Radiation Heat Transfer
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Lecture - 18
Optically Thick Limit

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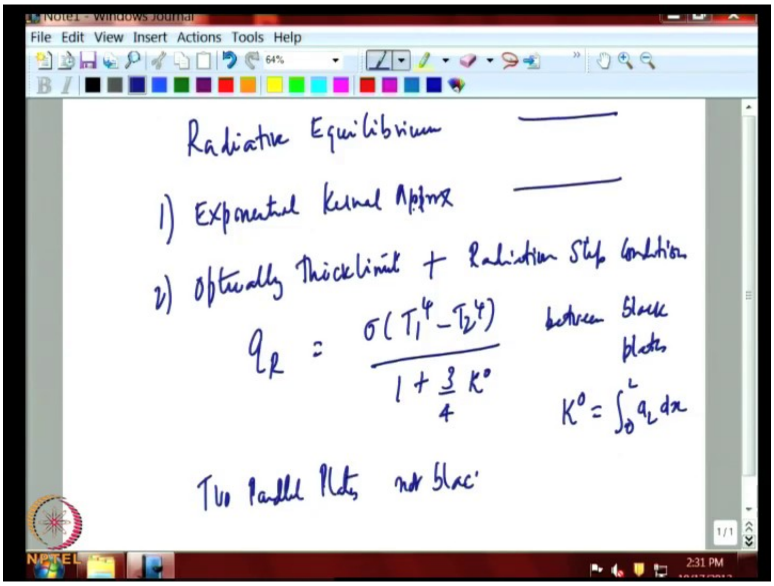

In the last lecture, we looked at Radiative Equilibrium in the presence of a gas which is absorbing. We solved this problems within two parallel plates. We first looked at the exponential kernel approximation and ensured that this gives fairly accurate results for both radiative flux as well as the temperature distribution. Then we look at the optically thick limit, and along with that we applied the radiation slip conditions. And again we obtained results which were consistent with the kernel approximation and the accurate solution. And the result we got was that radiative flux was equal to 1 plus 3-4.

This result is a very useful result. This is between black plates and this result shows that where K 0 is the absorption coefficient times the length of the gap between two plates. This show that in the optically thick limit, this quantity becomes very large and the flux tends to 0. In the optical thin limit is quantity is 0, so we are back to original result we had obtained earlier, for heat transfer between two plates without any absorbing gas in between. Now, today we want to extend the result between two parallel plates which are not black.

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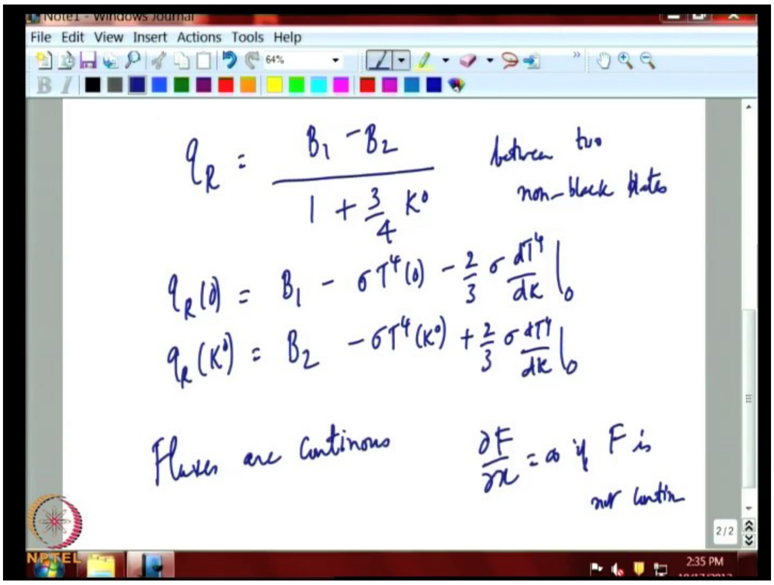

So, let us see how we extend it. If we call the result for heat transfer between two plates which is not black is that q of R is B 1 minus B 2. This is a general result obtained already in kernel approximation. We want to now write explicitly in terms of T 1, T 2 the two temperatures and the emissivity of the surfaces. To do that we recognize the following radiative heat flux at the bottom wall is radiosity minus emission from the bottom surface minus two-third. Similarly at the top wall; these are from the flux condition.

In radiate heat transfer we cannot assert that the temperature of the gas, next to the wall will be same as the wall temperature. That is we must permit the fact that, there is a slip between the gas temperature and the wall temperature. This is not normally invoked in conduction or convection heat transfer studies. In radiation we have to invoke it, because unless it is in the optical thick region, generally there is temperature difference between the gas layers next to the wall and the wall temperature.

Although, the temperature is not continuous between the gas and the wall, fluxes are continuous. Fluxes are continuous, because at discontinuous flux implies a violation of the first law thermo dynamics. Because if the flux is discontinuous then the divergence of the flux, will become infinity. So, for example in our case, it will be infinity if, F is not continuous. Therefore, we always invoke the continuity of flux and this result here is from this condition. Now, we also know that under radiative equilibrium this quantity is related to the radiative flux.

