Radiation Heat Transfer

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Lecture - 22
Total Emissivity method

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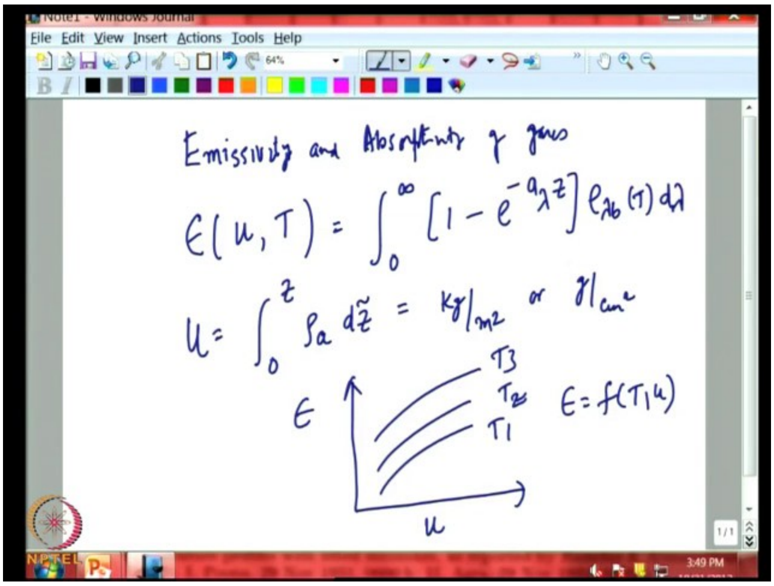

In the last lecture, we saw how we can calculate the emissivity and absorptivity of gases based on information from each of the absorption bands of the gas. Today, we will take an example of how we can solve real problem, given the emissivity absorptivity of gases. We say emissivity gas depends upon the path length of the absorbing gas, the gas temperature. It is defined as absorbing co-efficient times the length scale there.

This quantity the integration of wavelength of absorbing co-efficient has to be done by somebody. Here a path length nothing but the total amount the absorbing gas that is there along this path. This is usually measured in kg per square meter or gram per cm square in the older system units. If somebody can provide information about how this emissivity varies with path length for different temperatures then we can fit a function, so we can make epsilon function of temperature and u. Once this quantity is specified and a variable to u, we can then utilize this installing actual problem in a radiate heat transfer. Now, let us see how we convert the problem, which we had earlier defined in term of absorption co-efficient to one in terms of emissivity.

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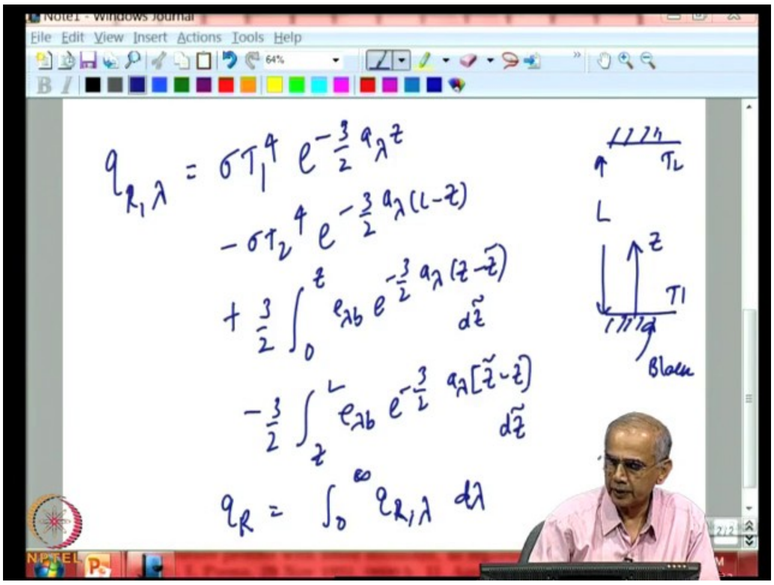

In terms of the spectral values if we recall, if we have, let us take a surface this is the distance z from the bottom. Actually you go up to some upper level the total length is L. Now, after using the Kernel approximation we can write down the assuming both surface are black, right now. So, will get sigma T 1 to the power of 4, this T 1 let us say this T 2. So, here e to power minus 3 by 2 after using the Kernel approximation is flux downwards to the top surface. Then you had the emission from all gas elements between 0 and z and finally, all the downward emission from all the gas elements above z, e lambda b e to the power of minus 3 by 2 a lambda into z tilde minus z.

Notice that always here showing that this thing is positive because it is a decay of the flux, due to absorption. Now, we want to convert this equation which is in terms of a lambda into one in terms of emissivity because we want to ultimately calculate q r, which is nothing but 0 to infinity q r lambda d lambda. We do the v length integration and we utilize a fact the total emissivity of a gas epsilon is 0 to infinity 1 minus a lambda z, e lambda b d lambda. Therefore, delta epsilon delta z would be equal to 0 to infinity a lambda e to the power of minus a lambda z e lambda b d lambda.

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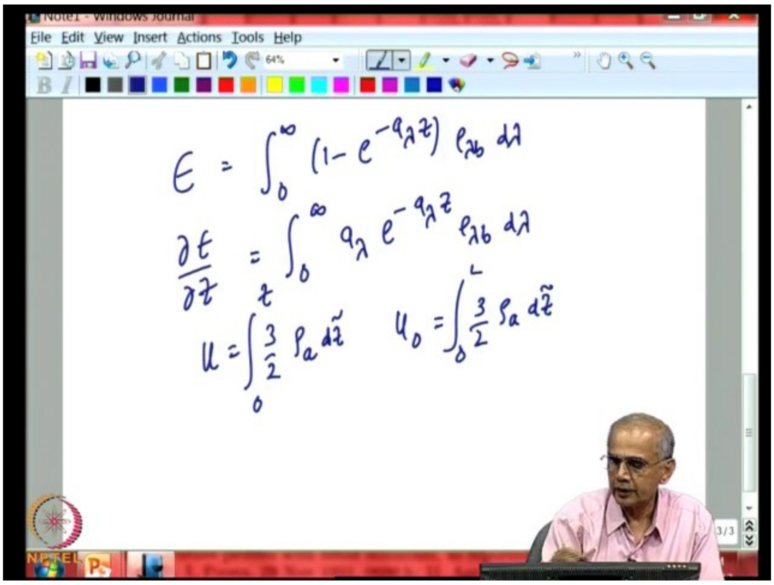

We go back to the pervious equation. We can clearly see that we can replace this by 1 minus epsilon after wave multiplication these can be replace by d epsilon d z. Finally, we are going to use u assuming homogenous medium, we can define this as typical the maximum amount of radiation. We can define this as these are total path length of the absorbing gas this is the, variable and these are parameter. Now, using this definition of emissivity the derivative of the emissivity path length, the total path length, we can rewrite the original equation, now as follows.

