Radiation Heat Transfer
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Lecture - 16
Diffusion approximation

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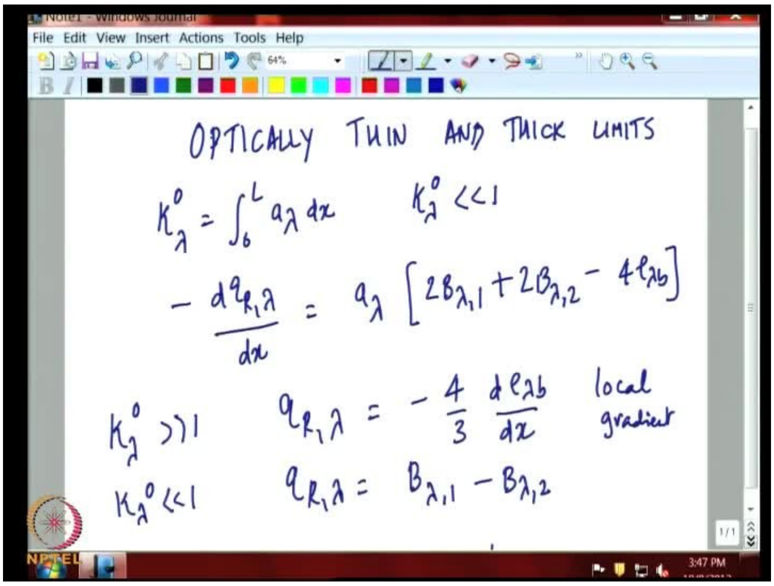

OPTICALLY THIN AND THICK LIMITS

$K_\lambda^0 = \int_0^L a_\lambda \, dx \qquad K_\lambda^0 << 1$

$$-\frac{dq_{R,\lambda}}{dx} = a_\lambda \left[2B_{\lambda,1} + 2B_{\lambda,2} - 4e_{\lambda b}\right]$$

$K_\lambda^0 >> 1 \qquad q_{R,\lambda} = -\frac{4}{3}\frac{de_{\lambda b}}{dx}$ local gradient

$K_\lambda^0 << 1 \qquad q_{R,\lambda} = B_{\lambda,1} - B_{\lambda,2}$

In the last lecture, we looked at the optically thin and thick limits. These limits provide a good insight as regards, what happens during radiation, when the photon mean free path is very small or very large. If you recall the total optical depth, is much less than 1, that is the photon mean free path is very large compared to the spatial scale of the system. Then we saw that the divergence of the radiative flux was equal to radiation absorbed from surface 1, from surface 2 and what is emitted by the surface.

In the thin limit we find that the interaction is between the gas and the two walls, that is absorption term and what the gas emits in both above and below the interface. In the thick limit we have derived and we got a result for flux, as a simple differential equation. These two look very different. Let us just also write a radical flux for the thin limit for comparison here. In the thin limit, if we recall the q R lambda is very simple, but difference in radiosity between the two walls.

In the thin limit the flux through the medium then depend on the gas, it merely depended on the radiosity of the two sides while in thick limit we have a differential equation, here we have a algebraic equations. The basic nature of the radiative flux expression has change dramatically. In one case the flux depends on the local gradient of the emissive power of the gas radiation while in the thin limit gas plays no role. The radioactive flux is merely the difference in radiosity of the two walls. So, it is important to understand this important difference in the two limits.

In the thin limit the gas plays no role, gas does not play any role at all. It allows the photons from one wall to go to the other wall and vice versa, the photons of the other wall go back to this wall. The medium is completely passive, it does not do anything. This is because the photon mean free path is so large that the photon is essentially untouched as it goes from one wall to another wall. So the gas plays almost no role in the flux, but if we look at the thick limit the radiative flux does not depend on what is happening in the wall.

Wall play no role in here, everything depends upon the local gradient of the black body emissive power of the gas emission. The thick limit at any point inside the gas the radiative flux depends upon only the local gradients of the black body emissive power at the gas temperature and has nothing to do with two walls. This is because the photon mean free path, in this case so small that photon which are emitted little far away from the interface you are interested in will not reach your location.

The photons emitted by the wall will not reach the gas. So, at any location the gas only sees the adjacent layers from where the photons come. If the layers are little far away, the photon cannot reach this layer. This is completely local control.

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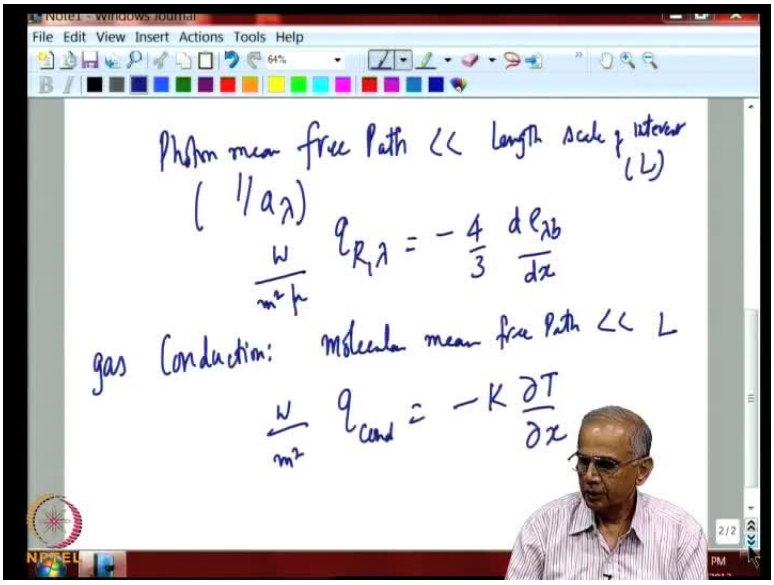

Photon mean free Path << length scale of interest (L)

($1/a_\lambda$)

$$\frac{W}{m^2\mu} \quad q_{R,\lambda} = -\frac{4}{3}\frac{d e_{\lambda b}}{dx}$$

gas Conduction: molecular mean free Path << L

$$\frac{W}{m^2} \quad q_{cond} = -K\frac{\partial T}{\partial x}$$

We want to appreciate the fact that this kind of phenomena where everything depends on local gradient is happening because the photon mean free path is very small compared to the length scale of interest which in this case is L, which in this case is 1 over a lambda. So, 1 over a lambda is very small compared to length scale of interest that is the distance between the plates then everything is control locally and we saw q R lambda is equal to minus four-third d e lambda b by d x.

We have to realize the same logic is applied to conduction heat transfer. In conduction heat transfer the molecular mean free path not photon, molecular mean free path. That is the distance travelled by molecule in a gas, this is a gas conduction. Conduction gases is small compared to the length scale of interest. Then you, all of you know that q conduction is minus K delta T. We want to see the parallel between these two basic situations. In radiation where the photon mean free path is small compared to the length scale of interest to us.

