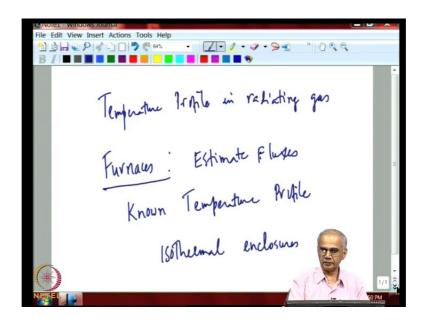
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Lecture - 23 Isothermal gas enclosures

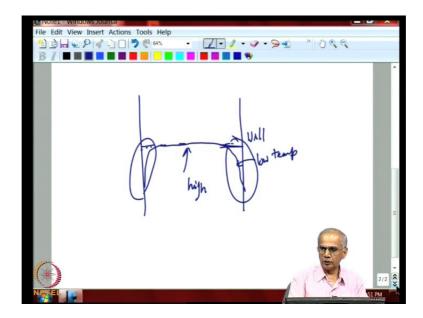
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In the last few lectures, we have been looking at how to estimate the temperature profile in a radiating gas. We saw that we need to solve an integral equation; we found various ways to solve the equation. We saw that we had introduced a concept of radiation slip, and also the solutions were very different in the thin and the thick limit. Now, we look at the totally different problem.

We consider furnaces, which are examples where radiation plays a very important role. Our aim will be not to estimate the temperature profile of furnace, but estimate fluxes. If we want to estimate fluxes in the radiation occurring in a furnace, then you must know the temperature profile. We must have a known temperature profile. Now, this could be either from measurements or from some other source or calculation. Now, the simplest assumption we can make is isothermal enclosure. That is, assume that the gas within the enclosures has a uniform temperature. This of course, is not strictly right as we can easily imagine.

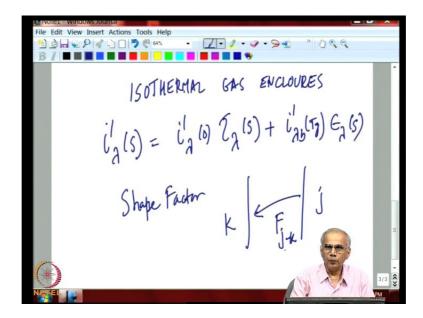
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But if we look at a typical furnace, temperature profile may be something like that. Gas is radiating here at a high temperature. This is the wall and near the wall there is cooling and so we have low temperature. Now as long as these regions are small, we can assume the gas to be isothermal. Because of usually strong turbulence in these furnaces and combustion chambers, there is a lot of mixing going on.

The temperature variation is quite small, except near the wall. If we are convinced that this small region near the wall, which is at somewhat low temperature, does not alter the flux calculation substantially. We can assume a uniform temperature across the entire furnaces. The only problem you will face is very close to two walls, our estimate of emission may be somewhat wrong.

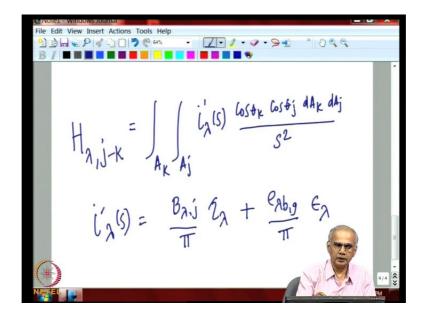
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Today we discussed isothermal gas enclosures. The advantage of isothermal gas enclosures is that, we can integrate the shape factor equation and estimate the intensity at any point S as that coming from some origin times the transmittance through the gas plus what is emitted by the gas; a function of gas temperature which is uniform, times the emissivity of the gas. This is the simple result we have.

Now, we would like to develop a concept of shape factor earlier; to calculate the fraction of radiation leaving one's surface, which lies at another surface. The derivation we did earlier was assuming that there was no gas between the two surfaces, one surface j here and one surface k here. We estimated the fraction of radiation emitted by j arriving at k, but we neglected the intervening gas absorption and emission. We want to include it. Now, we want to consider the fact that, the gas which is there between the two surfaces will absorb and emit radiation. Hence, we would like to consider that.

