system. So, even when you are solving this numerically, we will find it very convenient to have some good approximate solutions near psi equals 0, which we can use as an input, to more complicated numerical solution. So, even these days powerful computers can be used to solve complex problems. Even in such cases it is useful to have approximate analytical solution, which may be good starting point, or good guessing solution in an iterative technique are some other technique.

In a technique that depends on at good initial guess the good guess is the companion we have a intuition or it comes through your ability to generate this kind solution. Now, we substitute into this equation and we collect like powers of psi.

That is we match on either side of the equation, terms continuing the same power of psi because, they have to match otherwise equation is not valid. To get g 1 of psi is equal to minus 4 E 2 the tau g 2 of psi, we have 16 E 2 an tau 8 and when you solve this equation at the origin, g 1 of 0 will come order minus 4 and g 2 of 0 will come out as 12, this is to satisfy boundary condition at psi equals 0. With that we can get the final solution, for theta as 1 minus 4 psi plus 12 psi square and other higher order term if we can are patient to calculate the higher order terms and let just remind also what the solution looks like.

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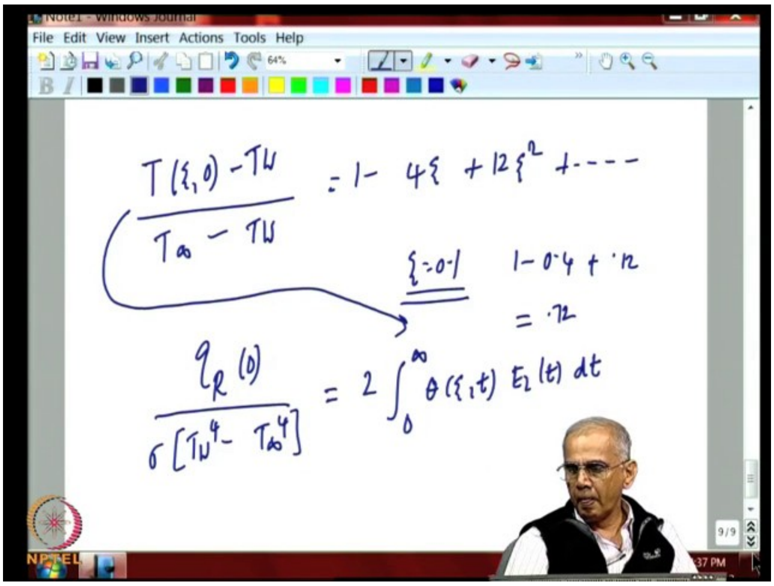

If T of psi and 0, we am talking about the value at the surface of the plate is T wall by T going to T wall finally, as come out as 1 minus 4 psi plus 12 psi square and may be some higher order terms, which we do not know yet. We know that this is saying that at psi equals 0 that is 1 which is the requirement of the problem and psi increases the temperature will decrease but, notice that this is strictly valid only for small psi.

Let us take psi is 0.1 for example, then we will get 1 minus 0.4 minus 0.1 to plus 0.12 right this is point is 0.1. This shows that the non dimensional temperature will decrease, to about 0.72 at a distance of 0.1 from the origin. Now, in radiation problems, we are just not satisfied with temperature we also want fluxes. In this case flux at the wall can be written as and flux is always non dimensional by the flux that will exist in the absence of the gas, that is the measure this can be easily be shown to be theta sine t E2(t) have t d t. Now, we know this solution, this solution can be substituted here and we can proceed further with the solution. When you do that, this is one of the solution you will get.

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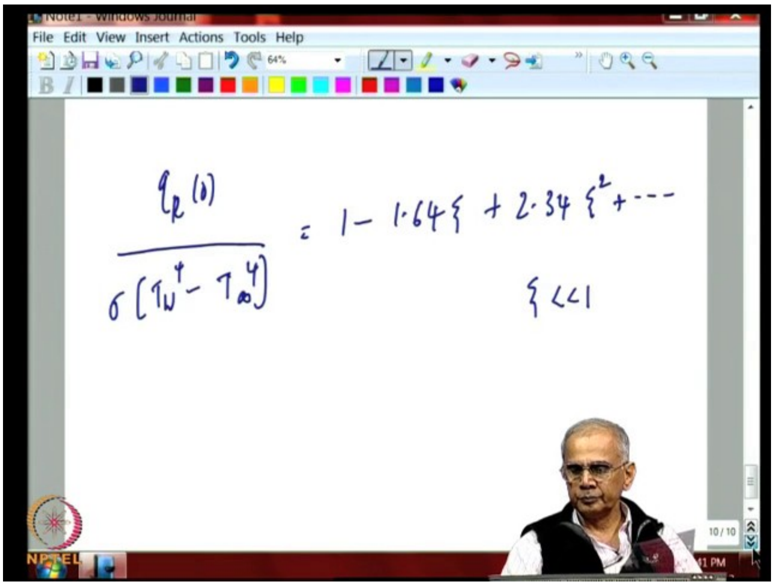

So, q R of 0 divided by the radiative flux, non dimensional value, as the ratio of the flux without any gas, will very along the x axis. We can notice this valid only for psi much, much less than 1. Again we take psi is around 0.1 We can see that when there is gas and there is convection playing a role, the radiated flux at the wall, will reduce it to small size, we cannot say anything at large psi because, this approaches not valuable flux psi.

