**Radiation Heat Transfer**
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**Lecture - 24**
**Well-stirred furnace model**

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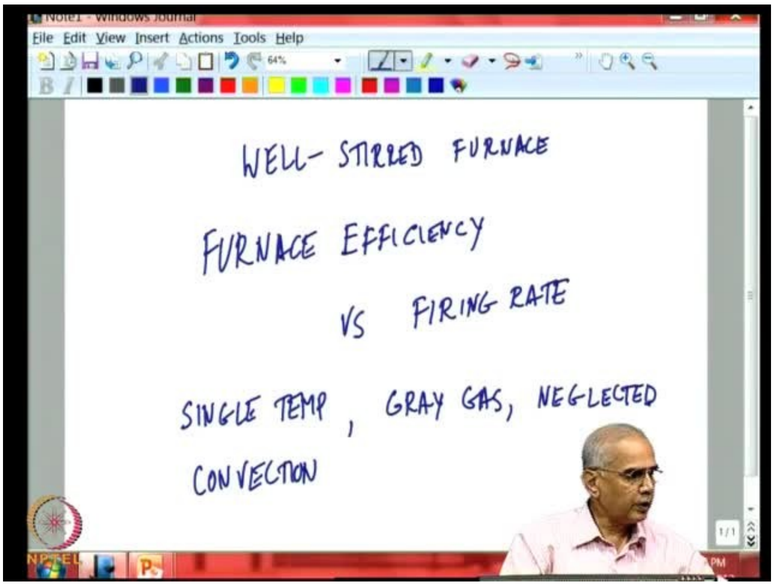


In the last lecture we looked at the well stirred furnace model in which we treated the entire furnace to be at a single temperature and we were able to relate the furnace efficiency with the firing rate and we saw that as the firing rate increases the furnace efficiency decreases. Although this is a very simple model with several assumptions; one of them is that, there is only one temperature of the gas, then we had assumed a gray gas and we had neglected convection. Hence you may wonder whether this kind of simple analysis has any value.

We must remember that this symbolizes only to get an idea of the, what are the major parameters which affect the efficiency of furnace. They were not meant to give accurate values of a real furnace. This can be thought about the teaching tool to give you an idea of what the important parameters are. Today with the availability of high speed computers we can always do a much more complicated analysis which will be an exertion of this really and get the more accurate answer, for those computer simulations will not give you any physical insight, will not tell us exactly why this happening. We need these simple models, not to give you accurate answers, but to give us an idea about what are the important parameters which influence the furnace efficiency.

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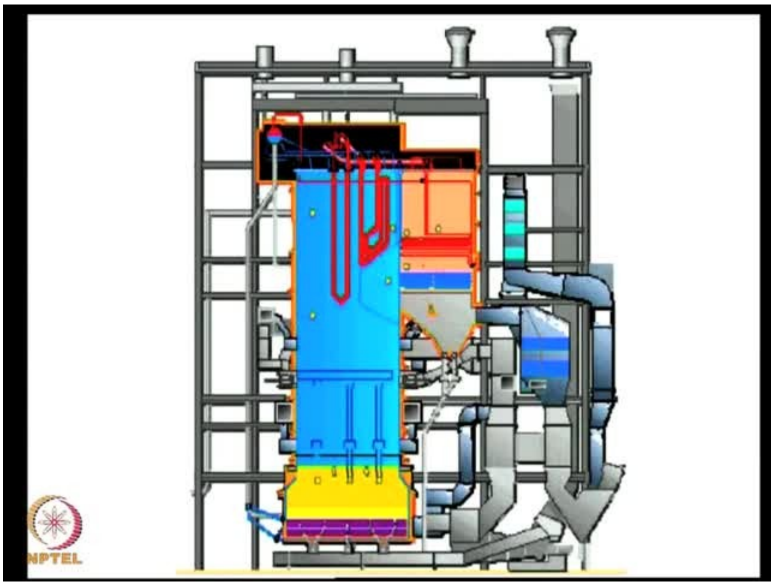

This is a typical power plant furnace and the region, which we were modeling, is this portion. So, call the radiant section the furnace and we treated this all at one temperature which is not true. There is a large gradient between where the flame is to where the gases are going out and we will see how to correct for these kinds of differences. But the point about this analysis is not to model this furnace accurately. The whole purpose of this model developed by Professor Hottel is to understand the relationship between efficiency and firing rate for a large class of furnaces because there are many, many furnaces which are quite different from this furnace and analysis will tell us.

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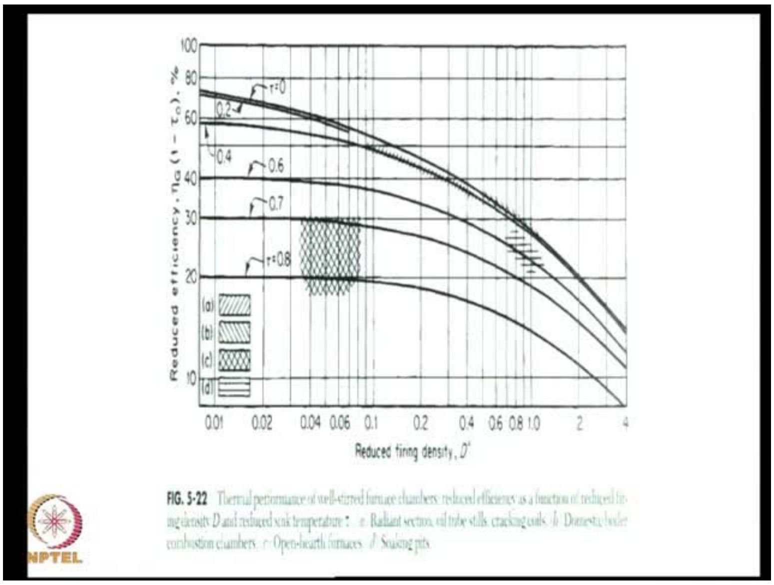


Let us now look at the result we got in the last lecture. We had the reduced firing efficiency on the y axis and the reduced firing rate or firing density in the x axis and for various values of the sink temperature, non sink temperature we saw the efficiency was going down. Now, the reason why this result is so important is because in this single chart, single figure we are able to show where the different furnaces exist in this two dimensional representation. For example ‘a’ is this region, it is the radiant section of cracking coils and oil tube stills used in refineries; ‘b’ is the domestic boiler which is used in many houses in cold countries to heat the houses; ‘c’ is open hearth furnaces used in metallurgy.

This is this region and finally soaking pits also used in metallurgical applications right here. Four different kinds of furnaces here are all represented in a single chart and it helps us understand why the soaking pits and domestic boiler are having a much lower efficiency than let us say the oil tube stills which have higher efficiency.  It also gives you an idea what are the highest achievable efficiency. The highest achievable efficiency at any firing rate is when the sink temperature is very, very low.

We can say that at low firing rates the highest achievable efficiency is of the order of 60 to 80 percent and as you go to higher firing rates. Here, efficiency does go down and at firing rate here of the order of two can say which is as slow as 20 percent. It gives you a very good handle on the important design issues that we are dealing with and as the cause of fuel goes on increasing and we need to make the furnaces more efficient, it shows clearly there is only one important way we can increase efficiency, by reducing the firing rate.

We have not much control over the non sink temperature because that is designed, determined by the application. In some application the sink temperature will be quite high. In others it will be low. This is not a parameter under our control. If the firing rate is under our control because for a given flow rate of fuel that is required we can have a lower firing density firing rate by increasing the area of the sink. That is a design parameter which we can alter in the design stage.

