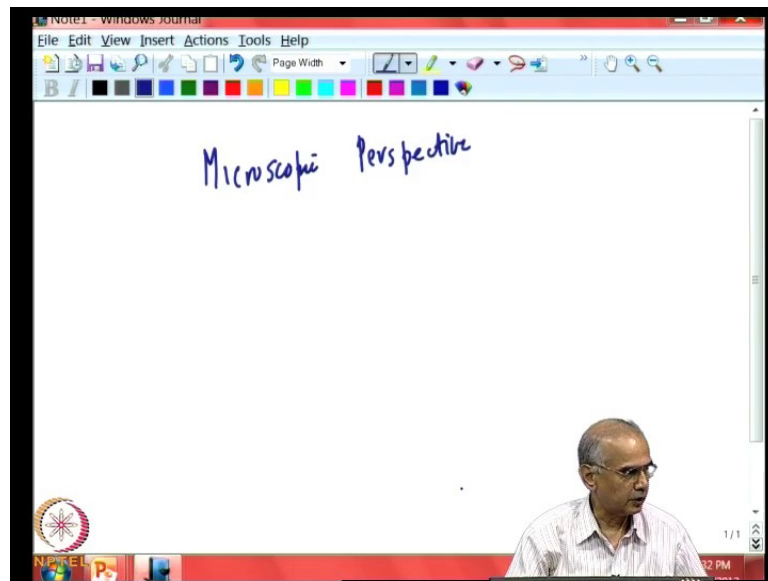


Radiation Heat Transfer
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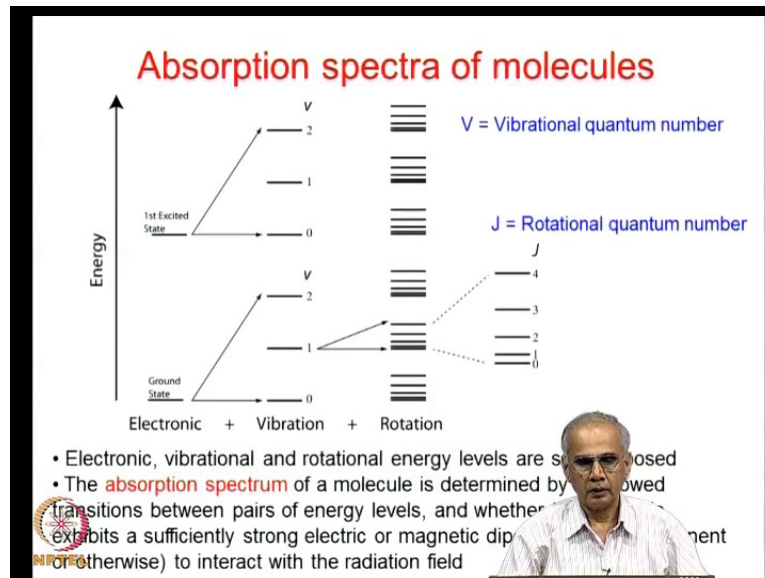
Lecture - 20
Isothermal gas emissivity

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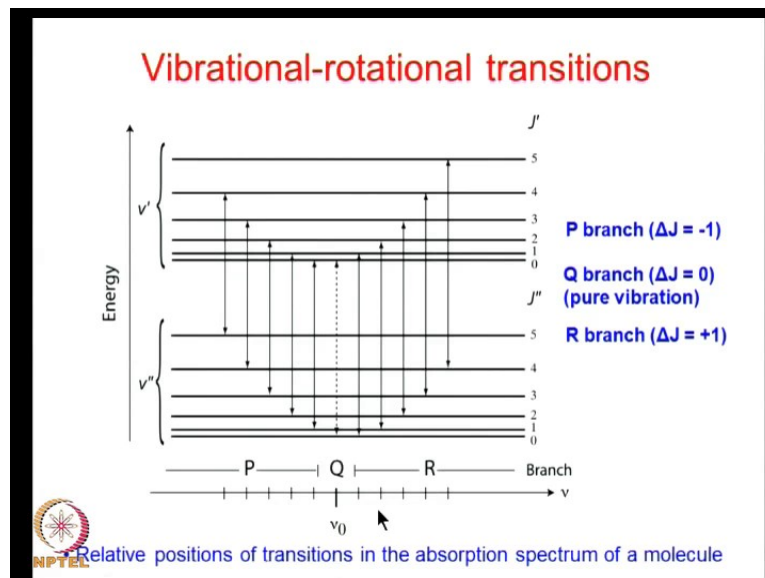
In the last lecture we were looking at radiative absorption in gases from microscopic perspective. We are trying to understand the role played by vibration-rotation in creating this complex structure of the radiation bands.

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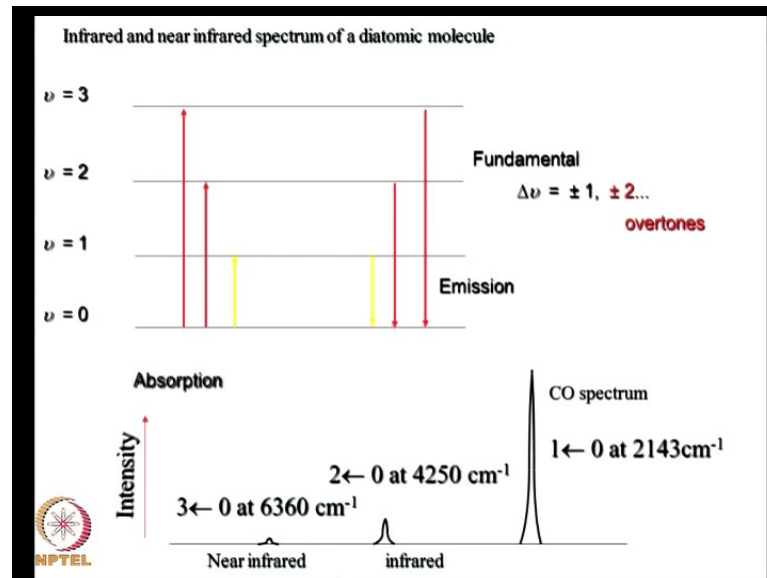
We saw that there are these vibrational transitions which come along with rotational transitions, which lead to three branches.

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The Q branch which is purely vibrational transition the R branch and P branch, and in R branch the frequencies are higher, because there is both positive vibrational energy change and rotational energy change. In the case of P branch, the vibrational transition is upwards and rotational is downwards. So, the frequency is lower.

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Then we saw that if gas temperature is high, the transition can occur instead of one level, it can be two levels and these are called over tones. These are important in high temperatures, especially in furnaces.

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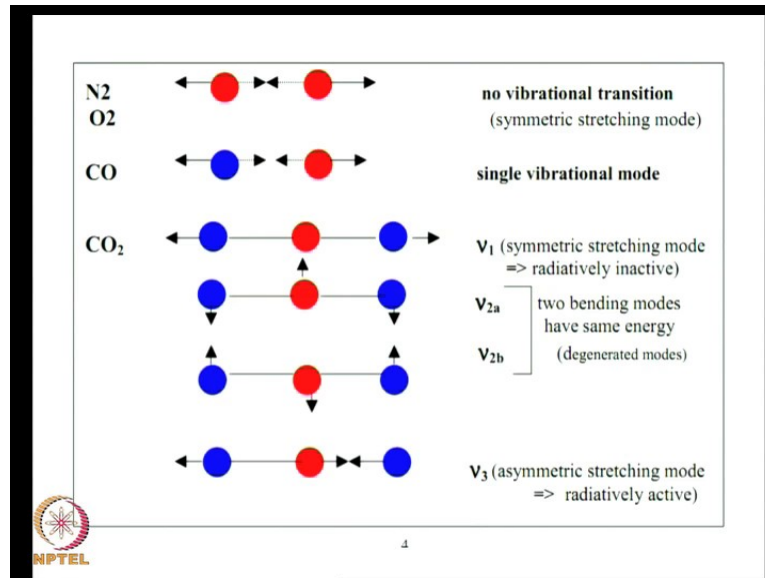
Molecule	Structure	Asymmetric Dipole?	Dipole Moment?
Oxygen		linear	No (magnetic dipole)
Nitrogen		linear	No
Carbon Monoxide		linear	Yes
Carbon Dioxide		linear	No
Nitrous Oxide		linear	Yes
Water		asymmetric top	Yes
Ozone		asymmetric top	Yes
Methane		spherical top	Yes

NPTTEL

We saw examples of various gases. We pointed out that symmetric diatomic gasses like oxygen, nitrogen have essentially no absorption bands because they have no permanent dipole moment. When they vibrate they do not have a dipole to interact with the electric field. They do not absorb or emit radiation.

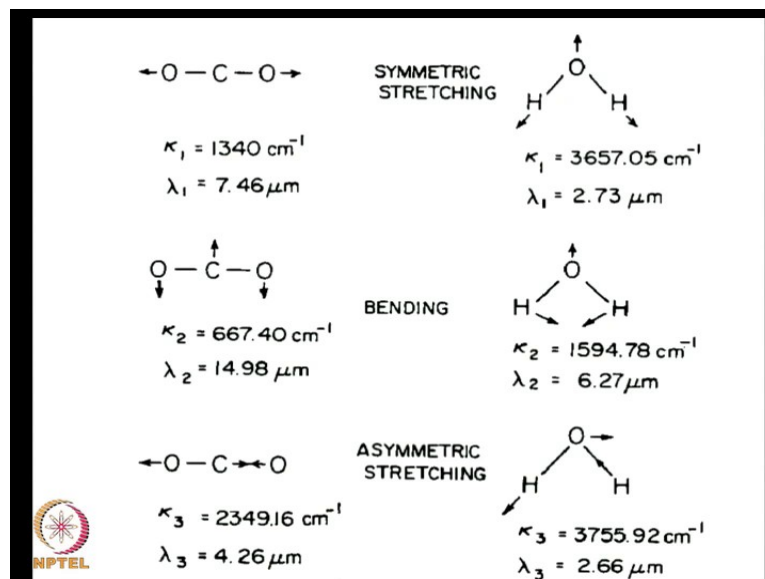
On the other hand, asymmetric molecules like carbon monoxide and molecules like carbon dioxide, which are linear tri atomic or nitrous oxide and water vapor which is non-linear molecule, ozone and methane.