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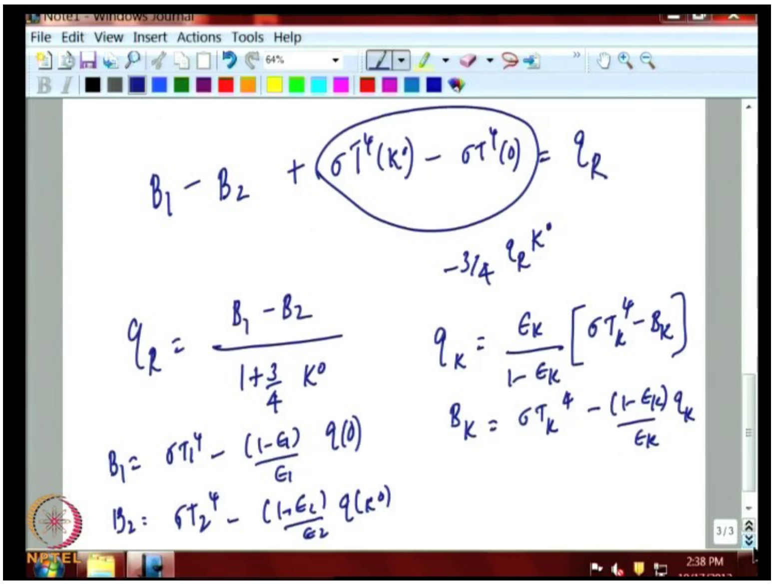

We can rewrite this above expression as, q R. There for we get this result that radiative flux 1 plus three-fourth. This is already obtained by correct approximation we are re deriving it for the thick limit. Question is what is B 1 B 2. Now this if it you call from the radiative trans enclosures flux on the K surfaces, epsilon K by 1 minus epsilon K.

This is derived as a general relationship in flux radiosity and the wall temperature. You use this result for the two walls. We can write this as B K equal to sigma T K 2 to the power of 4, minus 1 minus epsilon K epsilon K q K. This you applied to the two walls. This we have written as q of bottom wall to wall and if we recall the fact in relation to these two results which we have in front.

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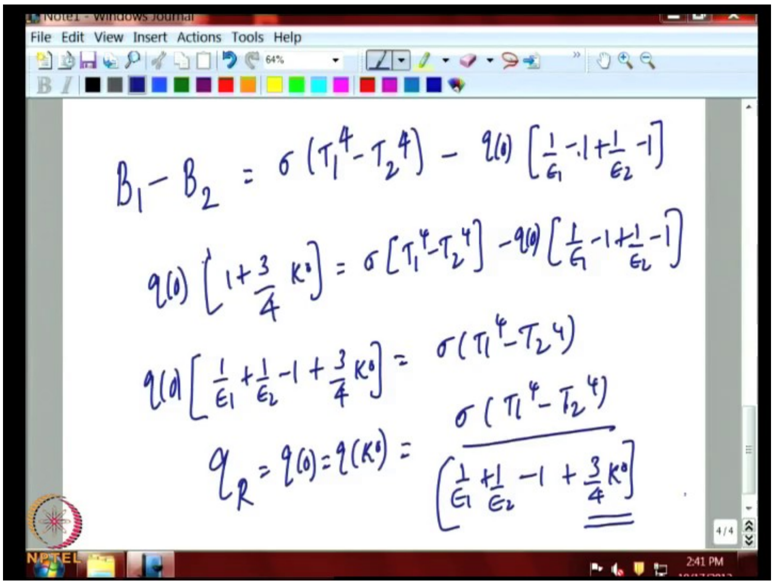

We now calculate, what is B 1 minus B 2 from this equation. And this comes out as, plus 1 by epsilon 2 minus 1. Now, here we use a fact at under radiative equilibrium, q of 0 is q of kappa 0. Now if you use the result that B 1 minus B 2 is nothing but q R into or q 0 into one plus three-fourth kappa 0 is equal to sigma minus q 0. Now we take all the q 0 to one side and again see that one 1 will disappear another 1 will remain. We will have q 0 into 1 plus epsilon 1, plus 1 plus epsilon 2 minus 1, from these terms plus kappa 0 equals. We get the final result which is q 0 or q R general, because q is the constant.

So, q R finally, which is same as q of 0 equal to q of kappa 0 is sigma. Now, this result is a general result and notice that when kappa 0 goes to 0, that is a thin limit. We recover old result for radiative heat transfer between parallel plates. And notice that in the limits is kappa 0 is become very large. If, we neglect these terms then the emissivity of the two walls becomes irrelevant. In the optically thick limit the gas hardly sees the two walls and hence the emissivity of two walls is totally irrelevant.

This result is a nice result clearly showing, when should we include gas radiation. For example, if you look at the insulation use commonly, made by large reflective layers, 100s of them. The emissivity of this surface may be 0.01. We can see that this will add up to 200. Unless the radiation optical depth of the gases between the reflecting layers is very large; this term is not important. The gas radiation becomes important only if, the two walls of black and so. This from is one and this quantity, let us say is as highest ten or so. Otherwise, the gas between two parallel plates is not important if the emissivity in the two walls is very, very small and this is best understood in terms of an electrical analogy.