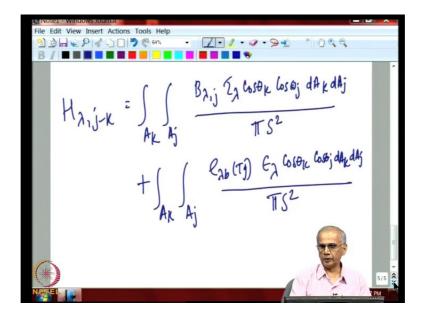
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We can easily see that the amount of radiation arriving in a given spectral interval leaving j and arriving at k is nothing but summation over k integral over k into j radiation arriving at that location, multiplied by the usual geometric factors which we looked at. This is the integration we had done earlier. Now, looking at the formula for i prime lambda S which we looked at earlier, it was written as surface j divided by pi; that is, i prime lambda zero times the transmittance of the gas plus what is emitted by the gas, times the gas emissivity.

That term consists of two terms; what is leaving surface j, passing through the gas and is transmitted and what is emitted by all the gas layers between j and k as the ray comes along. Now remember that in the absence of the gas, this is one this is zero. When we put this equal to one equal to zero and substitute that, you will get exactly what we had already derived for shape factor between two surfaces in the absence of the gas. Now, we are trying to introduce the role of the gas through the transmittance and emission by the gas.

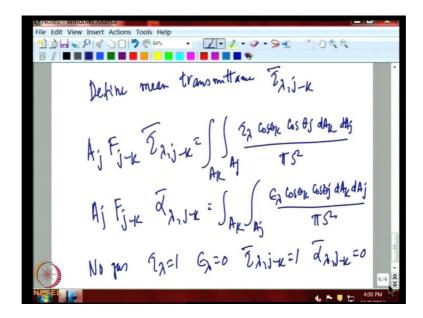
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If we substitute here in that equation, then we get H lambda j comma k is equal to integral over A k integral over A j B lambda j, what is leaving surface j transmittivity of the gas cos theta k cos theta j d A k d A j by pi S squared. This term is very well known already to you; because this term tau lambda is equal to 1 is the equation we have already derived in our earlier discussion of S lambda j. But, now there is one more term which involves emission by all gas paths that leaves along this path. Here we will have e lambda b of T g epsilon lambda cos theta k cos theta j d A k d A j by pi S squared.

So, a calculation of the radiation arriving at surface k from j is somewhat more complicated, more involved because of the transmittance of the gas and the emission by the gas. Now, since we are going to encounter these terms often, it is good to define some main quantities.

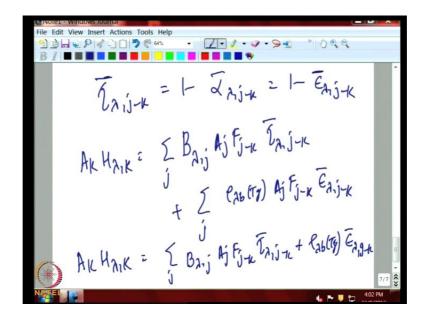
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We will define mean transmittance through the gas over lambda j k. The definition is as follows. The definition is so constructed that, in the absence of gas it collapses to the definition we had used earlier for shape factor. This mean transmittance is so defined that it will be equal to one in the absence of the gas; that will be equal to one. We are back to the original definition of shape factor, which we had already discussed earlier. Similarly, we have a new definition worth the mean absorptivity or mean emissivity of the gas.