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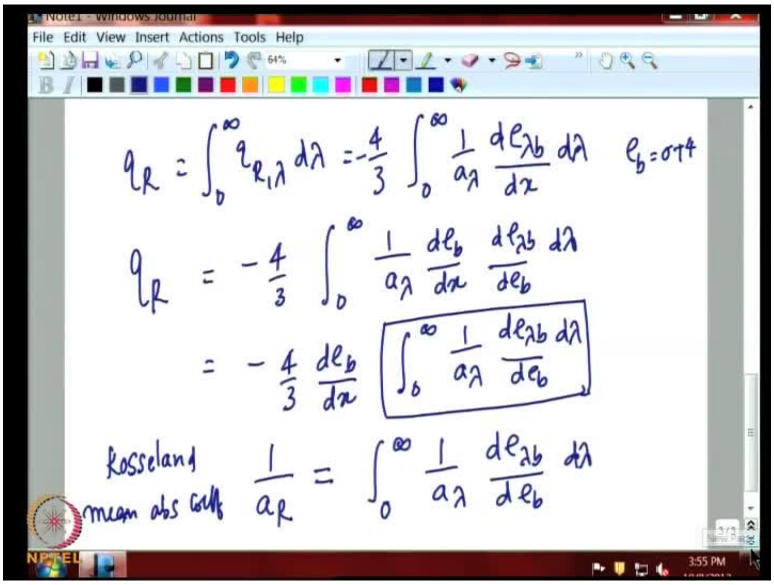

$$q_R = \int_0^\infty q_{R,\lambda}\, d\lambda = -\frac{4}{3}\int_0^\infty \frac{1}{a_\lambda}\frac{de_{\lambda b}}{dx}\, d\lambda \qquad e_b = \sigma T^4$$

$$q_R = -\frac{4}{3}\int_0^\infty \frac{1}{a_\lambda}\frac{de_b}{dx}\frac{de_{\lambda b}}{de_b}\, d\lambda$$

$$= -\frac{4}{3}\frac{de_b}{dx}\boxed{\int_0^\infty \frac{1}{a_\lambda}\frac{de_{\lambda b}}{de_b}\, d\lambda}$$

Rosseland mean abs coeff $\frac{1}{a_R} = \int_0^\infty \frac{1}{a_\lambda}\frac{de_{\lambda b}}{de_b}\, d\lambda$

Then everything is locally controlled and hence the radiative flux depends only on the local gradients of the black body emissive power. For the same reason in gas conduction heat transfer when the molecular mean free path is small compared to the length scale of interest to us then the conduction heat transfer is proportional to temperature gradient. These two expressions for fluxes are not really same. This one is Watts per meter square per micron and this is Watts per meter square.

We may be having some difficulty comparing these two. So, next what we will do is we will integrate this thing over all wavelengths to get the total paths so that comparison is more easy. So, q R is nothing but 0 to infinity of spectral flux into d lambda. This will be 4 by 3, 0 to infinity 1 over a lambda d e lambda b by d x. There is minus at reserved here, d lambda. So, to do this integration in easy way, we divide and multiply by e b. Remember e b is nothing but sigma T to the power of 4.

First multiply that so that you write q R. We have to multiply up and down by e b, you get d e b d x, we get d e lambda b by d e b. Here mainly multiply and divided by this quantity d lambda. Now, since this quantity is independent of wave length we can take it out and we are left with quantities which depend on wave length which we need to integrate may be numerically.

This quantity is a number to be calculated, once we know the variation of the absorption coefficient a lambda with wave length. There is a complex function and we will spend quite some time in later part of the course explaining how this task is achieved, but right now let us say we can do it and this is called the Rosseland mean absorption coefficient. This, this integration is known as Rosseland mean absorption coefficient and this by definition is 0 to infinity 1 over a lambda d e lambda d by d e b d lambda.

This quantity can be calculated once given the temperature the gas and the spectral properties of that gas. Today tools we have computers, we can calculate this and keep it for future use. This is the number which is function of temperature which can be calculated a priory.

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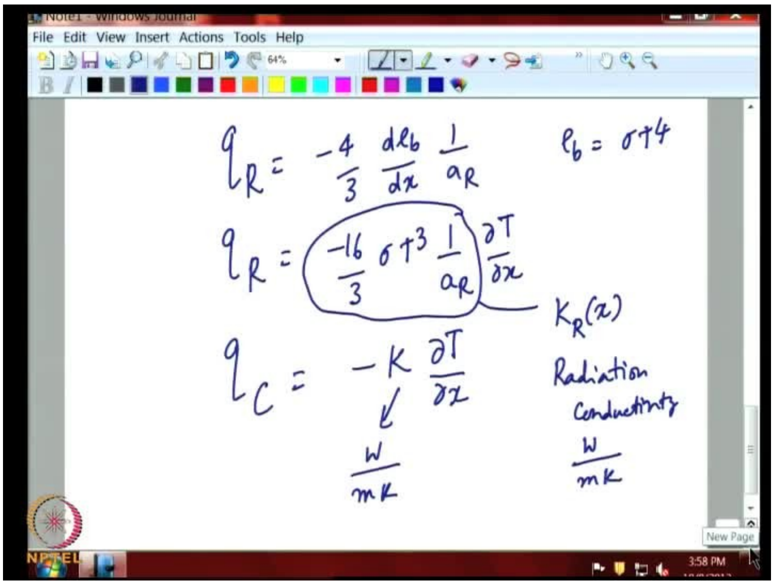

$$q_R = -\frac{4}{3}\frac{de_b}{dx}\frac{1}{a_R} \qquad e_b = \sigma T^4$$

$$q_R = \underbrace{\boxed{-\frac{16}{3}\sigma T^3\frac{1}{a_R}}}_{K_R(x)}\frac{\partial T}{\partial x}$$

Radiation Conductivity $\frac{W}{mK}$

$$q_C = -\underset{\frac{W}{mK}}{\underset{\downarrow}{K}}\frac{\partial T}{\partial x}$$

With this definition we can write down q R, which is minus 4 by 3 d e by d x into 1 over a R by definition. Now, e b all of you know is sigma T to the power of 4. We can write it in a most simplified form as minus 16 by 3, this becomes sigma T cube; 4 sigma T, 4 has been absorbed here, 1 over a R delta T /dx.

Now, we need to compare this with the Fourier conduction law. So, conduction heat transfer we saw was minus K delta T delta Z. We compare these equation with this equation, we can define a new quantity in the thick limit and it is this quantity, this is K R which is not a constant depends on x is called radiation conductivity and these are units of Watts per meter degree Kelvin.