In the design stage if we cannot increase the efficiency of furnace from somewhere here from 20 to 30 percent to 30 40 percent, we can clearly see that we have to almost double the sink area and one has to work out the economics of this issue. These kind of simple models are very useful to make broad judgments at the design stage, but finally, after the design is finalized the actual calculation of the furnace efficiency will have to use some much more elaborate and complicated model and today that is possible because of the availability of high speed computers.

We can easily extend the simple analysis that we have presented here to more complex situation. We will discuss that in today's lecture. Before we do that we want to highlight some issues related to this analysis and the first question is as regards the assumption of gray gas. Remember, that we have spent a lot of time explaining that the gray gas assumption is very poor for most situations because the gases that we deal with have an absorption co efficient which varies very strongly with wavelength. Hence we should not make a gray gas assumption, but we begin with a gray gas assumption. It is worth asking, what is the error that has been introduced here by making a gray gas assumption.

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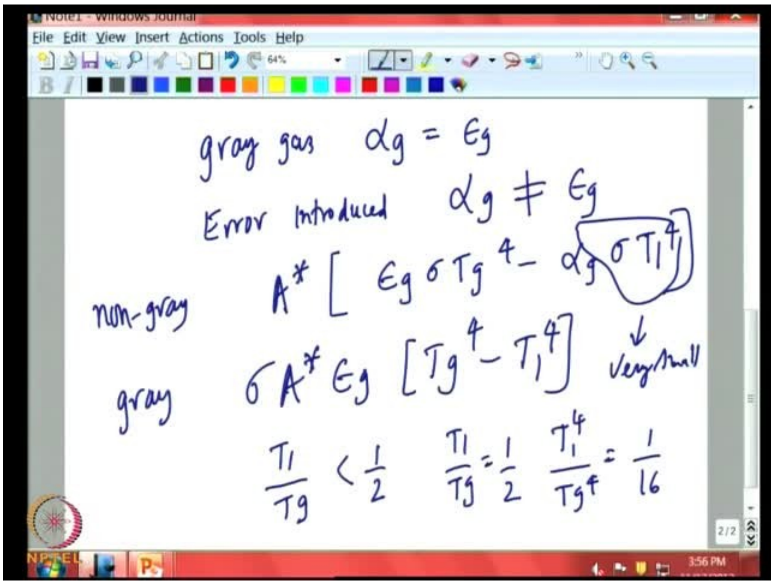


The key point of the assumption we made when we assumed the gas to be gray, it is that the absorptivity gas is equal to emissivity. This assumption is not very good. The question is what is error introduced because alpha g is not equal to epsilon g. In this case we know that for most gases absorptivity and emissivity are not same. If you look at this problem and look at the radiative heat transfer when these two are not equal, our wave transfer equation will look like as shown above. We have assumed these should be equal and taken it out.

We had used this equation in the last lecture which is not right. We should have used this equation, but interestingly in most application that we will deal with the T 1 by T g will be less than half, that is sink temperature typically in the application we are dealing with which was power plant furnace is less than half. Suppose if T 1 by T g is equal to half for our convenience then we see that T 1 to the power of 4 by T g to the power of 4 is 1 by 16. This term is very small.

So, although our emissivity estimate was quite accurate from the charts, our absorptivity estimates could have been wrong by 50 percent, but it does not matter because the second term of the equation contributes less than or around 5 percent or the first term. So, even if we make a 50 percent error in the second term it is not go into fundamentally alter our estimate of the efficiency of the furnace. We actually saw that when we increase the furnace efficiency from 0.38 to point to 1, the efficiency of increase was only 7 percent.

Any error in the second term in this equation because we assumed absorptivity be equal to emissivity, would not fundamentally alter the main conclusion. What is clear is that as long as the sink temperature is less than half of the gas temperature, we are not going to make a large error in assuming a gray gas. Although gray gas is poor assumption in this example where the sink temperature is small compared to gas temperature, it did not really make a difference.

Of course, there are other applications as we saw in the previous figure, if we are dealing with situations like where the temperature of this sink is very high, somewhere here, this is where temperature is low, here temperature is high like soaking pit, and there you may have to begin to worry about the second term. There we might want to incorporate the fact that absorptivity and emissivity are not equal. Now, let us move into other assumptions in the Hottle's model.

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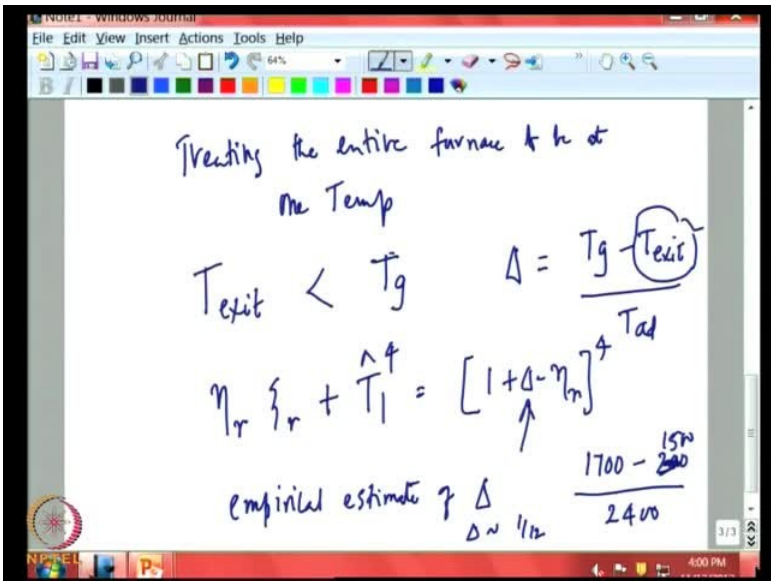


The others assumptions that has been made which could potentially cause large error is the fact that treating the entire furnace to be at one temperature. This is definitely not right because the exit temperature of the gas definitely will be less than the mean gas temperature of the furnace. This can be easily be accounted for if you assume that the difference in temperature between the mean gas temperature and the exit temperature divided by adiabatic term temperature.

This is the non dimensional measure of this effect, we can rewrite the, those equations we dealt with last class and that equation will change only slightly where delta by 0 last time we have got the delta. The only problem now is that we cannot estimate this delta apriory in a zero emission model. We need an empirical estimate of delta. For example, suppose the exit temperature was 100 degrees below the gas temperature, gas temperature was assumed was around 1700, the exit was like say 200 degrees lower and adiabatic temperature was around 2400.

T exit is 1500. The 200 by 2400, so it will be delta will be of the order of 1 by 12.1. This point 1 will not introduce larger, but this can easily be accounted for.  One can make correction to our estimate, if we have an in a real furnaces some estimate of how far the exit temperature is different from the mean gas temperature. One can make a correction, though this is a correction can be easily be incorporated. The next issue that you want to may worry about is neglect of convection.