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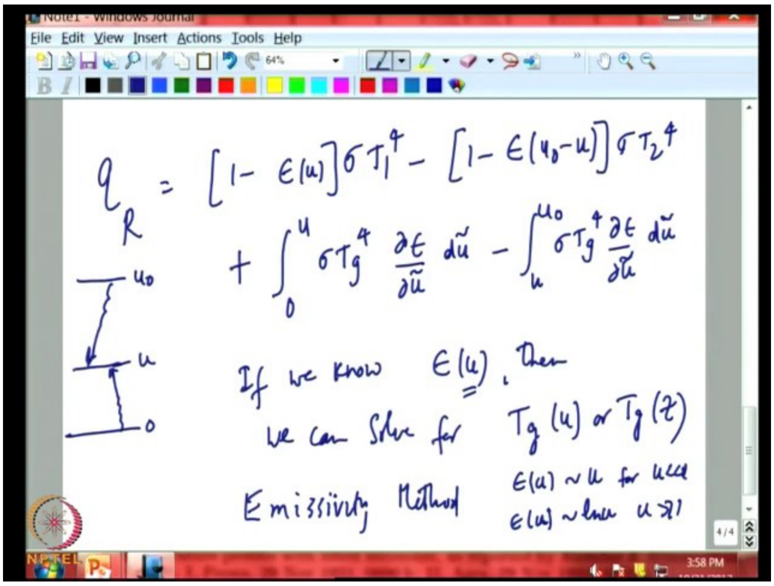

Only that argument in these functions is the distance over which, the photon has to travel in reaching a certain level. That quantity will be u for the lower surface and u 0 1 is u for the upper surface that is this is u 0, this is u this is 0. So, when radiant comes on surface 1 it will be distance is u and consent surface u 0 it is u 0 minus u. Then we have the gas emission terms.

Now, we find that the equation of radiative flux of the integration wavelength has been determined in terms of emissivity. The only task is if we know epsilon as a function of u. Then the problem is very well defined, then we can solve for T g as a function of u ultimately T g is function of z. The question is whether we have readymade data available for the variation of emissivity of path length. Now, the answer is that is not always available, but for certain well known gases like water vapor and carbon dioxide, people have taken the trouble to make laboratory measurements of the variation emissivity with path length and have fitted function to it and made available for the use by the every community.

If such data is available for the range of path length values that they are dealing with in a given situation. Then it is possible to use this emissivity method. So, emissivity method for solving non gray problems is feasible, if somehow we can get hold of exact nature of the variation of emissivity with path length. We are going to take and we known that emissivity varies linearly as u for small u and varies somewhat like log u or large U. Now, we already discussed that this are typical values in the limit of very low path length and very large pattern. We also saw there is an intermediate region where it varies correlative u.

So, knowing the given situation the range of u that you are going encounter, we can think of some function of this emissivity, which is either from laboratory measurements or we can also compute them from basic spectroscopic properties. Today because of the availability of high speed computers, we can take the actual spectroscopic data for the various lines in the vibration rotation band and do the integration and one of the method, we discussed in the last lecture for the co-related k method. So, any method can be used through band absorbance or co-related k and we can generate your own functional dependence of emissivity with you. There are various avenues available. Now, we will discuss one application of this problem.

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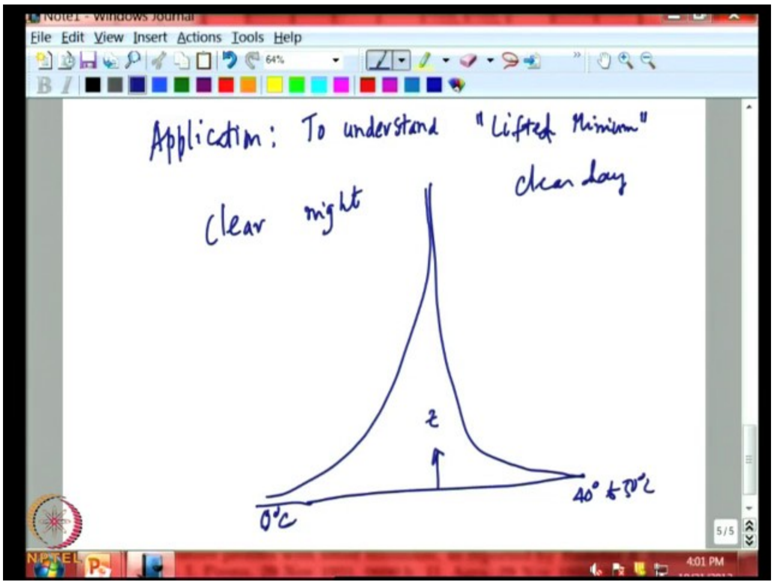

We will say the application is to understand phenomena called lifted minimum. This is a very interesting phenomena observed in the atmosphere a long time ago. And has had remained a mystery for long time because, it is a result which, when it was first discovered it was not believed. Now, let us look at a clear night when clouds are not playing any role. So, on a clear day on this side the surface temperature of the earth will reach a very high temperature, and you will see a sharp temperature. May be they will go to 40 or 50 centigrade the temperature will fall rapidly with height, this is the height here.

During the day if it is clear day, the surface of earth gets heated very much and can reach temperatures range of 40 to 50, but air next to it will be somewhat low temperature and will see a rapid fall of temperature with height. But as soon as sun sets, the air is clear, and then the temperature of the ground shots falling rapidly and ground is a good emitter, so it will cool rapidly. Air is a poor emitter and hence very soon the temperature of the ground will be much lower than, depending on the place you have it can even reach as low as 0 degree centigrade in winter.

The ground remains a coldest spot because it is very, very emitter and air which is not such a great emitter of radiation will not cool that much. This well known well known that the ground has the lowest temperature during clear nights and this has been documented in many places in the world. But around 80 years ago, a discovery was made in India which was counter to the simple understanding of the wave temperature varying near the ground.

Everyone assume that ground has a highest temperature in the day, which is not surprising because it absorbs air due to the sun's radiation. Also since ground losses heat rapidly, they accept the ground to have lowest temperature at night.

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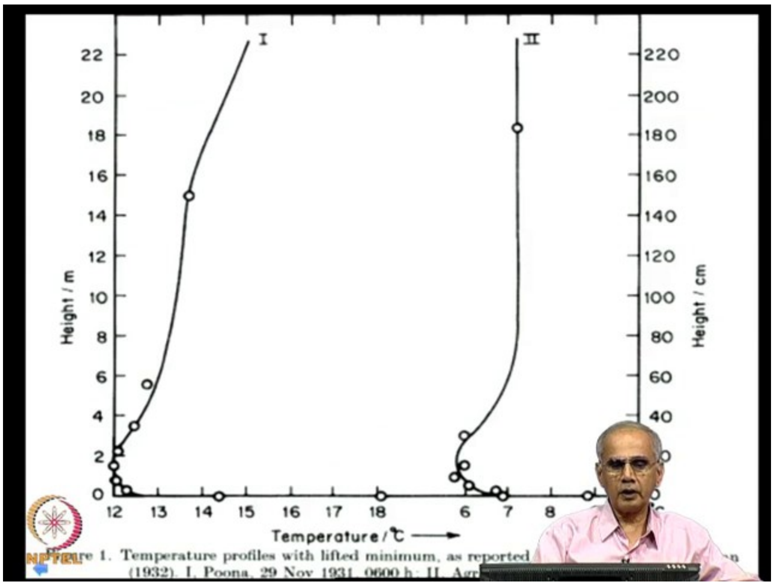

But the observing that were made both in Agra and Pune in 1931 by Ramdas and Athmannathan came as a great surprise, because when they made measurements of the temperature variation with height, in the lowest 20 meters of the atmosphere this region. They found that ground was at around 13 degree centigrade and the temperature fell by about a degree in the in the first 2 meters then started rising.

The minimum temperature was not at the ground, but about a meter or less above the ground and this was observed at two independent places one in Pune and one in Agra. This caused a great surprise, when it was first published because people asked how come the ground is warmer than the air. When ground is such a good emitter, why is it not able to cool rapidly to very low temperatures? This behavior is not seen on all days only a certain days, do they find the minimum temperature to be there above the ground or not at the ground.