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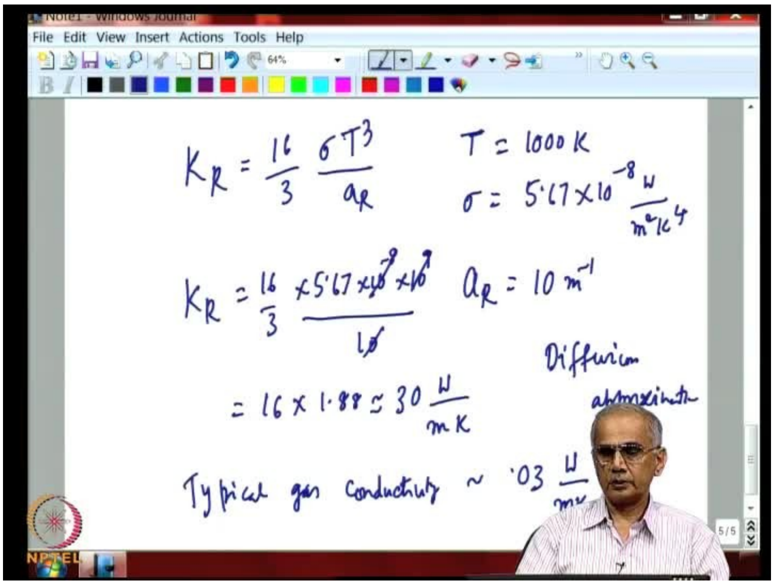

$$K_R = \frac{16}{3}\frac{\sigma T^3}{a_R} \qquad T = 1000\,K$$

$$\sigma = 5.67 \times 10^{-8}\ \frac{W}{m^2 K^4}$$

$$K_R = \frac{16}{3}\ \frac{\times 5.67 \times \cancel{10^{-8}} \times 10^{9}}{\cancel{10}} \qquad a_R = 10\ m^{-1}$$

$$= 16 \times 1.88 \simeq 30\ \frac{W}{mK}$$

Diffusion approximation

Typical gas conductivity ~ .03 $\frac{W}{mK}$

The conduction that you know already from a transfer background is also Watts per meter degree Kelvin. It is possible for us to estimate the radiative conductivity of a gas given the Rosseland mean absorption coefficient. Now, most of us should be aware that for a typical gas the conductivity, conduction heat transfer, conductivity is pretty low. We take air it will be around 0.025 Watts per meter square, Watts per meter degree Kelvin.

Let us see what kind of numbers you can get for the radiative conductivity. If we define radiative conductivity as 16 by 3 sigma T cube by a R. Now, let us take a simple example. Let us say gas is at furnace situation at 1000 degrees Kelvin. Let us say we know that from our black body formula this is universal constant. We just have to estimate a R. Let me take just for as an example, let a R be 10 meter minus 1, that the photon mean free path is about 0.1 meter. In this case let us calculate K R. K R will come out around 16 by 3 into 5.67.

Then 10 to the power minus 8 and this will be into 10 to the power of 9 by 10. So, all this will cancel out. This will be approximately around 30 Watt per meter Kelvin. We must recognize that in a gas this is fairly high heat transfer rate because typical gas conductivity not radiative, but molecular conduction is of the order of 0.03. What it shows is that if there is a gas or high temperature like 1000 degrees Kelvin and the Rosseland mean absorption coefficient is of the order of 10 meter minus 1 then the heat transfer by radiation is about 1000 times more effective than molecular conduction.

Clearly at high temperature we need to worry about the radiative heat transfer in this limit. Now, suppose instead of 1000 degrees Kelvin we assumed 300 degrees Kelvin that is one-third approximately. One-third cube will be around 27. This will come to around 1 Watt per meter degree Kelvin still it will be much larger than gas conductivity. What must be remembered is that if you are in the thick limit then the heat transfer by radiation its most often much stronger than heat transfer by molecular conduction and must be accounted for in any given situation.

We have to be sure that we are in the optically thick limit to apply this equation. This limit is also called diffusion approximation because in the thick limit radiation heat transfer behaves like a diffusion process. Photon is being absorbed at a very short distance and it is similar to what is happening in molecular conduction where the molecules undergo collisions very often so, diffusion model is valid.

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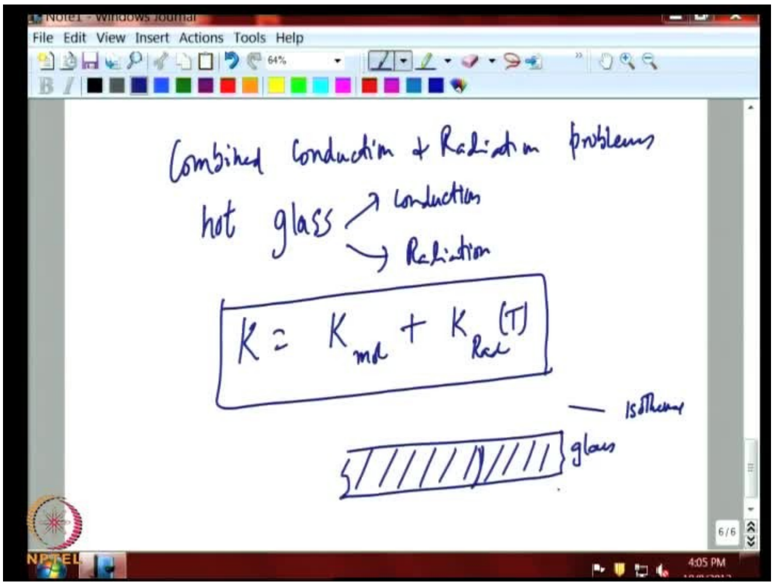

Combined Conduction + Radiation problems

hot glass → Conduction, → Radiation

$$\boxed{K = K_{md} + K_{Rad}(T)}$$

There are examples we will see in later lectures where this diffusion model can be used and you can imagine that especially in situations where you are looking at combined conduction radiation problems. It is convenient if we can treat both radiation and conduction as a diffusion process. Imagine heat transfer in glass. A hot glass in glass processing, then you have both conduction and radiation. Let us assume that the photon mean free path is quite short, so they can apply the thick limit.

Then the total conductivity will be just sum of molecular conductivity plus radiative conductivity. This we are talking about temperature very much and so in the manufacture of glass you need to know suppose this is glass being molded or processed. In order to ensure that during the process of molding glass sheets for example in making the television.

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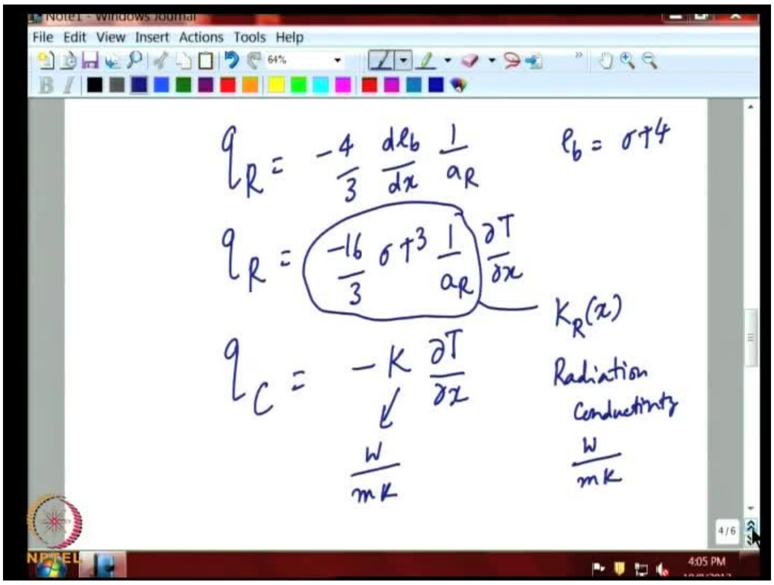

$$q_R = -\frac{4}{3}\frac{de_b}{dx}\frac{1}{a_R} \qquad e_b = \sigma T^4$$

$$q_R = \boxed{-\frac{16}{3}\sigma T^3\frac{1}{a_R}}\frac{\partial T}{\partial x} \quad K_R(x)$$

