Radiation Heat Transfer
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Lecture - 15
Plane Parallel Model

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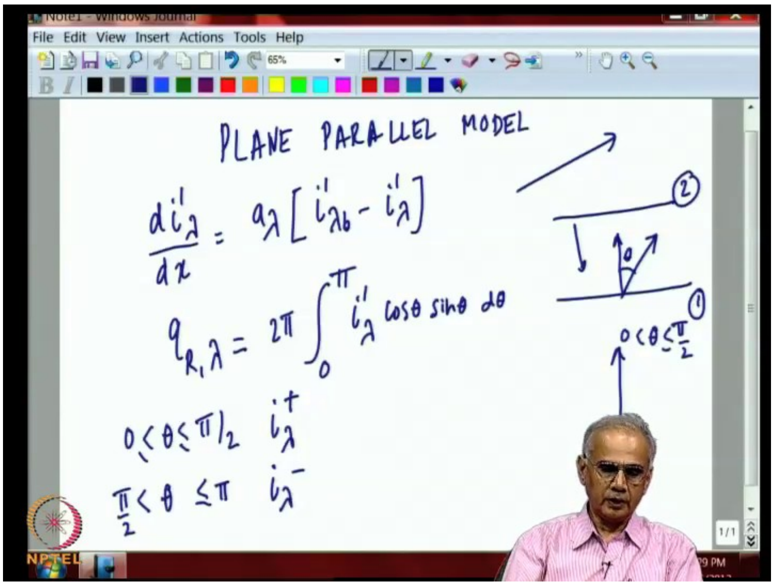

In the last lecture we looked at a Plain Parallel Model for radiation. This is essentially a one-dimensional model, but remember that in radiation even in one-dimensional model it has account of all the rays travelling in all the different directions. We recall that we started with the basic equation for radiative transfer, which was for non scattering medium which says that the change of intensity with distance is proportional to the emission by the gas and reduce by the absorption by the gas; second term absorption first term is emission. This is absorption coefficient which has units of beta minus 1. This can be integrated, so that along a given direction we get the intensity, but we are interested in fluxes not intensity.

We took a parallel plane parallel model, and we defined a normal and we looked at angle theta, we assumed everything is symmetry with respect to phi the azimuth angle. And from here we related the intensity to flux by the following relation. This equation has to be integrated and integration done in two parts. In this part upward going rays that is theta between 0 and pi by two or all rays which are moving upwards.

We solved this equation for theta between 0 and pi by 2 and that one we called as high lambda plus rays moving upwards and for the rays moving downwards from the top, that is theta greater than pi by 2 and up to pi, we call it i lambda minus. We have to distinguish between the rays going up and rays going down, because the lower surface 1 here will only emit rays going upwards and the upper surface 2 will only emit rays going downwards.

When integrating this equation for upward going rays we start from the bottom surface and go up, and when we look at the downward going rays we start from the top look at the emission of the upper surface.

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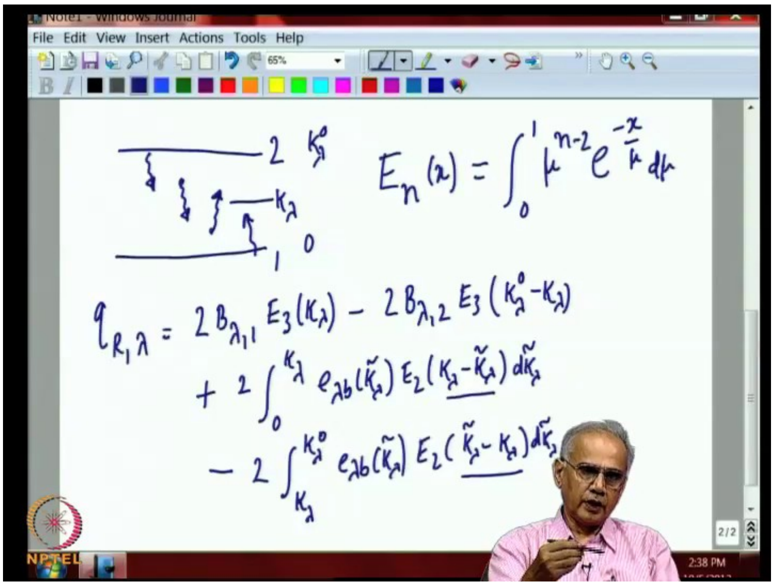

We saw that the four terms in the equation. Two terms involved emission by the surface and attenuation the gas and other two term involved emission by the gas upwards and downwards. We will write the flux expression. The flux expression had four terms. The q r lambda the net radiative flux between lambda and lambda plus d lambda consists of radiation leaving surface one and it is attenuated in all direction, minus radiation leaving surface two, minus because it is downwards, going rays and the difference between the total optical depth. This is 0 and this is kappa lambda 0 and any location here is kappa lambda. The distance traveled here for the second one is kappa lambda, 0 minus kappa lambda while for the rays going up; this is kappa lambda. This are the two terms which represents the boundary contribution, contribution from emission and deflection by the boundary which then moves through the gas.