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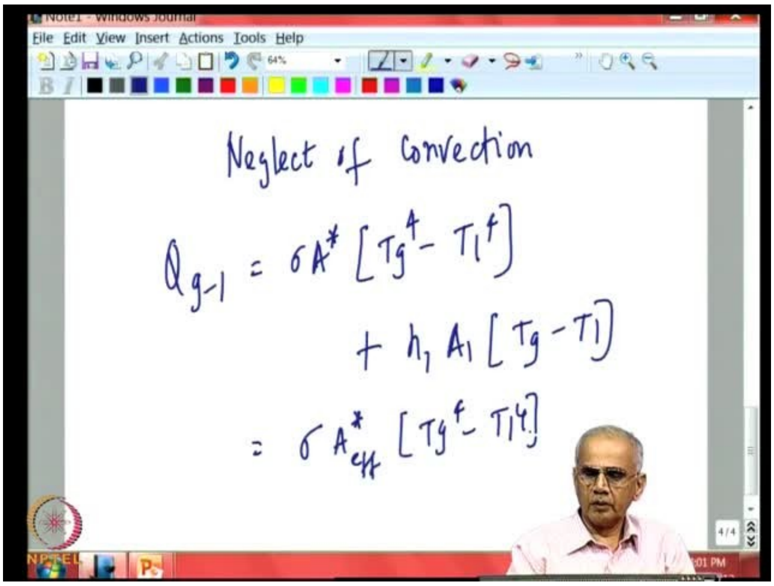


All of us are aware that in a real furnace there is a lot of mixing in the furnace due to turbulence flow. We  might wonder if that is going to alter the calculations. We can write down the gas heating heat transfer as the term which we had already included plus a term to account for convection heat transfer to the sink and the interesting point is that we can estimate this term and we know a rough estimate, but we can even redefine our A, we can define this as equal to sigma A star effective into T g power of 4 minus T 1 to the power of 4.

Where A effective star will be equal to 1 plus h 1 A 1 another A star into sigma A star T g square plus T 1 square into T g plus T 1.

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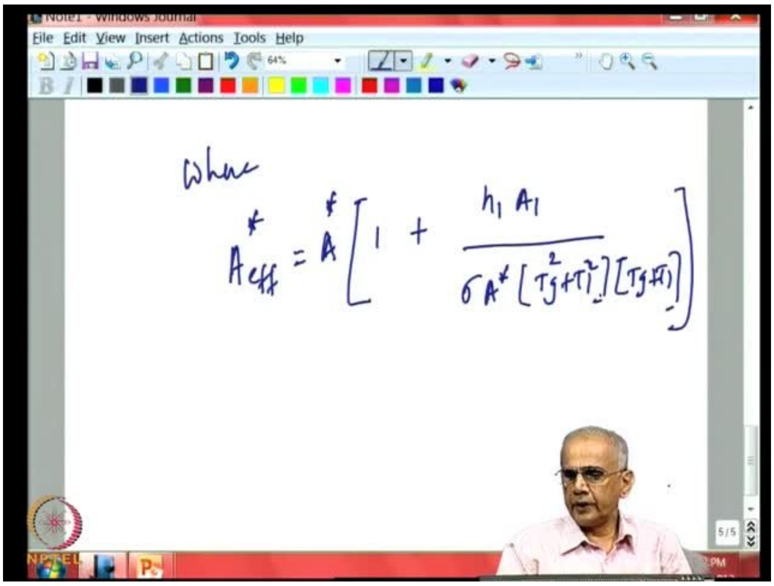


This will give you a rough idea of the additional heat transfer that is coming through convection. This similar to the way we took care of temperature gradient in the gas. These corrections for non gray behavior of gas, correction for temperature gradient in the system or according to convention all can be thought of as minor correction to the zero dimensional models, but of course, beyond certain point these corrections would not work. Now, let us take an example of furnace.

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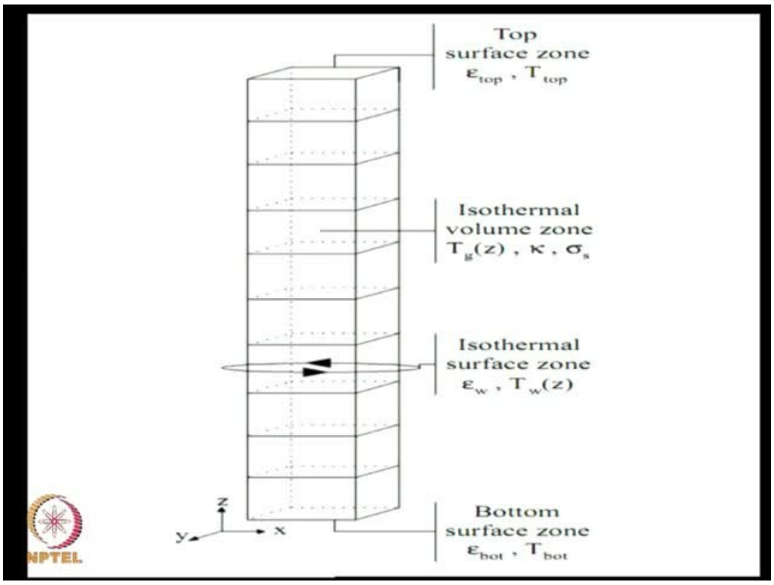


For example, take a furnace, a long furnace like this. This is the direction which furnace gases are moving. What we can do is we can divide the furnace into many, many zones and treat each zone as isothermal and apply the furnace model to for each and then link it to the next, next and next until it comes to the exit. This is an extension of the zero dimensional model of the Hottel, the Well stat furnace model to one dimension by having series of zero dimensional models linked together by common temperature.

Later we can see that if you want to take the real furnace which has two dimensions, actually three dimensions we can replace it by various volumes, each volume is considered to be isothermal. We can have multiple zones and so this has happened naturally. The Hottle's zero dimension well stat furnace model has been extended to one, two and three dimensions systematically and we will discuss how this can be done. We will not get into the nitty gritty details of how it is done, but broadly how this concept is can be naturally extended to multiple dimensions. But before we do that we would like to extend the analysis we have done so far to account for how the furnace will behave if there is ash deposit.

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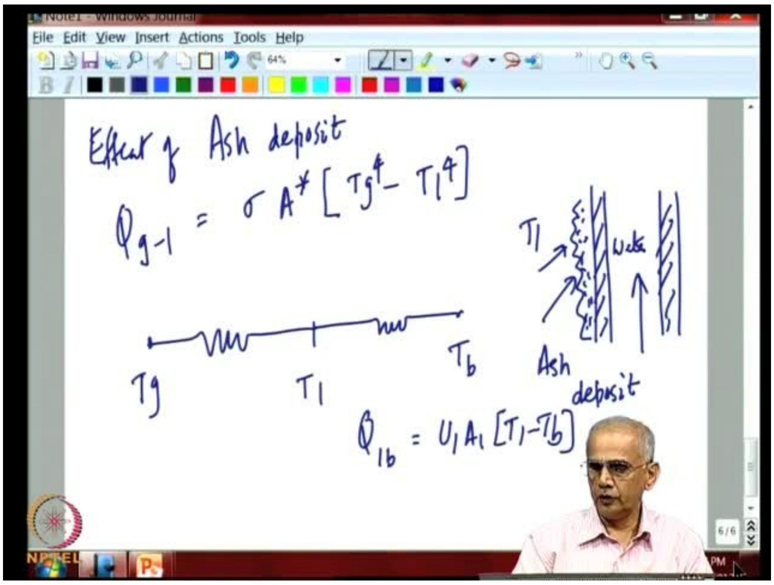


This is a common problem in power plant furnaces burning coal. The question is how you account for deposit of ash in a simple model. Now, remember that we had already estimated heat transfer from gas to sink as this is our model. Previously we have assumed the sink temperature as known, but if there is a tube in which the water is flowing, already be heated and the tube has a certain thickness and on that there is a deposit of ash. This is ash deposit. So, because of ash deposit this temperature T 1 will increase, but it is unknown quantity, we are to calculate that.