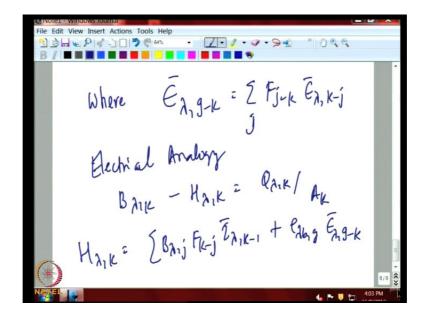
This is only for convenience. This is only a definition, nothing else. Instead of writing this expression each time, we are going to write this expression. We notice that if there is no gas, tau lambda equals one; epsilon lambda equals zero; tau by lambda j k equal to one and alpha bar lambda j k equal to zero. This is the limiting case which we had already discussed earlier, when we are dealing with radiation exchange between two surfaces in the absence of the gas. Now, we are putting the gas there, so we have the expression. Now, what we want to do next is to develop an electrical analogy, so that we can proceed further in order to simplify the problem.

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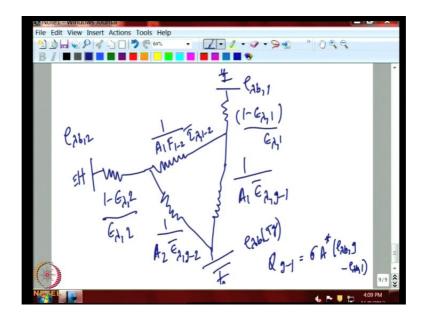
Now, let me just remind you that tau bar lambda j k; by definition, it has to be equal to one minus alpha bar lambda j k or one minus epsilon bar. We are assuming that the gas to be a diffuse isotropic emitter. We expect to do the spectral absorptivity and emissivity to be same. Now with this definition, we can write the total radiation arriving in surface k as sum over all j surfaces, radiosity of surface j times A j F j k tau bar lambda j k; that by our definition. Plus sigma over j e lambda b of the gas A j F j k epsilon bar lambda j k. Now for convenience, this is not a function of j and k. We can take it out. This whole quantity can be defined as emission of gas to k. This can be simplified as follows. It is a definition of gas to surface k emission, which is nothing but sigma j one to one A j of j k epsilon one to j k. That is our definition.

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So, let us now write it down, so that we are fully conversion with it; where, we define the mean emissivity of the gas to surface k as being composed of summation of all F j k's definition to keep our calculation simple. So with this, now we can develop an electrical analogy for enclosures with gas in them. Notice that the basic energy balance does not change. This remains unchanged. What is changing here is H lambda k, which now is H lambda H H is a lambda H H H is a lambda H H H H H is emissivity. So with this expression, we can now have an electrical analogy for enclosure with gases in them.

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There are three surfaces in an enclosure; three sources of heat. One is surface two, e lambda b two and surface one, e lambda b one. This could be thought of as the heat sink this the heat source in the furnace. Each of these surfaces has a surface resistance which we had discussed earlier with this standard. We have discussed that earlier in these earlier lectures about enclosure theory.

Then, we have now three resistances; we have to talk about. Here is, the gas source; this is the gas emission. Now, if we heard of the derivation of the analogy, this will come out as one over F 1 2 tau bar lambda 1 2. So, what is essentially is happening is the geometric resistance between surfaces 1 and 2, which was earlier defined as 1 over A 1 F 1 2, which is a purely geometric factor.

It is modified per transmittance term because the transmitter of the gas is not equal to one, the resistance to geometric resistance heat transfer is modulated by the gas absorption and the resistance is increased.

Now, for emission between gas and surface 2 by our own definition, this comes out as 1 over A 2 epsilon bar lambda E to two. This will be 1 over A 1. This completes the simple electrical analogy for an enclosure with absorbing gas in it. We see that there have been three modifications. The geometric resistance between two surfaces has been increased due to the presence of gas because gasses often resist heat transfer. It will increase the resistance term. There are two other terms which were not there earlier, which involves a gas radiating to either to sink or the source of the furnace.