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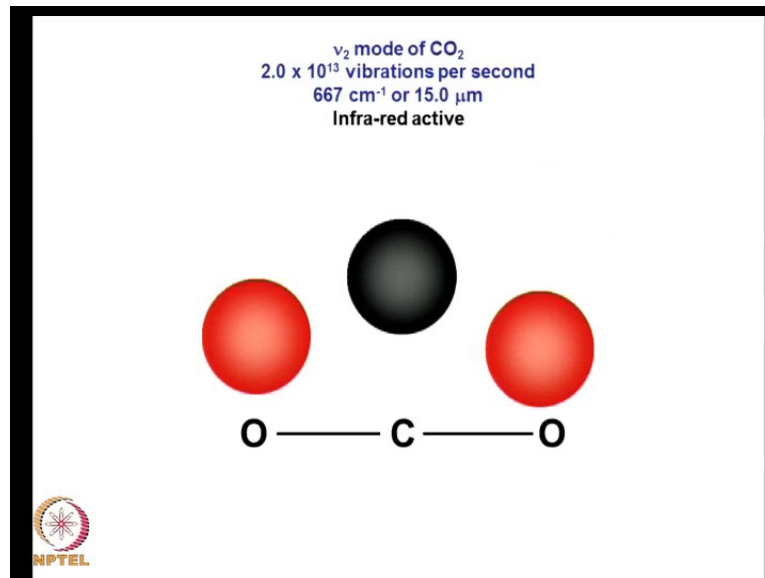
They all have absorption bands in the infrared. The way these bands are generated is based on either symmetric stretching or asymmetric stretching or bending.

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We saw the various frequencies at which these absorption bands occur.

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We saw examples of the symmetric stretching, asymmetric stretching and bending.

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Vibrational transitions have larger energies than 'pure' rotational transitions.

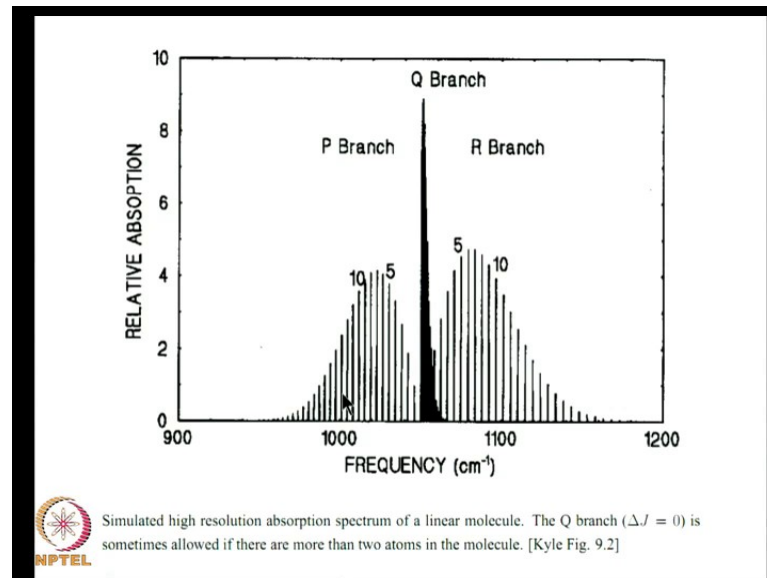
- **Stretching (symmetric and asymmetric)**
- **Bending**

Stretching frequencies are higher than corresponding bending frequencies



We saw that the stretching frequencies are higher because more energy is required in stretch than bend.

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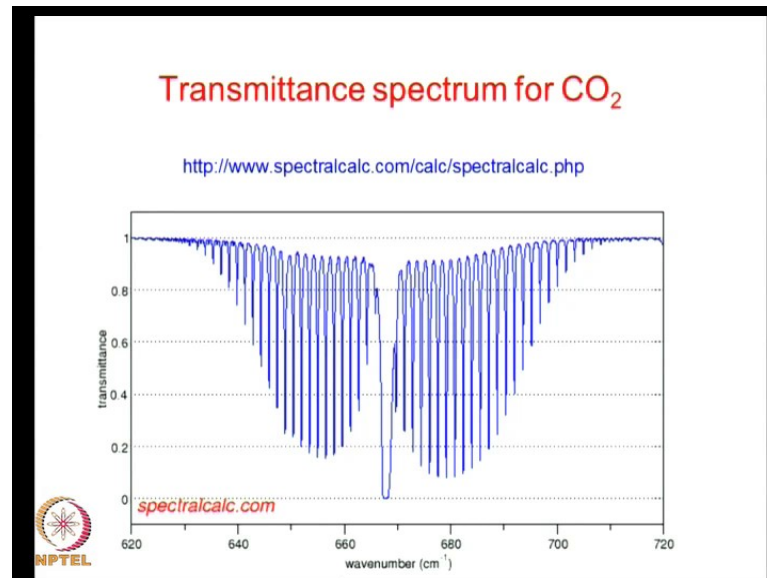


This is a typical vibrational-rotational band consisting of hundreds of lines. This is the problem one faces when solving problem in radiation that the absorption is the strong function of frequency or wave length and the variations are very rapid. For example, we have extremely high absorption at one wave length and immediately there is no absorption in the frequency between the two bands, between the two lines here.

This is the factor we have to contend with, that we have a quantity which varies extremely rapidly with frequency or wave length. When we integrate the radiative flux, the spectral radiative flux to get that total radiative flux, we have to use an extremely fine resolution to capture a strongly absorbing as well as the weakly absorbing region.

Now today, in principle this can be done because of the availability of computers. But still it is not done routinely; it is done more for basic research. We can integrate through all these individual lines by using extremely fine frequency spacing in the integration. We can capture all the radiation absorbed in these lines and the radiation which is allowed to be transferred between lines.

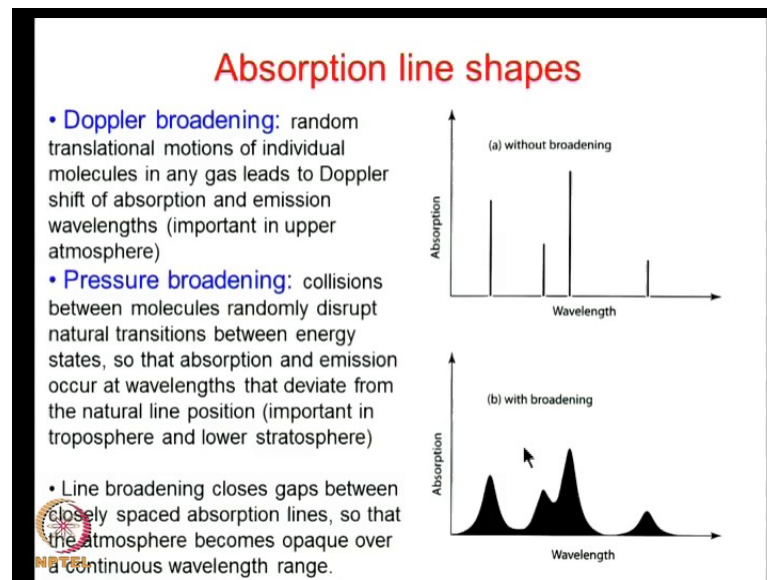
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But, our aim is to come up with a somewhat simpler method of solving; the so-called line by line integration. In principle, it is durable as long as we use the frequency step in the integration, which is smaller than the spacing between two lines. So, there are these lines and there is spacing between lines. As long as your integration step is smaller than the spacing, you should be able to capture this. But remember that one of these vibrations between bands has hundreds of lines and there are many bands. A typical example in the real world will have many gases; carbon dioxide, water vapor, methane, ozone. Each one of them has many bands, and each band has hundreds of lines.

The total amount of lines you have to integrate through is very large. Although in principle, we can do an integration over wavelength by taking extremely fine wavelength intervals. In reality, this is not done routinely.

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We saw the example of these lines as it exists and what happened to the lines when there is collision with the other molecules, which makes the line broader. The new feature of radiation transfer is, suppose we consider a medium containing carbon dioxide and nitrogen, carbon dioxide is the gas which absorbs infrared radiation; nitrogen does not.

Let us assume that there is a given amount of carbon dioxide in a container. The carbon dioxide and nitrogen combined total pressure is, let us say one bar. Then we increase the amount of nitrogen in the container by pumping nitrogen in and increase the total pressure by two bar. We do not change the amount of CO₂, but change the amount of nitrogen, which is a non absorbing gas.

We normally expect that addition of a non absorbing gas should not alter the ability of carbon dioxide to absorb radiation. But it does, because these additional nitrogen molecules will collide with the carbon dioxide molecules and broaden these lines substantially.

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Lorentz profile of a spectral line is used to characterize the **pressure broadening** and is defined as:

$$f_L(\nu - \nu_0) = \frac{\alpha / \pi}{(\nu - \nu_0)^2 + \alpha^2} \quad [7.1]$$

where $f(\nu - \nu_0)$ is the shape factor of a spectral line;

ν_0 is the wavenumber of a central position of a line;

α is the half-width of a line at the half maximum (in cm^{-1}), (often referred as a **line width**)



We saw that the typical collision broadening function is like this. In this function, ν_0 is the centre of frequency and α is the line width. This line width is a function of total pressure.

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- The **half-width** of the Lorentz line shape is a function of pressure P and temperature T and can be expressed as


$$\alpha(P, T) = \alpha_0 \left(\frac{P}{P_0} \left(\frac{T_0}{T} \right) \right)^{1/2}$$

where α_0 is the reference half-width for STP: $T_0 = 273\text{K}$; $P = 1013 \text{ mb}$.

α_0 is in the range from **about 0.01 to 0.1 cm^{-1}** for most atmospheric radiatively active gases.

The **Lorentz profile** is fundamental in the radiative transfer in the lower atmosphere where the pressure is high.

The collisions between like molecules (**self-broadening**) produces the large line-widths than do collisions between unlike molecules (**foreign broadening**). Because radiatively active gases have low concentrations, the **foreign broadening** often dominates in infrared radiative transfer.



The line width depends on total pressure. If you increase total pressure by adding a non absorbing gas, we will change the line width. Hence, change the ability of carbon dioxide to absorb radiation. This is unusual paradox which we do not find in many other fields. That is, a molecule which does not absorb radiation by itself is able to alter the radiative

heat transfer pattern by mainly colliding with the molecule, which can absorb radiation and changing the character of that absorption phenomena.

