Radiation Heat Transfer
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Lecture - 2
Blackbody radiation

In the last lecture, we had looked at the Planck's blackbody radiation.

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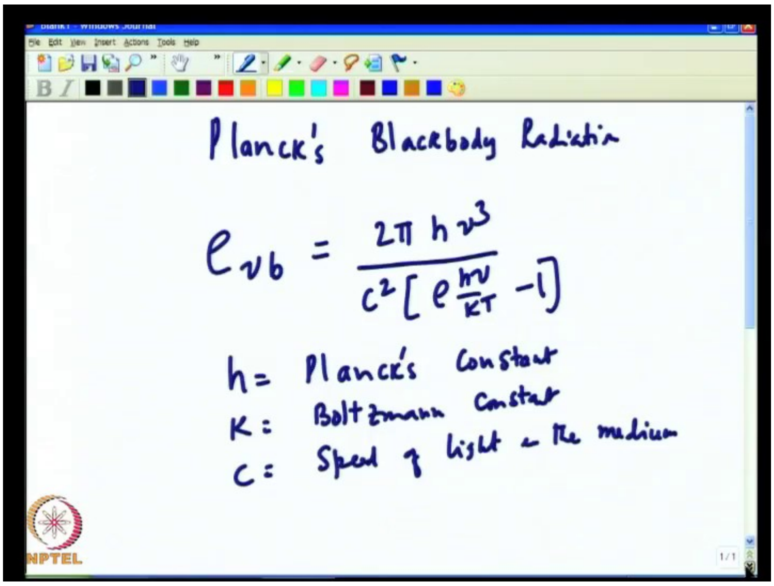

Planck's Blackbody Radiation

$$e_{\nu b} = \frac{2\pi h \nu^3}{c^2 \left[e^{\frac{h\nu}{KT}} - 1 \right]}$$

h = Planck's Constant
K : Boltzmann Constant
c : Speed of light in the medium

We saw that the Planck's blackbody formula for emissive power was 2л hv^3 divided by c^2 e into (hν by kT) minus 1, where 'h' is the Planck's constant, 'k' is the Boltzmann's constant and 'C' is the speed of light in the medium. This formula for the emissive power can be derived using principles in statistical mechanics and quantum mechanics. We went through another approach, proposed by Einstein which was easier and simpler to understand.

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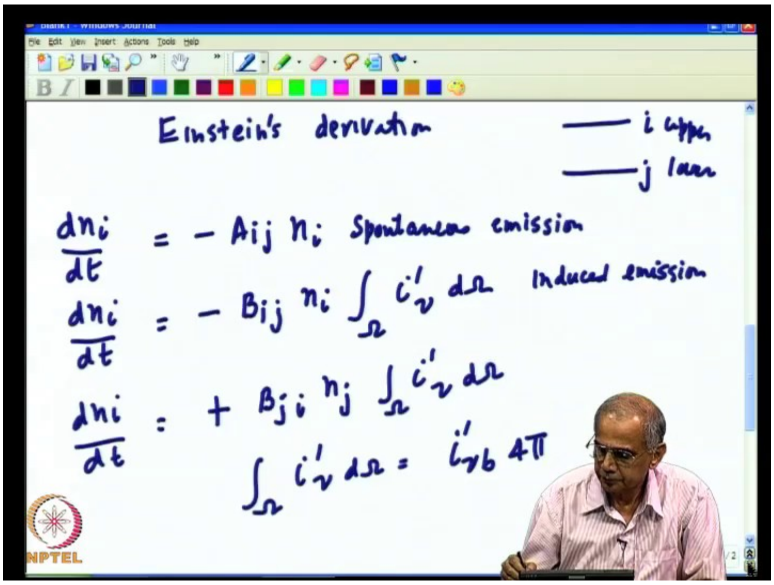

Einstein's derivation

— i upper

— j lower

$$\frac{dn_i}{dt} = -A_{ij} n_i$$ Spontaneous emission

$$\frac{dn_i}{dt} = -B_{ij} n_i \int_\Omega i'_\nu \, d\Omega$$ Induced emission

$$\frac{dn_i}{dt} = +B_{ji} n_j \int_\Omega i'_\nu \, d\Omega$$

$$\int_\Omega i'_\nu \, d\Omega = i'_{\nu b} 4\pi$$

We saw that the molecules in upper level i going to lower lever level j and we wrote down three expressions, dn_i by dt equal to minus A_{ij} n_i for spontaneous emission. Then, we looked at dn_i change in a number of molecules in time due to induced emission, which was the new idea that Einstein proposed. He said that there can also be emission of photons due to photons coming into the system. Finally, there is absorption, which has to be positive and which is from lower level j to upper level i. It depends on number of molecules in the lower level and the number of photons in the system. Einstein assumed that the incoming radiation from blackbody is isotropic in all directions He wrote down integral of i'ν dν as nothing but integral i'νB into 4π. This is because the radiation that is coming, is isotropic in all directions In equilibrium the sum of these three terms has to be equal to 0.

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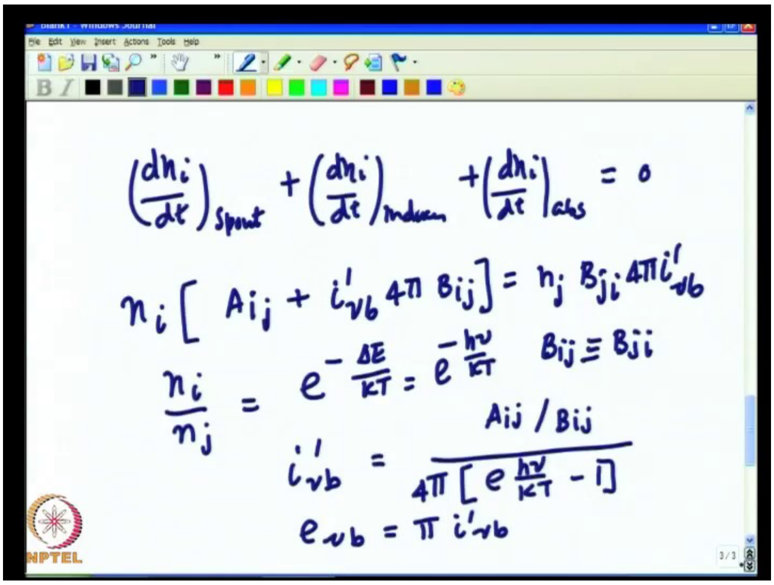

$$\left(\frac{dn_i}{dt}\right)_{spont} + \left(\frac{dn_i}{dt}\right)_{induced} + \left(\frac{dn_i}{dt}\right)_{abs} = 0$$

$$n_i\left[A_{ij} + i'_{\nu b}\, 4\pi\, B_{ij}\right] = n_j B_{ji}\, 4\pi\, i'_{\nu b}$$

$$\frac{n_i}{n_j} = e^{-\frac{\Delta E}{kT}} = e^{-\frac{h\nu}{kT}} \qquad B_{ij} \equiv B_{ji}$$

$$i'_{\nu b} = \frac{A_{ij}/B_{ij}}{4\pi\left[e^{\frac{h\nu}{kT}} - 1\right]}$$

$$e_{\nu b} = \pi\, i'_{\nu b}$$

We are going to ascertain that dn_i by dt (spontaneous emission) plus dn_i by dt (induced emission) plus dn_i by dt (absorption) is equal to 0. In steady state the number of molecules in the level does not change. When we simplify and calculate the formula n_i into Aij plus $i'\nu_b$ into 4π into B_{ij} is equal to n_j B_{ji} 4π $i'\nu_b$. From the arguments related to symmetry and reciprocity, Einstein assumed that B_{ij} must be equal to Bji. The process of emission on plot have to be symmetric and reciprocal. Then, we assume that the number of molecules at the upper level to that in the lower level has to go as $e^{-\delta e/kT}$.