Radiation Conductivity

$\frac{W}{mK}$

$$q_C = -K\frac{\partial T}{\partial x}$$

$\frac{W}{mK}$

The television, the front screen it is made of glass and we want that glass to be bold such that it is completely defect free. Then we have to ensure that there are no stresses in the glass during the process of freezing when the molten glass is frozen to get, give the given shape of the TV glass. It has to be stress free. To be stress free you have to keep the glass close to isothermal that is heat on the glass should be close to isothermal. We have to predict the temperature profile across this thickness and there you have to account both for molecular conduction and radiative conductivity. In this cases as you will have guessed this will dominate the heat transfer radiative conductivity.

So this model, the diffusion model using the Rosseland approximation is quiet common. We can see that this model of radiative heat transfer where radiation is treated like light conduction heat transfer, it is quite useful provided one can make the approximation that the photon mean path is very small compared to the length scale of interest. We are looking at the two limits of radiation heat transfer and we are able to do this analysis at the, at the spectral level and obtain the two limits, but in solving a full problem when we assume that we are not able to make the approximation of the thick or thin limit.

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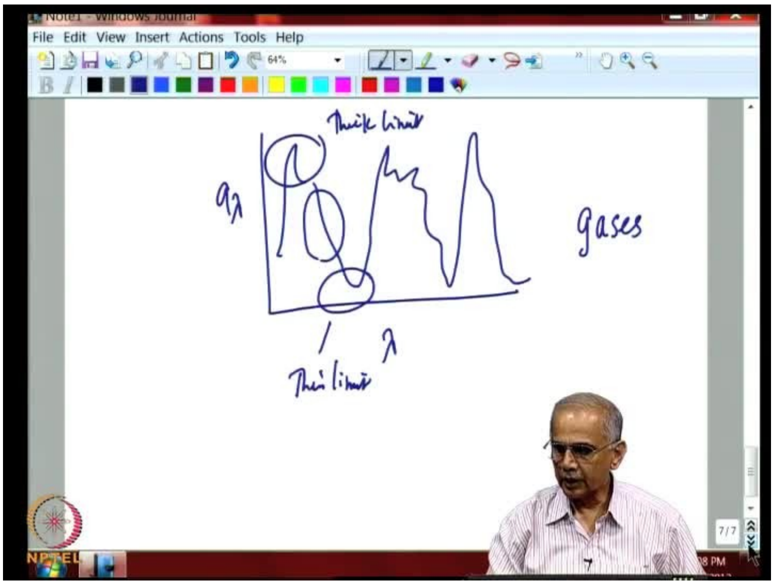

This is quiet common in radiation transfer problems. Because if we look at the absorption coefficient of any gas as a function of wavelength it has very complex behavior. So, where L lambda is low it may be the thin limit, where L lambda is large it may be thick limit. We cannot assume at all wavelengths the gas is in the thick or thin limit because there are regions where the gas remain thick limit, there are regions with thin limit in between. It will not be either thick or thin limit.

The limiting case we just now saw is useful only to get some physical insight in to the process of radiative heat transfer, but really not very useful in practice except may be in the case of glass where the thick limit is reasonably valid. But in the case of gases where the absorption coefficient of the function of lambda has such a complex structure, we cannot use the thick or the thin limit. We have to deal with a full problem. So, our next task is to ask how to solve the problem in the general case when the thick or thin limit is valid what do we do. Now, before we go on to that we want to just understand how the basic problem we have looked at so far is modified when we have scattering.

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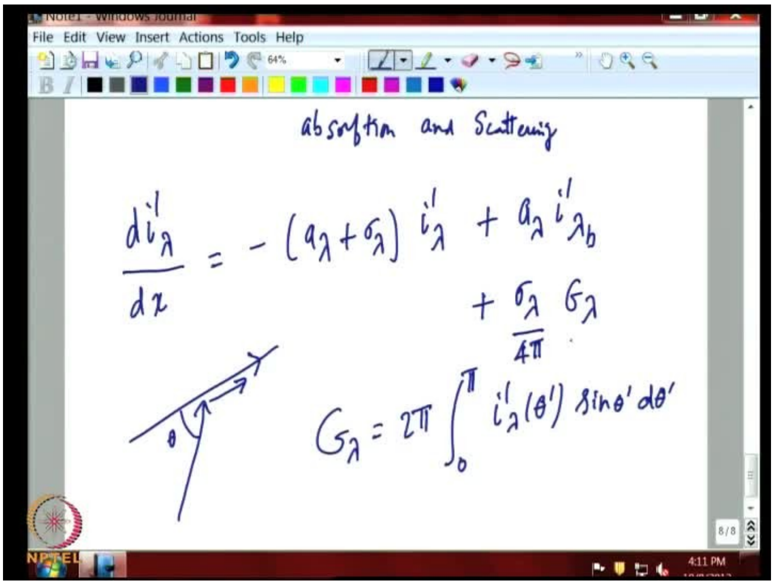

absorption and Scattering

$$\frac{di'_\lambda}{dx} = -(a_\lambda+\sigma_\lambda)\, i'_\lambda + a_\lambda\, i'_{\lambda b} + \frac{\sigma_\lambda}{4\pi} G_\lambda$$

$$G_\lambda = 2\pi \int_0^\pi i'_\lambda(\theta') \sin\theta'\, d\theta'$$

We will deal with scattering in little more detail in the later lectures, but right now it is good for you to know what is the difference in the basic equations. So, suppose we have both absorption and scattering. We will write down this equation for a plane parallel model. We have both absorption and your scattering coefficient. So, first part is the extinction that is the decrease in the intensity on account of both absorption and scattering.

Remember, in this case what is happing is along the path on account of either absorption, that is the absorb photon or photon scattering, we lose the photon in that direction, either photon absorbed or scattered into some other direction. That is the first term. The second term is gain in photon that is happening because of emission and finally one more term may come in which is photon which are scattered from other directions.

A photon coming in some other direction get scattered into this direction. It takes the intensity in all other directions and we assume, assume the azimuthal symmetry of our convenience. We assume to get 2 pi and then with respect to this angle theta, we have to integrate the intensity is all directions. The basic heat transfer which in the absolute scattering is the simple model we saw earlier gets a little more complicated in two terms.

In the extension term there is one more term added because the photon is lost due to scattering. Then the photon is gained due to scattering in of photons travelling in some other direction into this direction. We look at photons in all other direction and ask what fraction is scattered into this direction and that is calculated using this method. We should be able to see that this complicates the problem enormously because when this term was not there the directional spectral intensity was only related to itself and the direction of spectral intensity of a black body at the gas temperature.