In most days lowest temperature is seen in this wavelength at the ground, it will go this way. Only on certain special day does this unusual minimum above the ground called the lifted minimum appear. This result is counter intutive, because on all other days the temperature minima occur at the ground. Now, this issue is of course, of great academic relevant to understand phenomena, but it all has some relevant to the real world. If the lowest temperature appears not at a ground surface, but about a meter above it, then this has a round agriculture because if there are vegetables or other things at this level. If the temperature cause even low let say to around 0, then that can be floss formation and damage to the vegetable.

One important criteria notice is that, this feature of the lifted minimum appears only on clam clear nights. On windy nights, windy clear nights you get the traditional minima at the ground, but on clam nights this minima shifted to about 10 centimeters or 1 meter above the ground. The question is what phenomena control the occurrence of minima not at the ground, but some centimeters or meters above it. Initially various arguments were given. Some people argued that may be there the dust level near the ground, which enables the air to cool more effectively because of the whether dust but in many of the observations made both in India and also later abroad there was no indication existence of dust layer.

The other possible reason people thought was the role of turbulence they thought that on clam clear night turbulence are made very low. It could have let a very unusual behavior of the 80 diffusion coefficient of turbulence to have some unusual features, which leads to this minima. But this hypotheses was now well supported by observations because on clam clear nights. Now, believe one finds a little turbulence air is so still that turbulence generally does not exist on those days. So, turbulence was not important and dust had not been observed in these locations.

Then one must attempt to explain these phenomena through a combination of radiative cooling and heat transfer by conduction that is only possibility one can imagine. That there is some interaction between conducting heat transfer near the ground and radiation heat transfer, which somewhat leads to this interesting feature. This problem in order for you to solve it, you have to setup the basic equations for conduction and radiation.

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$$F^{\downarrow}(u) = \int_{u}^{u_{\infty}} \sigma T^{4}(u', t) \frac{d\epsilon}{du'}(u' - u)\, du',$$

$$F^{\uparrow}(u) = \{\epsilon_{g} \sigma T_{g}^{4}(t) + (1 - \epsilon_{g}) F^{\downarrow}(0)\}\{1 - \epsilon(u)\} - \int_{0}^{u} \sigma T^{4}(u', t) \frac{d\epsilon}{du'}(u - u')\, du',$$

$$\epsilon(u) = \frac{1}{\sigma T^{4}} \int_{0}^{\infty} \{1 - \exp(-\kappa_{w}(\lambda)\, u)\} B_{\lambda}(T)\, d\lambda$$

$$u(z) = \int_{0}^{z} \rho_{w}(z') \left\{\frac{p(z')}{p(0)}\right\}^{\delta} dz'$$

$$\epsilon(u) = 0.04902 \ln(1 + 1263.5u) \quad \text{for } u \leqslant 10^{-2}\ \text{kg m}^{-2},$$
$$= 0.05624 \ln(1 + 875u) \quad \text{for } u > 10^{-2}\ \text{kg m}^{-2};$$

Solve this problem, here it is shown how in this case we can write down the downward flux of the radiation from the atmosphere, integrate it between u and what is called as u infinity here. This is downward flux from the atmosphere, downwards and the upward flux if you assume the surface of the ground is not a blackbody, but a gray surface. Then you have to worry about the emission of the surface as per refection of the radiation coming to the ground.

This transmitted finally, to the level of interest this is the emission where all layers between surfaces the level u, where emissivity as usually defined in the traditional way. See a small difference in the way the path length is defined, a path length defined as low water vapor times d z. We also have added, what is known as scaling approximation. In the earth atmosphere pressure dust decreases the height total pressure, so that will effect on the line width of the absorption band. We want account for this non homogeneous path we are dealing with. The traditional definition of path length is modified by this case by these expression of p z prime p z per of the delta. This takes care of the non homogenous path followed by radiation as it goes from surface two very high altitudes.

Now, the emissivity that appears in these equations can be fitted into two results both involve logarithmic function and one is for values path length below 0.01 kg per meter square. The other is those who parts in above 0.01 kg meter square. Now, these equations and numerical values are obtained by fitting laboratory data to obtain these values.

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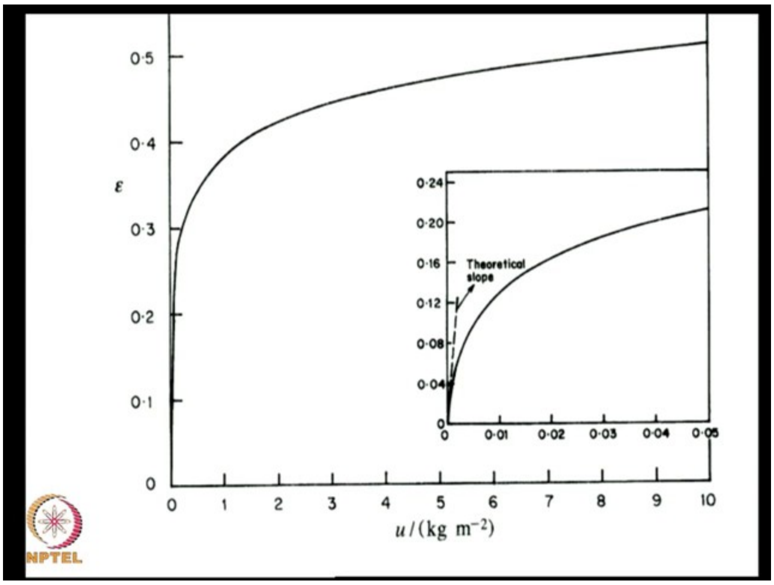

Here is a plot of the emissivity of water vapor as function of the path length the water vapor. Now, in the outer figure here you see what path length going for 0 to 10 kg per square meter, the emissivity rapidly increasing in the first few centimeters then gradually increasing to high value after a large path length. In the insert here we have shown variations over a small range from 0 to 0.08 kg meter square and we can see that the shape repeats itself, when we look at a smaller scale.

Now, think about this result for water vapor is that it is not a strong function of temperature. The emissivity of water vapor at room temperature is essentially independent of temperature; so that the single function to represent all cases that we are going to deal with.

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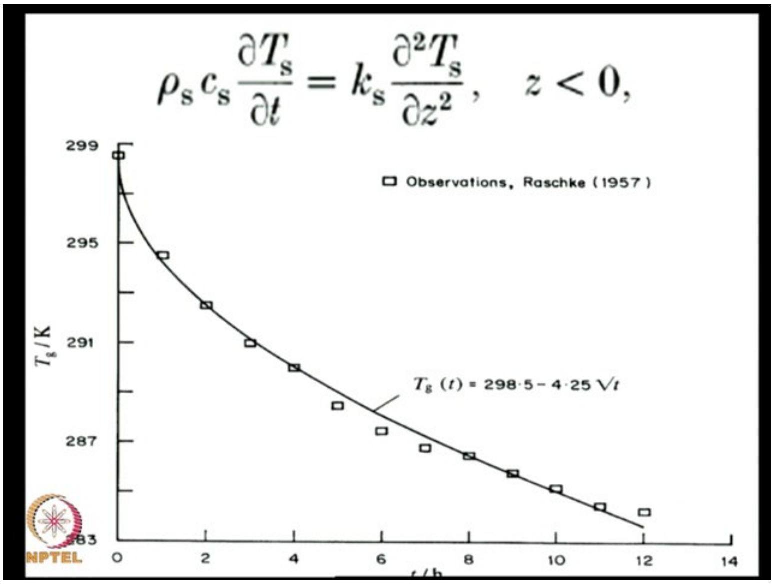

In addition to knowing the emissivity variation with path length, we also need to know how the ground temperature varies with height in the atmosphere. In the absence of radiation, the equation for temperature variation in the ground, the simple diffusion equation which many of you have seen. To solve this equation you will find that the temperature surface is going to decrease as square root of the time this is the time here. This was obtained from actual observation of the temperature of ground at night rapid of both for repeat for about 12 hours. This by solving the differential equation in the ground and this result will see later on is very useful. Now, let look at, how this equation is solved.