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Doppler profile is defined in the absence of collision effects (i.e., pressure broadening) as:

$$f_D(\tilde{\nu} - \tilde{\nu}_0) = \frac{1}{\alpha_D \sqrt{\pi}} \exp \left[- \left(\frac{\tilde{\nu} - \tilde{\nu}_0}{\alpha_D} \right)^2 \right] \quad [7.2]$$

α_D is the **Doppler line width**

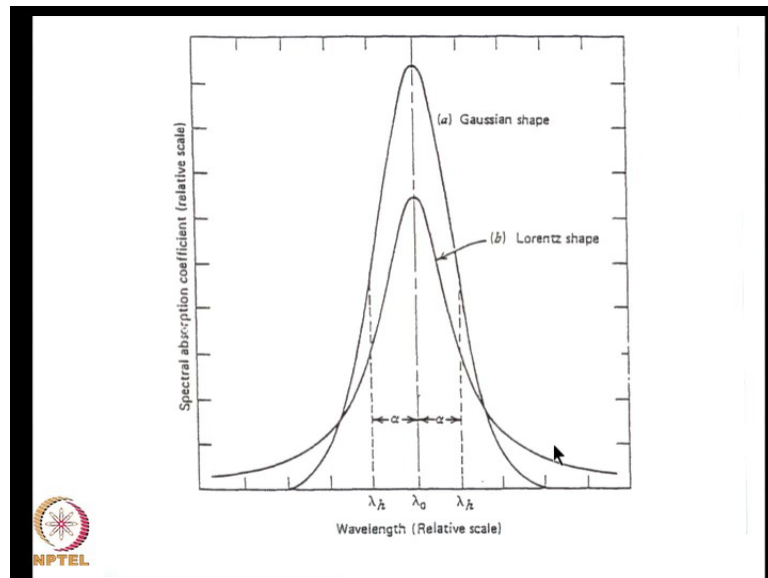
$$\alpha_D = \frac{\tilde{\nu}_0}{c} (2k_B T / m)^{1/2}$$

where c is the speed of light; k_B is the Boltzmann's constant, m is the mass of the molecule.



This is an example of an indirect impact of a gas like nitrogen. We also saw that in addition to collision broadening or Lorentz broadening, which is caused by collision between different molecules, there also broadening that occurs because molecules are traveling at quite high speed. The frequency of the emission by a molecule that we see in the laboratory is different from the frequency as we see sitting on the molecule. And, so this Doppler broadening depends strongly on temperature. We can see that the half width for Doppler broadening is α_D . And α_D is proportional to square root of temperature. So, at high temperature the width is much larger than at low temperature.

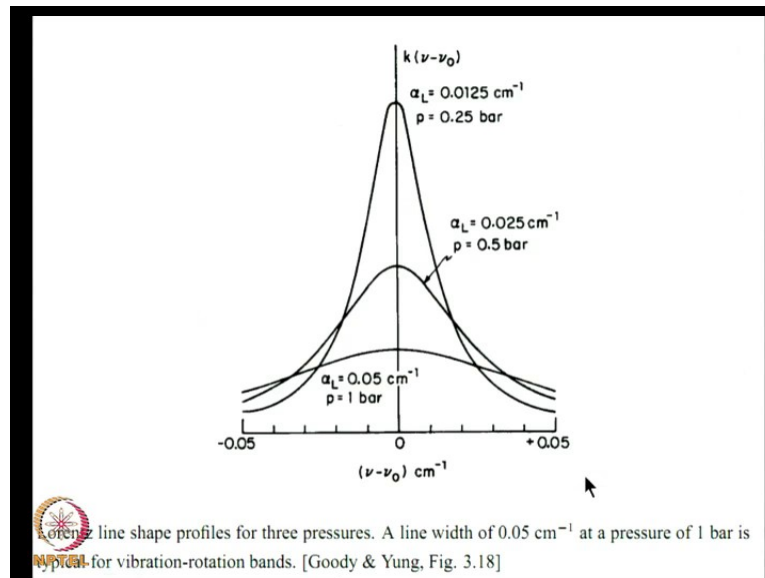
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We saw the typical profile of the Doppler broadening and Lorentz broadening. Doppler broadening has a Gaussian profile. Hence, it steeply falls to 0, while the Lorentz broadening goes 1 over 1 plus x squared. So, it goes down with frequency at a much slower rate. We can see that we can have situations where in the Doppler broadening dominates near the centre of the line, while at the edge of the line, Lorentz profile dominates.

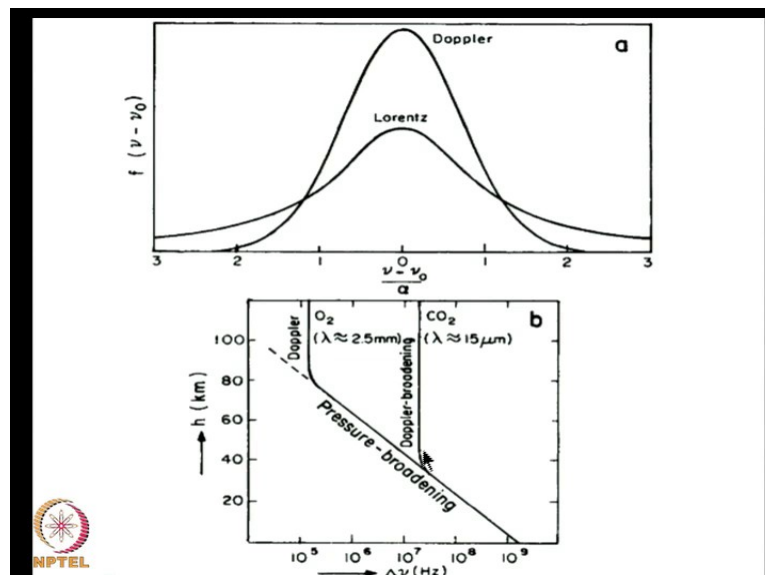
If such a situation occurs, then you have to use a more complicated shape which are the shape of the Doppler profile near the central line Then we pick the Lorentz profile. This is called white profile. It is used in those situations where the Doppler broadening dominates at the central of the line and Lorentz broadening dominates at the wings.

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This figure shows the important effect of pressure on the line width. We can see that the line width which is only 0.0125 centimeter minus 1 at low pressure like one-fourth bar, as the pressure is doubled, the line width is doubled. Again the pressure is doubled, line width is doubled.

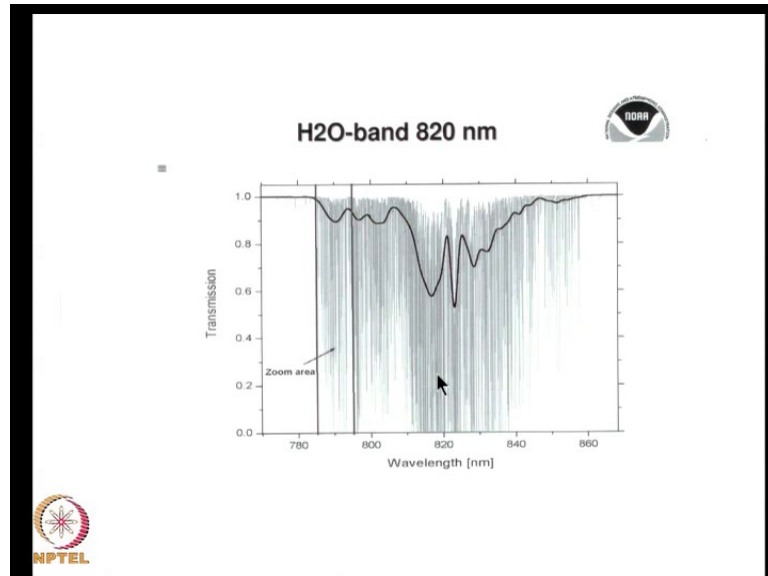
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We can see that line width in this case increases linearly with pressure. That is why in the earth's atmosphere Doppler broadening dominates at low pressure above about 80 kilometers, while pressure broadening is what we will see at the lower elevations for oxygen.

For carbon dioxide; beyond 30, 40 kilometers, We can see that Doppler broadening dominates and below 40 kilo meters, the pressure broadening dominates.

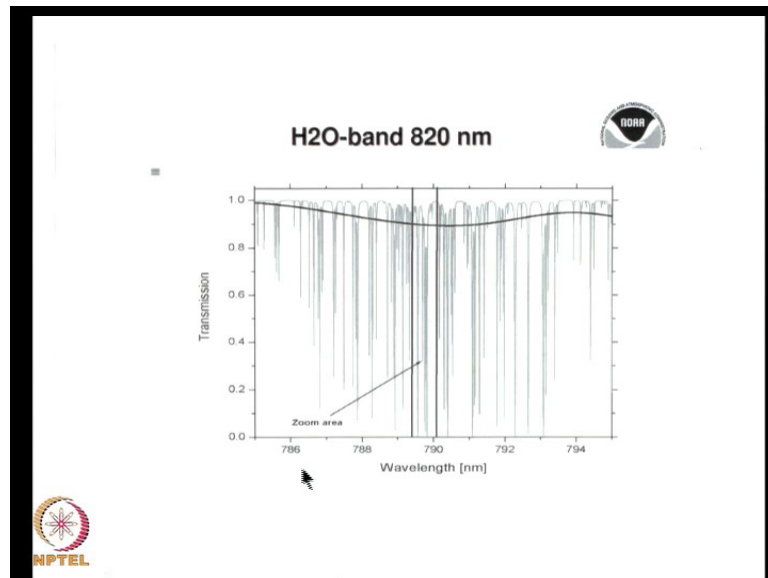
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Here is a typical example of water vapor absorption band at 820 nanometer showing presence of thousands of lines with pretty complex behavior. The smoothed picture is shown here. In the oldest spectrometer that we used thirty to forty years ago, we would only get that smoothed profile; we will not get the individual lines.