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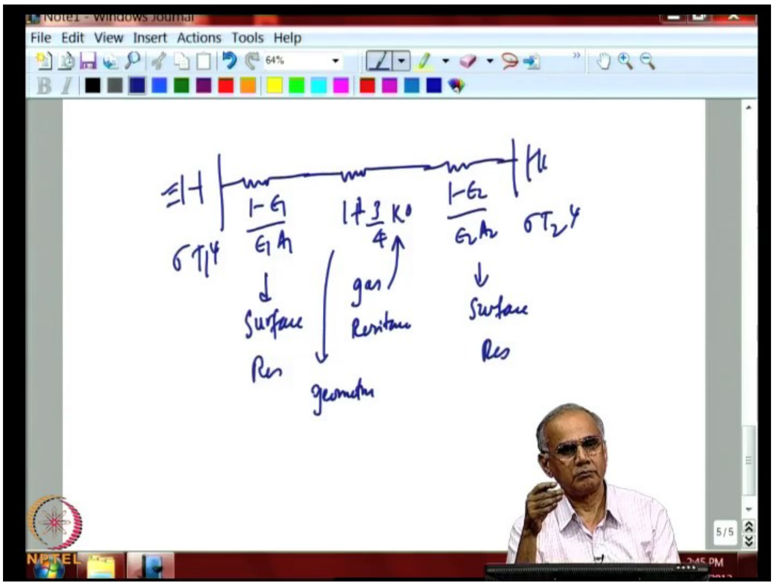

We can see clearly that there are two surface We can call this now a gas resistance is also has geometrical this is surfaces resistance; this is also a surface resistance. We see that if the optical thickness of the layer is very, very large, then this terminate this two not important. On the other hand if the emissivity's of very, very small, the surface resistance dominates. The geometric, factor and the gas distance is not relevant.

This kind of electrical analogy is useful to understand which particular mechanism offers maximum resistance to heat transfer. We should be focusing on the others or not that important. This is a good example of understanding the interaction between surfaces possesses like reflection, emission, absorption. And what is happening in the gas between the surfaces and how under certain condition surface absorption may be more important, while in other situation in gas abortion may be very important.

So far we have dealt with radiative equilibrium. Now, radiative equilibrium is a special case occurs if radiation the only process that is occurring another process or not important. But as most of us realize in any real world problem, radiation occurs in conjunction with conduction convection and other process. So, radiative equilibrium is not something we encounter very commonly. So, we have to deal with problems were radiative equilibrium is not valid.

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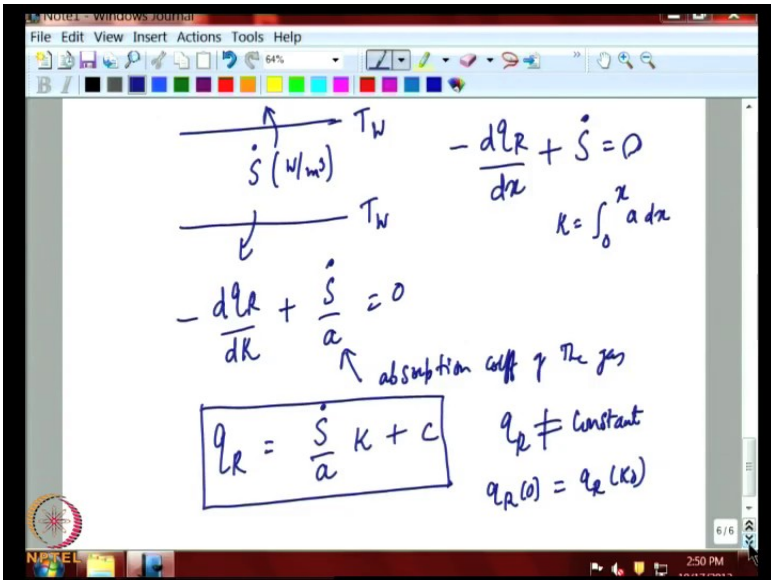

We will take of a simple example of a situation where radiation is not the equilibrium. And this example although very simply stated has relevance to furnaces. Imagine a furnace in which heat is released at S dot watts per milli cube by combustion. Imagine a furnace where combustion is occurring, heat is released at the rate of S dot watts per meter cube. For convenience we will assume the two walls are the heat sinks. They are at the same temperature T watt; is called them T watt. We have two walls at the same temperature removing the heat of all the furnaces where the heat generated.

Now, this is the very simple representation of a real furnace, where heat is a removed for combustion and the walls are the places were heat is removed by flowing water which is your may be your boiler may be the water wall of a power plant furnace or a super heater or some other place. This is a classic example of a standard problem in furnace or boiler design. The basic formulation is that the rate of change of radiative heat flux, has to be balanced by S dot this is the formulation. This is a case where radiative equilibrium is not satisfied, because the divergence of radiative flux is not 0; it is balanced exactly by the heat released by combustion.

Now this problem can easily be solved. First you assume that we can write everything in terms of an optical depth kappa, which in this case is 0 to x, a d x where a is the absorption coefficient. This can be written as d q R d x plus S dot by a is equal 0. Where a is the absorption coefficient of the gas. This we can integrate immediately to get q R is equal to S dot by a into kappa plus a constant.

Notice that in contrast to the previous examples where we dealt with radiative equilibrium. Here, q R is a not a constant, but varies with the optical depth. Now, we need information about constant and this is most readily obtained by realizing the fact, that the total heat released in the combustion chamber has to be removed from the two walls. By symmetry the two wall flux q R 0 and q R kappa 0 have to be equal.