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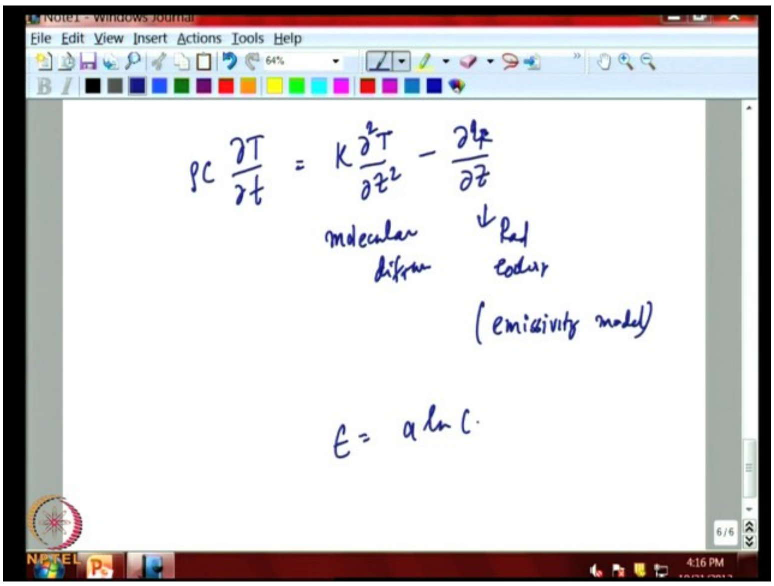

We solve the transient equation for the atmosphere. The transient for the atmosphere involve transient steady term, the molecular diffusion term and the radiative cooling term. For this calculation, we are going to use the emissivity model. We saw that where the expression for emissivity as a log 1 plus.

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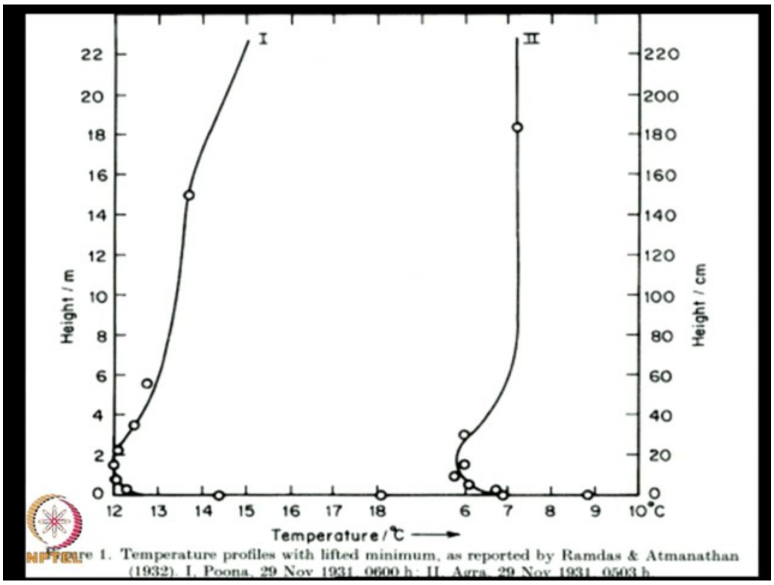

We here show again the expression for emissivity, now we knew that logarithmic function of emissivity, which we saw and will solve for this equation given this logarithmic function. Just as a reminder this logarithmic function are epsilon was of this kind and from laboratory data one could get values like epsilon u equals 0.049 log 1 plus 1263.5 into u for u less than and equal to 0.56.

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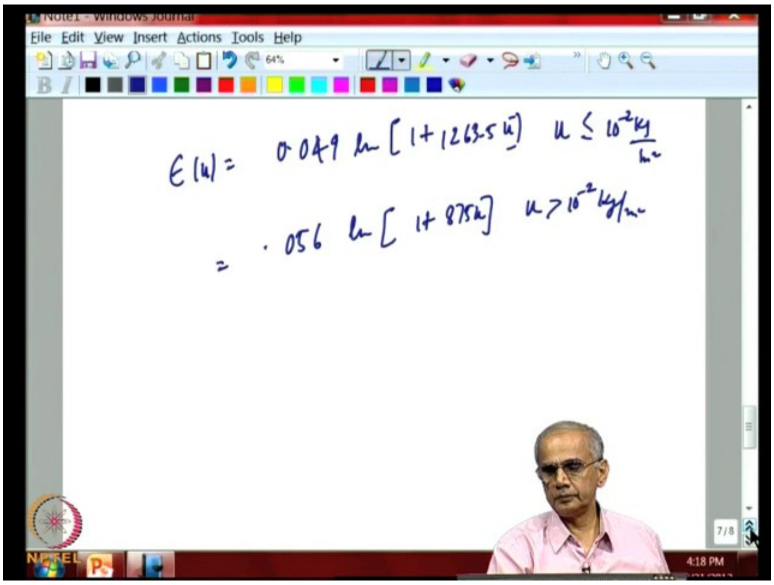

This is a piece wise continues function of emissivity at a certain value below u upon 0.01 when it 0.01, the slope changes, but not the function, function will be continues, but, the slope is not continues.

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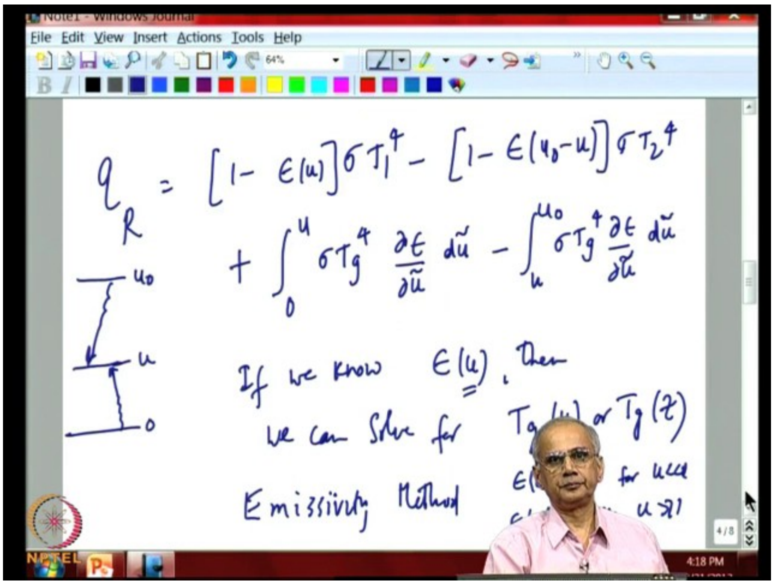

This was used in this equation, when the d q r d z was obtained from this equation, so we have d q r by d u calculated from this equation, emissivity has been specified.