This is equal to $e^{-h\nu/kT}$. We are assuming that the molecules have a distribution following the well known theory that comes from statistical mechanics, that the distribution of energy among the molecules has to decrease exponentially with height. If you put all these together we will get $i'\nu_b$ as A_{ij}/B_{ij} divided by $4\pi[e^{h\nu/kT}-1]$. To get the similar result as Planck's formula, we know that $e_{\nu b}=\pi$ $i'\nu_b$.

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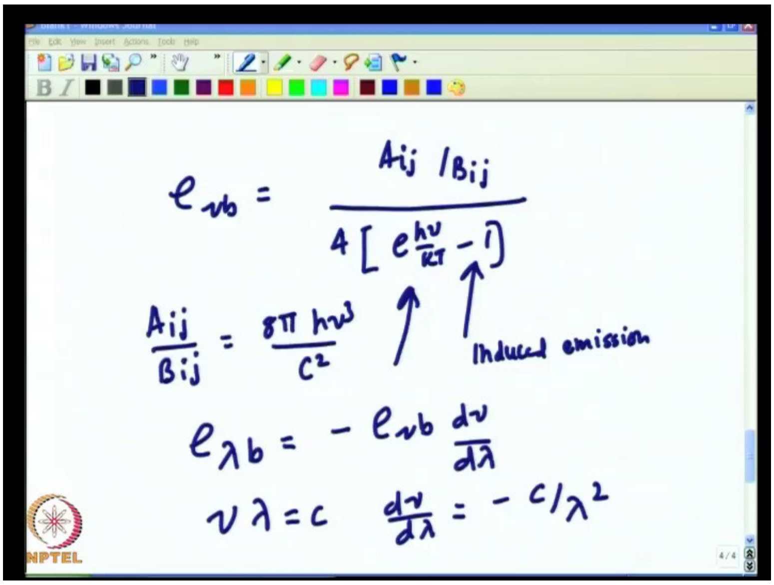

$$e_{\nu b} = \frac{A_{ij}/B_{ij}}{4[e^{\frac{h\nu}{kT}} - 1]}$$

Induced emission

$$\frac{A_{ij}}{B_{ij}} = \frac{8\pi h\nu^3}{c^2}$$

$$e_{\lambda b} = -e_{\nu b}\frac{d\nu}{d\lambda}$$

$$\nu\lambda = c \qquad \frac{d\nu}{d\lambda} = -c/\lambda^2$$

Therefore, we can write $e_{\nu b}$ from Einstein's derivation as A_{ij}/B_{ij} divided by $4[e^{h\nu/kT}-1]$. If we compare this result with the derivation done by Rayleigh jeans in the classical limit, one can show that A_{ij}/B_{ij} is nothing but $8\pi h_{\nu}^3/C^2$. We get back the result that is obtained by Planck, thereby showing that 1 comes from induced emission, which was not accorded for the either Rayleigh jeans or by ν in any derivation.

This term, is introduced by Planck, in somewhat adhoc fashion, while in Einstein's derivation, this comes out very naturally as a contribution of induced emission to the result. This derivation can be rewritten in terms of the other. This shows that, the term coming from the distribution molecules, shows a decline with the energy levels., Both these are clearly accounted for in this derivation. Now, this derivation done Both Einstein and Planck use frequency to calculate emission power . We also would like to know what are the value per unit wavelength obtained from these equations. Once we convert νB and dν in terms of wavelength from frequency, we can find the value in terms of wavelength also. We know that νλ is C, and that dν will be $\lambda s\text{-}C/\lambda^2$.

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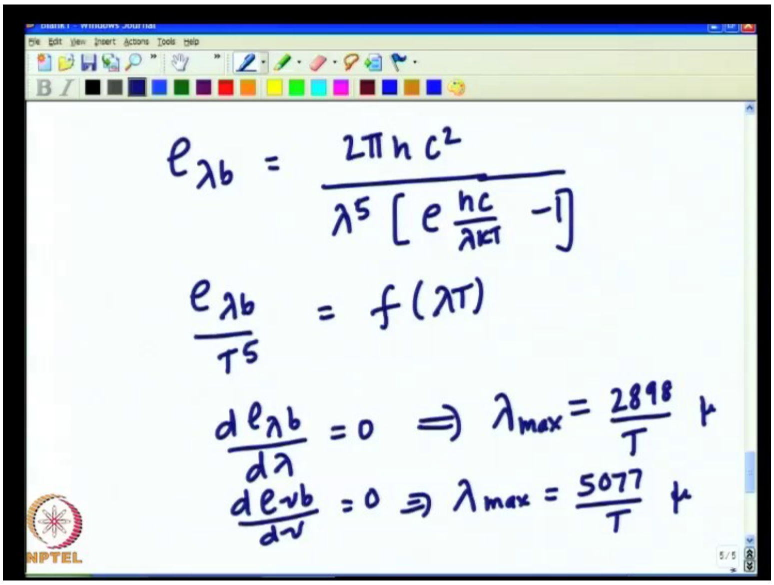

$$e_{\lambda b} = \frac{2\pi h c^2}{\lambda^5 \left[e^{\frac{hc}{\lambda kT}} - 1 \right]}$$

$$\frac{e_{\lambda b}}{T^5} = f(\lambda T)$$

$$\frac{d e_{\lambda b}}{d\lambda} = 0 \Rightarrow \lambda_{max} = \frac{2898}{T} \mu$$

$$\frac{d e_{\nu b}}{d\nu} = 0 \Rightarrow \lambda_{max} = \frac{5077}{T} \mu$$

$e_{\lambda b}$ becomes $2\pi hC^2/\lambda^5[e^{hC/\lambda kT}-1]$. This is an expression that is used very often. This expression has interesting properties, one is that if we divide $e_{\lambda b}/T^5$, the right hand side will be only a function of the product λT. This is very useful and later we will see how to exploit this particular feature of the function to advantage.

The next important issue is to know where the maxima of the Planck's blackbody function lies. You can obtain that by differentiating the above equation. We will get a transcendental equation, which we have to solve numerically to get the following result. The wavelength, at which it is the maximum, will come out as 2898 by T, where T is in degrees Kelvin. The result value is in microns (μ). This shows that at the peak of the Planck's blackbody emission curve, the emission power per unit wavelength occurs at the maximum wavelength.

On the other hand, if we look at the previous derivation with respect to maximum frequency, we derive the maximum wavelength given by 5077 by T micron This means that the maxima of the $e_{\lambda b}$ function occurs on a different wavelength when $e_{\nu b}$ function is at its maximum. This is because the $e_{\lambda b}$ function is $1/\lambda^5$ and $e_{\nu b}$ is ν^3.