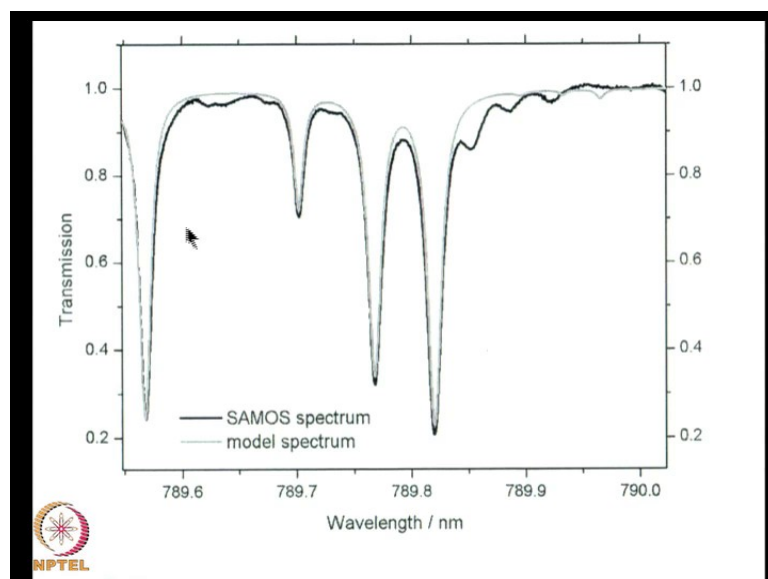
Today, We can identify these individual lines either using a high resolution spectroscope or by using a software which generates these lines based on the Quantum Mechanics and experimental data. Now, we are going to show this figure enlarged slowly. Now, this is going from 780 nanometers to 860 nanometers, 80 nanometers.

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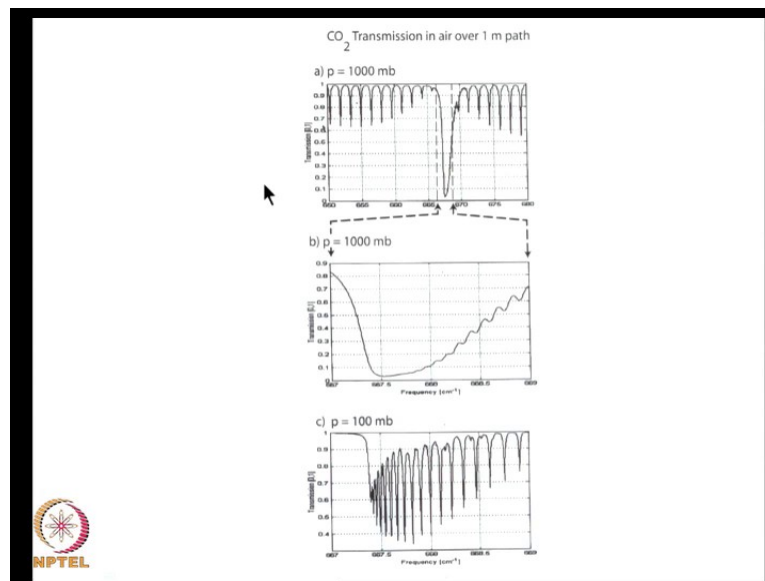
Now, we have a close up where now you look at only 8 nanometers. So, ten times we amplified the figure. Now we begin to see the individual lines and the spacing between the lines. This is where we see that we have to integrate very carefully because this graph of water vapor is almost transparent in a certain wave length and in the adjacent wave length, it is almost opaque. There is a rapid transition between opaque to transparent to opaque. If we use a very crude resolution, we will get this figure which is shown here. That can sometimes give misleading answers.

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So for accurate estimate of fluxes, it is important to use as high resolution as is practically feasible. Now we look at the same spectra again, instead of 8 nanometers looking at only 0.4 nanometers, very high resolution. This one comparing the absorbed spectrum with the simulation spectrum, showing that the simulations have reached fairly high level of accuracy. It is able to identify each line in this band and showing a large spacing between the lines.

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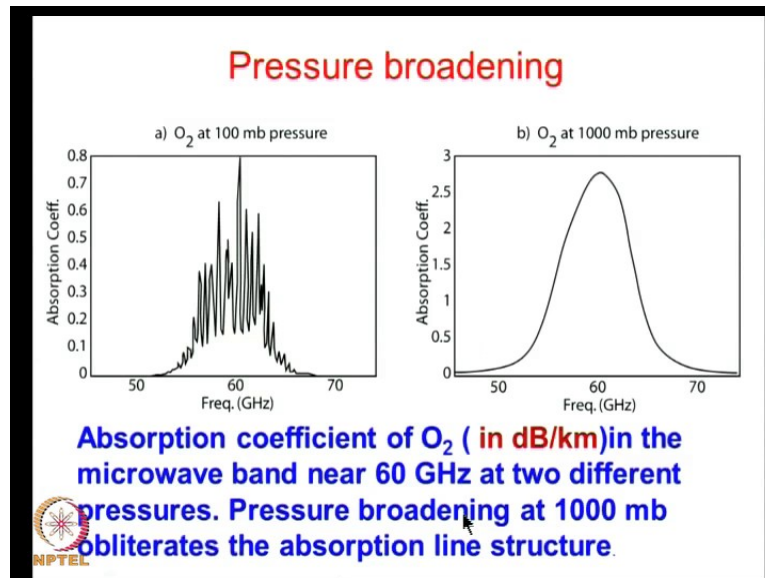


We show once again the impact of pressure; it starts with 1000 milli bar. We get the full carbon dioxide spectrum at 670 centimeters minus 1. Then we look at a close up of the absorption band right close to 667. We see that this thing has slight waviness. That waviness is there because of the lines. But these lines are not fully resolved.

If you now go to 100 milli bar where the pressure broadening is much weaker, these lines are visible clearly. These lines are visible at low pressure or not visible at high pressure because the line width increase so much that it completely blocks out the space between the lines and we get a fairly smooth function.

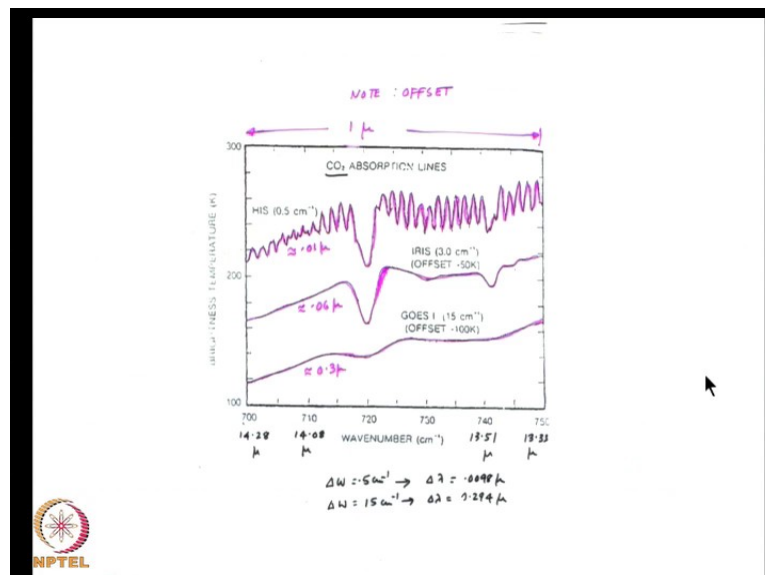
If you are doing this problem at one bar pressure, then of course we can easily calculate this by using a fairly crude integration. But if we are dealing with this situation, we have to be very careful how to integrate this transmission to get the actual result.

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One more example we have given of oxygen absorption coefficient in the microwave for low pressure and for high pressure. We can see that, in the low pressure you will see all the high resolution fluctuations; while at high pressure, all of these get smeared out and we are getting a fairly smooth profile here.

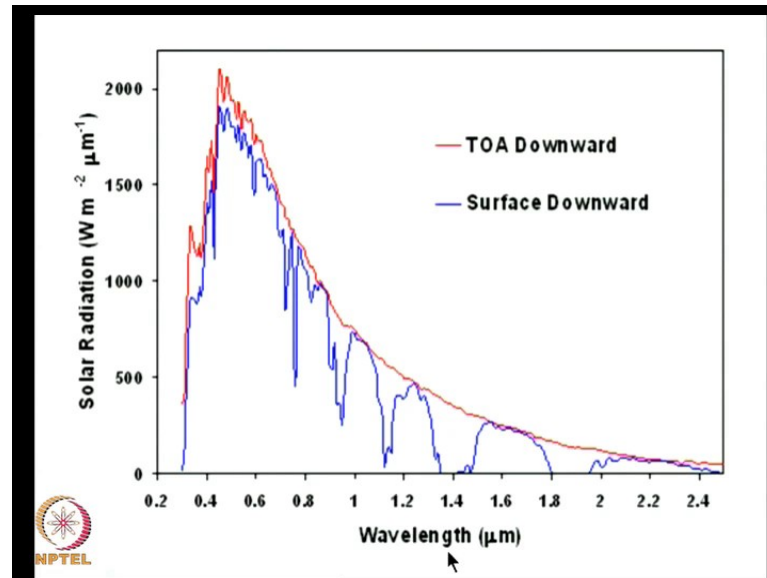
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Lastly, we will see how the atmospheric measurement of carbon dioxide emission by satellites. The older satellite had extremely poor resolution about 15 centimeter minus 1. So, could not really resolve the carbon dioxide spectrum in 720 or 14 micron, 15 micron

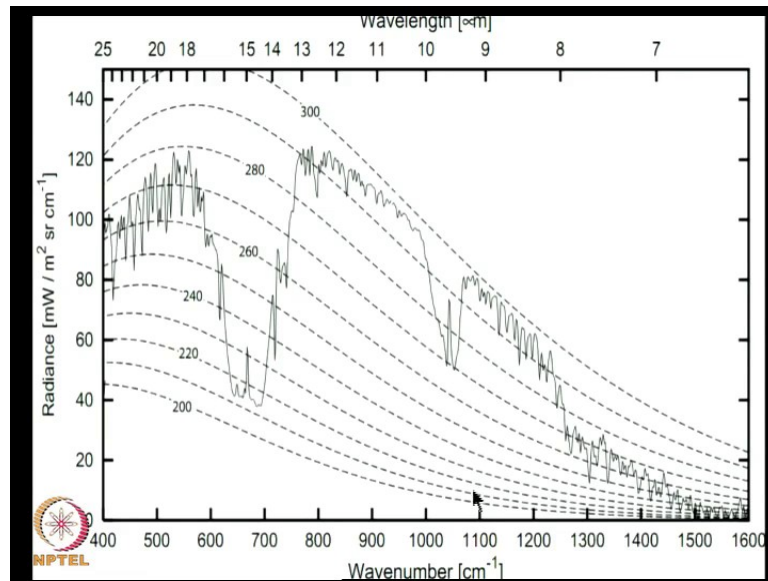
band. But the next satellite Iris has high resolution is able to identify two of the lines. Finally, we have very high resolution infrared sunsets. We are able to resolve many of the lines very clearly.