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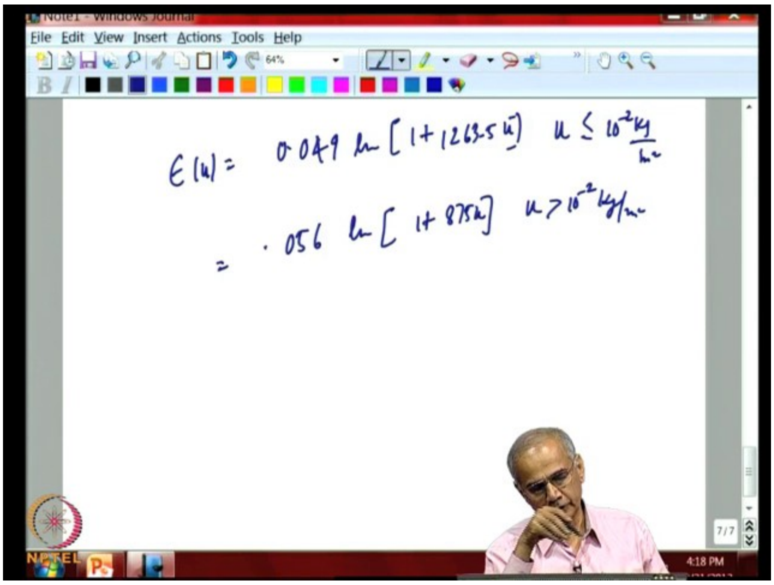

This differential equation is solved for temperature, so this is the purely step stepping procedure still some care is required by doing the numerical integration in showing that you maintain a finer mesh in a region where we expect to see the minima. This kind of equation is fairly easy to solve today, because availability of high speed computers.

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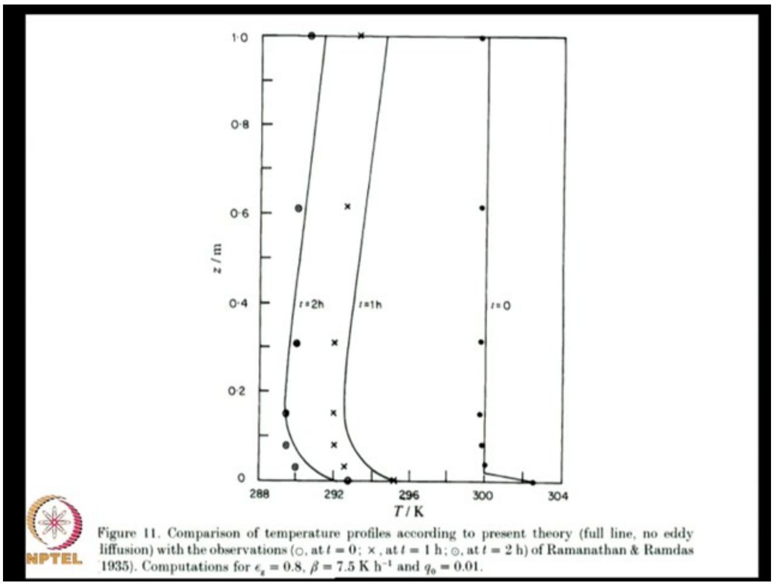

We take one example of this solution, starting with q r as system profile and time T equal 0. Then we integrate in time, remember that even time T is 0. There is a very sharp slip here this is not surprising because during the day the sun heats up the ground to a very temperature. At sunset we will expect to see a sharp temperature gradient between the gas elements away from the surface one those near the surface. But after about 1 hour we can see immediately that the temperature profile is has a minima at around the height of 10 centimeters.

The agreement between observation and simulation is not perfect, but the shape of the curve is if it is quite well. At 2 hours we can see that the temperatures air near the ground is in the lower than the surface temperature. So, what we showing is that we have been able to use this simplest model non gray model, which is the emissivity model to correctly get the variation of temperature with the height. Now, the question we want to ask is, now that able to reproduce that we need more understanding of this feature.

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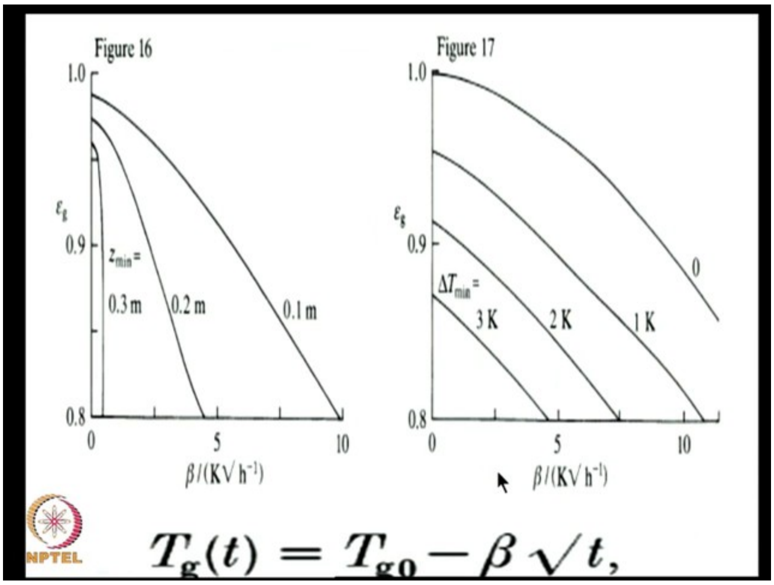

Now, here are results which indicate how the height of the minima and how it depends upon the cooling rate. If you look at the, there is very high cooling rate of the ground, then we are going to see much of minima because ground cooling will dominate the entire picture. On the other hand your ground cooling is low, than we can get a situation, where we can get a lifted minima.

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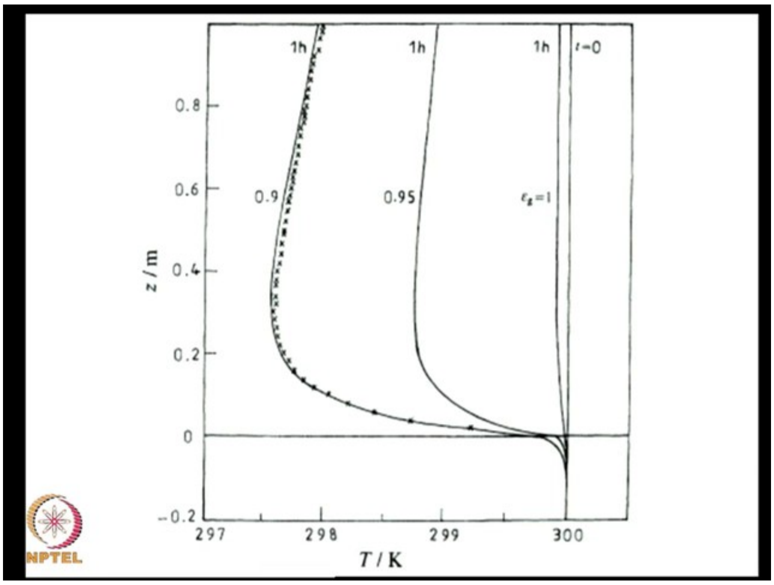

Now, to here to solve the problem exactly you need, solve the problem in the ground here along with the temperature profile in the atmosphere. It should be just let us state that briefly.