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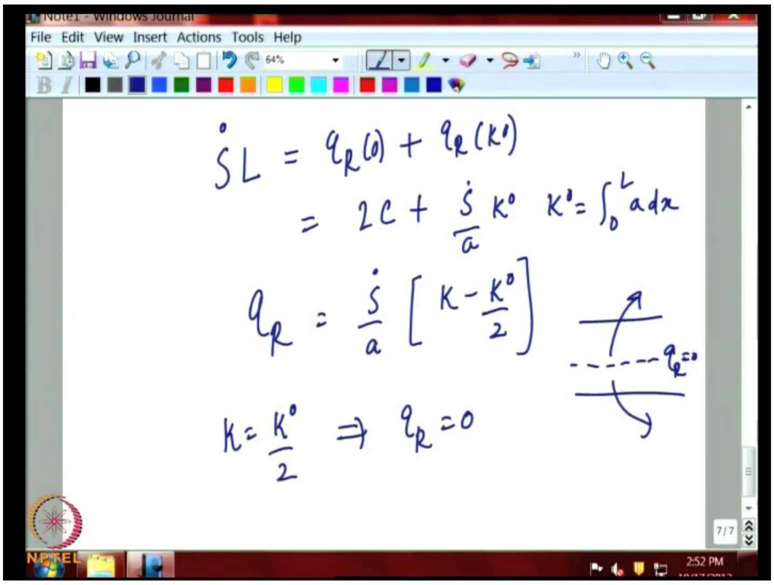

The total heat released, S dot into L has to be released in old combustion chamber has be equal to heat removed by the two walls. That is one requirement and by substituting values of q R at the two ends, you get this as 2 C plus S dot a kappa 0. Kappa 0 where is nothing but 0 to L a d x. And from there we can obtain a value of c.

Finally, we get expression for q R as nothing but S dot by a kappa minus kappa 0 by two. We can see that at kappa equals kappa 0 by 2 q R has to 0. This is because the heat released from the combustion chamber has to go out to both the walls. So, at the center of the walls q R as to be 0 and that is emerging here clearly in the expression.

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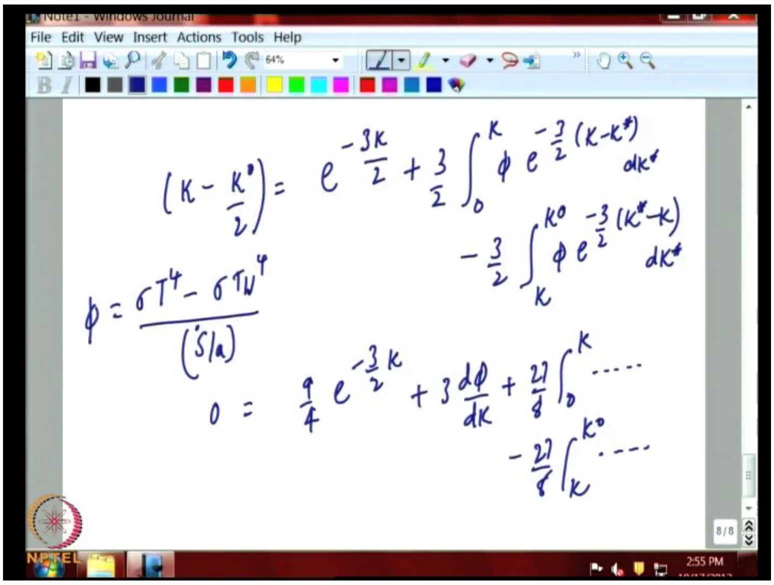

Now, we have to solve for the temperature profile. Our expression is q R S dot by a, kappa minus kappa 0 by 2, this be same as what we did in the previous two lectures. Right hand side is essentially same as before, except that we will define the phi. So, phi we should recall is to be defined this case, because the two walls are at the same temperature. We are going to define phi as T 1 to the power of 4 minus T wall to the power of 4 divided by S dot by a. This is the definition which will ensure that the temperature profile as a same, structure what we did earlier. Here, the basic driving force is the heat release of the wall.

Now, we can differentiate this equation twice like before. This will come from the Leibniz's rule for the limits. We realize the fact that this term and this term must similar except for the terms. We multiply this equation and subtract the two integral cancel out and will be left with the very simple differential equation.

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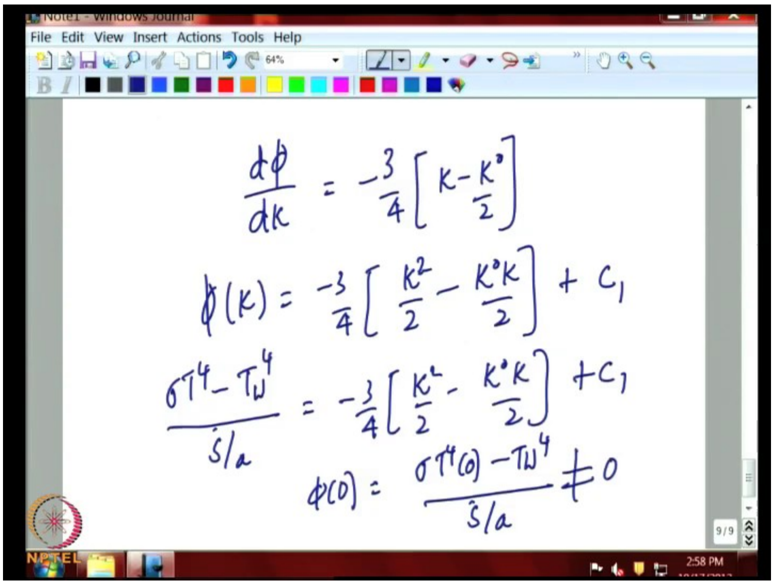

This simple equation has some similarities to the equations solved for the radiative equilibrium case, we integrate the equation we get, because a constant. Now, this constant is very important, because notice that this Phi is nothing but sigma T to the power of 4 minus, T wall to the power of the 4 divided by, S dot by a and so let me just complete this equation.