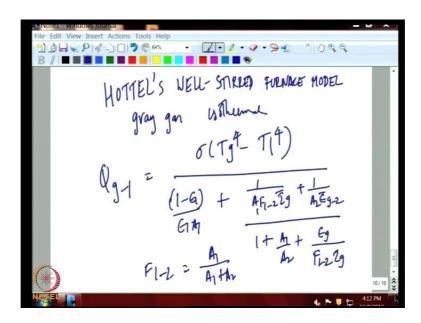
We must notice that in the absence of the gas these two terms are not there. This is zero; goes to infinity. This goes to one in your standard electrical analogy per heat transfer between two surfaces. Essentially the presence of radiation introduces what one might call the third surface; which represents the gas temperature. This is useful analogy to have. If we use analogy, then ultimately we can express the heat transfer from the gas to surface one, which can be treated as a sink. What we can do is we can draw all the resistances.

But, finally for convenience we will write the final expression forming heat transfer from, heat transfer from gas to surface 1 is, where your sigma times some effective area and e lambda b of g minus e lambda one. We can write equations to involve all these

terms. Now, a best way to illustrate the use of this example is to calculate this for a given situation.

Now, this work is done many years ago by professor Hottel from MIT, who is interested to have some simple calculation of the radiating heat transfer in furnaces. We need to apply simple models of radiation to estimate heat transfer from surfaces.

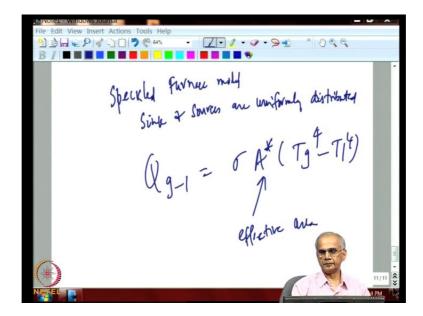
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Now, this is known as the" Hottel's Well- Stirred furnace model". This model treats the entire gas as being at one temperature and all the expressions we have done now and assume the gas is gray and isothermal. We do that when the heat transfers from g to one; for that electrical one, can be easily written.

This is the expression for the heat transferred from the gas to one for a gray gas in isothermal furnace containing two temperatures one and two by two surfaces minus sink. We see clearly that the role of the gas here is coming through the mean transmittance, the mean emittance of the gas and the array ratios. Now, one can think of two extremes in this case. The one extreme would be assuming that F one two is approximately equal to A one by A two.

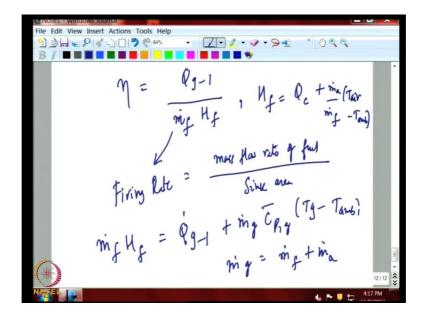
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This simple idea is based on a model which assumes that is called the speckles, which assume that the sinks and sources are uniformly distributed. Essentially we are assuming that sink is everywhere. Then the shape factor F one two is nothing but ratio of area two by area one and two. If the two surfaces are uniformly mixed, then the shape factor will be dependent only on the area ratios. With that assumption we can calculate the value of that quantity we had written down. Generally this simplifies to write down Q gas to one is sigma A star. Now, this is what is called the effective area of the furnace, which we can calculate for specific examples.

Now, later we will show how this calculation can be performed. But, right now our aim is to take up a specific example of estimating the heat transfer efficiency of the furnace. We are going to propose a very simple model of the furnace.