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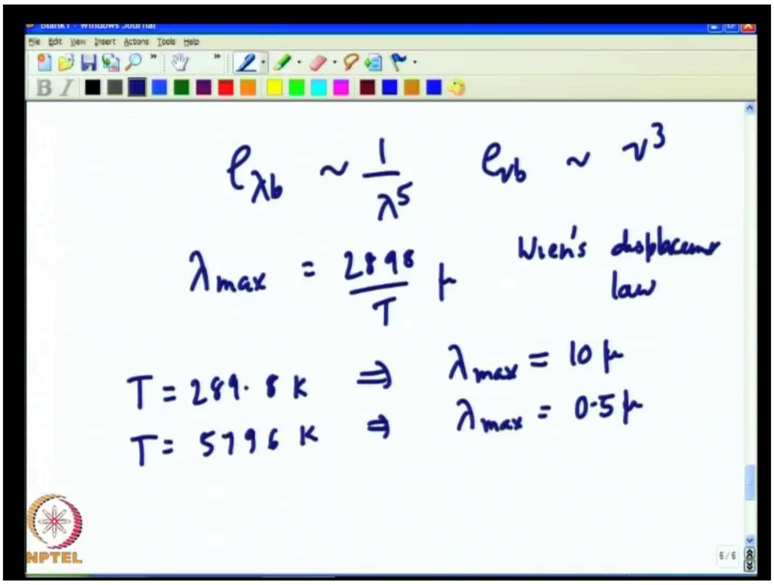

$$e_{\lambda b} \sim \frac{1}{\lambda^5} \qquad e_{\nu b} \sim \nu^3$$

$$\lambda_{max} = \frac{2898}{T}\,\mu$$ Wien's displacement law

$$T = 289.8\text{ K} \Rightarrow \lambda_{max} = 10\,\mu$$

$$T = 5796\text{ K} \Rightarrow \lambda_{max} = 0.5\,\mu$$

These are different kinds of functional dependence, so the maxima of these two functions will not occur at the same wavelength. When we coat the maxima of any blackbody function, it is important to mention which coordinate system we are working with, but most books coat the value that is given by $e_{\nu b}$ function. This is called the Wien's displacement law and this is very useful information as to where exactly the peak of the blackbody function lies.

Another quantity of great interest was, in addition to the peak of the blackbody spectrum there is also the total amount of radiation line. Let us look at a typical blackbody at the temperature of 298.8 Kelvin. We find its maximum radiation in the lambda B coordinate at 10 micron. It means that, if we look at bodies at room temperature and if they behave like blackbodies, then their peak radiation occurs around 10 micron. This is true for most objects at room temperature. So, the maximization occurs in the region at around 10 microns.

On the other hand, if you look at the Sun's emission, assuming that sun is a blackbody at 5796 Kelvin for convenience, then we find that the maximum radiation occurs at 0.5 micron. We can say that, there is a huge difference as to where the maximum radiation lies. For bodies around room temperature, the peak is around 10 micron while for bodies such as the Sun, which is at high temperature, the maximum radiation occurs at around 0.5 micron. This information will be very useful when we want to determine the amount of radiation absorbed from the sun. It means that you need to have a surface, which absorbs radiation very effectively in this region around the visible. In addition to where the maxima lies, we also would like to know, how much energy lies in a certain wavelength range.

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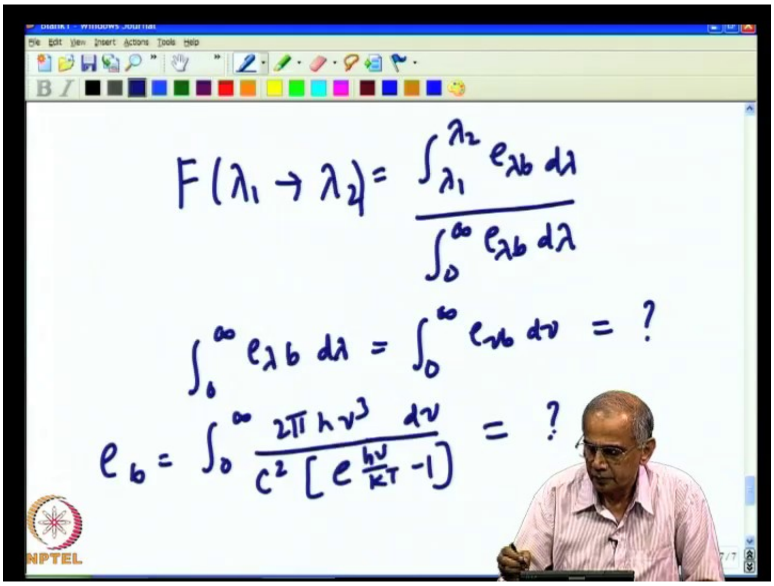

$$F(\lambda_1 \rightarrow \lambda_2) = \frac{\int_{\lambda_1}^{\lambda_2} e_{\lambda b}\, d\lambda}{\int_0^{\infty} e_{\lambda b}\, d\lambda}$$

$$\int_0^{\infty} e_{\lambda b}\, d\lambda = \int_0^{\infty} e_{\nu b}\, d\nu = ?$$

$$e_b = \int_0^{\infty} \frac{2\pi h \nu^3 \, d\nu}{c^2 \left[e^{\frac{h\nu}{kT}} - 1 \right]} = ?$$

We look at the total radiation emitted a blackbody and then compare it with how much it lies in a given wavelength range. This tells us what fraction of the radiation emitted by the blackbody lies in a certain wavelength range. We integrate $e_{\lambda b}\, d_{\lambda}$ function or $e_{\nu b}\, d_{\nu}$ function from 0 to ∞, we get the well known Stefan’s Boltzmann formula. Let us do a quick derivation of this. We use the Planck’s expression 'e_b' which is the total radiation frequency of wavelength .

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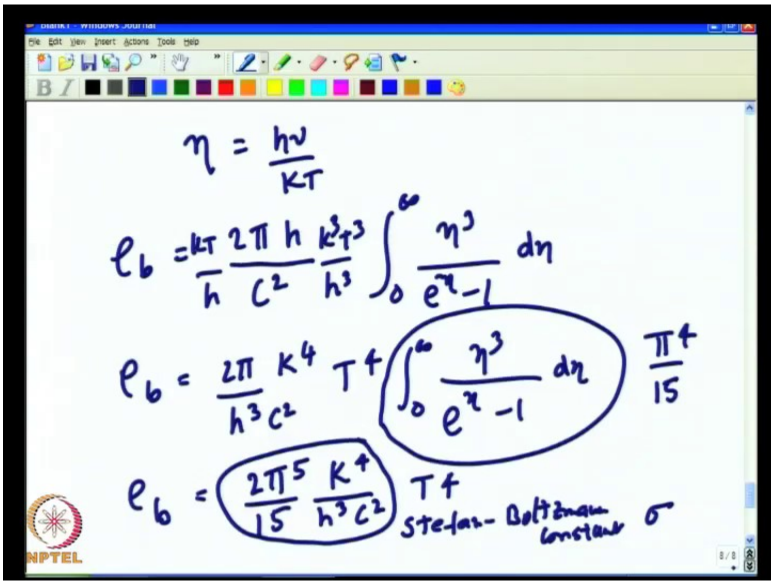

$$\eta = \frac{h\nu}{KT}$$

$$e_b = \frac{kT}{h}\frac{2\pi h}{C^2}\frac{k^3T^3}{h^3}\int_0^\infty \frac{\eta^3}{e^\eta - 1}\,d\eta$$

$$e_b = \frac{2\pi K^4}{h^3 C^2} T^4 \boxed{\int_0^\infty \frac{\eta^3}{e^\eta - 1}\,d\eta} \quad \frac{\pi^4}{15}$$

$$e_b = \boxed{\frac{2\pi^5}{15}\frac{K^4}{h^3 C^2}} T^4$$

Stefan-Boltzmann constant σ

We assume η (eta), which is h_v/kT and derive by integration from 0 to ∞, e_b to get . 2ȷ^5/15 into K^4/h^3C^3 into T^4. This integral is just a number.

We are more interested in knowing what is the dependence of the emissive power of temperature T^4. This kind of integral is studied when dealing with complex numbers by using MATLAB and mathematics. So, we do that you will have a simple The expression e_{b} = 2ȷ^5/15 into K^4/h^3C^3 into T^4 is called the Stefan Boltzmann constant.