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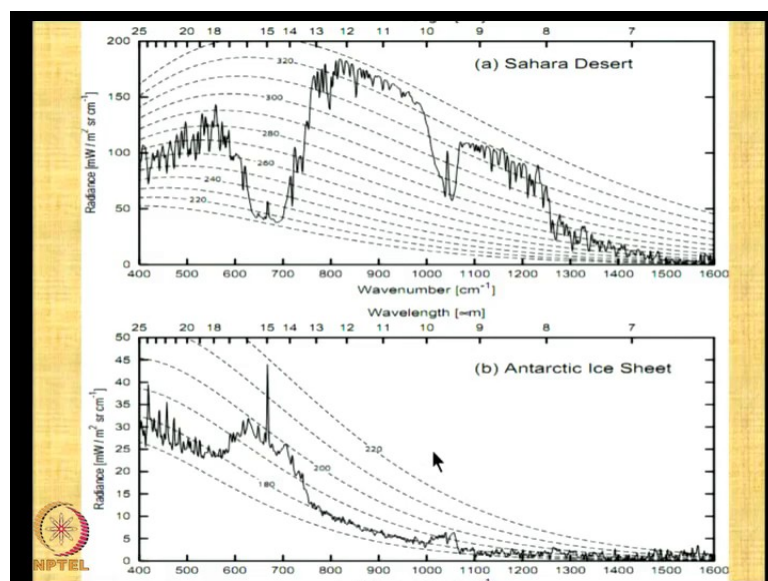
Compared to all the absorption on the solar radiation is shown clearly here. The orange line is the emission at the top of the atmosphere, which is fairly smooth; close to black body at 5018 degrees Kelvin. Now we will look at bottom of the atmosphere. The blue color is much more complicated because of the absorption bands of various gases like water vapor, which are there in the infrared in the solar region.

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More information is available about the impact of these gases by looking at the radiation measured from the satellite in the region around Sahara. We can see that, broadly there is a black body spectrum at 300 milli bar influenced by absorption by the carbon dioxide at 15 micron; ozone band at 9 micron. Other influence of water vapor here. We can clearly see that, carbon dioxide is such a strong absorber that, the emission from the earth in the carbon dioxide band is coming not from the surface over 300 K, but from the tropopause at around 200 K.

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Here we compare the difference between Sahara desert and Antarctic ice sheet spectra as seen by satellite. We see that the carbon dioxide dip, you see here in Sahara and it is seen here as a rise. Now, this is a very surprising result. It only gives you a hint that in Antarctica there is a very strong low level inversion. Hence, when we sense this region through the absorption band of water vapor, we are sensing the emission not from the surface, but from a certain height. If there is a inversion that high will have high temperature and hence there are high radiations.

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Table 7.1 Units used for path length (or amount of absorbing gases), absorption coefficient, and line intensity.

Amount of absorbing gas (path length)	Absorption coefficient	Line intensity
cm	cm^{-1} (volume absorption coefficient)	cm^{-2}
g cm^{-2}	$\text{cm}^2 \text{g}^{-1}$ (mass absorption coefficient)	cm g^{-1}
cm^{-2}	cm^2	cm
cm atm	$(\text{cm atm})^{-1}$	$\text{cm}^2 \text{atm}^{-1}$

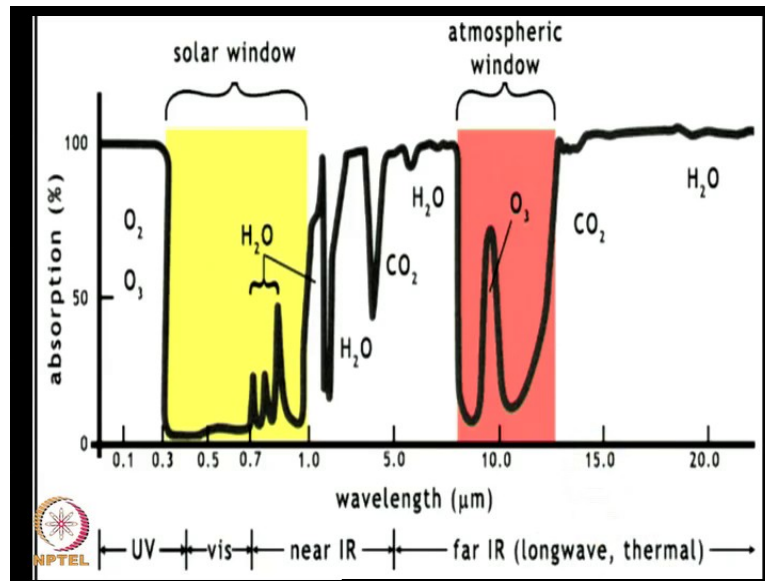
NOTE: mass absorption coefficient = volume absorption coefficient / density of the gas



Now, let us look at a few important nomenclatures that we use in the subject. The amount of absorbing gas can be expressed in many ways. We can express in terms of grams per centimeter squared. The amount of the absorbing gas available in a column; that is nothing but density multiplied by the length scale.

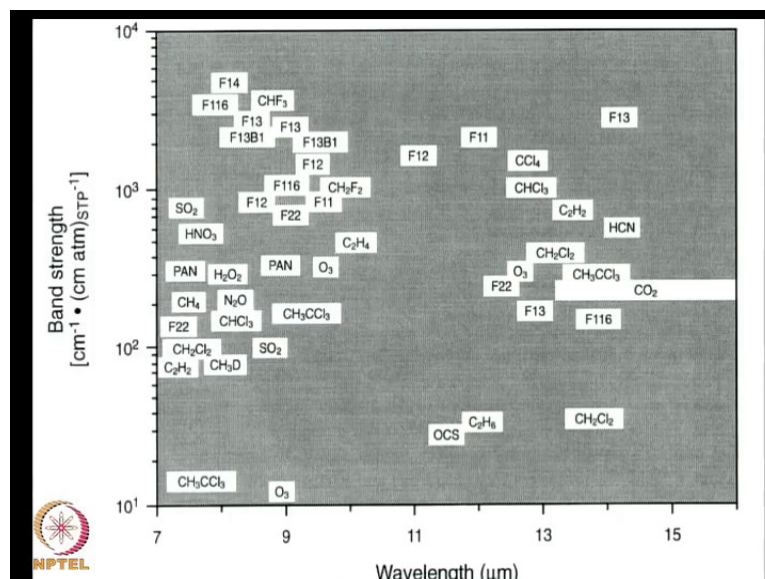
That is a fairly common method. The other method is centimeter atmosphere is called a pressure path length. We calculate the amount absorber by looking at the amount of absorber that is present along a path per unit pressure. These are common units used for the amount absorber as well as absorption coefficient.

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Now, this figure clearly illustrates the important difference between the solar region and the IR region. We can see in infrared region beyond around 0.7 micron, there is a lot of absorption by the atmospheric gases; H₂O, carbon dioxide, ozone and so on. On the other hand below around 0.7 microns, atmosphere is almost transparent. Of course, this is the basis of the green house effect.

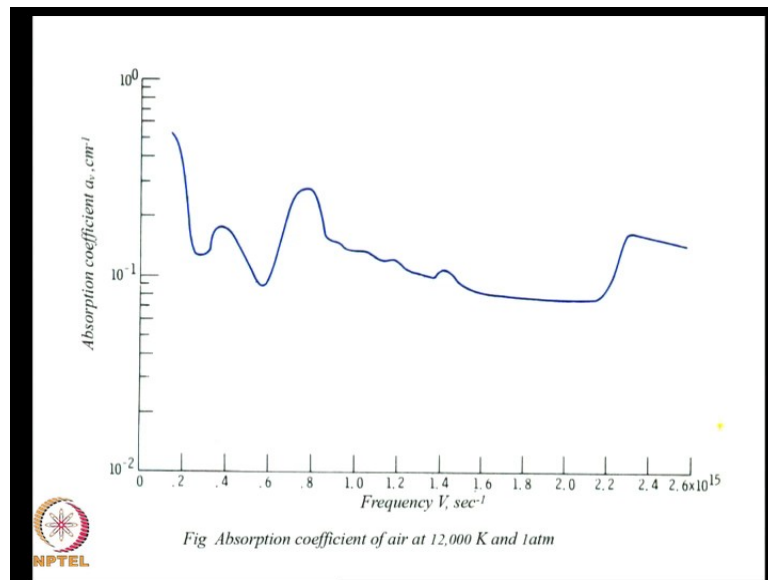
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Now, here is the sample of various molecules which absorb and emit radiation. The wavelength ranges from 7 microns to 15 micron. We can see that there are large number

of gases, mostly man made; which are been emitted during industrial activities and which remain in the atmosphere for certain amount of time. Within that period they interact strongly with the radiation leaving the ground. This has to be documented in order to firstly estimate the warming caused by these.

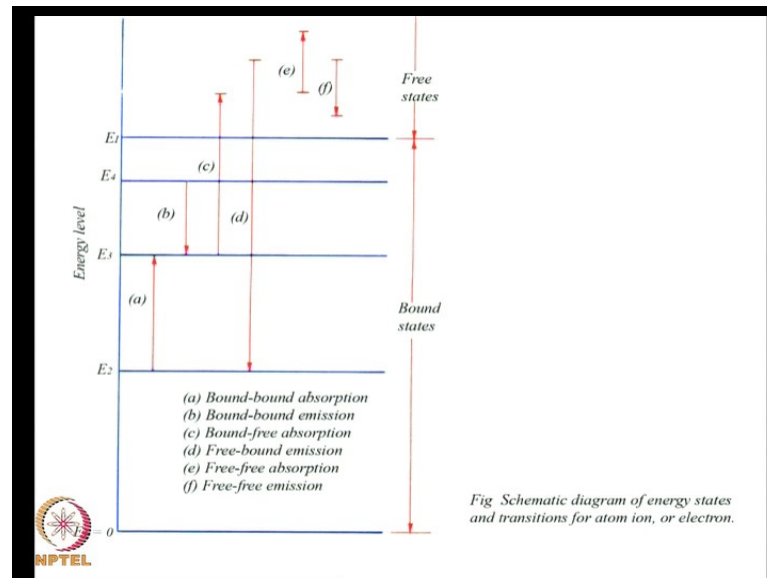
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We have got a general picture of how the molecules behave. Let us see a few examples to illustrate. Here is the absorption coefficient of air at 12000 degrees Kelvin. Notice that, in comparison to what we saw for carbon dioxide, water vapor and so on, where the absorption coefficient has a very complex structure with wave length, here it is a fairly smooth function.