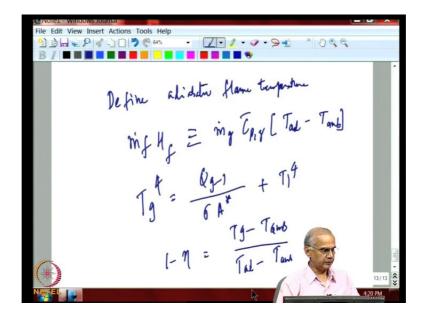
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First thing is what the efficiency of the furnace is. One thing is, how much heat is transferred from the gas to the sink divided by what is the fuel mass flow rate times the effective calorific value of the fuel. Now, this H f really consists of the measured tariff value and the additional heat that is coming in because the air is preheated. The air is heated well above the ambient then it is bringing extra energy into the system. That must be accounted for. In most real form, this may not be very large. But, it is included for the sake of completeness m dot f is known as the firing rate of the furnace. That is equal to mass flow rate of the fuel divided by this sink area.

The firing rate of the furnace is how much fuel we are burning per hour in relation to the total surface area available to absorb the heat. Now once you write that, the total heat released by the burning of the fuel will be m dot H f; has to be equal to heat that you transferred from the gas or sink plus heat that is taken away by the gas, as it leaves the furnace. If the gas leaves the surface has the temperature much above the ambient, you are losing heat that needs to be accounted for. Here m dot g is nothing but mass rate of fuel and mass rate of air.

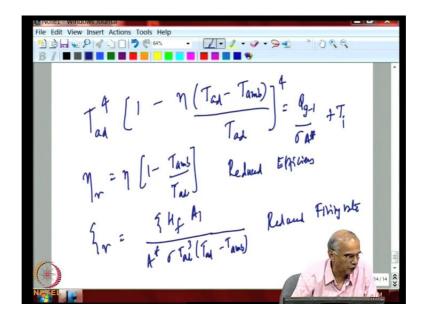
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With this simple definition we assume that, in the absence of this heat loss or any other heat loss, we define an adiabatic flame temperature as equal to this is a definition, so it cannot be really be questioned. This is adiabatic flame temperature. It is that temperature a gas will reach, if that has no heat loss from a gas to the surroundings, if all the heat is released is used to increase the gas temperature. This is, of course an upper limit; this is not really, actually encountered in any real life situations. Now, with this definition we can write down the temperature of the gas as heat transferred to the gas. We know one relation between T g and T one.

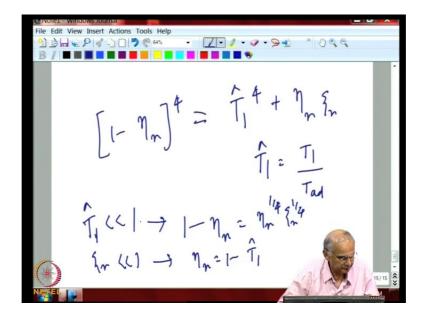
There is one more equation, and based on the equation we can show that one minus eta, efficiency of the furnace is T gas minus T ambient by T adiabatic minus T ambient. The efficiency of the furnace in some sense depends on how the gas temperature approaches the adiabatic flame temperature. If the gas temperature is almost equal to flame temperature, then your furnace's resistance is hundred percent. Of course, in the real world we do not really attain those high pressures. This definition is adequate. Now, we substitute this back into the energy balance equation.

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We will get the following expression; which will be T adiabatic flame temperature. So by substituting this, we can see that we can solve for the non-efficiency of the furnace in terms of the ambient temperature, the adiabatic flame temperature and the sink temperature. Now, it is suitable to modify this equation by defining a reduced efficiency. If we define the reduced efficiency as this, if you define reduced firing rate is now non-dimensional value, now it is T adiabatic here. Now, this is the expression we are using in order to simplify the final expression for efficiency. So, by using this definition of reduced efficiency and reduced firing rate, these are way to non dimensionalising the two terms.

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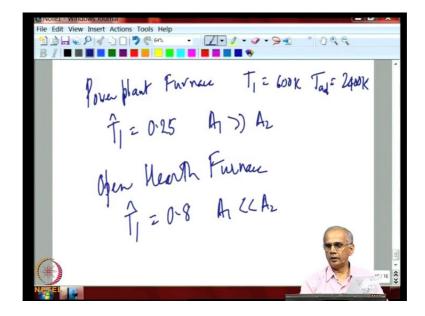


Once we know, once you do that, then the final expression for efficiency of the furnace comes out as one minus eta r to the power of four is equal to T one non-dimensional. This expression should be examined at some depth because this expression tells us a lot about what controls the furnace efficiency. This is expression; worth examining in some depth.