When we introduce scattering this last term here links the intensity in all other direction to this direction. That means you have to solve the problem of intensity in all directions simultaneously because intensity in some other direction theta prime is linked to i prime lambda. This is what makes scattering such a complex problem and so we will not be able to spend a lot of time in this course to look at this term, but we will touch upon it briefly, so that you have some idea about how come this scattering are tackled.

But at this point we want to develop appreciation that if we deduce scattering into radiation heat transfer problem, the problem because one magnitude more complicated because we cannot solve for the intensity in one direction in terms of just these three terms. But we introduce the term which connects the intensity in one direction to the intensity in all other directions. We will get back to this problem later, but let us now look at much simpler cases. We looked at a thin and thick limit and we pointed out that it is not always a possible to look at these limits for gases where the absorption coefficient is strong function of wavelength.

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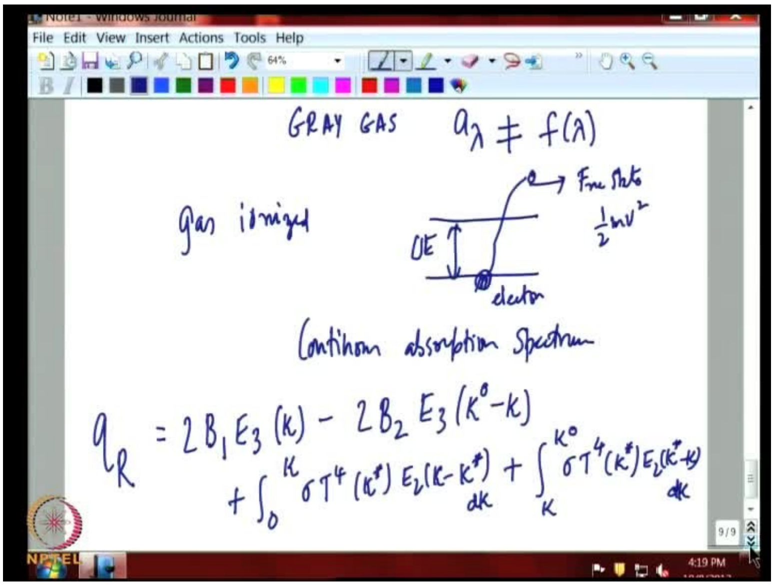

GRAY GAS $a_\lambda \neq f(\lambda)$

gas ionized

Continuum absorption Spectrum

$$q_R = 2B_1 E_3(\kappa) - 2B_2 E_3(\kappa^0 - \kappa) + \int_0^{\kappa} \sigma T^4(\kappa^*) E_2(\kappa - \kappa^*)\, d\kappa + \int_{\kappa}^{\kappa^0} \sigma T^4(\kappa^*) E_2(\kappa^* - \kappa)\, d\kappa$$

We will look at a very simple case. It is called gray gas. Gray gas the definition one where the absorption coefficient is not a function of wavelength. This rarely occurs in the real word, most gases have a strong dependence of the absorption coefficient on the wavelength, but we are going to use the gray gas merely as a teaching tool to tell you how the general equation for radiative heat transfer is solved. This is more a way to explain to you the technical solution.

If the real problem involves a non gray gas then the entire thing has to be solved on a computer and we have nothing to really show in the class room very easily how the problem is actually solved, but an appreciation of how the equation is solved in your computer program is useful for the purpose of understanding the solution provided by the computer. We will go through the gray gas model although we are fully aware that gray gas is an approximation which is not normally applicable in real situation. The only time gray gas model may work is when the gas is ionized.

The gas is ionized then it can absorb a photon and take the gas from the bound state and comes out and gas gets ionized and the electron with those originally bound, it reaches a free state. Such a electron can absorb any frequency because the kinetic energy of the electron can have any value between 0 and Infinity. When the photons are energetic enough to knock an electron out of the gas and it goes freely then such a gas can absorb radiation at all frequencies above a certain frequency.

This is delta E and it is continuous absorption spectrum. So, as you can realize in the case of astrophysics where they deal with stars which are at high temperatures. Most of the gases are ionized. In astrophysics the gray gas approximation is very useful, but in furnaces or in atmospheric science radiation gray gas is not a very good approximation, but still we are going to spend some time showing how equations are solved for gray gas, really as way to teach you how these problems are tackled.

When the gray gas is not valid we need to extend the methodology to allow for the complex function of a lambda but that is merely a more computation and that can easily be done today on the computer. So, our aim is that you understand a basic method of solution using gray gas as a as a useful a example, but it is not a practical example. In the case of gray gas the substitute lambda goes away, everything is substituted by lambda. We can write down the basic radiative transfer equation as follows.

The first term is the radiosity and the attenuation through the gas, integrated to all, all angles, angle is a built in to this exponential integral function, then you have second term which is the distance between the top plate at the given level. Then you have the emission from the gas below the level of interest to you and finally, a gas above the layer of interest to you. Of course you need d kappa. This is an equation with all the lambda subscript removed is the basic equation.

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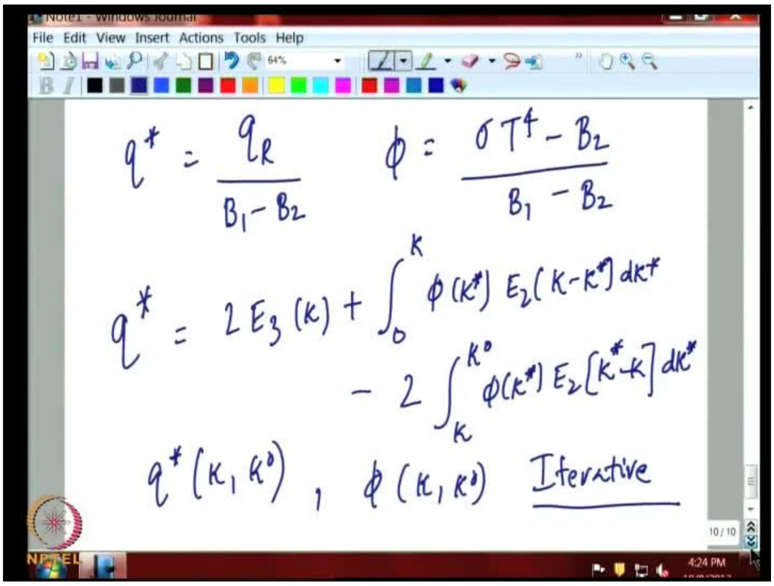

$$q^* = \frac{q_R}{B_1 - B_2} \qquad \phi = \frac{\sigma T^4 - B_2}{B_1 - B_2}$$

$$q^* = 2E_3(\kappa) + \int_0^{\kappa} \phi(\kappa^*)\, E_2(\kappa - \kappa^*)\, d\kappa^* - 2\int_{\kappa}^{\kappa^0} \phi(\kappa^*)\, E_2[\kappa^* - \kappa]\, d\kappa^*$$

$q^*(\kappa, \kappa^0)$, $\phi(\kappa, \kappa^0)$ Iterative

Now, before we solve the equation we can see, want to solve this equation to solve for some value B 1 and B 2 and will be nice if we can remove this B 1 and B 2 from this equation by a suitable scaling. We will define a q star, a non dimensional cube as q R by B 1 minus B 2. Throughout that in the absence of a gas q R equals B 1 and B 2 this is equal to 1. This is good thing to remember that q star is so defined that in the absence of gas it is equal to 1 to define a non dimensional temperature as sigma of the gas or the gas minus radii of surface 2.