The numerical value of this quantity have been derived even before Planck had made his derivation for the spectral quantity. In the laboratory, one can measure e_b as well as T and after that we obtain this quantity empirical. The Stefan Boltzmann constant comes with a symbol sigma today. We can evaluate it from three of the universal constants in physics, namely the Planck's constant, the Boltzmann's constant and the speed of light in vacuum where black body radiation is used for calculating vacuum.

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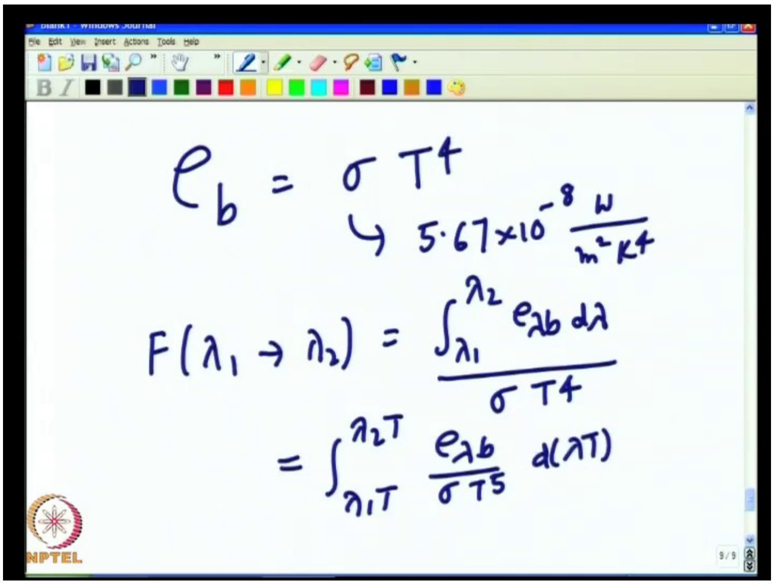

$$e_b = \sigma T^4$$

$$\hookrightarrow 5.67\times10^{-8}\ \frac{W}{m^2K^4}$$

$$F(\lambda_1 \rightarrow \lambda_2) = \frac{\int_{\lambda_1}^{\lambda_2} e_{\lambda b}\, d\lambda}{\sigma T^4}$$

$$= \int_{\lambda_1 T}^{\lambda_2 T} \frac{e_{\lambda b}}{\sigma T^5}\, d(\lambda T)$$

We take $e_b = \sigma T^4$ where this quantity is $5.67\text{x}10^{-8}$ W/m^{2}K^4. We now need to find how much radiation lies between the two wavelength.

We can rewrite this quantity as a function $F(\lambda_1 \rightarrow \lambda_2)$ and integrate $e_{\lambda b}$ d_λ from λ_1 to λ_2 by σT^4. This can be further simplified and rewritten as an integral of $\lambda_1 T$ to $\lambda_2 T$. $e_{\lambda b} / \sigma T^5$ d(λT).

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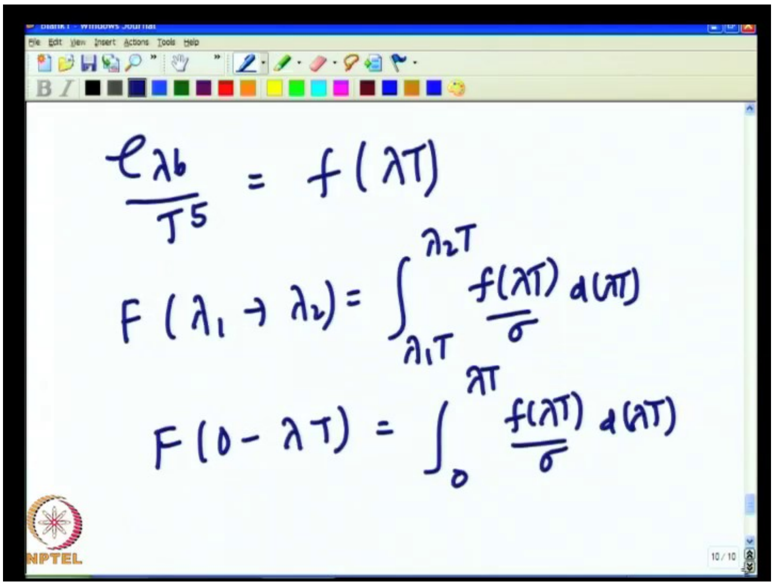

$$\frac{e_{\lambda b}}{T^5} = f(\lambda T)$$

$$F(\lambda_1 \rightarrow \lambda_2) = \int_{\lambda_1 T}^{\lambda_2 T} \frac{f(\lambda T)}{\sigma}\, d(\lambda T)$$

$$F(0 - \lambda T) = \int_0^{\lambda T} \frac{f(\lambda T)}{\sigma}\, d(\lambda T)$$

We pointed out that $e_{\lambda b} / T^5$ is a function $f(\lambda T)$. We substitute this value in the above equation for $e_{\lambda b} / T^5$. We need to know λT together in order to calculate this value. We can also write the equation, based on the simpler wave where λ_1 is 0 and λ_2 is λT and integrate from 0 to λT. . This expression is useful to evaluate numerically today using math law or any other standard integration procedure.

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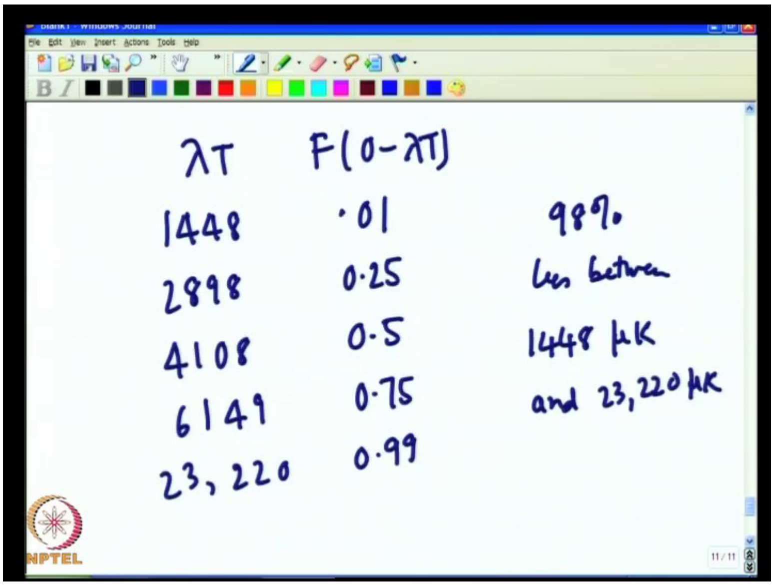

λT	F(0-λT)
1448	·01
2898	0·25
4108	0·5
6149	0·75
23, 220	0·99

98%
lies between
1448 μK
and 23,220 μK

Suppose, we look at λT value and what fraction of radiation lies between 0 and λT, then at 1448 micron Kelvin it is 0.01. At 2898 it is 0.25. At 4108 it is 0.5, at 6149 it is 0.75, at 23,220 it is 0.99. It means that most of the radiation emitted by blackbody lies between 1448 micron Kelvin and 23,220 micron Kelvin. In evaluating proper surfaces this information is useful because you need to know in which wavelength range the radiation primarily lies. Let us take a few examples to illustrate that.