We did this simplified analysis just to provide some insight, into the nature of the problem. Now, it is good to move on to more ambitious approaches. The one which was been very successful in our early examples of pure radiative transfer, as been useful in Kernel approximation. We can ask why solutions depend only on linear series; why not go for kernel approximations. We will attempt a solution to the problem we posed earlier, in the context of kernel approximations. The approximation, we has earlier used was series solution.

Now, the series solution has a problem with regard to the value of psi. It is valid only if psi is much, much less than 1. Now that approximation is not very useful is we want to go very far down. We will look at extension of these solutions by using a more complex method

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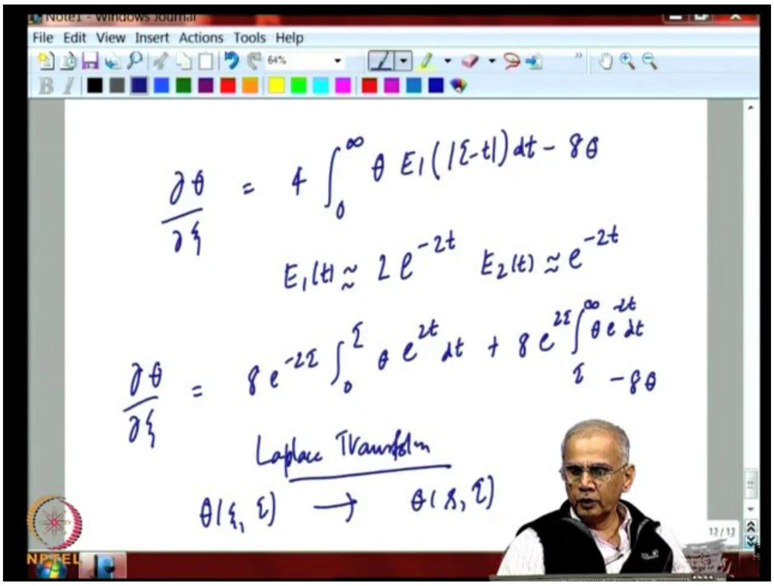

The equation we are trying to solve is delta theta delta psi is equal to 4 0 to infinity theta even of tau minus t d t minus 8 theta. This equation we are trying to solve if we recall the typical kernel approximation one can think of are one example of this is even if t is approximately equal to 2 e to the power of minus 2 t and E 2 t is approximately is equal to e to the power of minus 2 t.

Now, in this approximation what we have ensured is that E 2 (t) is exactly at t equal to 0 the t is equal to 0 t 2 t is one which is exact value, well E1 of t actually goes to infinity where t is equal to 0, but we have exact value of 2, but remember E1(t) appears inside integral. It is not as important that we get, the value of E1 correct at all locations, as long as we get the answer correct in integral sense, our approximation is not that bad.

Now, with this approach we can write down the non dimensional change in temperature as equal to 8 e to the power of minus 2 tau 0 to tau. Now, we expand the integral theta e to the power of 2 t d t plus second term tau to infinity. Now, we are assuming everything going up to infinity in the t direction here minus 8 theta.

This equation is more complicated than what we had done earlier. Now, the way to solve this is problem is by Laplace transform and from definition we are going to transform d theta of psi and tau to theta of s and tau where psi coordinate is transform to the s coordinate.

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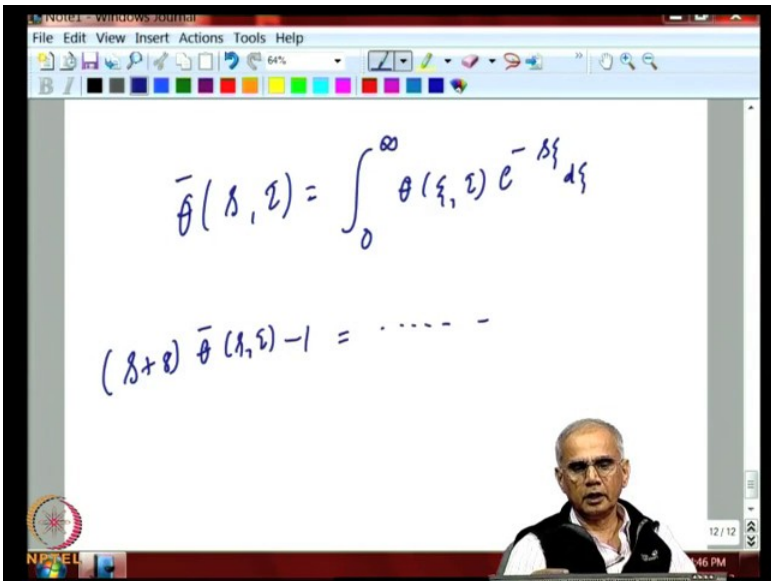

Now, we are recall that the Laplace transform by definition is 0 to infinity theta of psi and tau into the power of minus s psi d psi. We are transforming the non dimensional theta from psi tau coordinate to s tau coordinate where s is a transformation variable here, we will do that here and substitute the equation, you get the following equation s plus 8 theta bar(psi and tau) minus 1 derivative term. The left hand side will be essentially be same, that we change then we applied this approximation, we will discuss this in the next lecture.