Then there are two more terms involving gas emission first is from upward 1. It is equal to 0 to from here to here all the upward emission. It involves the black body emissive power of a gas, times a function which of science angular integration. The last term is again minus the downward emission by the gas between this point at the top and that involves the contribution from the surface 1.

This contribution from all gas elements between this surface and kappa lambda, this is where we calculated flux. This is the downward emission surface 2; this is downward emission by all gas elements between kappa lambda and kappa lambda 0. Now you should look at the argument here. We can see that we have written this, such that it is always positive. This is by convention that the argument here has to always be positive. In the region between 0 and kappa lambda, kappa lambda tilde has to be less than kappa lambda we write it this way. In the second term kappa lambda where is between kappa at kappa lambda 0 that for it has to be reversed, kappa lambda remain kappa lambda. This is a next question which we are going to use a lot.

In this expression, there are two terms called the exponential integral function and the general definition is E n affects is equal to 0 to 1, mu to the power of n, n minus 2 e to the power of minus x by mu d mu. This accounts for all the angular integration that one has already done and these tabulated functions available in books on heat radiation transfer usually in the appendix. We can look it up in any time and in today's world this can be computed, fairly easily by any software.

So, what we have done essentially is in problems related to heat transfer, there are three levels of integration involved. You have to integrate over a angle; you have to integrate over wave length and you have to integrate over space. We have essentially separated this three actions the angular integration is built into this function E n of x. Spatial integration is involved in this equation here and we need to do somewhat later the wavelength integration. The major challenge we face, in not the angular integration. The problem is straight forward and can be done by any computer very easily. A major challenge we face in solving the equation is a fact that, if you want to know the temperature distribution of a gas in an enclosure, then the unknown temperature variation is inside the integral.

This is a peculiarity in radiative heat transfer. In both conduction and convection heat transfer you encounter differential equations. There is a large arbitrary of techniques available to solve differential equations of many kinds. That whole methodology has been used very effectively, in solving large number of problems in conduction an convection heat transfer. In radiated heat transfer in gases the problem has been more difficult, because unknown temperature does not appear as of the part of differential equation, but is a unknown as a part of an integral, this is an integral equation.

We do not as have many techniques for solving integral equation analytically as we have for differential equations. So, most of solutions that will be normally done will be numerical. But in subsequent lectures will do a few solutions which are analytical, because analytical solution give you a great physical insight with regards to what is happening in a given problem. So, all though will be dealing with fairly simple and somewhat not realistic real world situation, ideal situations. Still we believe that the simple cases are very useful to help you understand the phenomena of radiative heat transfer. The key point to remember here is that because in this equation we have integral equation essentially what you see here in radiation is something very different from either conduction or convection.

Conduction and convection are governed by differential equations so that the temperature of a given region is influenced only by the temperature of the surrounding regions; immediate surrounding reasons. That is why we are able to write everything in terms of derivatives in differential equation. On the other hand in the case of radiation the temperature, in the middle of a gas in enclosure may depend on what is happening at the wall. Because some of the photons that is emitted at the wall can go right through the medium without being absorbed and directly interact with the region you are concerned with. This could be thought of as action at a distance.

The radiation problems are essentially are non local in character; that is, temperature of a given region is influenced not only by the temperature of surrounding regions but also influenced by temperatures of surfaces or gases far away from the region of interest. We will see that this equation which is more general kind, can be shown to ultimately become as a differential equation in certain limits. This formulation will be general and when it takes of limiting case limiting condition then we will show that this equation can be become like an equation in conduction heat transfer, but in the real world situations which we encounter in radiation, this is in the furnaces or in the atmosphere or in metallurgy we do not always have situation where we can convert this into a differential equation. We must know how to solve integral equations.

We will illustrate a few examples in the next few lectures so that you are familiar with the techniques of solving integral equations. But you must realize these are only as teaching tools to enable you to understand phenomena when you solve any real world problem involving radiation, we will solve it numerically. The entire power of high speed computing can be brought in to solve fairly complex radiation heat transfer problems. Those are solved by large software packages, but those will not give you any real insight into what is going on in this radiation phenomena. We need simple analytical solution; just so that you appreciate what is going on in this mode of heat transfer, in contrast to either conduction or convection. We want to compare the way in which radiation transfers heat. With that we already know what occurs in the case of conduction or convection.

Now, before we go further to solve this equation we not only need the flux but also need the divergence of flux. Because all of us recognize that the divergence of the flux is what is important in solving for temperature variation in a medium. But ultimately we are applying the first law of thermodynamics and in the first of thermodynamics on the right hand side you have the divergence of all heat fluxes. That includes conduction convection and radiation. So, ultimately we must know how does divergence of the radiative flux looks like and we derived expression, but differentiating this expression for the divergence of the radiative flux.