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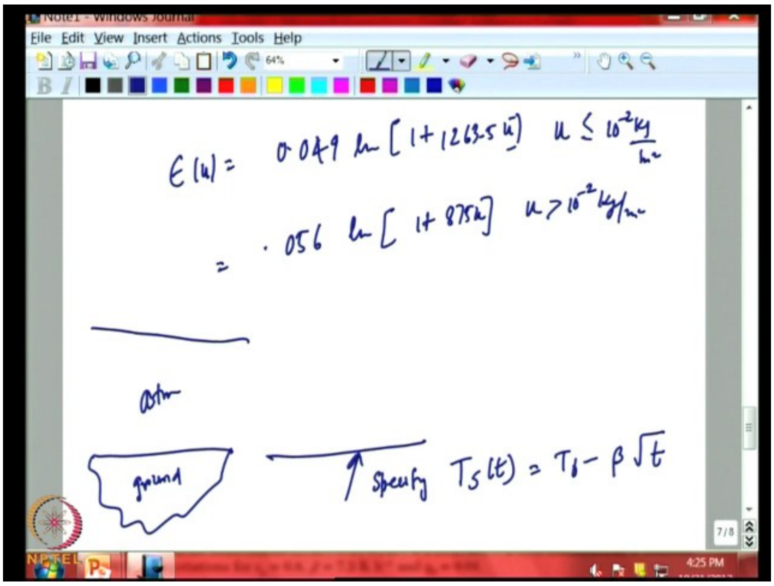

The point is that we can solve the problem two ways. Either specify the radiation surface temperature with time and one way that has been earlier is, to assume that the ground temperature falls at square root of T is a quite a enough solve a approximation. Or if you do not want to that, you solve the ground an atmosphere as a coupled system. It solved both for the temperature of the ground as well as of the atmosphere. This is a more realistic solution, and we see an illustration of this coupled solution.

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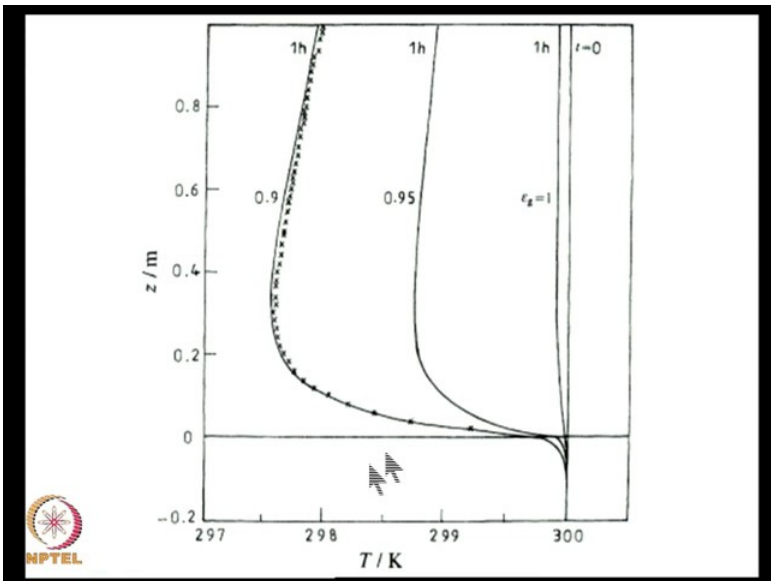

So, here is the ground about 0.2 meters deep ground as been showing here, along with the atmosphere. These are various solutions for the atmospheric temperatures of the various ground emissivities. So, one can see that if ground emissivity is quite large, close to 1, and then we did not really see the real picture any more. Other the hand if we assume emissivity of 0.9, we can get temperature profile in which minimum temperature is around 1 to 2 degrees below the ground temperature.

We are able to reproduce the lifted minima phenomena in the computer simulation and compares well with simulation in which the ground temperature is not calculated. We can solve for both ground temperature, air temperature and the atmosphere temperature near the ground together. Then we see that temperature will be very strong and visible only if the emissivity is 0.9 below at 0.9 the near occurs that 0.3 meters.

We can see clearly here all the temperature ground is not quickly rapidly. Now, well air is cooling more rapidly now. So, both air and ground started 300 raise Kelvin almost is thermal atmosphere here. Once air start cooling near the ground is such some very stable layer above, and cooling is control mainly by what is happening near the ground.

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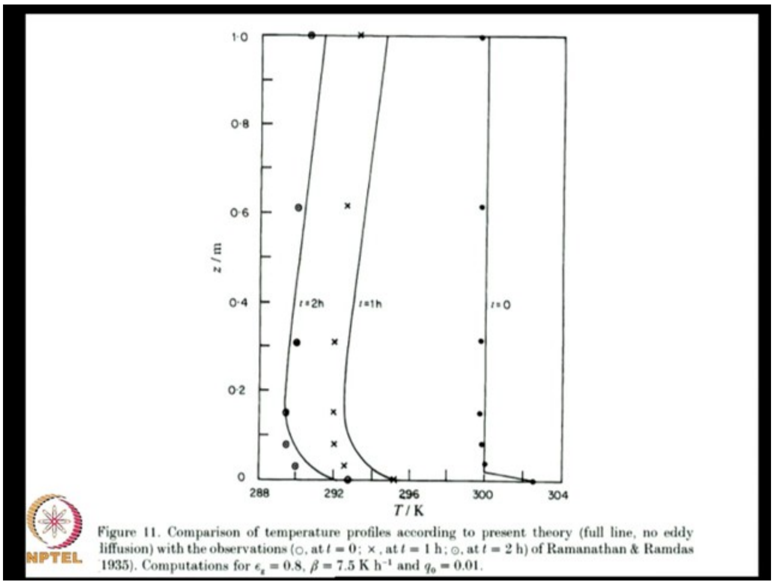

Here are few more examples showing that the observed temperature profile, near the ground does have a minima, and our numeral solution is able to capture some if not all features of this phenomena. So, precise agreement cannot be expected, but generally shape of the curve follows quite closely what is the observed in the field.

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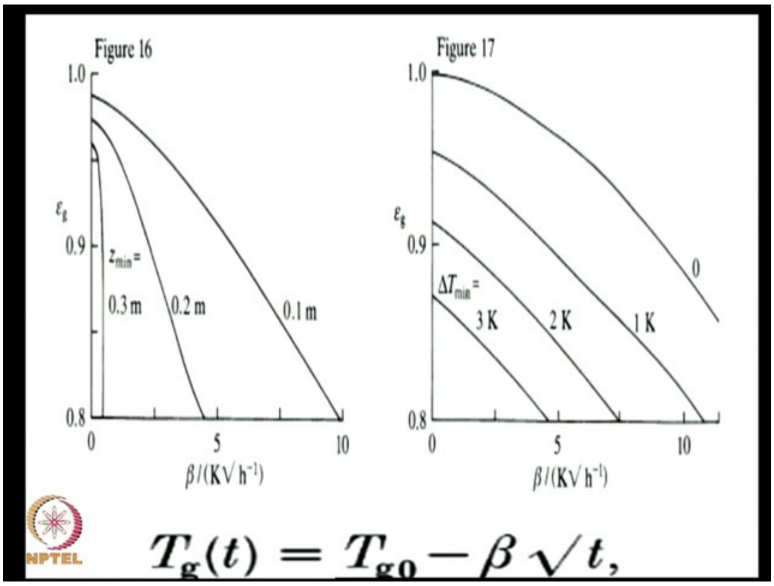

Now, next we will look at how some of the parameters show interest to us, vary with ground temperature. This is the two variables, which we will assume as shown above which is as you saw in a water vapour bad approximation, but they were also calculated for various degree of minima. But the key point we notice here is that as the ground cooling rate becomes larger and larger, its minima approachs 0.

So, lifted minima can occur if and only if your clam clear night if the ground cooling rate is not very large, then we can get a minima about pervious certain below, that of the ocean surface, while the cooling rate is very high at the land. This solution of the combined conduction radiation problem told has was that the existence of minima, depends both on the ground cooling rate as well as the emissivity of the ground. We will write down the important conclusions as follows.