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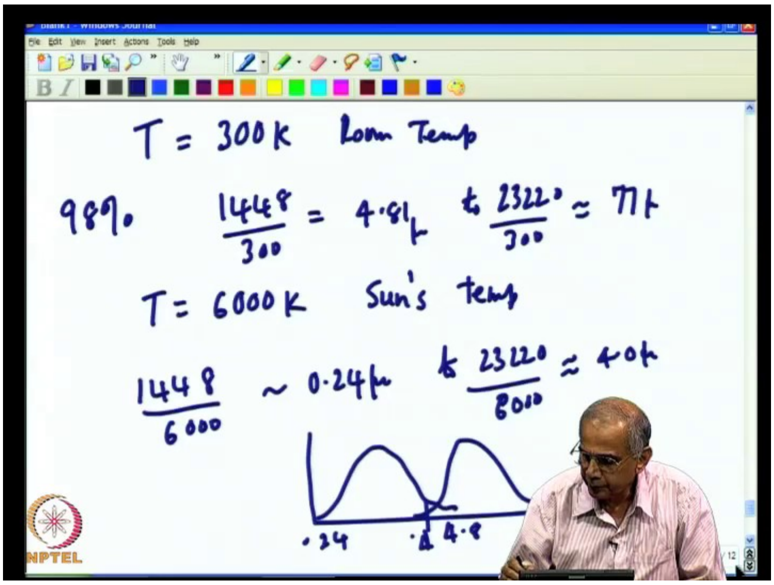

$T = 300\,K$ Room Temp

$98\%\quad \frac{1448}{300} = 4.81\mu$ to $\frac{23220}{300} \simeq 77\mu$

$T = 6000\,K$ Sun's temp

$\frac{1448}{6000} \sim 0.24\mu$ to $\frac{23220}{8000} \simeq 4.0\mu$

Let us take the surface at 300 K Kelvin. So, 98 percent of the radiation lies between 1448 by 300, which is nothing but approximately 4.81 micron, and 23,220 by 300, which is approximately 77 micron. One can say that most of the radiation emitted by blackbody at 300 Kelvin around room temperature, is somewhere between 4 micron to 75 micron. On the other hand, if the temperature of the object is closer to 6000 Kelvin, it is closer to Sun's temperature.

Then 98 percent of the radiation lies between, 0.24 micron to 4 micron. We can say that most of the radiation emitted by blackbody closer to Sun's temperature, lies between 0.25 and 4. This shows that, the two are almost non overlapping.

They are completely non overlapping wavelength domains. This becomes very useful to know when you want to evaluate properties of surfaces. The properties that are relevant to calculating the absorption of Sun's radiation, will be very different from the properties that we will have to use to calculate the emission of the surface.

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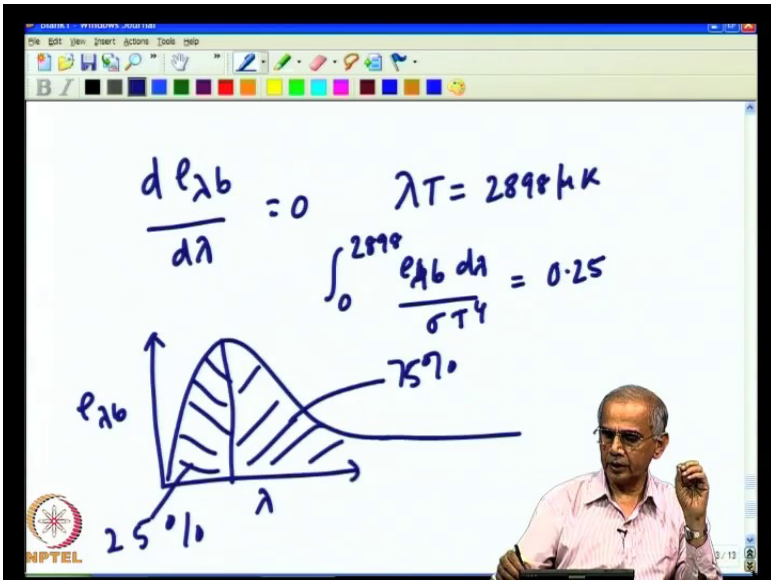

$$\frac{de_{\lambda b}}{d\lambda} = 0 \qquad \lambda T = 2898\,\mu K$$

$$\int_0^{2898} \frac{e_{\lambda b}\, d\lambda}{\sigma T^4} = 0.25$$

The second information is, we noticed that $e_{\lambda b} / d_{\lambda}$ was 0, where λT was 2898 micron Kelvin. We also saw that the integral from 0 to 2898 (λT), $e_{\lambda b}$ d_{λ} by σT^4 is 0.25. If we plot $e_{\lambda b}$ with λ, , then 25 percent of the radiation, lies before maxima and 75 percent lies above the maxima. What this implies is that the blackbody function is not symmetric like our Gaussian distribution, it is asymmetric.

This is the property of a blackbody, when we are dealing with actual real surfaces and not blackbodies itself. We need to now look at the properties of real surfaces. Then compare this real surface with the blackbody. One important property we should remember is that, blackbody is the best emitter and the best absorber. When we compare a real surface with an ideal surface like the blackbody, subject to the same temperature, the emission by real surface will always be lower than that of the blackbody.

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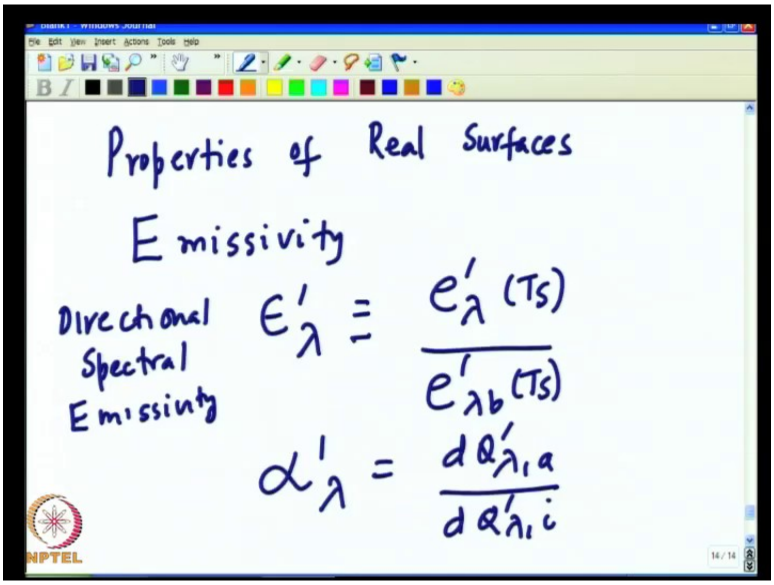

Properties of Real Surfaces

Emissivity

Directional Spectral Emissivity

$$\epsilon'_\lambda = \frac{e'_\lambda(T_s)}{e'_{\lambda b}(T_s)}$$

$$\alpha'_\lambda = \frac{dQ'_{\lambda,a}}{dQ'_{\lambda,i}}$$

We define what is called Emissivity which talks about properties of real surfaces. Real surfaces are different from ideal surfaces. Emissivity compares the emission of a real surface to that of a black body at the same temperature. For example, the directional spectral emissivity of the surface, is the emission in a given direction, given wavelength and by the actual surface to that of the blackbody at the same temperature.

These are very fundamental definitions of what a directional spectral emissivity is. Similarly, we can define directional spectral absorptivity. In this case we define the directional spectral absorptivity as the amount of radiation absorbed in a given direction, given wavelength to the radiation incident in the same direction and same wavelength. Emissivity is defined as a ratio of the actual emission to blackbody emission, while absorptivity is defined a ratio of radiation absorb to the incident.

At the microscopic level emission, absorption is called reversible. That is, if a certain object has a certain property of absorption, then the same object will have similar property to emit radiation, because, when there is emission molecules fall from the upper state or lower state and emit a photon. If we send the photon back, then there is an equal property, that the molecule will absorb the radiation and take the molecule from the lower state to the upper state.

