Radiation Heat Transfer
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Lecture - 14
Introduction to Gas Radiation

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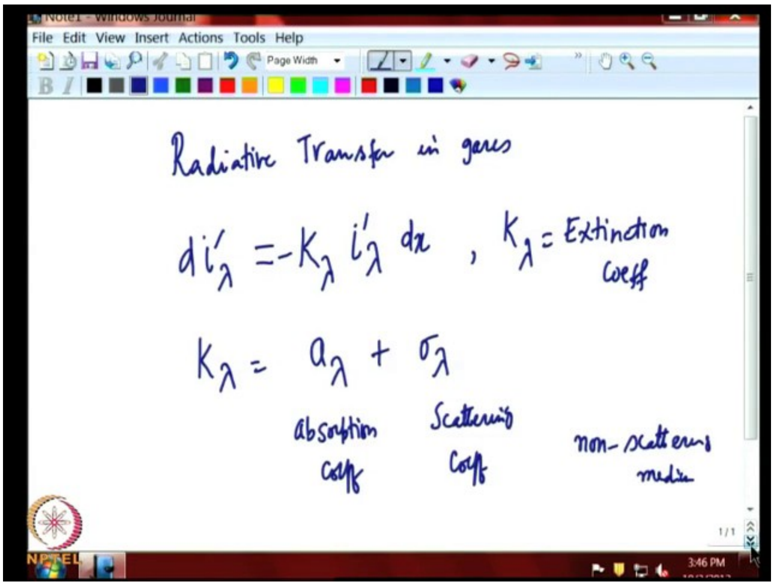

In the last lecture we started our discussion on radiative transfer in gases. We saw that because gases absorb radiation, they can also scatter radiation. The directional spectral intensity in a gas is not a constant, it changes. We saw that a simple expression relates the change to the incoming intensity, the thickness of the layer and are constant of proportionality called the extinction coefficient.

The extinction coefficient has two parts, absorption coefficient and the scattering coefficient. What this equation says is that the number of photons going in a given direction is reduced by either absorption or scattering. In the additional lectures we will focus only on absorption. We will look at a non scattering medium to simplify the discussion and we will have scattering later.

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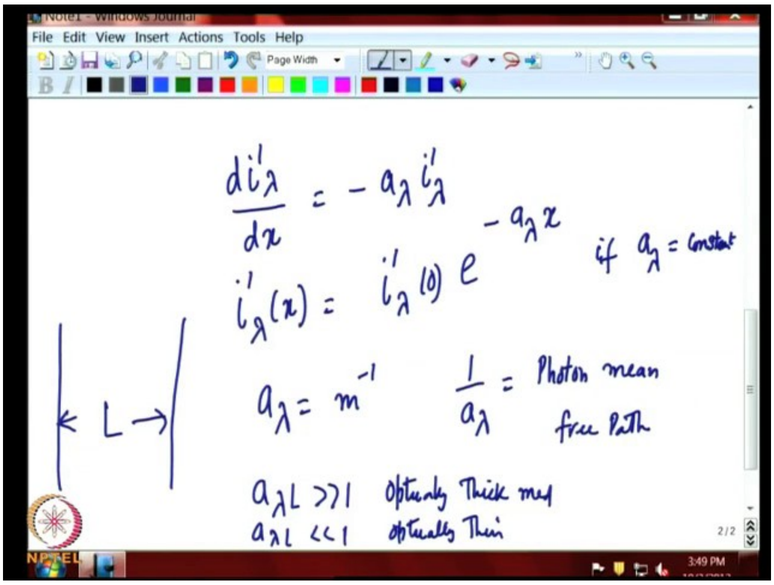

In a non-scattering medium, you see that the rate of change of intensity with x is nothing but minus a lambda i prime lambda. This is a simple first order differential equation. Although we can solve it and the solution will say that i prime lambda at any x is equal to i prime lambda at the origin into e to the power of minus a lambda x, if a lambda is a constant, a lambda is not varying along the x. We can see that a lambda has to have the units of 1 over length, and 1 over a lambda can be thought of as the photon mean free path.