We notice that at phi of 0 otherwise phi of kappa 0, extract phi 0 first. This will be sigma T four at the gas of the wall minus of all temperature. Now, notice that this not equal to 0, because this is slip. Because, the temperature of the gas at the wall is not equal to wall temperature. This slip has to be calculated. Now, the accurate we have doing this should be substitute this back, in the integral equation and satisfied at one locations C kappa equals 0 we can calculate C. We will do a different way; we will use a radiation slip condition.

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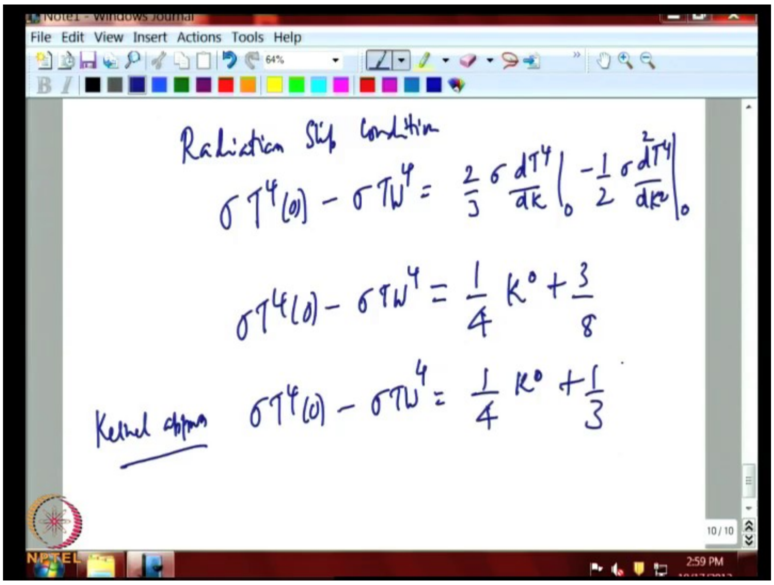

This is a short cut easy to do and this we recall from the last lecture. This is what we obtained by Taylor series expansion for radiation slip. In our case we know these two quantities. We can calculate the radiation slip as being equal to one-fourth kappa 0 plus 3 by 8. If we deduce the kernel approximation and substitute this is almost one-third.

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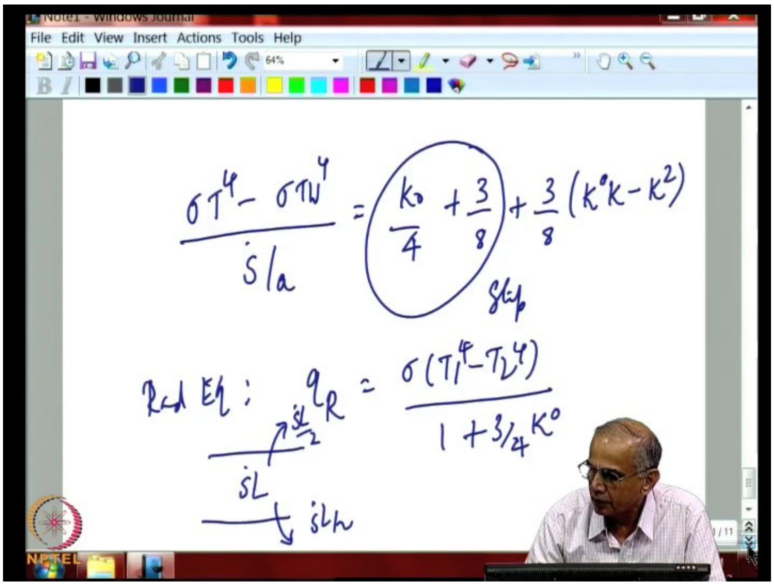

The final expression for radiative heat transfer gas will be. This result the general result you got here and that is our interesting features which we want to recognize. We can see that both at kappa call 0 in kappa 0 that is a slip, which is can be quiet large and notice that in this case this slip never goes to 0. This slip is always that it is very small when you are in thin limit, but as you go to thick limit this quantity does not go to 0. So, whenever there is radiation heat transfer this slip exists always. This is first important thing we see.

The other condition is in the optically thick limit this quantity dominates over this quantity and again there slip can be quite large. This results defers from the earlier result we got for the radiative equilibrium in which case we got, the condition that radiative flux was there were kappa 0. We saw that kappa is very large heat flux goes 0; heat cannot go to 0, because heat is being continuously generated inside the system and this heat has to transferred out. So, at no time even the thick limit, flux cannot go to 0. We can easily see that S dot by L is the heat is generated and so half of it has to go this way, half of it has to go from the bottom plate.

This is a requirement and because of that there has to be a temperature gradient between the center and the wall and this temperature gradient will go on increasing as you go to higher optical thickness. That is why the, this slip we can see is proportional to this one as a we can take the case where k 0 is half.