We calculate the heat transfer. The same heat is transferred by radiation from… We have a simple resistance model. This is the temperature of the boiling water. This is the sink temperature, this is the gas temperature. Now, the resistance heat transfer here comes from this equation. This one from T 1 to T b you write another equation Q 1 to b will be some overall heat transfer coefficient of the area of the sink into T 1 minus T b and in steady state, these two must be equal. Hence you find the unknown sink temperature. This equation has a fourth power temperature, this is linear. Strictly speaking we have to do this numerically, but one can do as approximation.

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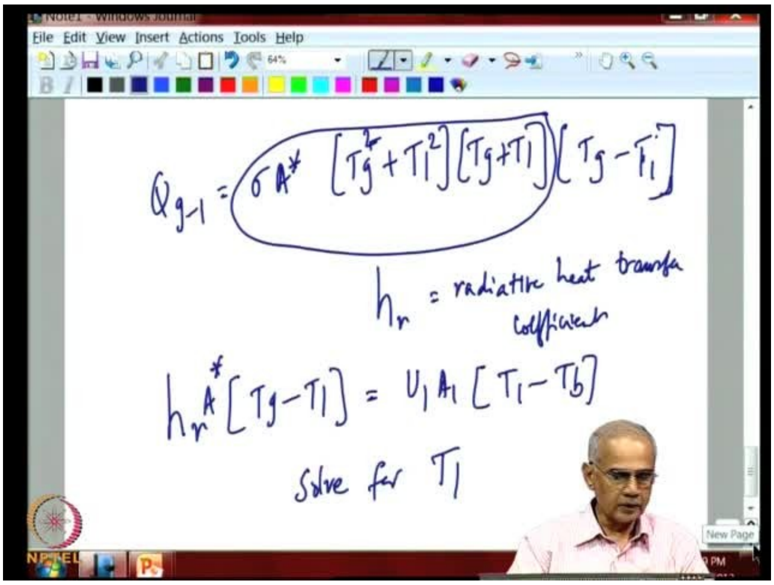


This Q g to 1 can be written as above. We can treat this quantity as h r radiative heat transfer coefficient. If you do that then you are writing h r into T g minus T 1 h r into A star is equal to U 1 A 1 T 1 minus T b. We can now solve for T b T 1 rather because T b is known, the boiling temperature of water and T g is known from the other radiation calculation. If you do that we will get the following result for T 1.

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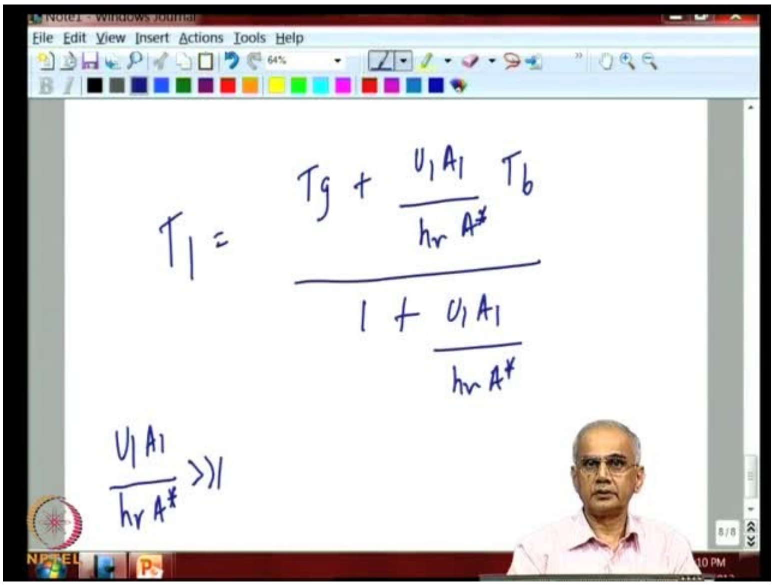


T 1 will come out as T g plus U 1 A 1 by h r A star into T b divided by 1 plus U 1 A 1 by h r A star. It is an approximate calculation. We can do a more accurate one numerically if we want to, but this highlights very nicely what the dominant issues are. Suppose the heat transfer resistance between the ash deposit and inside the water is much more effective than the radiate heat transfer from gas to the ash deposit, actually this is much bigger than one.

This term dominates; we can see T 1 approaches T b. So, this is a case without really any ash deposit. On the other hand if  this is quite small than the sink temperature will approach the gas temperature. That can be quite an important influence on the efficiency of this.

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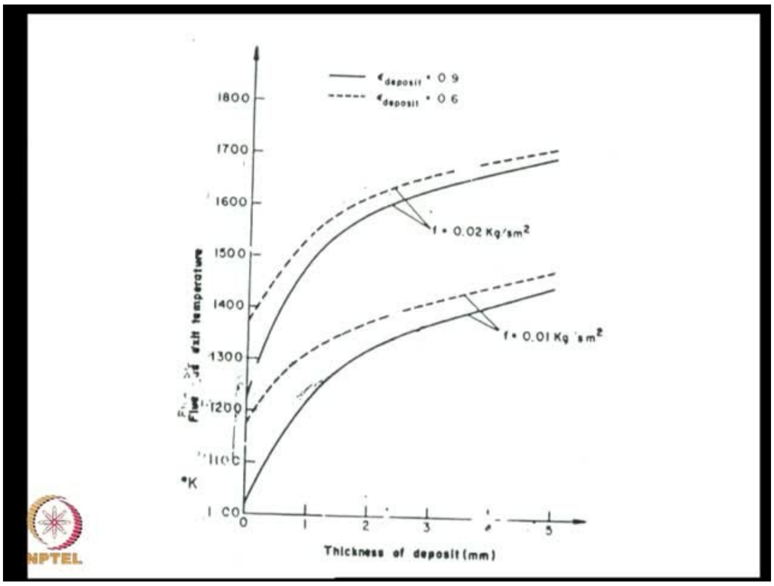


We can calculate this and we are going to show you the results of this computation through the result of the temperature of the gas as a function of thickness of the deposit. Now the deposit that is made outside of the tube has two influences.

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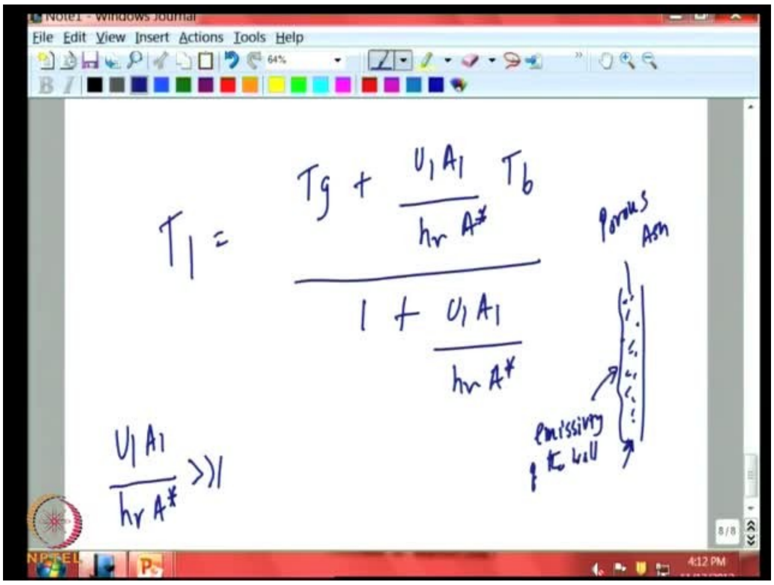


Here is a wall and here is the deposit. So, deposit offers higher conduction resistance because deposit is porous ash. First it will offer louder resistance to heat transfer by conduction. Secondly, it will change the emissivity of the wall. It will change the wall emissivity. Both these issues have to be addressed. In the calculation that is shown here we have taken two different emissivity of the wall. One is 0.9 which would be the case with ash free tube if the ash deposit has light color so it will level low or the other 0.6.