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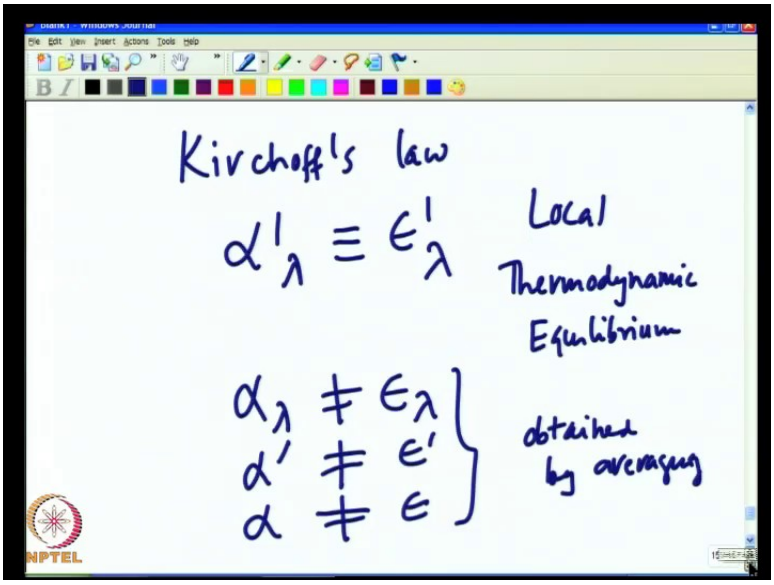

Kirchoff's law

$\alpha'_\lambda \equiv \epsilon'_\lambda$

Local Thermodynamic Equilibrium

$$\left.\begin{array}{l} \alpha_\lambda \neq \epsilon_\lambda \\ \alpha' \neq \epsilon' \\ \alpha \neq \epsilon \end{array}\right\}$$

obtained by averaging

Kirchhoff proposed that under most conditions these two quantities are equal. The only requirement that he imposed, was on the equality of directional spectral absorptivity and directional spectral emissivity that is, the object must be in local thermodynamic equilibrium. In local thermodynamic equilibrium, the ability of an object to absorb radiation at one wavelength and one angle will be same as a ability of that object to emit radiation at this same angle and same wavelength.

Kirchhoff's law only relates the directional spectral quantities although this is always true. The hemispherical spectral absorptivity is not always equal to hemispherical spectral. Similarly, directional total absorptivity is not always equal to directional total emissivity or the hemispherical total absorptivity is not equal to hemispherical total emissivity.

These three things one must remember because many books claim that this Kirchhoff's law is not true. Kirchhoff's law only connects the most basic quantity that is directional spectral quantity between absorption and emission. It does not say anything about quantities which are obtained by averaging. Once we average absorption and emission, then things change substantially. For example, ά will not be equal to έ.

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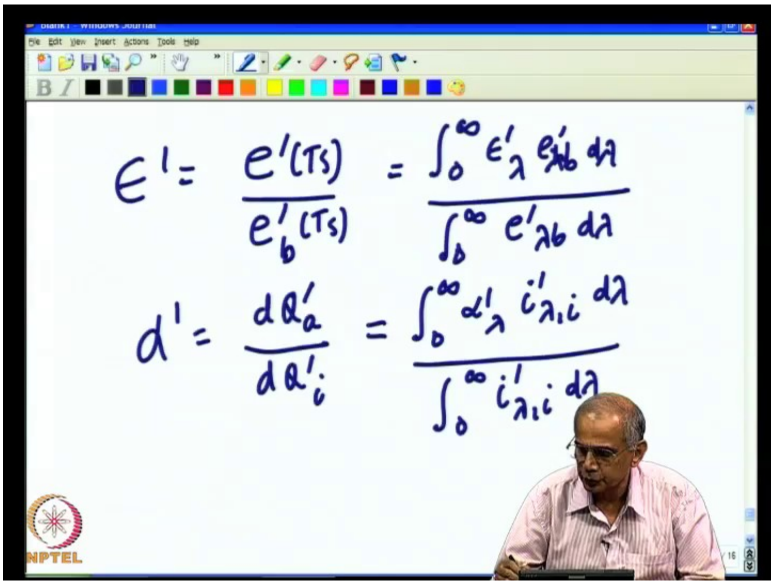

$$\epsilon' = \frac{e'(T_s)}{e'_b(T_s)} = \frac{\int_0^\infty \epsilon'_\lambda \, e'_{\lambda b} \, d\lambda}{\int_0^\infty e'_{\lambda b} \, d\lambda}$$

$$\alpha' = \frac{dQ'_a}{dQ'_i} = \frac{\int_0^\infty \alpha'_\lambda \, i'_{\lambda,i} \, d\lambda}{\int_0^\infty i'_{\lambda,i} \, d\lambda}$$

By definition, it is the directional total emission by the object at a given temperature by the directional total emission by blackbody at the same temperature. This can be defined as follows. We obtain the directional total emissivity from the directional spectrum emissivity, by weighting the directional spectral emissivity by the black body function at the temperature of the surface, which we study the radiation. On the other hand, we define the directional total absorptivity as the total radiation absorbed at that angle with the total radiation in symmetrical time.

We see that the directional total absorptivity does not make any reference to any blackbody function. We rewrite this as an integral of 0 to ∞ from the definition of intensity. When integrating the directional spectral emissivity, the weighting function is that of the blackbody at the temperature of a surface by the directional spectral absorptivity. We integrate with the amount of radiation incident on that object from some source, which may not be a blackbody.

This study shows that the absorptivity obtained for any substance depends very much upon what is incident on that surface. If Sun light is incident on a surface, its absorptivity will be of a certain value, while if radiation from some other source is incident on the surface, it will have a different value. ά will differ substantially from one situation to another because incident radiation is different. We compare these two and see when we can apply Kirchoff’s law for this kind of situation.

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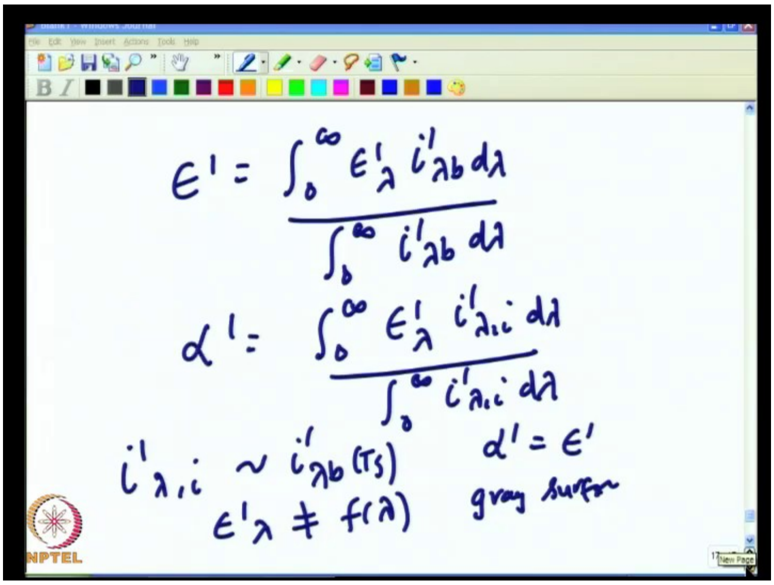

$$\epsilon' = \frac{\int_0^\infty \epsilon'_\lambda \, i'_{\lambda b} \, d\lambda}{\int_0^\infty i'_{\lambda b} \, d\lambda}$$

$$\alpha' = \frac{\int_0^\infty \epsilon'_\lambda \, i'_{\lambda,i} \, d\lambda}{\int_0^\infty i'_{\lambda,i} \, d\lambda}$$

$i'_{\lambda,i} \sim i'_{\lambda b}(T_S)$ $\alpha' = \epsilon'$

$\epsilon'_\lambda \neq f(\lambda)$ gray surfac

We will first write έ as an integral of0 to ∞ έ$_{\lambda}$ ἰ$_{\lambda b}$ d$_{\lambda}$ / integral of 0 to ∞ ἰ$_{\lambda b}$ d$_{\lambda}$. Then using Kirchhoff's law, we write ά as an integral of 0 to ∞ έ$_{\lambda}$ ἰ$_{\lambda bi}$ d$_{\lambda}$ / integral of 0 to ∞ ἰ$_{\lambda bi}$ d$_{\lambda}$. Though the above two equations look somewhat similar, they are not the same because, in the case of emissivity, we are varying it with the blackbody intensity at the temperature of the surface.