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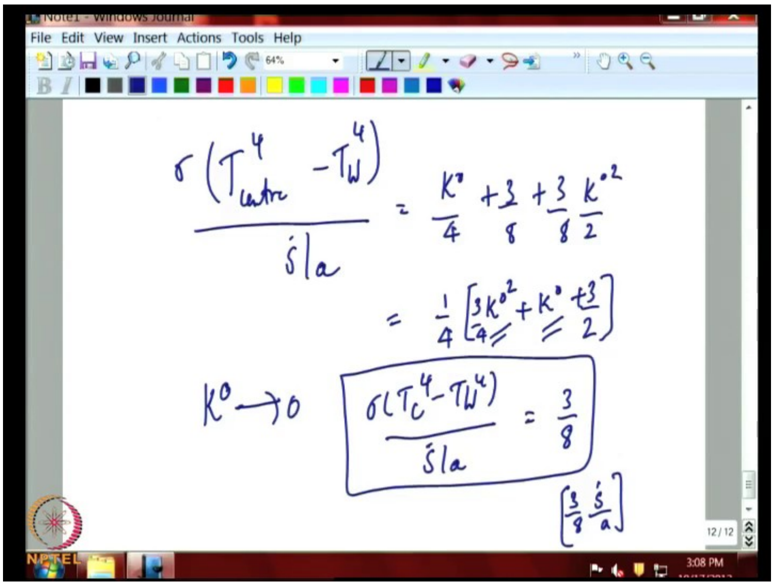

T is center line and this will come out as, this K 0 by 2. We get K 0 square by 2. This is all can retain in a more useful way, as one-forth k 0 square, 3 k 0 is square it will be 3 by 4 plus k 0 3 by 8. This what shows clearly that a center temperature goes on increasing, as the optical techniques is increases, because the gas must get it off the heat generated inside the furnace continuously; whatever the optical thickness. Hence the temperature reference between the center line and the wall has go on increasing. Other interesting limit is k 0 tending to 0 the thin limit. That we see that this center line temperature minus wall temperature becomes nothing but 3 by 2 here.

This is the in a sense the minimum temperature difference that, has to be maintained between the center of the furnace and the wall even in the thin limit. This result is rather interesting, because the temperature being proportion to 3 by 8 into S dot by a; and of course has limit a goes to 0 the different temperature becomes quite large. Again one says that the temperature difference between the center and the wall is proportional to the heat released which is not surprising and inversely promotional to the absorption coefficient, which might look a bit the surprising except that once you realize a fact that the efficiency of radiative heat transfer from the center line to the wall is partly determined by the absorption coefficient.

This absorption coefficient is quiet small then the efficiency of transfer becomes quite poor. Hence we need what are the difference between the center line and the wall. So, from these exercises we have understood the role of internal heating as in this case or the role of wall emissivity which we saw earlier in the lecture. We have done two problems in this simple as gray gas case.

One involving radiative equilibrium and the other involving heat release in enclosure which has to be removed from the gas by radiative heat transfer to the wall. In both them we saw the simple analytical solution provided useful information, about the nature of interaction between radiation and the boundary condition.

Now, we took up the gray gas case first, because that is a case most convenient in terms of solving a problem. We realize a fact that the gray gas approximation is not a useful approximation for radiative transfer and gases in many real life situations, because most gases which absorb and emit radiation not gray gas. In most gases the absorption coefficient vary substantially with wave length.

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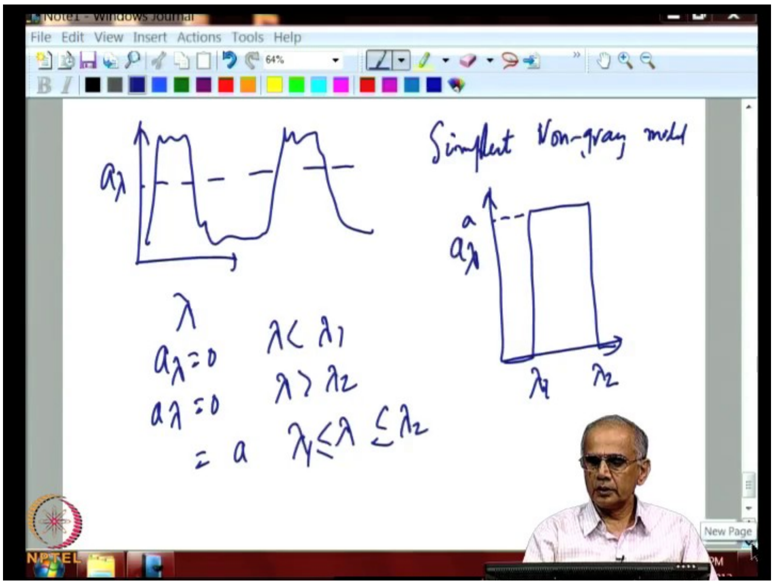

This character of the absorption coefficient; very strong variation, with that is a certain wave length the gas can completely be absorbing and in certain other wave lengths the gas is strongly absorbed. So, for such a situation in rather difficult to define an appropriate mean absorption coefficient to solve the problem. It is better to recognize how to solve problem which are not gray.

Now we look up what we would call as a simple non gray model. The simplest non gray model involves x would be fact that a lambda is not a constant. We take a very simple case a lambda is different from 0 in narrow range of wave length, lambda 1 to lambda 2. Essentially what we say is a lambda is 0, for lambda below lambda 1, a lambda is 0 also for lambda greater than lambda 2 and is equal to constant value for lambda between lambda 1 and lambda 2.

There is a narrow region in which the gas is absorbing radiation and outside this region, the gas does not absorb radiation. This is of course very crude representation of the real world, but it is a good teaching tool to explain, how we have taken to account the non gray characteristic of gas radiation.