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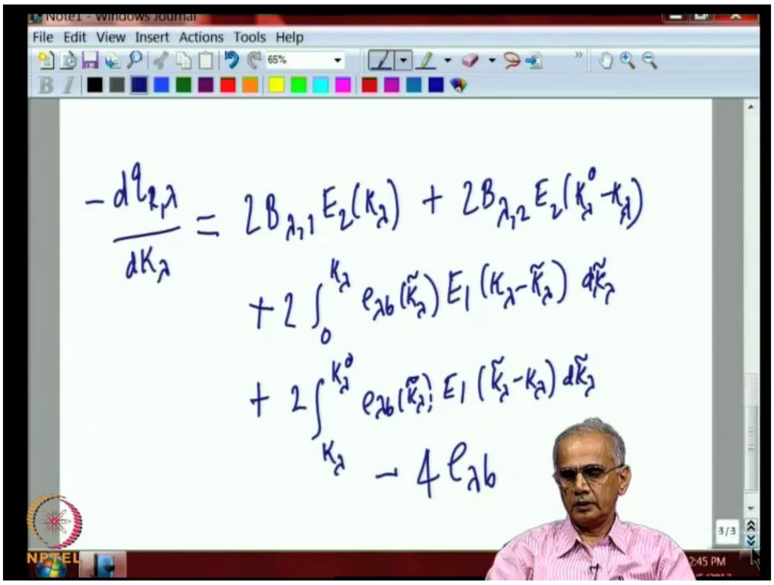

For convenience we write as minus. We write a minus because minus divergence represents the energy added to the system. So, each term are right hand side, that we want to write now, can be thought of as the term representing energy deposition or energy removal from this system. The first term is the radiation leaving surface one and absorbed by the gas. So this is heat addition to the gas, heat absorption.

The second term is energy leaving the top surface and reaching the level kappa lambda and being absorbed. So these two terms represent the gain by the medium on account of absorption of solar radiation. Then you have a term representing the gain from all other gas elements below kappa lambda. These are emissions by layers below the level kappa lambda which are absorbed at that level and similarly we have terms which represents all the emission by gas elements, above these layers radiating downwards and absorbed by the layer.

We must be able to do this from the previous equation by q r lambda, but differentiating by applying the Leibnitz rule for differentiation of an integral which was discussed in the last class. We have all do it and look at the first half representing the absorbed radiation on the top surface, second term absorbed radiation from the foreign surface, third term absorbed emitted by all gas space below this layer and the last term radiation absorbed by the layer of all the emission downwards so they will above kappa lambda. Finally, we have the emission term.

This is a important term this represents in four five directions, how the radiation is emitted in all directions by the gas. This is a expression for radiative heat flux. Now this is in the spectral domain. We want the total radiation.

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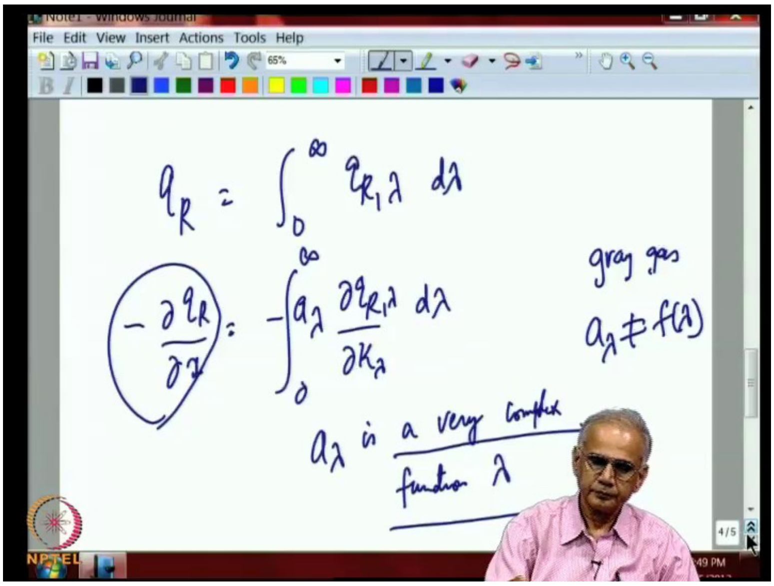

We know that the total radiation flux is 0 to infinity q r lambda d lambda and therefore, one can write as this quantity and declare it over all wavelength. So, ultimately the quantity of radiative flux is this term minus that represents the energies total energy added to the system by radiation.