These are convenient definitions, it is done such that the final equation, the non dimensional one now is nothing but q star equals 2 E 3 kappa plus 0 to kappa phi of kappa star E 2 of kappa minus kappa star d kappa star minus 2 kappa to kappa 0 phi of kappa star E 2 of kappa star minus kappa d kappa star. We notice that this equation is much cleaner has no parameters; B 1 and B 2 have been absorbed in the definition of q star. This equation is function only of kappa and kappa 0.

Our aim in a given situation, given all the boundary conditions and other constraints of the problem is to find how q star varies with kappa and kappa 0 and of course, how does temperature vary with kappa and kappa 0. There will be one parameter kappa 0 independent variable kappa which is the location space and non dimensional temperature and non dimensional flux. We have brought down the problem into very simple framework in terms of number of parameters.

Now, today this equation can be solved on the computer, there is no difficulty. We have to do is unknown quantity is here and for a given radiative flux, let us say a specified radiative flux, we can find the temperature distribution by assuming some function here, integrating it to see if it is equal to this value, if it is not true then you adjust this continuously. In iterative fashion one can solve this equation fairly rapidly on the computer. So, all you require is some approximation, may be some polynomial approximation per phi of kappa star integrated phi equal to q star which is specified. Then you go back. On the other hand if in a given problem the temperature distribution is given to you.

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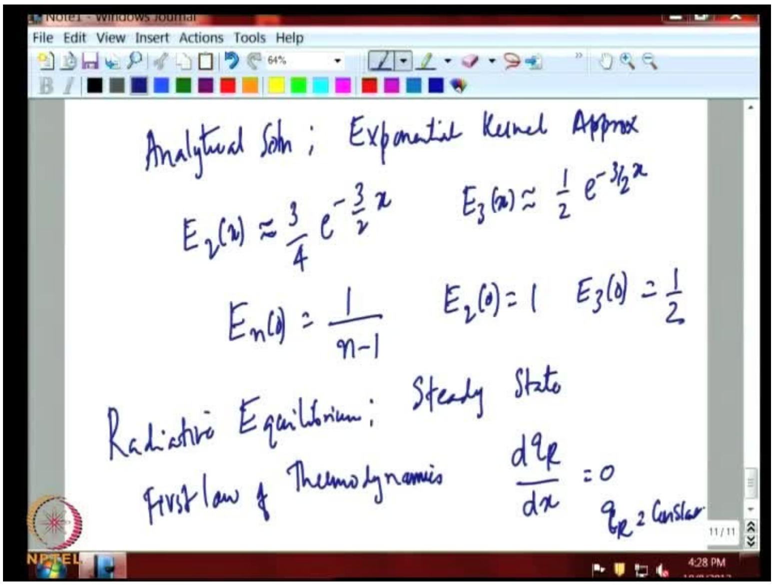

Analytical Soln ; Exponential Kernel Approx

$E_2(x) \approx \frac{3}{4} e^{-\frac{3}{2}x}$ $E_3(x) \simeq \frac{1}{2} e^{-3/2 x}$

$E_n(0) = \frac{1}{n-1}$ $E_2(0) = 1$ $E_3(0) = \frac{1}{2}$

Radiative Equilibrium ; Steady State

First law of Thermodynamics $\frac{dq_R}{dx} = 0$

$q_R =$ Constant

Then you know this quantity that is very easy to integrate this to get the flux. The basic problem here is either the flux is given and you calculate temperature distribution or the temperature distribution is given and you calculate flux. We can calculate both. One has to be given; the other has to be found. The more difficult problem is given the heat flux, radiative heat flux, how to calculate the temperature distribution. That is integral equation whether the unknown is inside the integral.

The only method available is iterative method. We guess on temperature, integrate, find the flux, the flux does not match, go back and adjust your assumed temperature distribution. Now, our purpose here is not to discuss the numerical solution which can be done these days quite easily. To get an analytic solution you need to make some approximations, there is no choice. We will adopt what is known as exponential kernel approximation.

In this approximation you are simplifying the angular integration part of the radiation equation and by making the angular integration simpler we are able to solve the problem analytically and the particular kernel approximation you are going to adopt is that E 2 of x is approximately equal to three by four e to the power of minus 3 by 2 x. And E 3 of x which is the other one which comes in the equation is approximately equal to half. If we look at this approximation, actually we know E 2 0 should be 1 because in general, if we recall E n of 0 was 1 by n minus 1, so that means E 2 of 0 is 1 and E 3 of 0 is half.

In this approximation we have ensured that the kernel approximation is exact at 0 for E 3, but approximate for E 2. E 2 should have been 1 E 2 0, but it is three-fourth. If we look at the equation for q star E 2 is appearing inside the integral, E 3 is appearing outside. In case of E 3 it is important that we correctly get behavior at x equal 0. Because of E 2 although one may not get the behavior of x equal to 0 right, as long as the integral of this quantity is roughly right then it is okay because this appears only inside the integral.

That is speed approximation. There are other approximation available, you can look it up in various text books, but all them gives similar answers, answers are not very different from the kind of approximation that others make. Now, before we proceed further we must decide what problem we have to solve. We will now look at the problem of radiative equilibrium. Recitative equilibrium is a case wherein you are dealing with a gas in which radiation is the only process we are neglecting conduction, convection on any other heat transfer process that may be occurring.

We think the radiation is the dominant process and so we will only look at radiation as the only as the mode of heat transfer and then we are assuming steady state. No change with time. If you assume these two that there are no other processes occurring in the gas except radiation and the gas has a steady state, then from the first law of thermodynamics we conclude that in steady state with radiation as the only way of transfer d q R d x has to be 0.

In steady state the divergences or a flux is 0 and the only flux of relevance we are saying is radiative flux. That means q R has to be a constant.