As a matter of fact, one look at this one can assume that air at 12000 Kelvin to be a gray gas with the constant absorption coefficient. We will not make too much error. The question is, why does air over the smooth dependence on wave length or frequency. While the other examples we saw, they are all much more complex. To understand that, we need to know the kind of transition that is going on here.

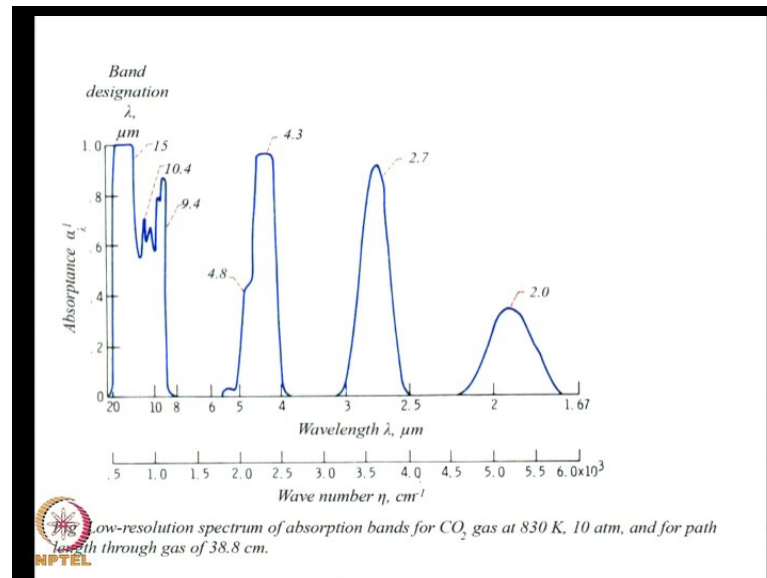
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In the case of gases, the vibration-vibration transitions that we saw are all bound-bound transition. That is, it goes from one bound state to another bound state. The energy absorbed is precisely fixed by the equation ΔE is equal to $h\nu$. On the other hand, if the temperature is sufficiently high, molecule can jump from bound state to free state. In the free state, the kinetic energy of the molecule can be anything. Hence, we absorb radiation continuously with no gap in between.

This is of course advantageous from the point of view of analysis because if the gas undergoing bound free transition, then the assumption of a uniform absorption coefficient is a very useful technique; does not demand too much computed time. We can actually check, when even if we did complicated non-gray assumptions the final answer we will get is not very different from what we obtained by assuming this to be a gray sample.

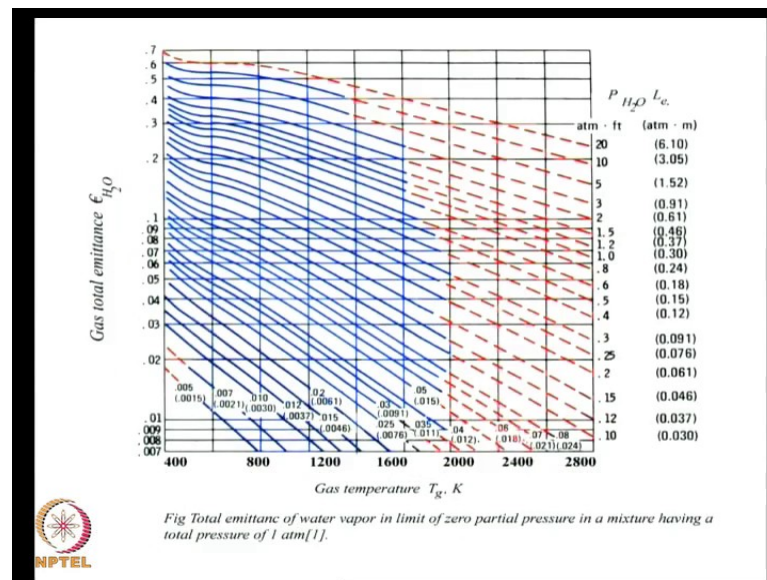
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Now, let us take an example, of carbon dioxide. We saw that air is continuous spectra. Now let's look at the absorption band of carbon dioxide at a fairly high temperature 830 Kelvin; typical situation kind of furnace at a fairly high pressure and the gases at ten atmospheric pressure and in a container of the length scale 38.8 centimeters.

We can see that there are four major absorption bands. Some of them are not visible because they are overlapping. Actually, in this figure it is 10, 15, 10.4, 9.4, 4.8, 4.3, 2.7, and 2.0. So, of these, we can see the three of these bands are overlapping here and two bands are over lapping here. It is a fairly complicated problem because if two bands are overlapping, then you have to correctly account both the absorption by each band.

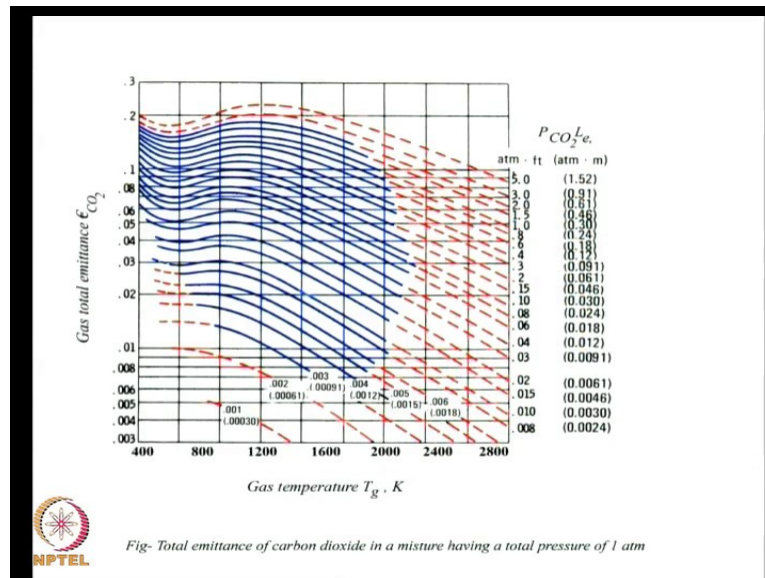
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This is a typical behavior of carbon dioxide. Now, finally we need to estimate emissivity of a gas. We will ultimately integrate all wave length and get an idea of the emission. That is best done by using charts. But it is still a very useful tool to have a chart because we can do one sort of hand calculation to ensure that you have a good understanding of what is going on in this problem.

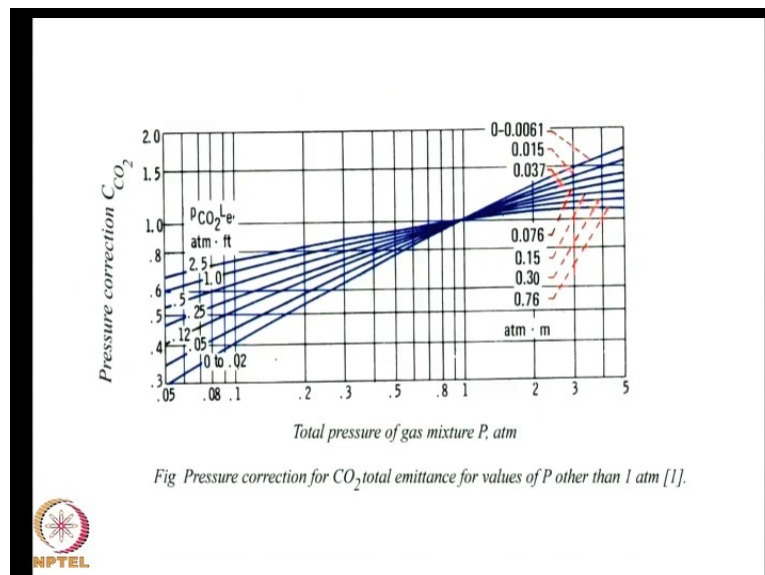
Here is the total emissivity of water vapor as a function of gas temperature for various path length. The path length is in atmosphere centimeter. We can see that as the temperature increases, the emissivity of the gas goes on decreasing. We notice that even at the highest path length, the emissivity does not cross 0.7.

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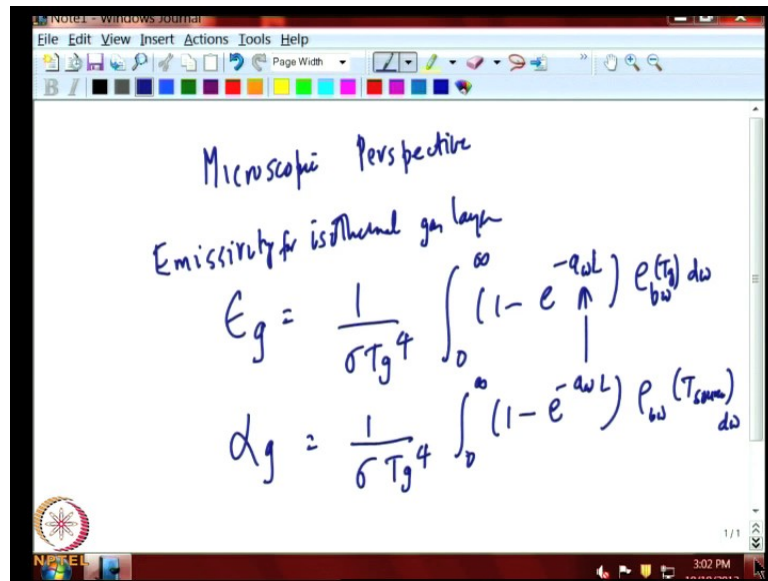
Here is the emissivity of carbon dioxide has a function of gas temperature. These are various path lengths going from 0.01 to 0.006. We can see that there is the region where the emissivity is not varying much, but there is another region where emissivity is falling fairly rapidly.

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Now, one of the challenges in dealing with radiation in furnaces is the problem of overlap between lines; between a line and another line.