Now, this is a step which is most difficult step in this whole procedure, because a lambda is a very complex function of lambda. This is what makes life very hard to integrate this, because the typical absorption band of a gas. That it is a millions of lines and you have to account for absorption at each line and the lack of absorption between lines. All those said, should be accounted for and this is no mean task and people have been at it for the last almost 100 years. This problem was first looked at by astronomers. In astronomy absorption coefficient is not a strong question of lambda. In astronomy they are willing to adopt a gray gas model that is assuming a lambda is not a function of lambda. This does remove one complication that is the integral of wave length, but still have to deal with angular integration and spatial integration

We go back to the divergence of radiative flux and recognize the 4 absorption terms the absorption from the bottom surface, emission of the top surface, absorption emission by all gas that remains below a level kappa lambda and absorption of radiation emitted downwards by all gas elements towards kappa lambda and minus is for e lambda emission.

Now, this equation is quite complicated and they primary complicate that arises that highly pointed out we want to, repeat that so that it is understood. The main problem is unknown is inside the equation and then makes it very hard. Now before we set out to give example the how to solve this equation, we want to look at two limiting cases.

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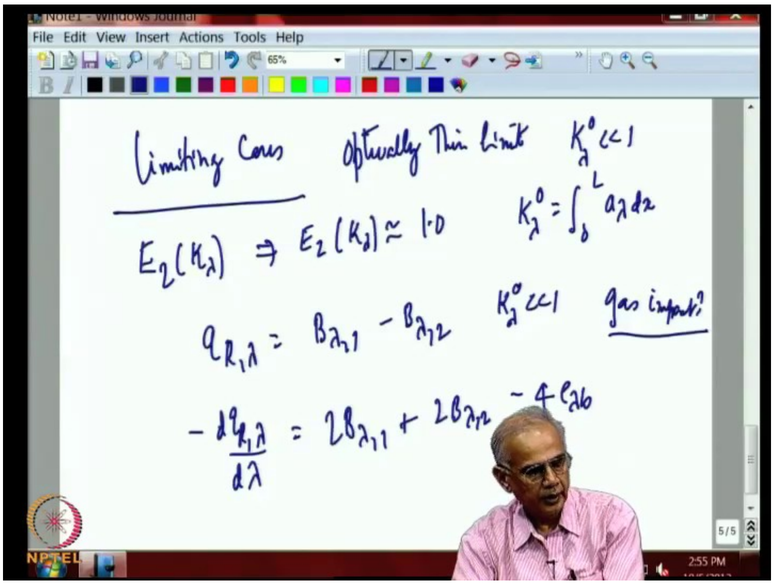

Limiting cases are always good to look at because they provide some insight regarding the behavior of the system. Because if we understand the behavior of the system in certain limits we better appreciate what the system does when not in the limit. The first thing we look at what is known as optically thin limit that is the total optical depth. Now, this does occur in real world because we take the atmosphere in which below. It most contains Nitrogen, Oxygen and Argon. All these three gases hardly absorbed any radiation. All the radiations absorbed in the atmosphere by minor gases like water paper Methane and Carbon dioxide. These are in such small amounts on Earth's atmosphere that we can treat them to be in the optically thin domain. There are examples in real world where the medium is an optically thin domain and we can almost avoid the integration in this case.

Now we take the three limits if the following conditions are satisfying. One can show that for small value is of kappa lambda which is going to happen because this total value is less than 1, now that will imply that E 2 will be approximately 1. This useful approximation in the thin limit. Essentially it is saying that there is no absorption by the gas. Gas absorption is so weak and the dimensional of the enclosure are so small, because you remember kappa lambda 0 by definition is 0 to l A lambda d l.

Therefore, kappa lambda 0 can be 0 l is very small a l lambda is very small. In this situation, in most places in the enclosure we can assume a 2 is equal to 1 and if you do all that we can show that q r lambda becomes minus.

In the thin limit, both the expression for radiative flux and the expression for the divergence of radiative flux are very simple. In the case radiative flux you see no impact of the gas. The fluxes are being exchanged between surfaces a l lambda 2 without measurable deviation. The second expression shows that, even after we take the thick limit that there is some absorption. This expression is the expression for absorption by the gas of the radiation leaving surface 1. The second one is absorption of the gas of radiation leaving surface 2. What it shows in the both cases little absorption there is in the gas, that the intensity of radiation that comes to the layer kappa lambda is not affected by the passage to the medium, because medium is very weekly observing.

We have the absorption term and the emission term. Now, optical thin limit is not very common which is understandable, because we pointed earlier, one of the features of the absorption coefficient is that it varies very strongly as function of wave length. We have one wave length it will very strongly absorbing and just the adjacent wave length we go to it is absorbed top.

In dealing with these minor gases, that are there both in furnaces and in atmosphere, we have to recognize the fact that thin limit is rather rare. Still it is a very useful approximation because you all dealing with essentially algebraic equation. We do not have either analytical equation. This is something which we can solve very easily.