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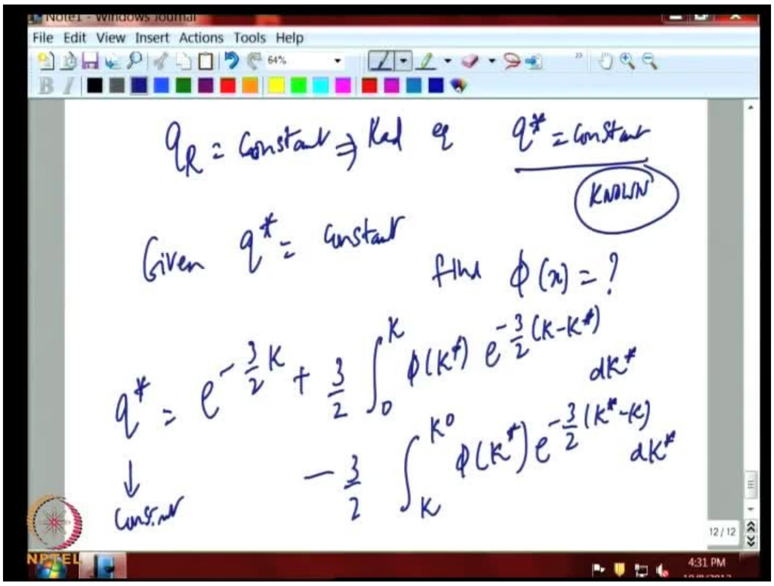

q_R = Constant $\Rightarrow$ Rad eq $\quad$ q^* = Constant

KNOWN

Given q^* = Constant $\quad$ find $\phi(x) = ?$

$$q^* = e^{-\frac{3}{2}\kappa} + \frac{3}{2}\int_0^{\kappa} \phi(\kappa^*)\, e^{-\frac{3}{2}(\kappa-\kappa^*)}\, d\kappa^* - \frac{3}{2}\int_{\kappa}^{\kappa^0} \phi(\kappa^*)\, e^{-\frac{3}{2}(\kappa^*-\kappa)}\, d\kappa^*$$

q^* $\downarrow$ Constant

This approximation we are going to deal with this limit. So, q R is the constant in radiative equilibrium In our case a non dimensional q star is the constant and we are treating this as a known. This is given to you a priori and given this q star we also calculate phi. For given q star is a constant to find the temperature distribution. This is the standard problem in pure radiation heat transfer in steady state, one dimension. There is no other mode of heat transfer.

Therefore, d q x is 0, so q is a constant. We made the kernel approximation which already, had been mentioned. So, with that we will get the equation which says q star e to the power minus 3 by 2 kappa plus 3 by 2 0 to kappa, unknown temperature distribution e to the minus 3 by 2 kappa minus kappa star d kappa star minus 3 by 2 kappa to kappa is 0 and 3 by 2 kappa star minus kappa d kappa star. In this equation q star is a constant, remember that unknown function is inside the integral.

Now, having made the kernel approximation that is replacing the complex kernel by an exponential function, we exploit the fact that exponential function is repeatable and if it is repeatable then you can easily differentiate this equation twice and all of you know that e to the power of minus x when differentiated twice will give you the same answer. So that is the feature we want exploit in this formulation and so we will do that. Let us now go and differentiate this equation once and then twice.

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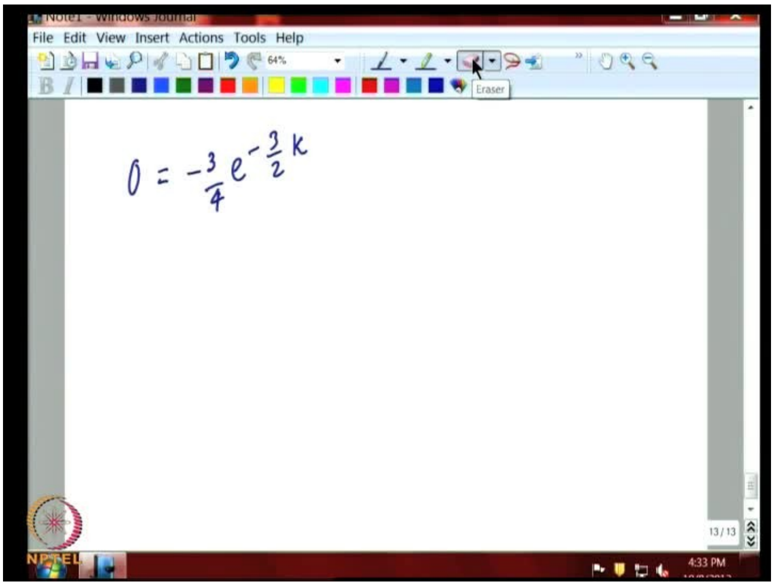

$$0 = -\frac{3}{4} e^{-\frac{3}{2}\kappa}$$

Remember that q star is a constant. So, d q star e kappa is 0. We write down the left hand side of equation is 0 because d q star d kappa is 0, then the second term, we have is 3 by 2 half e to minus 3 by 2 kappa will become minus three-fourth e to the power minus 3 by 2 kappa. If we differentiate this twice, and use the Leibnitz rule. Leibnitz rule is important and we have discussed that earlier in the context of shape factor calculation.

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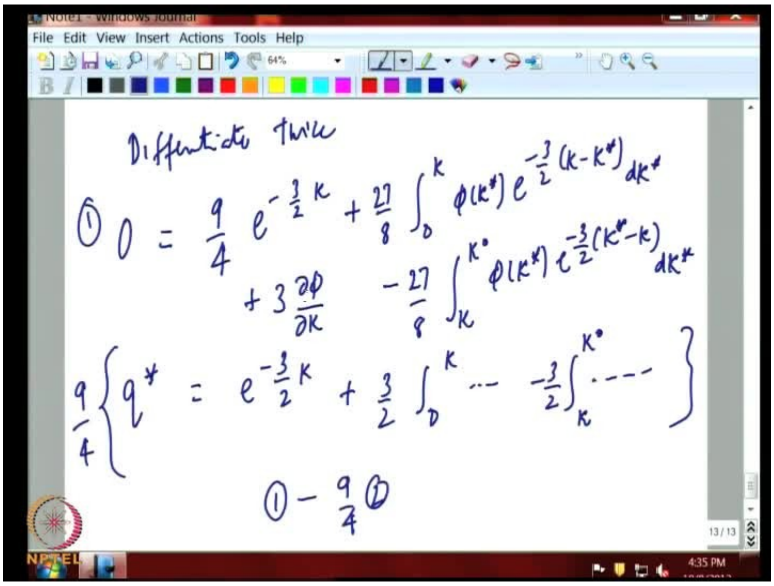

Diffentiate twice

$$① \; 0 = \frac{9}{4} e^{-\frac{3}{2}\kappa} + \frac{27}{8}\int_0^{\kappa} \phi(\kappa^*) e^{-\frac{3}{2}(\kappa-\kappa^*)} d\kappa^* + 3\frac{\partial\phi}{\partial\kappa} - \frac{27}{8}\int_{\kappa}^{\kappa_0} \phi(\kappa^*) e^{-\frac{3}{2}(\kappa^*-\kappa)} d\kappa^*$$

$$\frac{9}{4}\left\{ q^* = e^{-\frac{3}{2}\kappa} + \frac{3}{2}\int_0^{\kappa} \cdots - \frac{3}{2}\int_{\kappa}^{\kappa_0} \cdots \right\}$$

$$① - \frac{9}{4} ②$$

We differentiate twice, so that left hand side is anyway 0 because can q is also the constant and you will get after two division 9 by 4 e to power minus 3 by 2 kappa plus 3 by 2 0 to kappa phi of kappa star e to power minus 3 by 2 kappa minus kappa star d kappa star minus 3 by 2. Now, when we differentiate twice it will give you 27 by 8 minus 27 by 8 because 3 by 2 into 3 by 2 twice will be 9 by 4 kappa to kappa 0.