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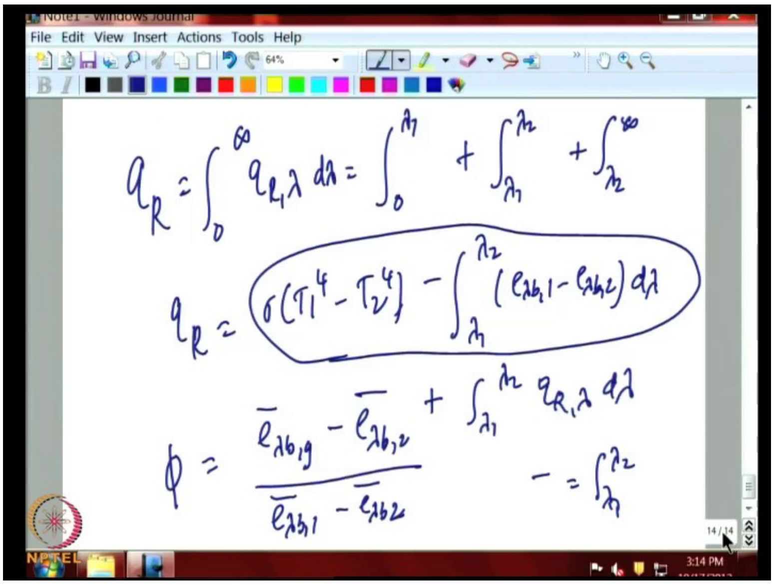

When we calculate the total radiative flux which involves integral over all wave length of the spectral radiative flux. We divide it into three parts; in the first and the last part there is no interaction with gas. We can easily write this as sigma T 1 to the power of 4 minus T 2 the power of 4 minus integral over this wave length where radiation is very important.

Essentially we have done the work that involved the integration. This term is the term which is always easy to calculate involves no gas radiation, but this term is very complicated the second term, because we have to allow for the fact that radiative flux will vary in a said narrow range in this wave length. So, now with this problem, we defined the non dependent temperature somewhat differently, because not gray gas.

We take the temperature of the gas in the wave length range of interest to us that is lambda 1 and lambda 2. These bars are averages over the wave length in which gas absorb radiation. So, once we have done this, we can rewrite the radiative flux divergence terms in a simplified way.

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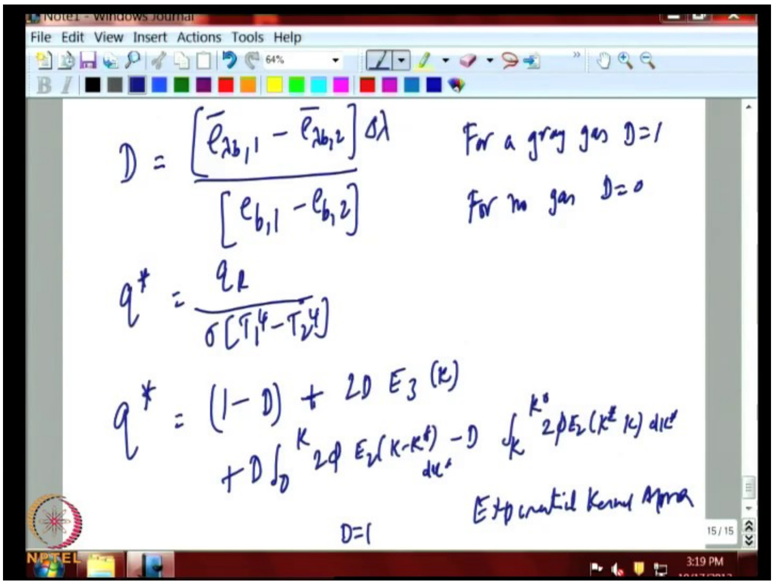

We now write down the parameter called D, which represents the radiative flux in the wave length range where the gas absorbs and emits radiation. This can be thought of as the amount of radiation exchange in the absence of a gas, between surface 1 and surface 2 in this wave length interval delta lambda, compared to the radiation that is transferred from 1 to 2 at all wave length. This crudely a measure of the radiation lying in the lambda order range compared to the total.

We should recognize the fact that once we treat the gas as gray this quantity becomes 1. For a gray gas and for no gas. With that we write down the expression for radiative flux and as before we define q star as q R. This is and definition between two parallel black plates and with that we get the expression for q star as 1 minus, D plus 2 D E 3 of kappa plus D. So, what you see here is that, when you put D equals 1 that corresponding gray gas this terms of solved this term becomes two, one and one this is identical to the gray gas expression that we have to use earlier.

Extending the gray gas to non gray gas, it is fairly simple as long as you have talked about only one absorption brand of the gas and that interval is defined through the factor D. If D equals to 0 there is no gas. So, q star is 1; that we all know. In the absence of gas between the two plates the ray transfer between the two black plates is nothing but sigma T 1 T 2 to the power of 4; that is captured here. The second thing is this term becomes 0, this is two, this is one. We get back the equation for a gray gas. This is a generalization of the gray gas approximation and again we can make the exponential kernel approximation same way and solve the equation differentiate twice everything is identical.