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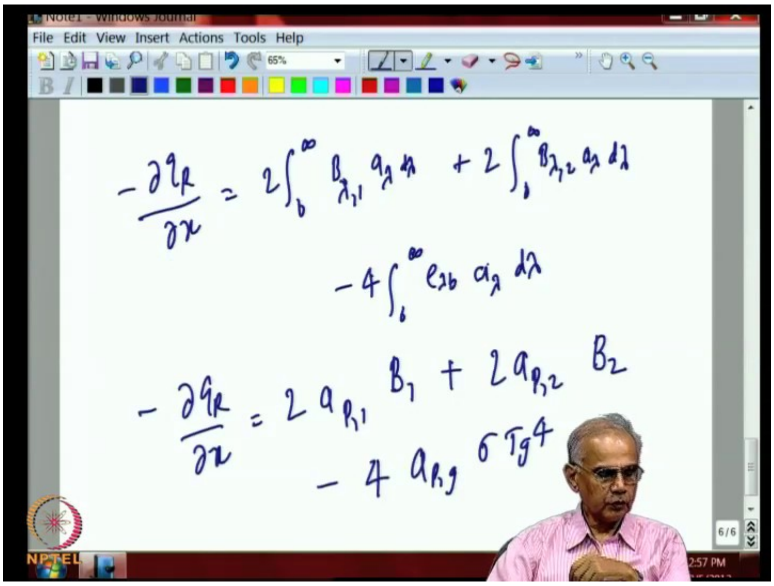

Now to illustrate how this can be solved we take the thin limit and ask ourselves what is d q R by d x. We have to integrate the previous expression over lambda to get this. This expression becomes fairly simple. We are integrating over wavelength. This is the first term. This is the second term and this is the third term. What we have done here is integration of wave length and ultimately this will be performed on the computer.

Suppose let us assume that, we had access to computer and it did all this calculations for gas typical of fondness containing some carbon dioxide, some water paper and knowing the property has affected all this. Even if we do that it will be useful to write this equation in a more understandable form. This simple expression shows there are three terms called the plank mean absorption coefficient.

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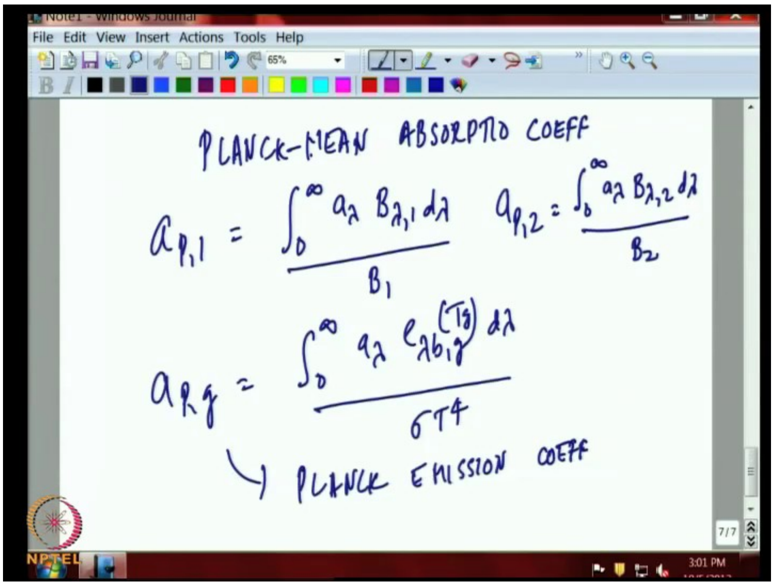

The first expression is for a p 1. Similarly a p two. So, each of this functions is weighted by the wave length variation of the radiosity or the corresponding surface and finally, for the gas. The purpose of defining these three absorption coefficient one for radiative surface 1, one for the radiative surface two and the third for the gas is that in some situations, these can be calculated a prior before we solve the problem. So, once you have tabulated the values of a p 1 a p 2 and a p g, as function of temperature we can solve most problem that one encounters in this optical thin limit.

Now, this limit is not that useful as pointed out, because there are very few gases which satisfy the condition that they are optically thin at all wave lengths of interest. That is very rare. There will be situations and the certain wave length, certain gases will absorb radiation strongly. We may be in position to define the plank mean coefficient and use it. We have mention one more thing, last quantity a p g is a plank absorption co efficient weighted by the black body function at gas temperature, which should be really be called plank emission coefficient.

This is because this coefficient is weighted by the black body function at the gas temperature, so it has all the required condition satisfied we call it an emission coefficient. But historically this has been called absorption coefficient. These two are absorption coefficient, because they represent the integration over wave length of the incoming radiation at one, there are two, they could be very different wave lengths. We need to run separately, but the plank absorption or emission coefficient, the last term, that depends both on the specific gas and it is a lambda is absorption as well as the weighting function brought in by in this case as a lambda b.

The thin limit is applied still for certain applications, because this expression is a differential equation. This expression is plugged into the general energy equation and there will be other terms and this will be going along with them. So, some of those cases, where people solved this equation, they will be extremely happy that the expression for the divergence radiative flux is fairly simple. If we know the temperature, a problem becomes even simpler, but normally temperature is an unknown, it needs evaluated.