We have shown how as the thickness of the ash deposit increases, how the temperature of the gas leaving the furnace varies for two different firing rates 0.02 and 0.01 k g per second per square meter and we can see clearly that by the time we have 5 millimeter deposit of ash which can happen quite quickly if the coal that is being used is has high ash content like in India. Then we can see the temperature at the exit can go from 1000 to 1700 this will have a very serious impact on the efficiency of the furnace because furnace is not able to transfer enough heat to the heat sink, this case by water.

This clearly illustrates how often you need to clean the walls of the tube to ensure that the ash deposit kept to within about a millimeter or so because as it goes on increasing it does reduce the ability of the gases that transfer heat to the sink. Again we see that a simple model of the kind derived by Hottle has given us a rough guide as to what could be the impact of ash deposit on the furnace performance. That is the usefulness of this kind of analysis.

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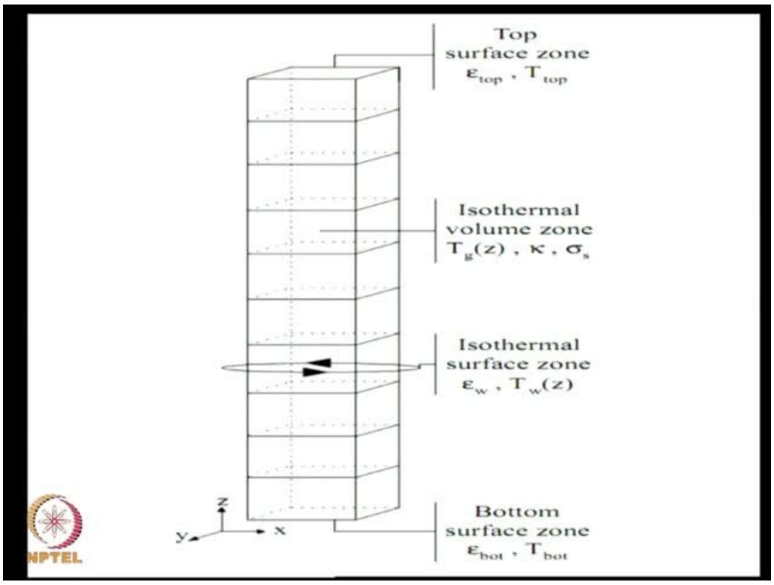


But, as the problem gets more complicated and there are more dimensions then of course, the single dimension module will or zero dimensional model will not work very well. Now, let us get to more complex situation.

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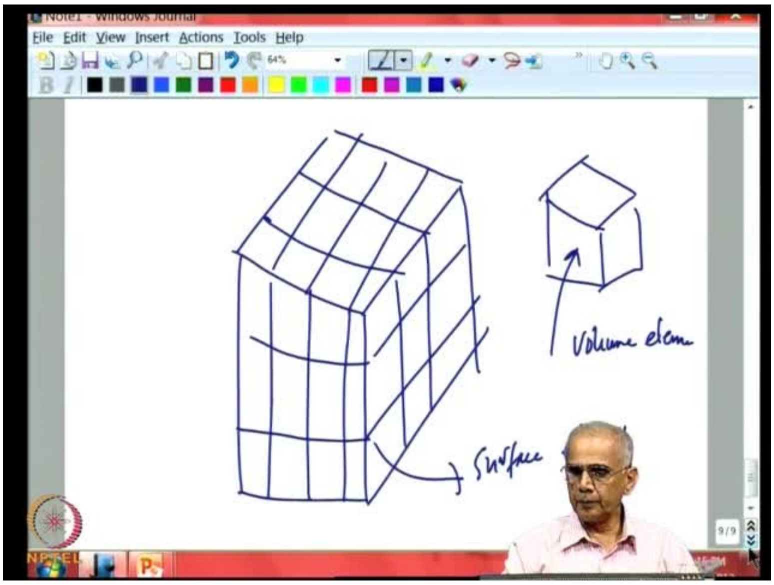


We look at the full furnace design. So, problem is truly three dimensional. If you think of this full three dimensional model, we can see that the furnace composed of large number of volume elements as well as surface elements. We need to extend the single zero dimensional model of Hottle to multiple dimensions this is called zoned model. Now, it is routinely used in furnace design and let us now see how this is achieved. So, remember that there are variation exchanges between two surfaces from surface to volume and also volume to volume. There are three different, let us try to put that down.

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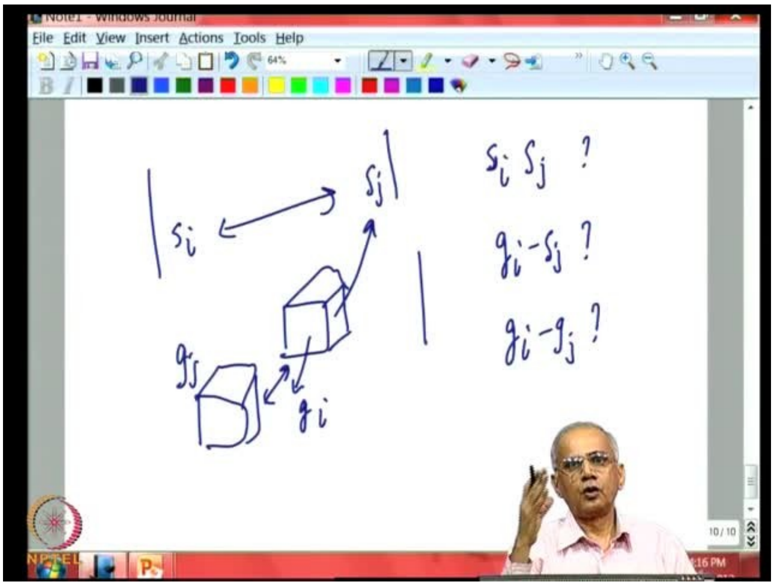


There is surface in the furnace. This is surface S i, this surface S j. We have to worry about S i to S j. Then we have a gas volume here which we will call as g i and the surface S j here. This we talked about this exchange. This exchange is exchange from volume to surface, recalculate that. Finally, we have to calculate volume to volume exchange between one gas element to another gas element, that is g i g j. We need to define these three parameters, surface to surface, volume to surface, volume to volume. So, let us define each of these three.