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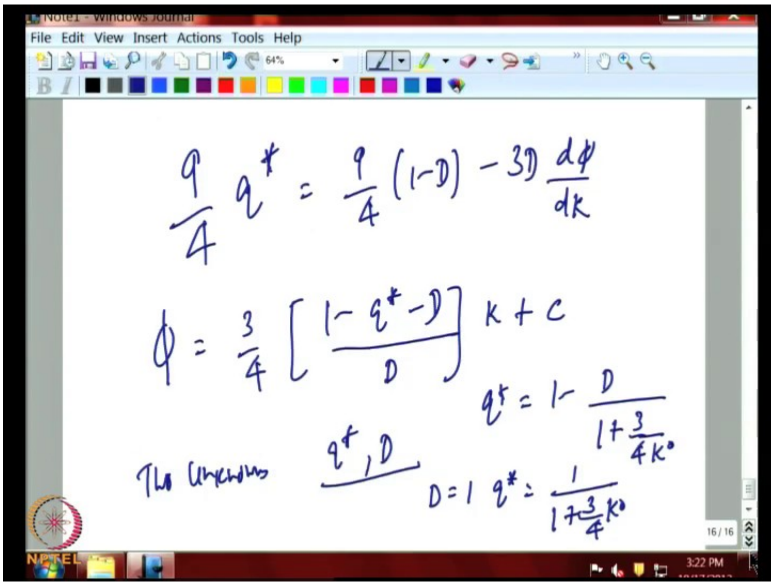

This expression again we want to remind you is similar to the gray gas expression. We put D equals to 1, once more you recover the gray gas limit. So this stage we see that the gray gas approximation is built in to the solution.

The general expression for phi is, we have two unknowns they are constant and they are q star and D. Once that relation between C and q star D is known we have the complete solution of the problem. We can show that q star is 1 minus D. This is quite similar to what recovered a gray gas where D was equal to 1. There is a slight difference here which we have to be very careful about. When D goes 0 of course q star is one we know that result. Now this is a very important result and we would like to understand and explain result we obtain earlier for the gray gas and the result that we obtained can be compared in various ways.

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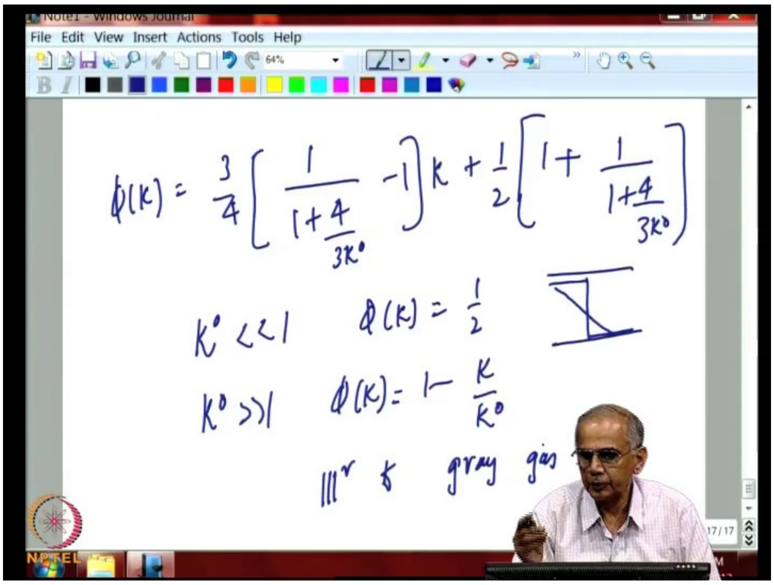

We will put down the temperature distribution. That is explained for temperature profile and notice that this result is in depended of D that parameter which defines the fraction of radiation lying between lambda and lambda 2 that does not expression will come this equation. For of course the q star the radiate flux depends on D and notice that in the limit as kappa 0 is much, much less than 1; phi of kappa becomes half and that is most is usually you seen and the drop this term out and drop this term out. Phi equals 0, because we write this as half minus three-fourth kappa and be here we do this here.

We can see this result is that the as kappa is going be very large; this is goes to 0, this term drops out and as kappa 0 is very large, this goes to 0, this is half. The gas temperature we saw earlier is like this two slips and when kappa 0 becomes very large; we have phi of kappa, is 1 minus kappa by kappa 0. We can see that this is linear distribution which we saw earlier and it is more like that, but there we between two walls that is bound occur and what we do is kappa is becomes very, very large; this terms drops out this is gone kappa, this term drops out. This expression can be then used similar to what we had for gray gas. The temperature distribution between the two walls does not depend on whether is gray or non gray but the flux expression was very different. It is flux expression which gives it some unusual, features which are different in the gray case non gray case.

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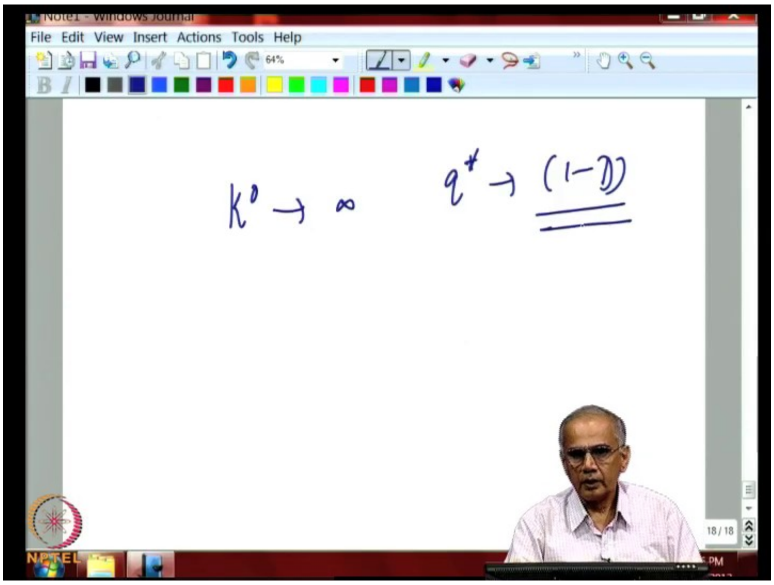

The key point is that as kappa tends to infinity q star now tends to 1 minus D. This is very important result and we will take this up in the next lecture.