If by chance the incoming radiation does happens to be proportional to the blackbody intensity at the surface temperature, then we can say these two are equal. But in real world, this is very rare when the radiation that is sent on a surface, has a wavelength distribution proportional to that of the black body of surface temperature.

If the quantity έ$_{\lambda}$ is not a function of λ, then both έ and ά are equal. This is called as gray surface. We can assume that the directional total absorptive rate is equal to the directional total emissivity.

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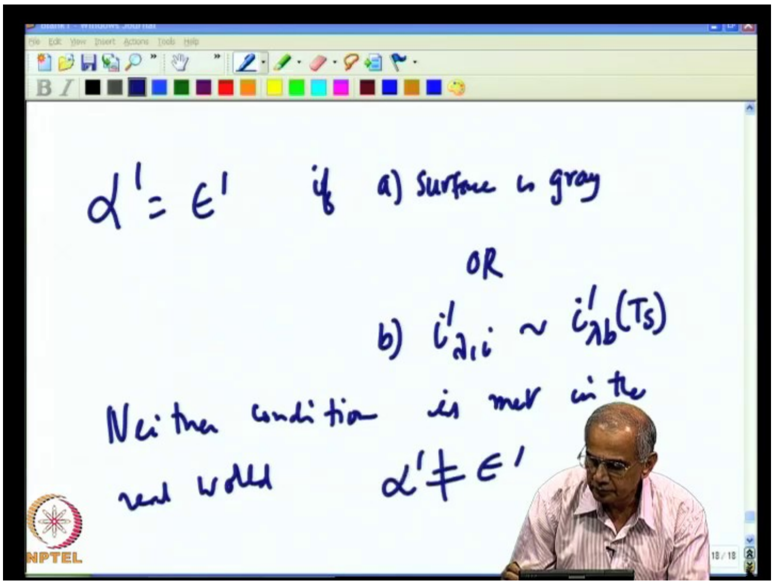

$\alpha' = \epsilon'$ if a) surface is gray

OR

b) $i'_{\lambda,i} \sim i'_{\lambda b}(T_S)$

Neither condition is met in the real world $\alpha' \neq \epsilon'$

This is equal if the surface is gray or the incoming radiation is proportional to blackbody radiation at the temperature of the surface both these occur very rarely. Most surfaces that we encountered have strong wavelength dependence of absorptivity or emissivity. The assumption over gray surfaces is not very accurate. Secondly it is a rare occasion, when the radiation coming on object happens to be having a wavelength dependent on surface temperature. , Kirchhoff's law only depicts the relation between absorptivity and emissivity for directional spectral quantities and not for directional total quantity. The validity of this is specified at the gray surface or the incoming radiation proportional to the blackbody intensity at the temperature of the surface. Neither conditions are met in the real world and hence these two are not equal. Suppose instead of integrating the wavelength, we integrate the angle to find out when the hemispherical spectral emissivity is equal to hemispherical spectral absorptivity.

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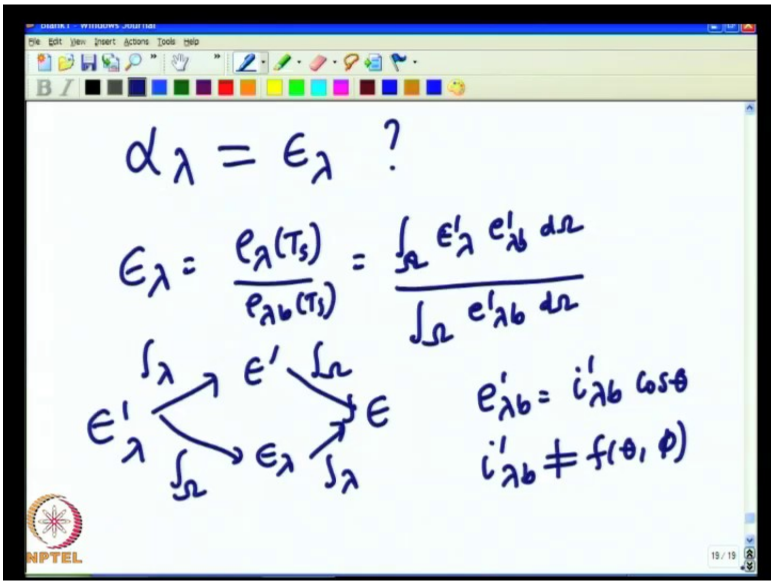

$$\alpha_\lambda = \epsilon_\lambda \; ?$$

$$\epsilon_\lambda = \frac{e_\lambda(T_s)}{e_{\lambda b}(T_s)} = \frac{\int_\Omega \epsilon'_\lambda e'_{\lambda b} \, d\Omega}{\int_\Omega e'_{\lambda b} \, d\Omega}$$

$$\begin{tikzcd} & \epsilon' \arrow[dr, "\int_\Omega"] & \\ \epsilon'_\lambda \arrow[ur, "\int_\lambda"] \arrow[dr, "\int_\Omega"'] & & \epsilon \\ & \epsilon_\lambda \arrow[ur, "\int_\lambda"'] & \end{tikzcd}$$

$$e'_{\lambda b} = i'_{\lambda b} \cos\theta$$

$$i'_{\lambda b} \neq f(\theta, \phi)$$

To do that, we define ϵ_λ e_λ (T_S) / $e_{\lambda b}$ (T_S) as an integral of Ω $\acute{\epsilon}_\lambda$ $ẻ_{\lambda b}$ $d\Omega$ / integral Ω $ẻ_{\lambda b}$ $d\Omega$. . We obtain a emissive quantity by averaging over angle.

Ultimately we will get ϵ, which is the total hemispherical emissivity. Now, remembering the fact that $ẻ_{\lambda b}$is nothing but $\dot{i}_{\lambda b}$ $\cos\theta$, on basic definition of intensive emissivity and the fact that $\dot{i}_{\lambda b}$ is not a function of θ and φ we rewrite the above equation for ϵ_λ.