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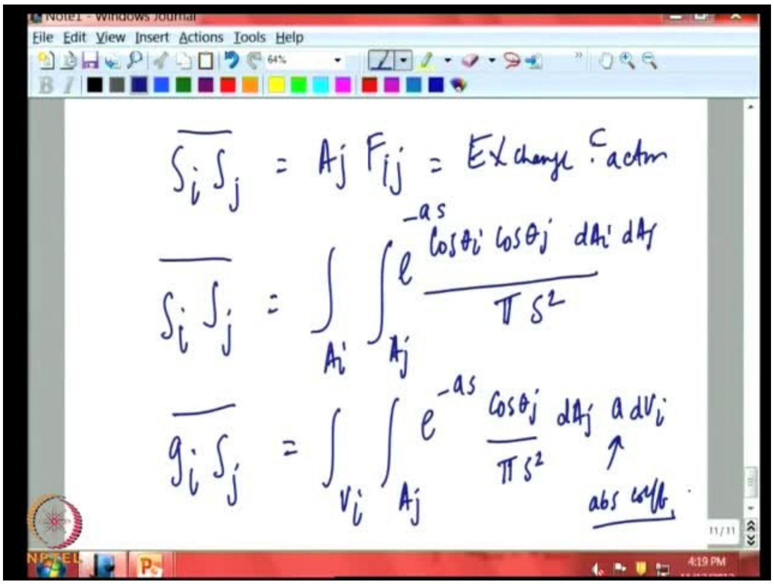


Now, the connection between what Hottel define as surface of a exchange here is nothing but A i F i j. We use a term shape factor and for your convenient to define it will definitely non dimensional, but Hottle for convenience defined an exchange factor which has units of area. The exchange factor between two surfaces, is by definition 1 over A i 1 over a j cos theta i cos theta j d A i d A j by pi. This we have done already many times. This is the standard definition of shape factor we have to of course, have it is for a minus absorbed common times the lens scale. This will affect the gas.

This is a quantity which defines the fraction of radiation leaving i and going to j influenced by geometry of the problem as well as the gas. Those are units of 1 over square meter. Now, let us say how we extend this idea for a volume of the gas to surface element. Now, here we see exactly what we are to do, integrate of the volume of the gas and of course, you have to integrate of the area of the element. This part is common for the element j cos theta j is there and pi s square is there. That is in common, but what you will do is that, you have d A j and further gas we have A into volume element. The main difference is we have replaced d A i by A times d v i, but A is the absorption coefficient, the gray absorption coefficient. This is a simple extension of the idea of exchanging of radiation between two surfaces to exchange of radiation between a gas volume and a surface area.

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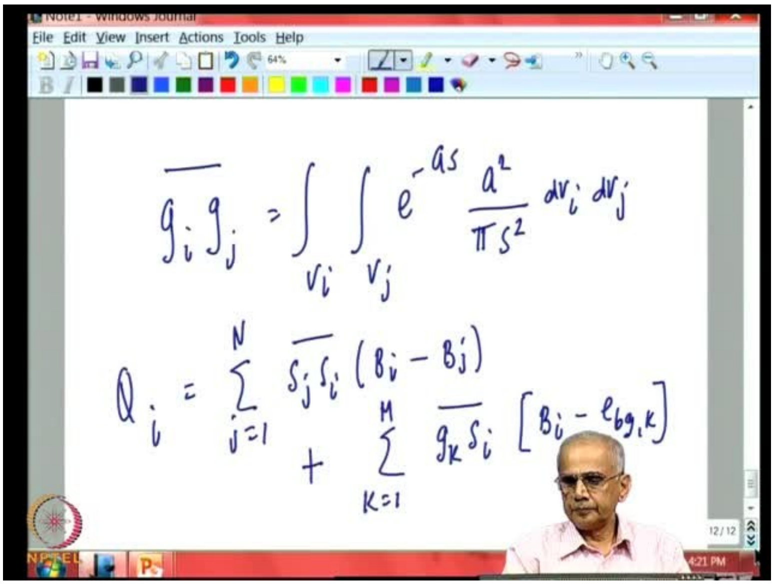


We have another definition which is the volume to volume integrated exchange. Here it will be integrated over two volume, it will have the gas absorption term, we will have a square by pi s square we have d v i d v j. We have to think about how the concept of shape factor, which we originally developed in the absence of gas can be easily extended to concept of exchange factor between two surface elements, further to exchange factor between a volume element of the gas and the surface element on the walls and finally, from volume over gas to another volume of the gas.

Suppose we want to write down how much is heat arriving at surface Q, we sum over all j surface elements and go for S j to S i and multiply by radiosity difference to get the heat arriving at surface i. Now, how much is the arriving surface i from the volume elements. We go from 1 to M, there are M volume elements and N surface elements and we calculate this g K into S i. This will be B i minus e b g of K.

We are able to calculate the heat arriving at surface element i, net heat to be added to surface rather to take into account exchange between surface i and surface j, between surface i and volume element K in that gas and finally, from the energy balance arguments.

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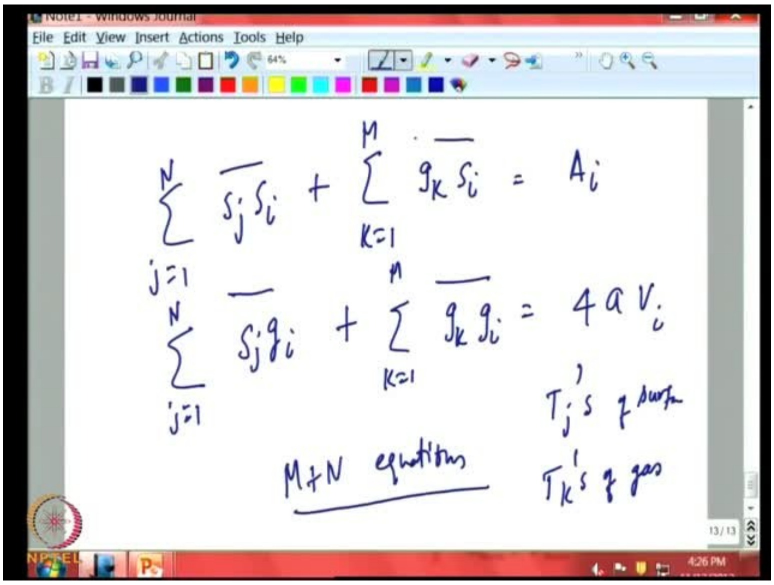


Our energy balance will tell us j 1 to N surface elements plus the M gas elements has to be equal to area of the surface A i and similarly, taking all end surface elements and interaction with gas elements and also interaction between all gas elements as equal to over a v i. These two equations put a constraint on the value of these exchanged factors and so we can solve for M plus N equation that we will have energy balance and solve for all the T j's of surfaces and T K of gas volumes.

We invert a matrix series to solve for all the surface temperatures and the gas temperatures. Look at the full solution. This is now routinely done and almost all organizations which have to estimate radiative heat transfer in furnaces. This is natural extension of what was discussed in last two lectures with reference to modeling radiative heat transfer furnaces. So, today with the availability of high speed computers one can do for example we can divide the surface of the furnace to hundred surface elements and may be hundred gas elements.

We have told the equations and inverting them does not take any time at all. The accuracy of the final result is not really sensitive to how you do the computation, but much more sensitive to how you calculate the properties. The emissivity of gas, the emissivity of surface those are the inputs in which there is a large error as indicated earlier emissive surfaces in that, which is available and depends on the surface condition and the history.

Those numbers have a certain amount of inaccuracy unless we know this state of the surface area accurately. The final accuracy wall is calculations is not sensitive to what technique we use here, but is much more sensitive to the accuracy of the input data. In order to get a more accurate estimate of furnace efficiency we need more accurate data input and that is where there is challenge. It is not in the analysis part, that is all very well developed and settled, but much more sensitive. Now, to give an example of a work done for a coal fired boiler.