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These are issues which we have to deal with, when we want to deal with the nature of medium. Now, first we will define emissivity of a gas. Emissivity of the gas can only be defined for an isothermal gas layer. If the layer is non-isothermal, we will have great problem in defining emissivity. Emissivity as we have already defined for a surface has a same logic; is 1 by T_g to the power of four integral 0 to infinity 1 minus e to the minus $a\omega L$ $d\omega$. Once we have given the value of $a\omega L$, then we calculate this function, then integrate over the wave length of the wave number. That is worth remembering that, since $a\omega L$ is a very rapid and complex function of frequency or wave length. We have to make a decision, how we calculate this quantity in a given situation.

Now, we remember that we have to have $e_{b\omega}$ there. This is $e_{b\omega}$ by T_g ; gas temperature. On the other hand, if we want to calculate the absorptivity of radiation coming from a black body, then we need to know the source temperature. Then we can ask how much of these radiations emitted by that source at that temperature is actually absorbed by the gas.

This is similar to what we have already done for surfaces. When we call emissivity, it was only function of the surface temperature. In this case, the emissivity will be the function of a gas temperature, but absorptivity will be the function of both the gas

temperature as well as the temperature of the source. The source will determine how much gas will absorb.

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The image shows a screenshot of a software application window titled "NOTES - Windows Journal". The window contains handwritten mathematical notes. At the top, the equation $E_g = f(\rho_a, L, T_g, b/d)$ is written. Below it, the equation $U = \int_0^L \rho_a dx$ is enclosed in a box, with $\frac{kg}{m^2}$ written to its right. To the right of the boxed equation, the definition $\frac{b}{d} = \frac{\text{line width}}{\text{line spacing}}$ is written. At the bottom, the equation $\alpha_g = f(\rho_a, L, T_g, \frac{b}{d}, T_g, T_{source})$ is written. The software interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with various drawing tools, and a status bar at the bottom showing the time as 3:05 PM.

Now this emissivity depends on, the amount of absorbing gas in a region; the appropriate length scale will depend on gas temperature and it depends upon the exact vibration of radiation line structure, which we might say as b by d being line spacing by nominal lines such as line width by line spacing.

What we see is that the emissivity of the gas depends not only on the amount of matter along the path, which can be amount of result being typically k g per meter squared. This tells us how much molecule; absorbed molecules are there along the path of interest to us. It also depends on gas temperature.

Our aim now is to ask, if we do not want to integrate the original spectral data line by line, is there alternate way to formulate the problem in terms of emissivity and obtain this function either through experimental data or by using data from a line by line code and creating a emissivity data base. Depending upon the situation, either of these two methods is used. In the same token here, the absorptivity of the gas is the function of the absorbing gas, length scale, line structure parameter b by d and the gas temperature as well as the source temperature.

So absorptivity as expected is a bit more complicated than emissivity. Emissivity on integration depends only on the temperature of the gas; while, absorptivity depends both on the temperature of the gas and temperature of the source from which radiation is emitted. That also influences your absorptivity of the gas. Now, how do these emissivity charts look like.

We will have a look at the kinds of emissivity chart that are available. Here is an example of emissivity chart of water vapor. X axis gives you the temperature in degrees Kelvin; the y axis gives you the emissivity going all the way from 0.007 to 0.7 and a factor of 100, increase here. It is shown for various amount of the carbon dioxide gas available. That is, water vapor available; water vapor parts to pressure in the length scale.

As one notices that for a given length scale as you increase the temperature, the emissivity of the water vapor comes down. This can be understood in terms of the fact that as a temperature is increased, the peak of the black body function starts shifting to a lower wave length according to the Wien's law. At 400 degrees Kelvin your peak of black body substance is around 7 microns. But as we go to 2800, we will have 1 micron. So, as a band shifts towards to a lower wavelength, there are not too many absorption bands of water vapor. Hence the emissivity comes down. This can be understood in very simple terms based on the knowledge of the data.

Here is the other example of carbon dioxide which, whose emissivity goes from 0.003 to 0.3 and temperature founded to be K. Once more we see that there is a region where gas temperature does not influence the emissivity too much at very high temperature, once for all the radiation from the black body goes to lower wavelength regions. Hence the emissivity starts falling because these bands which we discussed today did not really able to utilize that database that might be available. Now, if we are dealing with pure water vapor or pure carbon dioxide or mixed with nitrogen, which can then absorb, then one can presently use this chart.

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Mixture of gases CO_2 , H_2O

$$E_g = C_{\text{CO}_2} E_{\text{CO}_2} + C_{\text{H}_2\text{O}} E_{\text{H}_2\text{O}} - \Delta E \text{ (overlap)}$$

Correction for $P \neq 1 \text{ bar}$

So, now we ask ourselves what if we have a mixture of gases, which is quite common in furnaces or in the atmospheric application. Let us say the mixture of gases containing both carbon dioxide and water vapor. Then we cannot get the gas emissivity merely by adding the carbon dioxide emissivity and water vapor emissivity.

We must account for the fact that the various absorption lines of water vapor and carbon dioxide can overlap. Hence, we have to subtract the contribution due to overlap. This is inevitable. In addition, if we are going to use a chart which is the standard pressure conditions and we have a situation such that, the pressure in the furnace or other appliance is not the standard one on the pressure. Then we have to add correction terms.

This is correction for not being equal to one bar. That is what is shown in the other figure here. We see here the correction chart for carbon dioxide, when the pressure is other than one atmosphere. If the total pressure goes to as low as 0.05 atmosphere, correction can be as low as 0.3; while we go to high pressure like 5 atmosphere, the increase in emissivity can be 50 percent.

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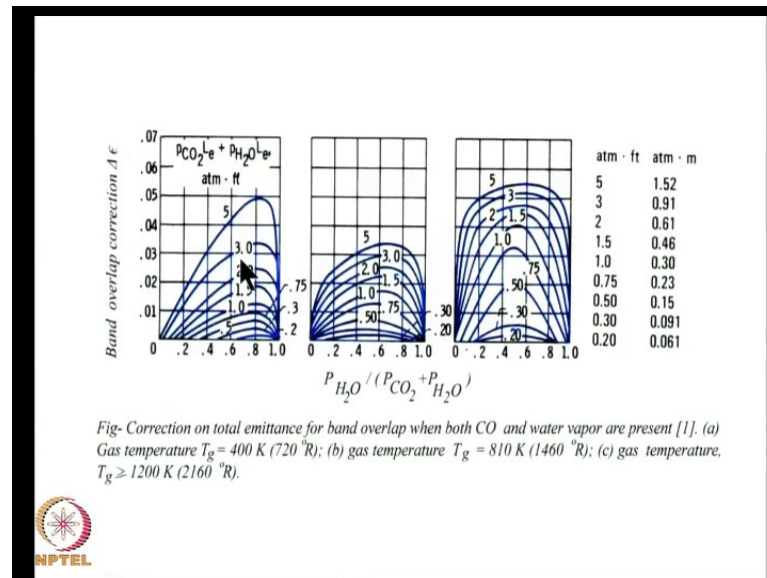
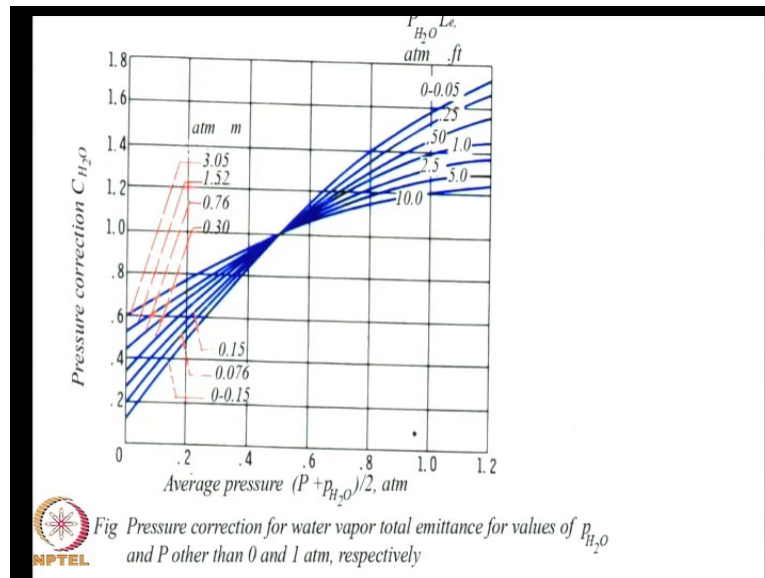


Fig- Correction on total emittance for band overlap when both CO and water vapor are present [1]. (a) Gas temperature $T_g = 400 \text{ K}$ (720 °R); (b) gas temperature $T_g = 810 \text{ K}$ (1460 °R); (c) gas temperature, $T_g \geq 1200 \text{ K}$ (2160 °R).

Both these situations will occur in the real world. There are furnaces with pressures below atmosphere. There are situation occurring in diesel engine and other devices, where the pressure goes to very high values. Now, there is one more correction here due to overlap; that delta epsilon term. And, here in example how you do the overlap correction at various temperature; 400, 810 and 1200 K. We can see that the correction for the absorption of the order of 0.01 to 0.05 and it depends upon the total amount of absorption molecules in the path. In either extreme of course, the correction is not there because you have the pure H 2 O or pure CO 2 here.

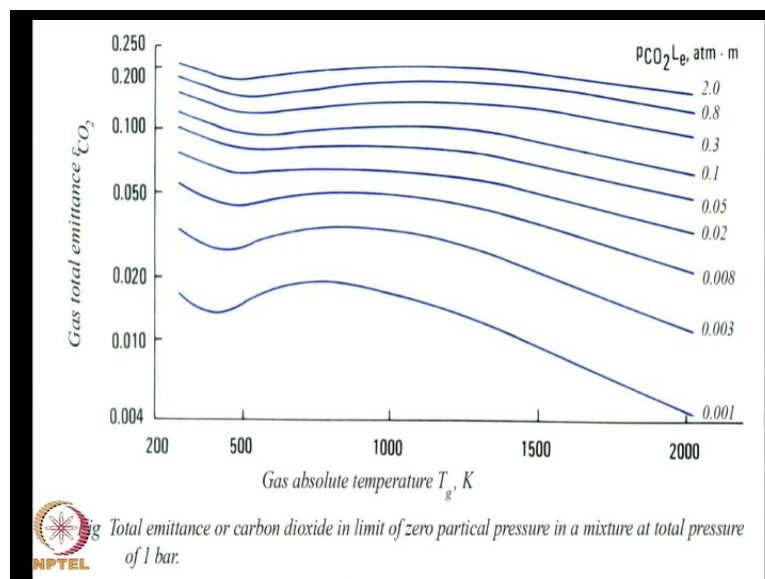
These are charts having routinely used by engineers to calculate performance of furnaces and boilers because today these will be now available in the form of equations or software and not charts. But charts are still used. Good to use because they will give we a physical insight into the kinds of accuracy and inaccuracy that may encounter in calculation.