That is, it is the average distance that a photon travels at that wavelength, before it is absorbed. It has lot of similarities to the concept of molecular mean free path, which most of you are aware from kinetic theory. This is important quantity and you can also imagine, if we have a gas layer of dimension l, a lambda l if it is much, much greater than 1. We call it a thick, optically thick medium that is it is highly absorbing because you can see that you can put a lambda l here, this quantity will become very small. It is a very rapid attenuation of the radiation. On the other hand a lambda is much; much less than 1 we call it optically thin. There is hardly any attenuation of the radiation.

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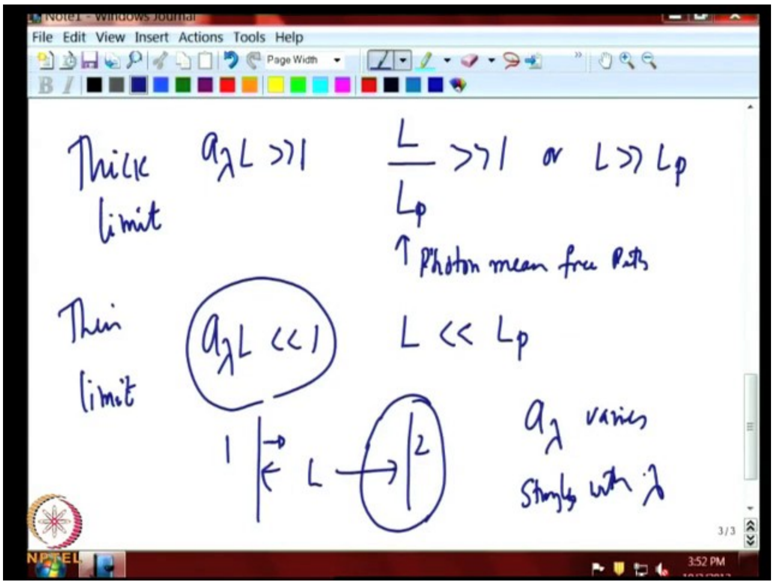

We can also look upon this as a lambda l much greater the 1 is equal to saying that length of the medium compared to the photon mean free path is much, much greater than 1. That is the medium is much larger than the typical mean free path of the photon. In the optically thin, limit the length is small compared to photon mean free path. We can see that the way radiation behaves in the thick limit and in the thin limit will be very, very different.

Let us take again this gas of length l, so the photon which is emitted by this surface 1 will not reach 2 at all. If it is the thick limit, because the photon is very quickly absorbed right here, gas may emit another photon here, which is to travel again a short distance. So, the surface 2 here will not actually see radiation emitted by surface 1, but really we will be only looking at those photons, which are within a distance l p of this layer. We can imagine that the situation the radiative heat flux arriving at the surface is controlled by what is happening in this region and is independent of that region.

On the other hand in the thin limit than the photon will travel from almost 1 to 2 without much attenuation. So, 1 and 2 will see each other and this is a case that really covered earlier, when we neglected absorption by the medium between two surfaces. We assume that the radiant leaving the surface is equal to that arriving at the surface. That is valid in the limit when this quantity is very small. Now, the very unusual differences of behavior, in the thick and thin limit are very important to understand the problem in real world situations. Now this is also complicated to the fact that a lambda varies strongly with lambda. So, as you vary wavelength this a lambda will fluctuate in a wild way.

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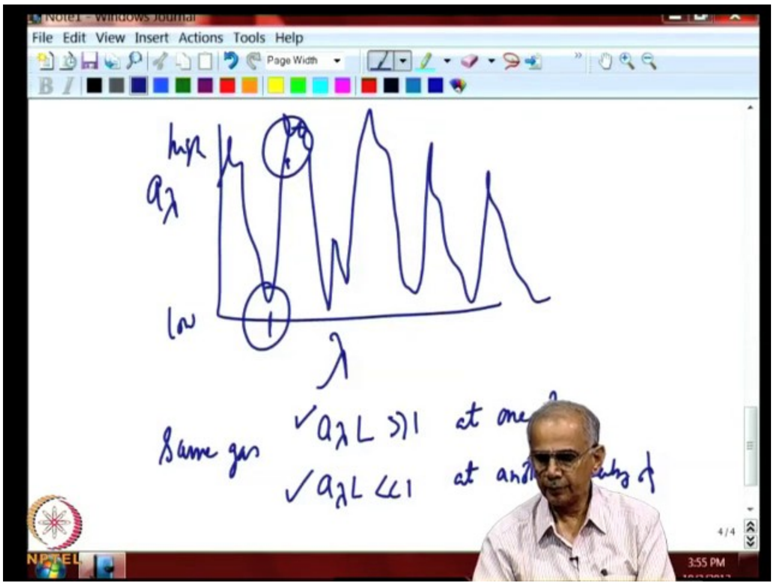