So, here the non dimensional temperature T one is T one by T adiabatic. Notice that this expression is very simple and it is very elegant for a given firing rate. We can get the efficiency eta r here, for a given sink temperature. So, what we see is that there are integral limit. Suppose the sink temperature is very low, then this term drops out. Then we get one minus eta r will be eta r to the power of one-fourth and tilde r to the power of one-fourth. This is one limit. In the other limit, where the firing rate is very small, this term can drop out. Then, we get a very interesting result that eta r is equal to one minus. These are the two limiting expressions that we have one for low firing rate.

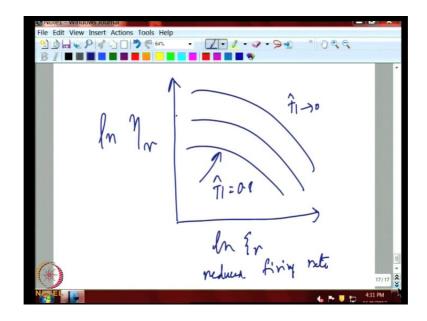
So, low firing rate is what really matters is the sink temperature. If the sink temperature is 0.8, then efficiency is only 20 percent. On the other hand, if the sink is at very low temperature approaching zero, then efficiency of approach can be one. This is what this tells you that the efficiency of furnace is strongly depending upon how much heat it is losing to the sink, which cannot be really be recovered. So, any attempt to increase efficiency of furnace must look upon how to increase the sink temperature.

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Now, in a typical power plant furnace; coal fired, natural gas fired, which we will discuss in some detail. In typical power plant furnace, T one is of the order of 600 k; T adiabatic is typically 2400 k. In which case T one tilde is of the order of 0.25 and the sink area is much more less than the areas which are hot. The other extreme is the open hearth furnace used in power plants or used in metallurgy. Here T one tilda is of the order of 0.8 and here the sink temperature is very small compared to the hot surfaces, which radiates heat. We have a range of possibilities from a very low sink temperature to very high sink temperature. All these options are there. Now, this result has been solved by Hottel.

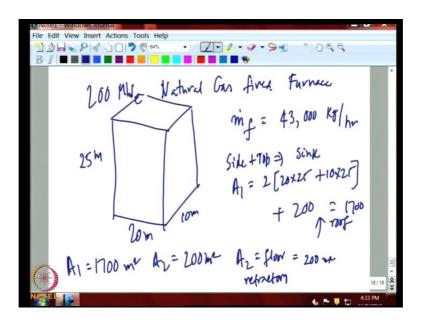
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One can easily show that the logarithm of the reduced efficiency, if we plot the logarithm of the reduced firing rate, we will get cross like this. This is T one tilda to zero this is for the open hearth furnace. Now, the key point to be noticed here is that at very low firing rate, we saw that the reduced efficiency of the furnace. It was only on the sink temperature T one tilda. That is clearly visible here. While at high firing rate, efficiency comes down substantially. So, obviously one of the ways to control the efficiency of the furnace, radiative efficiency furnace, is to keep the firing rate low; that is, providing enough sink area to dispose off the heat or keep the sink temperature high.

If we keep the sink temperature high, then we can attempt to reduce the heat losses. Now, to illustrate the usefulness of this kind of simple model, we will take an example of a real furnace. Only that will give you an idea about what really happens in a furnace.