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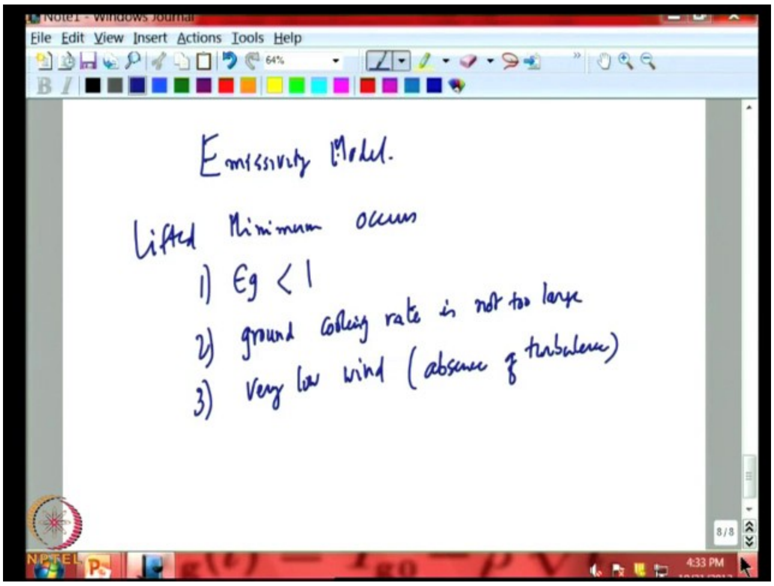

The emissivity model told us the following. They told us that the lifted minimum occurs one ground emissivity is less than one. Two ground cooling rate is not too large. This means that if the ground cooling rate is large the minimum temperature will be the ground. But if the ground is cooling slowly than is possible for the atmosphere to cool more rapidly than the ground and to reach at temperature, which is a lower than the ground that is the requirement. The second important point that came out was that the emissivity of the ground as to be below 1.

Now, this is a result which was initially not forcing and this result shows the important role that the curve in emissivity plays in this phenomena. This also partly explains the situation that occurs, why the situation occurs some what rarely. The lift minima phenomena not observed all over the world, is observed, only in certain condition. Of course, one of them we discuss already is very low wind because slight amount of wind will create turbulence and mix the different layers so that the minima will disappear. This has been shown when minima is occurring on a clam clear night.

If we take a stick and just wave it in front of you the minima will disappear, because minima can occur only in the absence of turbulence. So, even the slightest amount of turbulence will mix the layers up and remove the minima.

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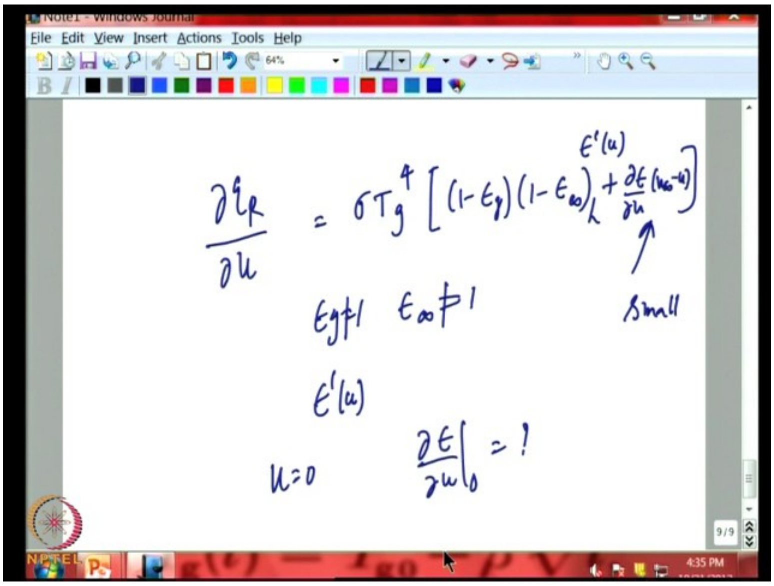

To understand the role played by the ground emissivity we must look at the expression for radiative divergence, which can be shown to be of this form, in this equation for the divergence of the flux. It is now clear that divergence of flux has to be quite large for the lifted minima to occur. So, epsilon g is 1 this term is small this plays no role in our analysis. Now, if this term as to be large, then epsilon g equal to 1 means there will be no cooling.

The requirement is epsilon g should not be equal to 1 and the total emissivity of the whole column also should not be equal to 1. That means, there must be some scope for the layers near the ground to cool to space through the regions, where atmosphere is fairly transparent. The most important thing is the role played by the derivative of emissivity. Now, near the ground this is the quantity of interest. If we go back and look at the figure we saw earlier.

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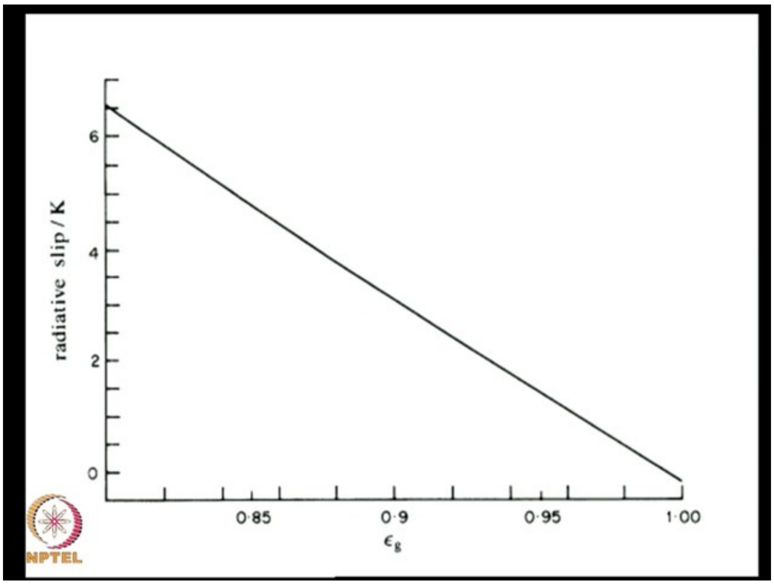

That there near the ground there is slip, which is phenomena is occurs in radiation. The slip depends on the emissivity, if there easily close to 1 there's no slip near the ground. As emissivity decrease slip becomes larger and larger this slip is a basic requirement for minima to occur, the ground as to be warmer by few degrees with reference to the atmosphere for the minima to manifest for itself. This shows the important role played by ground emissivity in controlling radiation slip.

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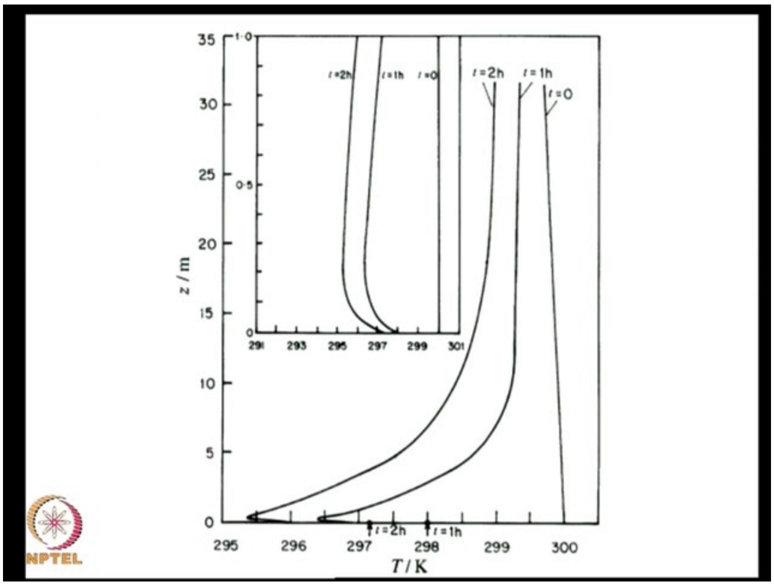

Now, we can see that the radiation slip is somewhat modified by the conduction heat transfer. So, very close to ground this phenomena which is shown in greater detail here. This slip is ultimately defused by the molecular conduction to give you a minima at a height of anywhere between 1, to less than meter in this case. Starting with isothermal atmosphere we are able to get atmosphere where minima is somewhere in this case within a 0 and 50 centimeters.