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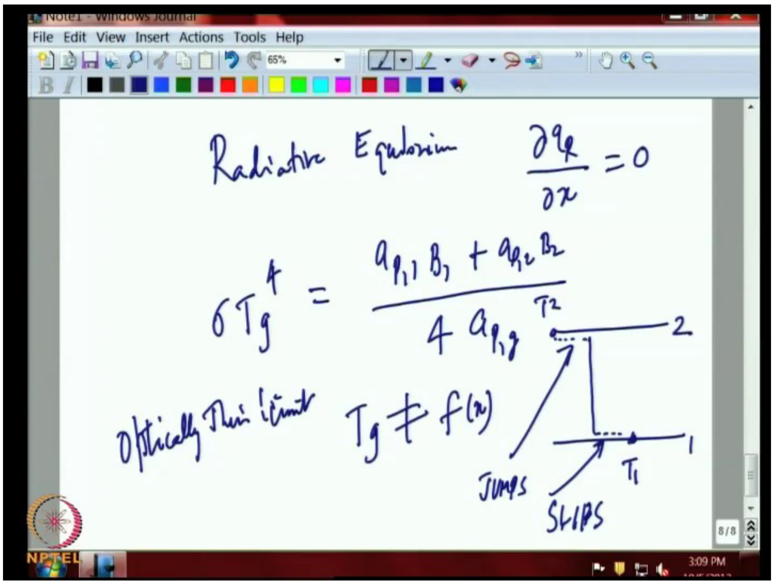

We can ask a simple example to illustrate this. Suppose the system is in radiative equilibrium. By this we mean system for which that divergence of the radiative flux is equal to 0. This is a useful starting point, because if you look at the energy equation assume steady state and neglect all other phenomena, like conduction convection other phenomena and focus on the radiation. Then you do expect that in steady state, that is from the energy equation, in a steady state everything will be in equilibrium. That is the case, than one can see from first row that the divergence of radiative flux has to go to 0.

For the case of the upper thin limit, this implies that the temperature of the gas then enclosure with, in which you invoke the optically thin limit, will come out as, a very simple expression for the temperature of the gas, because the equations are algebraic. All we have to do is from tabular information available in most text book in radiation. We have to calculate a plank mean absorption coefficient with a limit surface and surface two, multiply them by corresponding radiosity values, calculate numerator and dividing the denominator by the accepted monthly mean radiation value. In principle in thin medium, we can estimate temperature as function x y and z. If we have information available on the right hand side and if not, we should be doing other kind of computation which are much more demanding.

Now, the next step is after having defined the thin limit. We notice that in the thin limit, the equation for radiant heat transfer becomes algebraic equation. It is easy to solve the equation either analytical or numerically. One would, like to invoke the thin approximation so that instead of dealing with the full integral equation we deal with a simple equation and solve algebraically. Now, if you look at radiative equilibrium condition we can get the temperature.

So, computing this is fairly straight forward and values of the plank thin absorption coefficient are tabulated, this has to be four a p g. Once we have an estimate of a p 1 a p 2 and a p g, then based on the information of radiosity of surface one and surface two, you get T temperature in the gas.

We will notice that in this limit T g is not a function of x. And this is the peculiarity of the thin limit and one can easily show that is, this surface one and this surface two and surface two has that one temperature let's say T 2 and surface one other temperature T 1. We will find the gas will have only one temperature and there will be two jumps. These jumps are called slips for it says is that in the optical thin limit the gas has a single temperature invariant with distance x, but at both of boundaries there is the change the temperature that is, a temperature of the gas at the boundary is not same as the wall temperature. There is a slip.

This is the very special future of radiation heat transfer, which is not encounter that often in the case of conduction and convection heat transfer. In both of which it is normal practice, to assume that the temperature of the gas next to the wall, is same as the wall temperature. But this approximation is not valid. For now in this thin limit the problem becomes there are trivial, because the temperature not varying in the gas is uniform and there are two jumps which can be evaluated.

There are not many situations in the engineering practice where you might encounter this kind of condition. This may be partly true in small metallurgical furnaces, where due to furnaces that are small the ability to absorb radiation from the walls in somewhat limited. But what we notice that, from the radiative equilibrium conditions we are able to get temperature on the gas. But when we calculate fluxes, this is what we did, first in the case of fluxes; there is no influence of radiation. So as for as radiative fluxes in this enclosure is concerned, they are unaffected by the presence of the absorbing gas, but when it comes to divergence of the flux they are promptly there and they have to be accounted for.

Now, this might puzzle some people, as to how in the flux calculation we can neglect the role played by the gas, but in the divergence of the flux calculation gas properties come into play in a prominent way. We must remember that ultimately whether you really taken account the variation or not, depends upon the importance of variation which are ways, other modes of energy transfer; that is conduction and convection.