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Here is the correction chart for water vapor, if it is not at the total pressure or one bar.

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Now, there are other charts available by various other investigators. We must mention that these values from different charts can differ by as much as 10 percent. This is quite common. We can see that these overlap correction can be of the order of 0.05. These can be as low as 0.3. So, ultimately we will get an estimate of epsilon g. And, depending on which chart to use or which source to use, there can be variation of the

order of 10 percent. Today, of course all these are available through software and they are calculated.

There is one more approach available. This approach, we calculate emissivity of gases. It is an old approach, but with more accurate data available for various bands of various molecules. Now, there are attempts to have this done little more accurately. We will give one example here.

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The image shows a screenshot of a software window titled 'Windows Journal' with a toolbar and a whiteboard area. The whiteboard contains the following handwritten mathematical derivation:

$$\begin{aligned} \epsilon_g &= \frac{1}{\sigma T_g^4} \int_0^{\infty} [1 - e^{-a\omega L}] e_{b\omega}(\tau_g) d\omega \\ &= \sum_i \int_{\Delta\omega_b} \left(\frac{e_{b\omega}}{\sigma T_g^4} \right) (1 - e^{-a\omega L}) d\omega \\ &= \sum_i \frac{e_{b\omega_i}}{\sigma T_g^4} \int_{\Delta\omega_b} (1 - e^{-a\omega L}) d\omega \end{aligned}$$

The term $\int_{\Delta\omega_b} (1 - e^{-a\omega L}) d\omega$ is circled in red and labeled $A_{B,i}$ below it. The software interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with various drawing tools, and a Windows taskbar at the bottom showing the time as 3:17 PM.

We saw that the emissivity of the gas goes as integrate over wave number. This integration over all wave numbers can be split into integration over specific bands of, let us say carbon dioxide and calculate these quantities per band. Then average over all bands. Now the reason why this is popular is, if the given band is not too wide, then this quantity can be taken out. This will not vary a lot, unless the band is very wide. We can write this as, at the center of the i th band then we have a simple integration only of the absorption coefficient.

This integration is easy to perform because the blackbody function has been separated out because it is slowly varying. This is available in many sources. This is called a band absorptance of the i th band.

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$$A_{B,i} = \int_{\Delta\omega_i} [1 - e^{-\alpha\omega L}] d\omega$$

$$E_g = \sum_{i=1}^N \frac{\epsilon_{b\omega_i}}{\sigma T_g^4} A_{B,i}(\rho_{aL}, \beta_i, T_g)$$

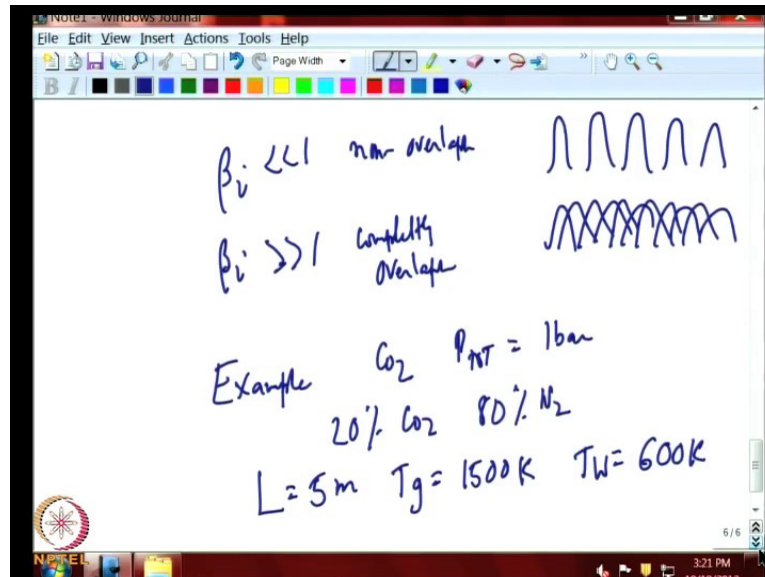
N bands $\beta_i = \frac{\text{line width}}{\text{line spacing}}$
 $\beta_i \ll 1$ non-overlapping $\beta_i \gg 1$ completely overlapping

Band absorptance of the i th band is equal to integration over the i th band over the absorptivity or emissivity of that band only. Once we do that, we calculate emissivity as sum over all the bands i one to n . We will say that there are N bands. We calculate the black body function at the centre of the i th band. We have the $A_{B,i}$ which will be the function of the partial pressure of gas, time, length scale involved, something called the line structure parameter which we will define now and gas temperature.

The line structure parameter β_i is nothing but the line width of the i th band by line spacing. This will play an important role in getting the absorptivity of the gas because if the line width is small compared to the line spacing, that is, β_i is much less than 1, we have essentially a non-overlapping lines; while if β_i is much greater than one, then we have completely overlapping lines.

We can see that in the second limit, which we can expect will occur at high pressures, the detailed structure of the lines will be smeared out. The problem will become simpler. In the other extreme here, when the lines are becoming non-overlapping the line spacing is a very important parameter.

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Beta i is much less than one is non overlapping. There are lines which are individual and separate. If beta i is much greater than 1, it is completely overlapping. So, it will be like this. We expect the behavior of the gas to be very different in these two extreme cases. We can illustrate this result through some examples that we can take up. We will take an example right now to make this point by looking at the carbon dioxide gas.

So, let us take an example to illustrate these issues. We have carbon dioxide gas; total pressure is one bar; it contains 20 percent CO_2 , 80 percent N_2 . This is a typical situation in the case of a furnace after combustion. Mostly it is CO_2 . Let the length scale be about 5 meters; that is, the distance between the walls of the furnace. Let the gas temperature be of the order of 1500 degrees Kelvin. Typical furnace is example, and a wall temperature which is 600. All these numbers we have given are once which occur quite commonly in furnaces. The wall is kept cool; call off the water boiling. And, the typical temperature of the furnace gas is 1500 K. Now, in this case how will this calculation for this various quantity look like.

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$$E_g(T_g) = \sum_i \frac{e_{b\omega_i}}{\sigma T_g^4} A_{B,i}(p_a L, T_g, \beta)$$
$$\alpha_g(T_s, T_g) = \sum_i \frac{e_{b\omega_i}(T_s)}{\sigma T_s^4} A_{B,i}(p_a L, T_g, \beta)$$

We can calculate epsilon g; which is equal to sum over all the bands of e b omega i by sigma T g to the power of four in to A B i, which is the function of the pressure path length of carbon dioxide, gas temperature and the line structure parameter beta.

If we want to calculate absorptivity for a radiation of the source, then we will sum over all the i bands. But we calculate the temperature of the source by sigma T s to the power of four. This will be i, I am sorry, of T s; so this is the fraction radiation coming from a source. We still look at A B i, this factor is same. The key point to note is that, absorptivity depends both on the source temperature and gas temperature; while, the emissivity depends only on the gas temperature.

This we want to remind you similar to what happened in metals. Metals also we saw the emissivity function of T g, while, absorptivity depend on both T g and T source. This comes out nicely. Now, let us calculate this; for this example, we are just now taken up. We will deliberately do all the calculation for various wave numbers and so that, we understand the contribution of various bands.

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band (μ)	ω_c (cm^{-1})	A_B	$\frac{e_{b\omega}}{\sigma T_g^4} A_B$	$\frac{e_{b\omega}}{\sigma T_s^4} A_B$
15	667	312	0.014	0.12
10.4	960	210	0.016	0.186
9.4	1060	170	0.015	0.089
4.3	2349	343	0.067	0.081
2.7	3660	384	0.062	0.013
2.0	5220	49	0.006	0.000
			$\epsilon_g = 0.18$	$\alpha_g = 0.41$

This is the centre of the band; it is centimeter minus 1. This is the band absorptance of the band. And, the contribution $e_{b\omega}$ by σT_g to the power of four into A_B . Then you have $e_{b\omega}$ by σT_s to the power of four A_B . These two are the contribution for absorbed emissivity. Bands that are considered are 15 micron band, 10.4 micron band. Band is represented by the wave length by micron.

We have considered here about 3 plus 3, 6 bands. The centers in the wave number space are 667, 960, 1060, 2349, 3660 and 5220. The band absorptance at these wave numbers are; for these proper condition 312, 210, 170, 343, 384 and 49. Now straight away from this particular column, we see that the strongest bands of carbon dioxide are the 4.3 and 2.7 micron band; for this condition of temperature and path length. But how much are contributed to emissivity and absorptivity is very different. We have to multiply this by this fraction; for emissivity, we get following contribution. We add all these up; you get 0.18 as the gas emissivity.

So, emissivity of carbon dioxide gas, in spite of a strong absorption in the 4.3 into 2 percent micron band is only 0.18. That is typical of gases. Even due to a lot of gas in container, the reasonable path length, we will not get emissivity like 0.8, 0.9. Now, the same situation when we ask what is the absorptivity, you have to worry about the source temperature, not the gas temperature. All the numbers will change. Here it goes up ten

times; this also goes up ten times. This is also higher, this band is also higher This band is lower This band is nothing.

This is because when we look at the radiation from the source temperature, which in this case is 600 Kelvin water wall of the furnace, so the peak emission from that 6 centigrade Kelvin source will be around 4 micron, 5 micron, something like that. Hence it will fundamental alter the character of your result here.

If we look at the absorptivity, it is more like 0.41. Ok. We see that the absorptivity of the gas was as the emissivity; they are very different parts of two because the contribution from various sources is very different. Now, we can see that in the case of emissivity, the major contribution came from here; in the case of absorptivity, the major contribution came from here.

That is a fundamental difference between the two situations. This clearly illustrates the fact that absorptivity and emissivity of gases can be very different. Hence, this; to treat them as a gray gas is not a good approximation and non-gray model is always better. But we saw non-gray model is difficult. Hence, we need to make some approximations and these will be discussed in the next lecture.