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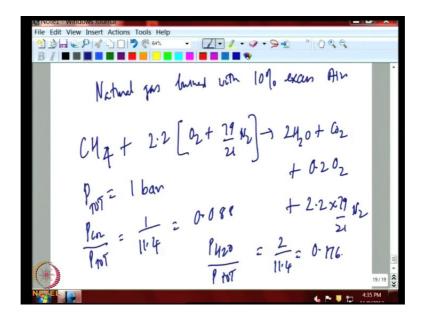


Let us take a very simple example of 200 megawatt electrical natural gas fired furnace. It is a clean case because we are not burning coal. A typical dimension of that furnace; typical limits or height is 25 meters and the plane is 20 by 10 meters and a typical flow rate given of the fuel is 43000 kilogram per hour.

Then, we assume all these areas on this side plus top of the sink where water is boiled. The sink area will be twice; 20 to 25, this area; 10 to 25, this area. They have to or both have to be doubled because there are two walls plus the roof area. Roof also contains heat sink. The A two is the floor area, which is a refractory which looses no heat. And,

that has an area 200 meter squared. They will come out as 1700. We have furnace with sink area of 1700 square and a refractory area of only to the floor, which normally contains ash in a coal fired furnace. Now, let us look at how to estimate the emissivity gas of the furnace.

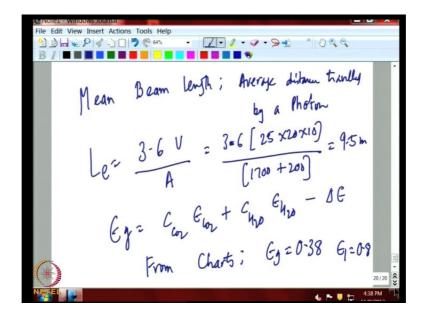
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For that, we have to assume a chemical reaction. Assume natural gas is burnt with ten percent excess air. The equation becomes methane plus ten percent excess air, which contains oxygen and nitrogen, is equal to assuming complete combustion. This is the excess oxygen going out because not all the oxygen is required. Now, this is the nitrogen which is going out. Let us assume the total pressure of the furnaces is of the order of one bar, generally placed slightly below a bar; because in the case of an explosion, we do not want the furnace to explode outwards. We want it to implode.

That is why pressures of the furnace are kept below the ambient pressure. Now, what is the partial pressure of carbon dioxide. The total pressure; this is from ideal gas law. We have to count the number of moles. The total number of moles is 11.4; part of which is to be used one. It is 1 by 11.4 is equal to approximately 0.088. This is the mole fraction of Co 2. The mole fraction of water vapor; there by 2 by 11.4 is 0.176.

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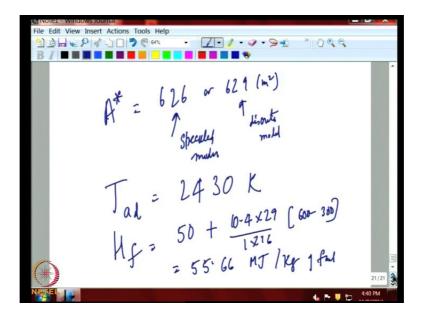


Now in order to calculate the emissivity of these gasses, we need to estimate approximate distance travelled by the photons within the enclosure. This is called the mean beam length. This is the average distance travelled by a photon in the furnace. Now, an approximate estimate of this value is L e is 3.6 V by A. We will look at how this expression arrived in subsequent lectures. This turns out is 3.6 into 25 into 20 into 10 the volume of the furnace by the total area. And, this comes out to be around 9.5 meters.

What this means is that, a typical photon travels approximately 9.5 meters before it is absorbed. This is the mean value. We realize that, photons are different wavelengths will travel different distances. But, this is a typical mean value. Now, how do we estimate emissivity of the gas. This is done as we have discussed earlier using this following formula, where the first term is emissivity of C o 2; the second term is correction for Co 2 because the pressure is not equal to 1 bar.

If the pressure is not equal to one bar this is the emissivity of H 2 o and the correction part of H 2 o minus this is the overlap factor. For this example, we can calculate this using charts we will get a number somewhat equal to. We may not get exactly this number; depends on how you interpret the charts, but you must get a value around 3.7 to 4.4 of the typical number. We will assume that the sink has emissivity of 0.8.