Notice this very, very close to ground, that is why many earlier measurements made with somewhat crude instruments, which did not quite get to the lowest 10, 20 centimeters may have missed the existence of this minima. So this minima to be discovered you need fine instruments, which will go all the way to the ground to identify that feature.

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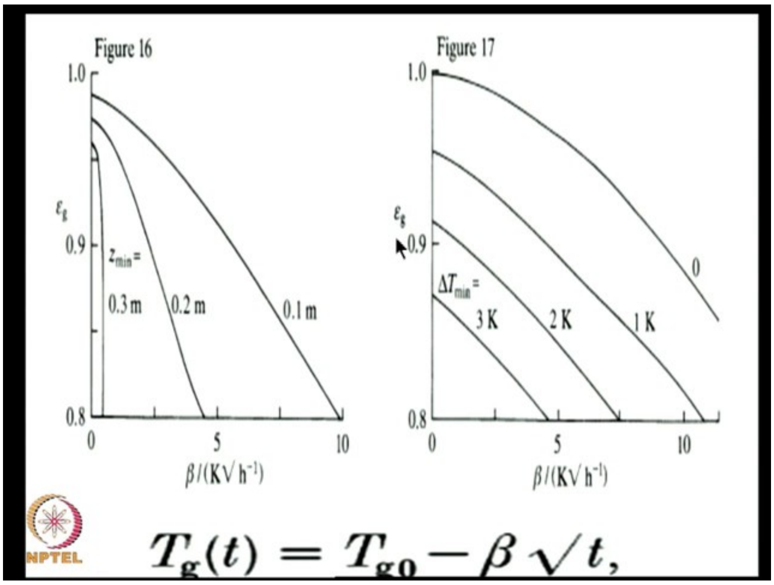

We see that the two requirements are very clear here. That the emissivity as to be realizably low to create a large slip the ground cooling rate if it is too large, that also essentially destroys the minima. The ground cannot cool very rapidly, that is why the conductivity of the ground does play a role. The ground, if it is a poor conductor of heat, then it will cool rapidly, then you will get this kind of rates. On the other hand, the ground is a good conductor heat, then it will not allow the surface to cool rapidly.

Hence it will keep the value of this beta around this value and then we can get pretty high minima. The second parameter we saw was the height, at which minima occur, that also depends upon the ground cooling rate a lot. Of course emissivity, as emissivity increases the minima moves closer to the ground. And as the cooling rate increases the minima come near to the ground. Ultimately it disappears, it goes into the ground. The height of the minima as well as the degree of the minima, how much cooling occurs depends both on the cooling rate of the ground and the ground emissivity.

This is an example which showed how a simple emissivity model of the atmosphere can explain a subtle phenomena like the, lifted minimum. But we must remember that this kind of result can be obtained provided, you get accurate information about the ground emissivity. Earlier one did not have high quality measurements of the ground emissivity.

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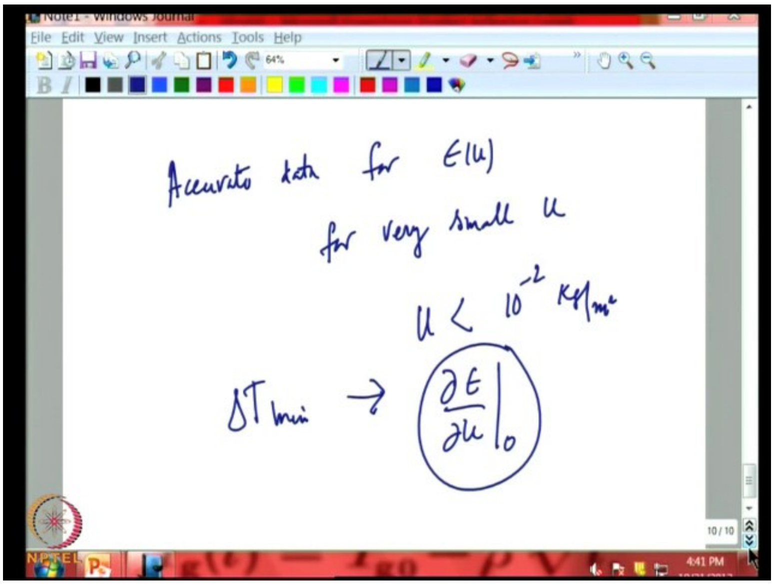

We need accurate data for epsilon of u for very small values of u. Remember u where typically of the order of 10 to minus 2 kg per meter square. This kind of data where not easily available. Hence, people used emissivity values, which are not as accurate as that has been shown here. If we do not do that you would not get reason for the minima. For example, we saw that the delta T mean is very much depended upon the slope of the emissivity curve near the ground, this is the crucial parameter.

Many of the older correlation where emissivity that where available, did not have sufficiently high values of this quantity. That led to the impression that lifted minima cannot occur. So, not only one at use emissivity model to unravel this phenomena, one also has to use extremely accurate data for emissivity.

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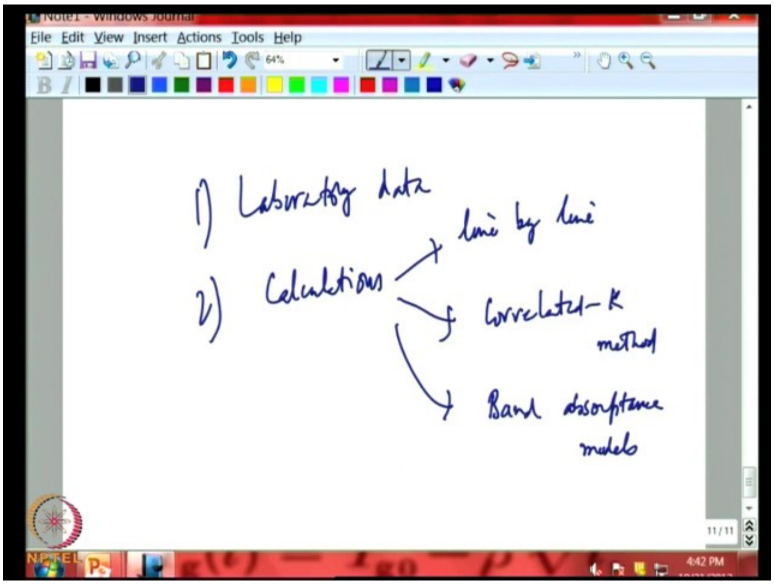

Today of course either we can depend on laboratory data, which are not easily available because they require very tedious measurements of emissivity of vary a path length and secondly is calculations. These calculations can be from line by line codes or can be from correlated k method, we discussed in the last lecture. Band absorptance models, all these can be used to obtain emissivity, for the range of path length that you encounter in a given problem.

In this problem we discuss in today's lecture, the path length that encountered very small and one has to look for very special data to get information. We illustrated to the role of emissivity, this model. In the next lecture, we look at other application of emissivity data that is relevant to furnaces. One important applications of radiation heat transfer in engineering is to furnaces. We will show how emissivity data for gases can be used in estimating the efficiency of a power plant a boiler furnace.