When we took the thin limit and look at the derivative, those may be in small if the gas is probably thin, but we must compare this radiative flux, with that due to conduction and convection. Only then we cannot say these quantity of small and is we can neglected, we have to compare this quantities however small with the competing quantities coming from condition heat transfer and convention heat transfer. Only then we can be absolutely sure that these terms are really small.

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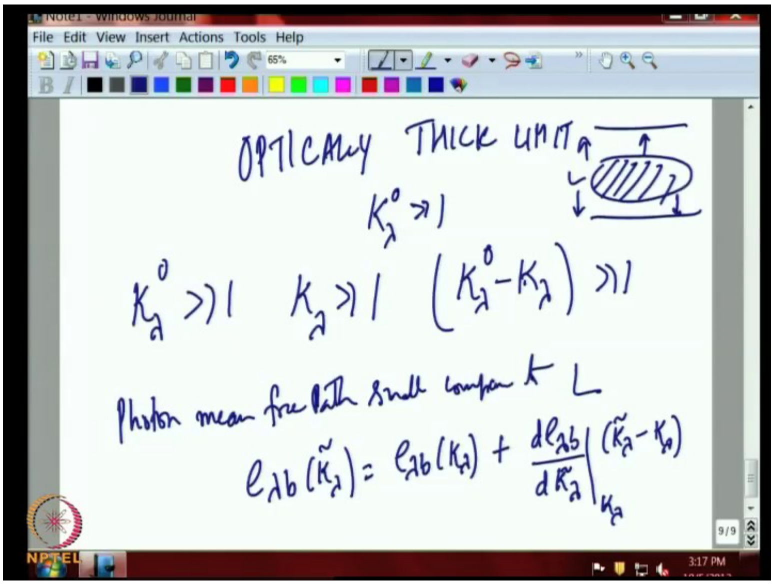

Now, the next limit we consider is the optically thick limit. The major feature of this limit is this is that the total path length is much greater than one. This is the exactly opposite of the thin limit. The thin limit quantity is very small, so we could make very simple approximation, but now we are dealing with a case where this quantity is very large. Now, in this limit if you want to look at the radiative transfer equation you do lot of work, because in this limit the expression we get is derivable. But not in a way that you have done for thin limit, where the form can very trivial, because you are able to essentially neglect gas absorption.

Not only is the total optical depth much greater than 1. We also invoke this fact that we also will look at only those point which are angel high optical depth, which translates to the factor our kappa lambda we do not look at, that defense in the wall are sufficiently for if from the wall that we can invoke the thick limit. So, when we get a thick limit will get a simple equation, but remember that somehow you have get the information about the value of kappa lambda, without that we can proceed further.

Now, once you assume this it automatically it follows that this is also true. We have kappa lambda 0 much greater than 1, kappa lambda much greater than 1 and also kappa lambda 0 minus kappa lambda also will be much greater than 1. Essentially what is saying, in that in the two parallel plates we are confined to the interior and not going to near this wall or this wall calls again go near a wall. Then we are going to violate the condition of optical thick limit. Once we have done that in the optical thick limit remember the photon mean free path is very small, L that is minimum plates. So, photons are getting observing quickly or a very short distance compare to the distance between the plates.

In that limit the gases in the interior will only see adjacent layers from there the emissions are occurring. They will not see the two walls, because they are very far and the radiation between wall is absorbed right next to the wall so the elements which are in the interior do not normally see the two walls.

That be in the case we can expand the black body function, that we encounter in the integral equation it is most difficult one, in terms of local value and a derivative. This expand want to do of the emission by a gas element kappa lambda delta, in terms of the emission at kappa lambda and the derivative of this quantity revaluated at kappa lambda.

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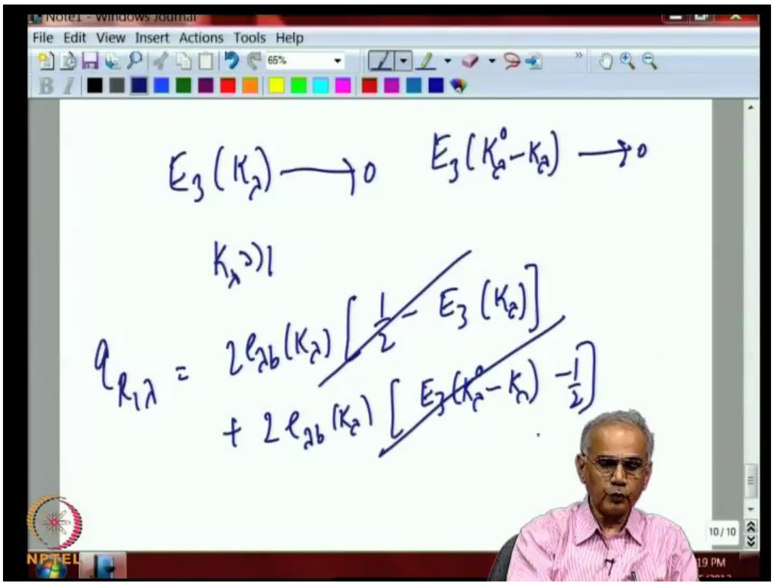

This information is plugged into the equation. In addition because kappa lambda 0 is very large, we can say that E 3 3 of kappa lambda for kappa lambda 0 is approximately equal to half. That is E three 0 half we know that. We are assume it is approximately going to half. Because it is very large, kappa lambda 0 is very large, this quantity tends to 0. It also tend to 0. So that means, you are for away at the two walls we the radiation of the two walls are acknowledged so much that you do not see it. And the approximations mentioned in the previous slide will lead to the following expression by q r lambda.