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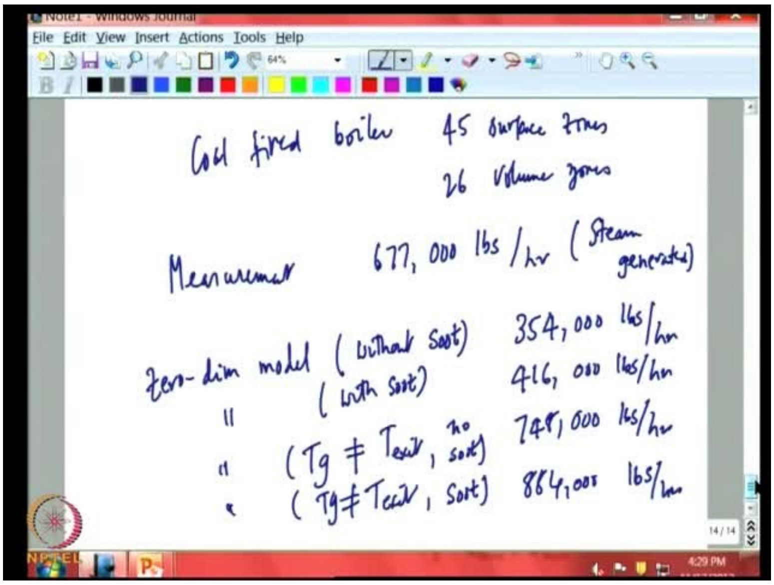


One example that is taken from published paper, they divided boiler into 45 surface zones and 26 volume elements and the estimated the amount of the steam generated. The measurement indicated the steam generated was around 627,000 Pounds per hour. This is in British units here because that is work done for a British boiler. They keep British units. Now, the question that we can ask is what are the estimates from the various models are used.

The first model is the zero dimensional model of the kind we discussed in the last lecture and we neglect soot because soot is not easy to model. Then we get a answer of 354,000 Pounds per hour. We can see the very large error in the zero dimensional model, almost half of the actual measurement. We may think that zero dimensional model is useless because it is giving completely wrong answer, but if you take the same model and include soot and increase the emissivity of the gas, this number goes up slightly, but not much, still its way beyond this number.

Suppose we take to the account in the same model, the fact that temperature is not equal at the exit and do not include soot, you get an answer like 748. It is very clear that your estimate of the steam generation is very sensitive to what number we use as the exit temperature of the gas and then you also account the exit temperature, not being equal to gas temperature and include soot, number goes even higher. Quite clearly the zero dimension model either severely underestimates the steam generation or it severely, over estimates the steam generation. We need long furnace model.

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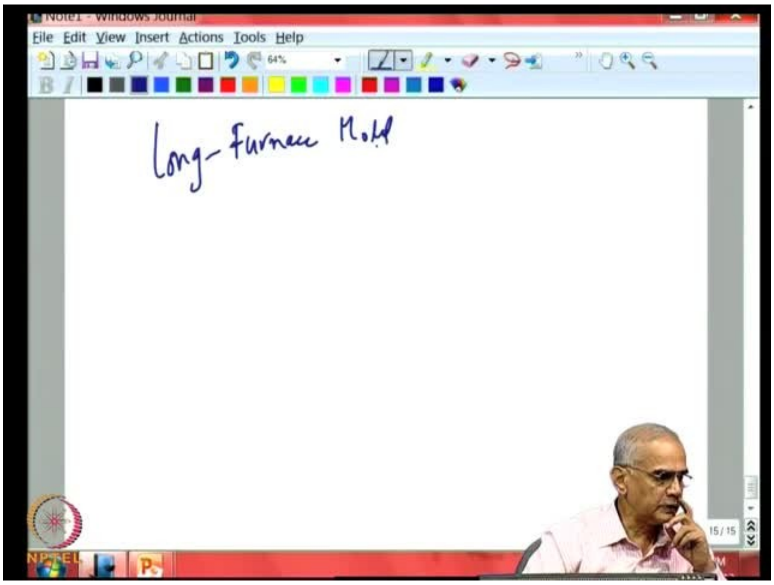


The next model is long furnace model of the kind discussed.

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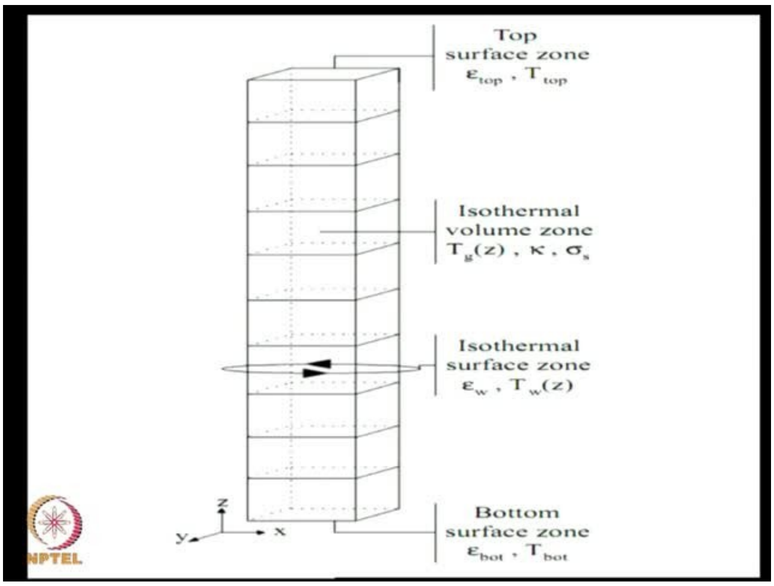

This is the model in which a series of volumes is of zero dimensions and gas slowly looses the temperature as it goes up, we solve this equation first and get the exit temperature, solve this equation so on in the vertical direction. This is one dimensional model, which will definitely be more accurate than the zero dimensional model, because accounts for the variation temperature along the path.

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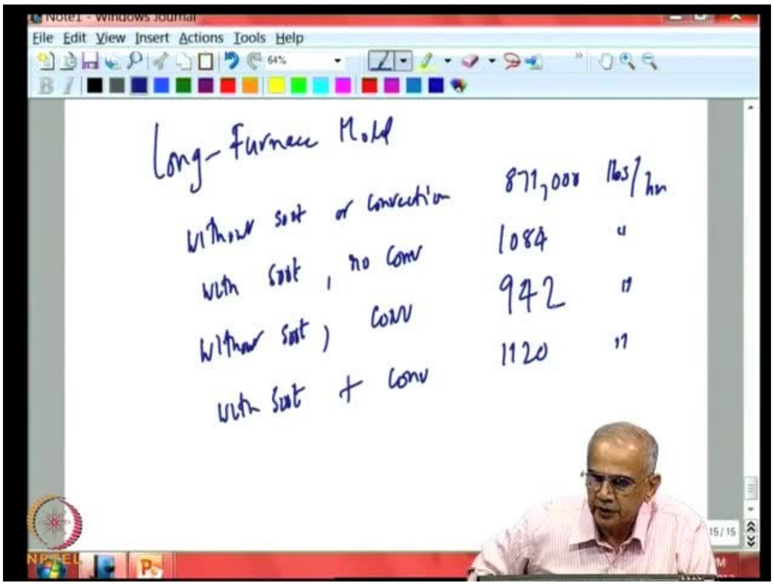


If use this model without soot or convection which is simplest case you get 871,000 pounds per hour. On the other hand if we include soot, but no convection then the number goes even higher, when you decide not to include soot, but include convection the number goes to 942 and finally, we include both soot plus convection the number again goes high. So, all these numbers are ready to high. What this shows is that the one dimensional model really did not improve the accuracy of our estimate, it tended to routinely overestimate the steam generation.