This has to be compared with the equation we wrote down earlier, which had q star. What we see is that in order eliminate these two integrals, between these two equations you have to multiply this equation by 9 by 4. If we multiply this equation by 9 by 4 and subtract that equation, so this is lets us call it as equation one and this is equation two. So, equation 1 minus 9 by 4 equation 2, we will eliminate all these integrals.

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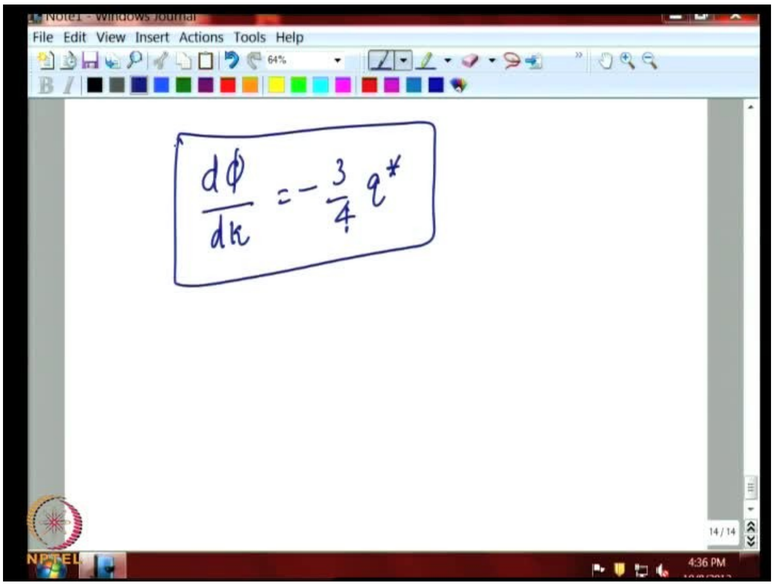

$$\boxed{\frac{d\phi}{dk} = -\frac{3}{4} q^*}$$

We take this equation subtract. These terms are all will cancel out, they will all have 27 by 8 and so you are left only with these three terms. Let us write down that equation, which finally is d phi by d kappa three by four q star. Finally, we go back and clarify what we have done. This term will all go away and we are allowed with this term and this term, and 1 3 will cancel out, and so we are left finally with this equation.

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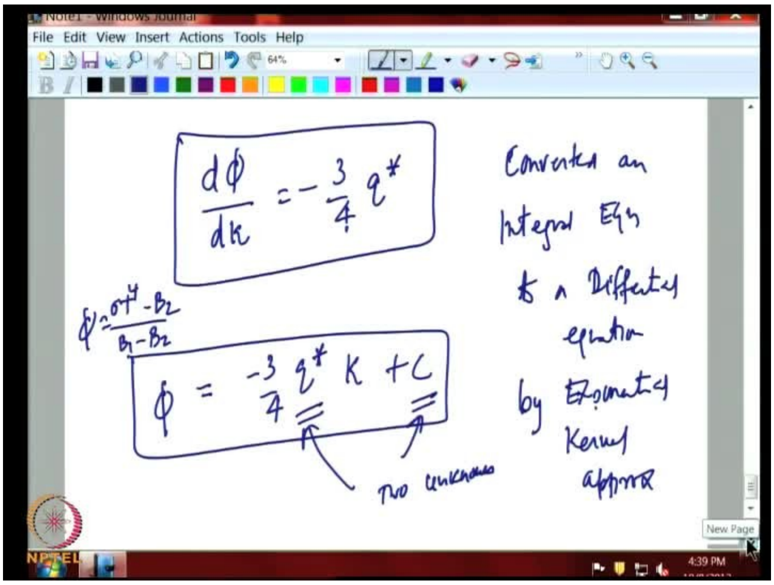

$$\boxed{\frac{d\phi}{d\kappa} = -\frac{3}{4}q^*}$$

$$\phi = \frac{\sigma T^4 - B_2}{B_1 - B_2}$$

$$\boxed{\phi = -\frac{3}{4}\underline{\underline{q^*}}\,\kappa + \underline{\underline{C}}}$$

Two unknowns

Converted an Integral Eqn to a Differential equation by Exponential Kernel approx

We have converted an integral equation to a differential equation by exponential kernel approximation. This approximation has been very useful to convert the original integral equation to a differential equation. This can be integrated very easily to get phi is equal to minus three-fourth q star kappa plus a constant. This non temperature profile is linear function of kappa, but remember that does not mean that temperature is a linear function of kappa that is because phi if you recall is defined as sigma T to the power 4 minus B 2 by B 1 minus B 2. So, phi depends as T to the power of 4, so it is T to the power of 4 which is linear in kappa not temperature.

We need to find the constants. To find two constant, of which, you can assume that this is provided. This is really unknown; this can be assumed as specified. We want to get the final answer for the heat transfer in terms of kappa. We plug these into the integral equation and say kappa equals 0; and when you do that, the constant will come out as shown below.

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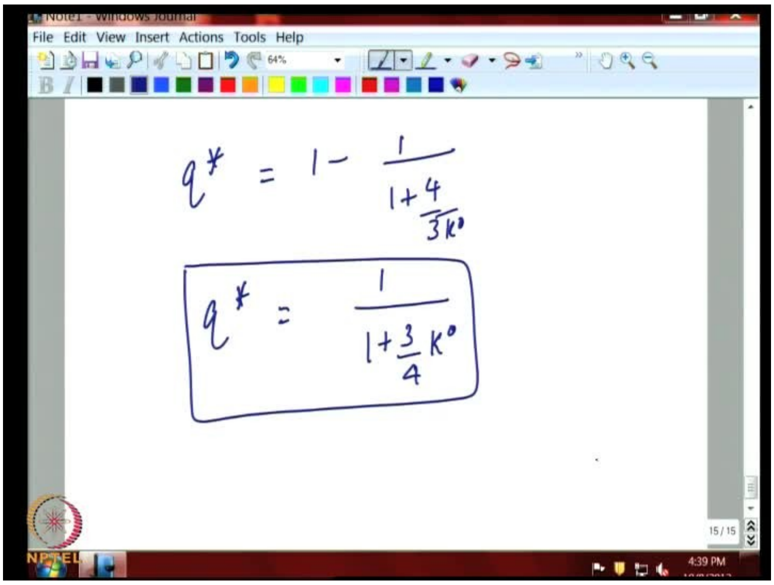

$$q^* = 1 - \frac{1}{1+\frac{4}{3\kappa^0}}$$

$$\boxed{q^* = \frac{1}{1+\frac{3}{4}\kappa^0}}$$

This is the solution; once we have found the constant, q star come out as 1 minus 1 by 1 plus four-third kappa 0, which can be simplified to q star is equal to 1 plus three-fourth kappa 0. This we will discuss in some detail in the next lecture, because this is very important result, which relates the heat flux to the optical depth of the medium; and we will discuss the meaning of this in the next lecture.