Now, we notice that many of this, will go to 0 because that is what assumed and this two will adopt. this will two also get cancelled out; half and minus half. This two expression will not be there both are gone to them because they are smart is to because they cancelling.

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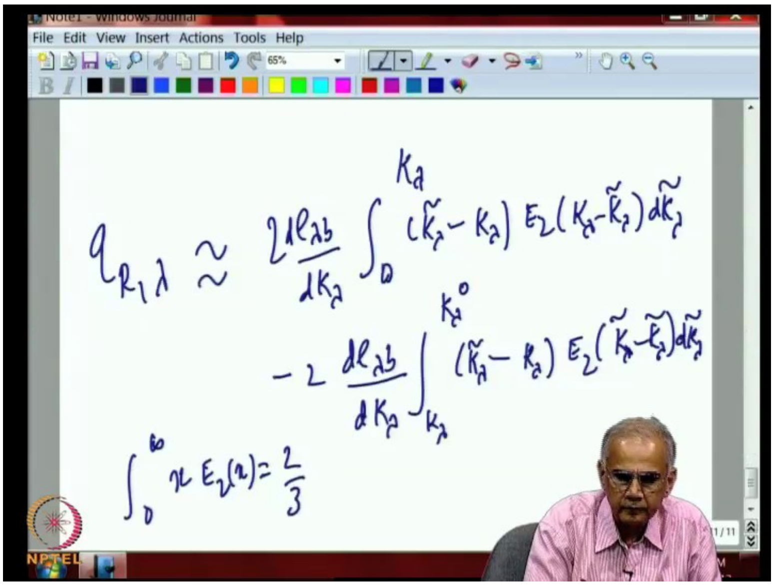

We left only with two term involving gas radiation, which is q r lambda is approximately equal to, the two expression representing gas emission. Now in the limit which is optically thick, this goes we 0 to infinity. This can also be re written in terms of kappa lambda 0 minus kappa lambda to make 0 to infinity and using the fact that 0 to infinity, x E 2 of x is equal to two-third we can simplify the expression to write the following very simple and elegant result which is negative heat flux is equal to minus 4 by 3, d e lambda b, d kappa lambda for thick limit.

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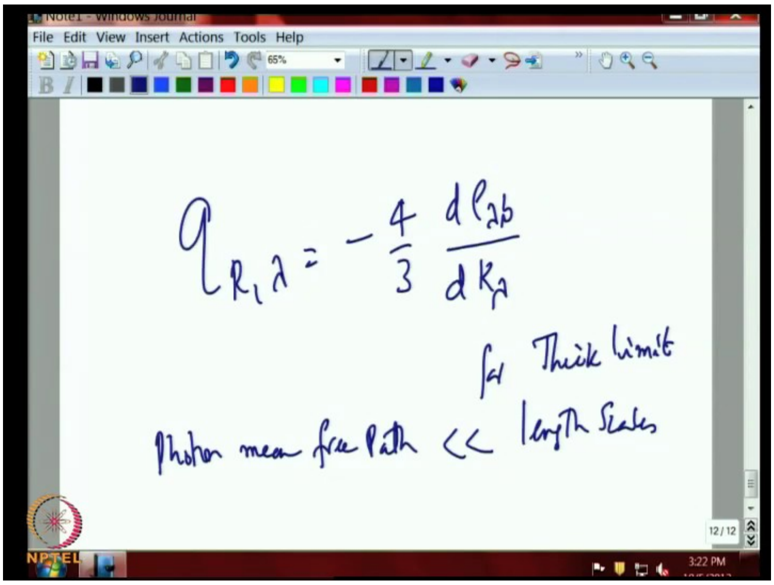

This is a very useful and powerful result, because we have converted to the basic complicated expression for radiative flux which was an integral equation in to a differential equation. This has happened because of photon mean free path is much, much problem than the length scale of interest to us. That means that photon hardly travels any distance and that means that everything is controlled locally, in the local gradient plays an important role. This what is happens also in conduction and convection which many of few of studied. We will continue this discussion in the next lecture and we will show how this result is a very useful result; all though you do not encounter many situation in engineering where in either thin or thick limit is actually valid, but still these limits provide an insight about a nature of radiation heat transfer.

Thank you.