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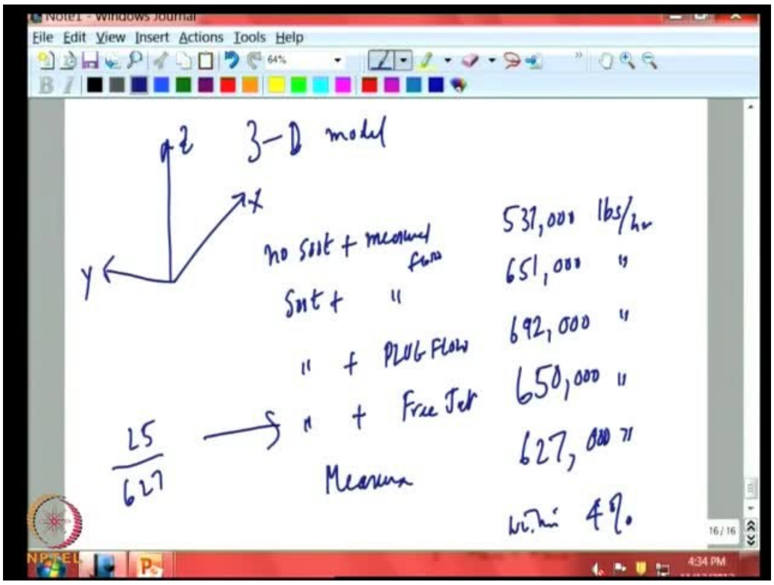


Quite clearly we need to go to the full 3 D model which accounts variation in the x y and z direction. So, all the three dimensions you account for, volume limits and so on. Then in this 3 D model if you have no soot and no convection, you get 531,000 pounds per hour. When we solve it, but no convection the value does go up to 651,000. This is plus measured flow.

We measure the flow and incorporate the convection heat transfer, same here then we assume soot as a simple plug flow model with uniform velocity we get 692 and finally, use a model with soot and assume the flow to be a free jet which is what is true because air coming from the fuel burner and taking in the parameter can add and will flow like a free jet, then we can get 650 and recall that measured value was 627. It can be satisfied that the prediction based on three dimensional model with soot and free jet gives you result accurate to 25 by 627 which is accurate to within 4 percent.

This is quite impressive, but quite clearly you need a 3 D model, you need also either the measured or some estimate of the nature of the free jet that develops in the boiler. What we have seen from the zero dimensional model to the three dimensional model, We can see that if you want accuracy you need to go to full three dimensional application, but if you want a understanding, the best understanding comes from the zero dimensional model of Hottle and it was really the building block for the development of the 3 D model.

We have given some idea about how furnaces are modeled in the real world and how the simple idea that we had discussed in the last lecture are useful to interpret the results of real furnaces and as pointed out, if we want to accurate information about the performance of the furnace we cannot hope to do so with this simple zero dimensional model. Zero dimensional model serve the purpose of a tutorial. So, finally the ideas from the zero dimensional model were utilized in three dimensional model to estimate the steam generated in the boiler to within about 4 percent.

Now, we ask ourselves one of the issues we raised in this discussion. If we recall in the calculation emissivity of the gas in this model we needed to use an empirical formulation about emissivity because in a real furnace of three dimension the emissivity depends upon the three dimensions and we have not actually discussed how this should be done.

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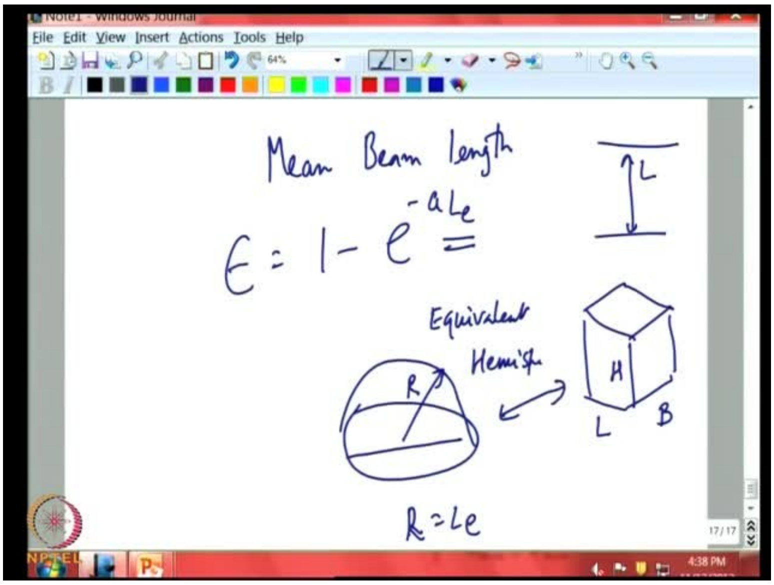


Let us discuss what is called the mean beam length. Mean beam length is a concept used by engineers to estimate the typical distance, effective distance in a three dimensional furnace that will give an accurate estimate of the emissivity of the gas. If a simple parallel plates the dimensional of course, is this length, but when we go to three dimensions there are three, that is there is length, breadth and height and we should know how we should combine these three length scale to get the effective length scale to be used for calculation of emissivity.

We write emissivity as 1 minus a into L e, we must know what that L e is. Now, the way that is done is first to ask yourself if we can replace it by an equivalent hemisphere and what is that radius of this field. So, mean beam length is that imagine hemisphere of a radius L e which gives the same radiation incident on the surface as a real three dimensional furnace. We would like to know the relation between this L e and all these dimensions of the furnace.

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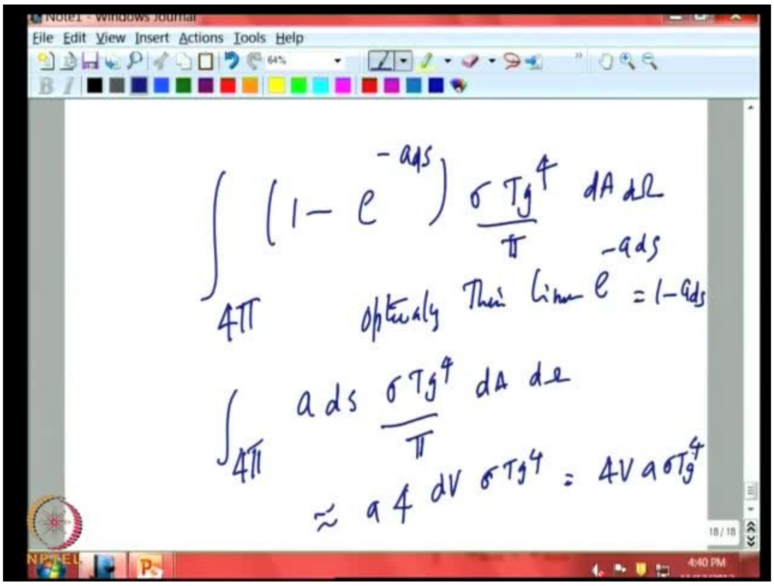

So, first thing we do is to ask what the radiation is falling from a hemispherical furnace radiating to the wall. The volume element let us say small, so we call this as d s and in the thin limit, optical thin limit, this can be written as, we expand this as 1 plus 1 minus a d s, substitute that this becomes a, we are dealing with gray gas here let us just write this as sigma T g to the power of 4 by pi and in this limit this becomes a d s into d a into d omega and will finally, approximate this as 4, because this will give you a 4 pi. So, 4 will cancel out, d A d s is taken d v and integrate all volume will give you 4 v a similar to the power of 4. We will continue this derivation in the next lecture.