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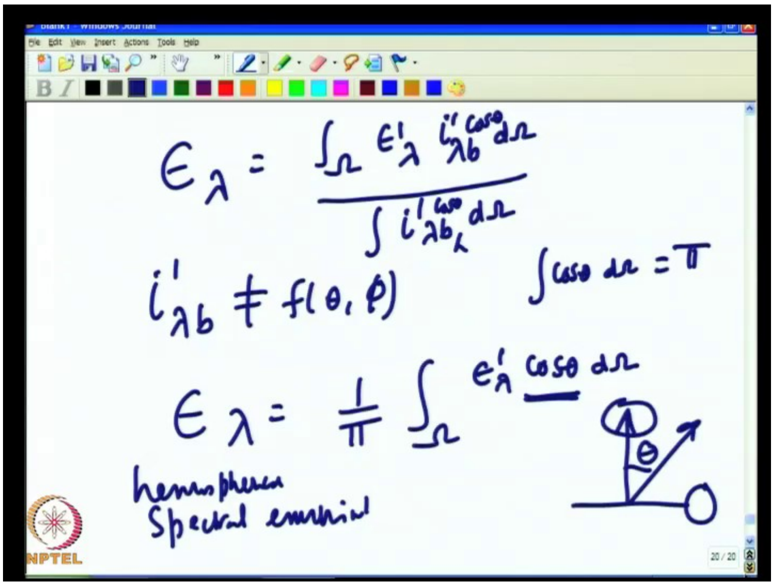

$$\epsilon_\lambda = \frac{\int_\Omega \epsilon'_\lambda i'_{\lambda b} \cos\theta \, d\Omega}{\int i'_{\lambda b} \cos\theta \, d\Omega}$$

$$i'_{\lambda b} \neq f(\theta, \phi) \qquad \int \cos\theta \, d\Omega = \pi$$

$$\epsilon_\lambda = \frac{1}{\pi} \int_\Omega e'_\lambda \underline{\cos\theta} \, d\Omega$$

hemispherical
Spectral emissivity

We write ϵ_λ as an integral over Ω as $\acute{\epsilon}_\lambda$ $i_{\lambda b}$ $\cos\theta$ $d\Omega$ / integral Ω $i_{\lambda b}$ $\cos\theta$ $d\Omega$. The definition of intensity is such that, the directional spectral emissivity of blackbody is not a function of angle that, was built into the definition.

Solving the integration the result will come out as φ. The final expression for ϵ_λ is, $1/\pi$ integral of Ω $\acute{\epsilon}_\lambda$ $\cos\theta$ $d\Omega$. . We do not obtain hemispherical value by merely taking the directional value and averaging it over arithmetically. We have to weight it with $\cos\theta$ where weight age is given to emission closer to the normal.

The region where emission occurs will have higher weight age than normal. The directional quantity when integrated, . the $\cos\theta$ term ensures that more weight age is given to radiations appearing at lower values of data.

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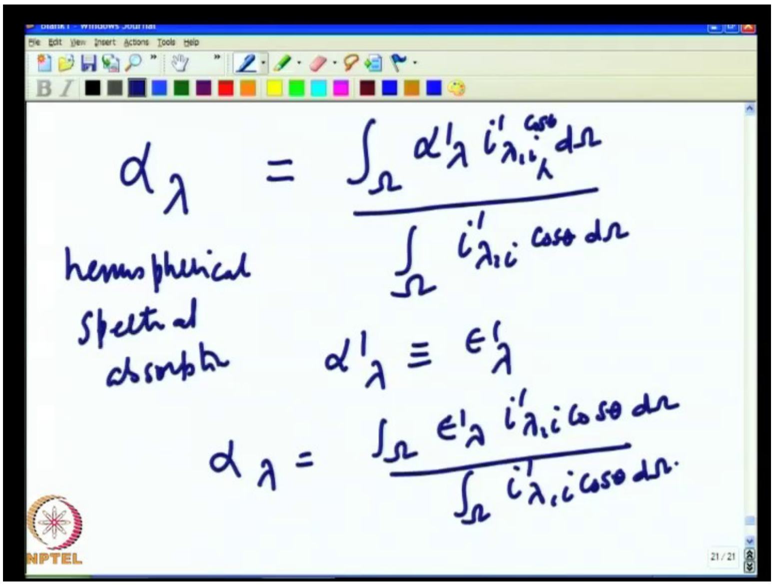

$$\alpha_\lambda = \frac{\int_\Omega \alpha'_\lambda \, i'_{\lambda,i} \cos\theta \, d\Omega}{\int_\Omega i'_{\lambda,i} \cos\theta \, d\Omega}$$

hemispherical
spectral
absorptn

$$\alpha'_\lambda \equiv \epsilon'_\lambda$$

$$\alpha_\lambda = \frac{\int_\Omega \epsilon'_\lambda \, i'_{\lambda,i} \cos\theta \, d\Omega}{\int_\Omega i'_{\lambda,i} \cos\theta \, d\Omega}.$$

Moving on to absorptivity, the hemispherical spectral absorptivity is defined as the integral Ω $\acute{\alpha}_\lambda$ $i_{\lambda bi}$ $\cos\theta$ $d\Omega$ / integral Ω $i_{\lambda bi}$ $\cos\theta$ $d\Omega$, This comes from the basic definition of absorptivity, which is the radiation incident in the solid angle and the fraction that is absorbed due to radiation. If you assume Kirchhoff's law, which is always valid, then we have α_λ is equal to ϵ_λ.

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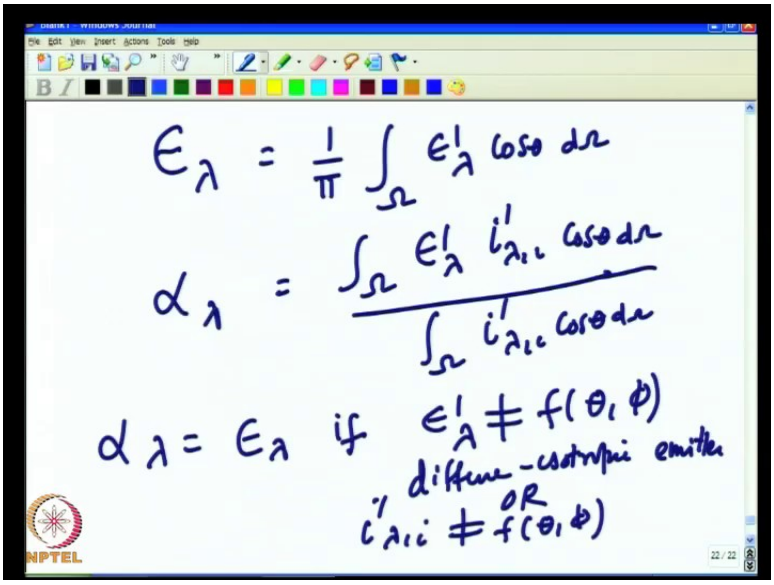

$$\epsilon_\lambda = \frac{1}{\pi}\int_\Omega \epsilon'_\lambda \cos\theta \, d\Omega$$

$$\alpha_\lambda = \frac{\int_\Omega \epsilon'_\lambda \, i'_{\lambda,i} \cos\theta \, d\Omega}{\int_\Omega i'_{\lambda,i} \cos\theta \, d\Omega}$$

$\alpha_\lambda = \epsilon_\lambda$ if $\epsilon'_\lambda \neq f(\theta, \phi)$

diffuse-isotropic emitter

OR

$i'_{\lambda,i} \neq f(\theta, \phi)$

Now, let us compare the two expressions called emissivity and absorptivity. We saw that ϵ_λ is $1/\pi$ integral of Ω $\acute{\epsilon}_\lambda \cos\theta \, d\Omega$. α_λ is somewhat different. We can see that in general these two are equal, if $\acute{\epsilon}_\lambda$ is not a function of angle that is the emissivity. The surface emits diffusely and is symmetric. The other possibility is that the incoming radiation is not a function of θ and φ. So, any incoming radiation is emitted equally in all directions. In this lecture, we have focused on properties of blackbody radiation and the properties of real surfaces. We define emissivity and absorptivity at its most basic level. Then we involved Kirchhoff's law, which says that the directional spectral emissivity equals directional spectral absorptivity. And we went on to show under what conditions one can assume equality between hemispherical spectral emissivity and hemispherical absorptivity. We saw that there are only equal under very special conditions. We will continue this discussion further because there is a lot of misunderstanding about, how to invoke Kirchhoff's law. So, we will spend some more time highlighting the misuse of Kirchhoff's law.