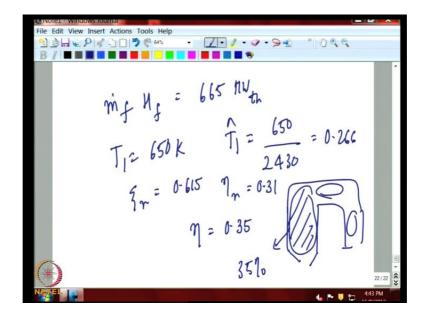
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Once we have estimated that, we can estimate the A star effective area of the furnace from the formula which you have derived. By using this formula, we may get 626 or you will get 629 meter squared. This is for the spectral model; this is for the discrete model. We can say that there is not much difference between these two. It is quite reasonable to assume that the sink surfaces are uniformly distributed around the furnace. Now, the next question is what the effective calorific value of the fuel is and what the adiabatic flame temperature is.

For methane, the adiabatic flame temperature is 2430 Kelvin and the effective calorific value including the 50 mega joule per kg for the methane fuel and the additional heat carried by air. This is the additional energy because the air is not at ambient temperature. This comes out to be 55.66 mega joules per k g of the fuel. This is the quantity which represents how much energy is coming in with the fuel along with the air.

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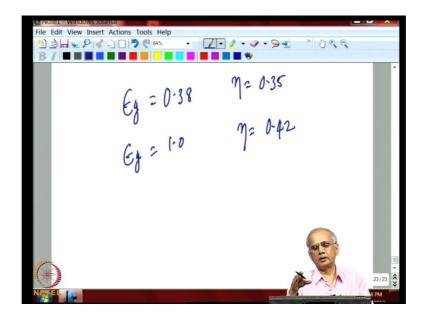


We can calculate for the given mass flow rate, 48000 k g per hour. We can estimate that so much of mega watt of heat is going into the furnace. If we take the furnace sink temperature to be 650 k, not a real assumption of power plant furnaces. Then, we will get the non dimensional value as 650 by 2430 adiabatic fuel temperature this is around 0.266. This, you substitute into the expression. From this, you get the reduced firing rate as 0.615.

Hence, the reduced efficiency as 0.31 and from there you calculate actual efficiency to be 0.35. What we find is that, for this simple calculation that we have done for the efficiency of a power plant furnace to transfer heat from the calorific value of the fuel to the water walls and heat sinks, which take that heat away; the efficiency around 0.35.

Now, this might seem somewhat low to many of you. But, remember that in a power plant, not all the heat is transferred from the gas to the water in one location. What it is saying is in the main portion of furnace, which can be sketched as follows. In this region, yes, efficiency is of the order of 35 percent. But, remember in the power plant we are recovering heat also here and here. So finally, the efficiency of heat transfer can be quite large. But, we only get concerned with how much heat is transferred in the main radiating zone of the furnace. That is coming out as 35 percent. But, we must also have questions whether the answers obtained is sensitive to some of the assumptions, we have made.

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For example, we assume the emissivity of the gas to be 0.38. Depending on how we use the charts and assumption you make; suppose you assume emissive one, which is rather high. But, this can happen if there is a lot of soot in the furnace during complete combustion. Even if we do that, the efficiency cannot increase; but, not to a very high value. Even if we increase the emissivity of the gas by 250 percent, the efficiency here goes on only by 7 percent.

What this shows clearly is that, the efficiency of the power plant furnace is not that sensitive to the emissivity of the gasses. Hence our rather approximate estimate of the efficiency of the emissivity of the gasses is not that critical. The charts we have used and estimation made are not that sensitive. The 40 percent efficiency is the maximum we are getting for emissivity one and for emissivity of that re estimate is around 35 percent. This is only the efficiency of the main furnace. In next lecture we will continue our discussion on the power plant furnace efficiency and see whether we can come up with some estimate of the errors that we may have introduced, due to assumptions we have made.

Thank you.