Let me give you as that of a quality picture, suppose you plot a lambda with lambda for a typical gas, it will have wild variations. It will go from very high value to very low value, for a very short distance. So, the same gas can have a lambda l much greater than 1 at 1 lambda, a lambda l much lesser 1 at another nearby lambda. The gas behavior is very different at two adjacent locations like this and this here it is optically thin, it is optically thick.

This is a real challenge; we face in radiation heat transfer. The fact that the absorption coefficient in most gasses, is a very complex function of lambda rapidly varying with lambda. If we want to make any approximation such as the thick or the thin limit, you got to be very, very careful because in examples in engineering there are very few situations in which, you can really apply one of the two limits. These limits are useful as a pedantic tool to understand the behavior of gases.

But real gases have such complex dependence of the absorption coefficient with wavelength, that they may simultaneously be in both thin and the thick limit, depending on which wavelength we are looking at. This issue we will keep repeating and reiterating because this is a fundamental problem in solving radiative heat transfer problems in gases because in contrast to liquids and solids, whose emissivity and absorptivity behave in a somewhat smoother fashion. In the case of gases the spectral variation of absorptive coefficient is very, very strong.

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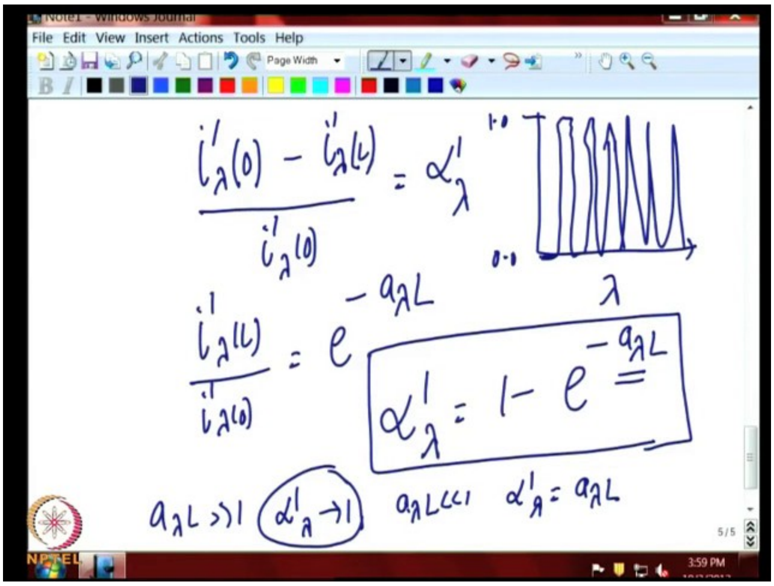

Let us make this point anymore clear, which you had mentioned in last class. We will repeat it, if we look at what is emissivity or absorptivity of the gas. If we look at the intensity, we want the intensity at the surface minus intensity other way divided by the intensity at 0. This is the directional spectral absorptivity of the gas, so this is the attenuation; this divided by the initial intensity and this is absorptivity. From our earlier solution of the equation, we know this quantity is e to the power of minus a lambda l, if a lambda is a constant along that line.

So with this we can see that the directional spectral absorptivity of gas is 1 minus e to the power of a lambda l. It is a very important result to understand connect what we learnt for solids and liquids to gases and we can see clearly that in the thick limit, the directional spectral absorptivity approaches 1. So, when a lambda l is much greater than 1 alpha prime lambda approaches 1 and when a lambda l is very, very small, alpha prime lambda is equal to a lambda l.

This is very important result that in a thick limit all gases their directional spectral absorptivity, approaches immunity and at the limit it is very small equivalent to a lambda l, if a lambda l is very small it is approx to 0. Now, notice that we have already mentioned that a lambda is a very, very complex and rapid variation with lambda, which means alpha prime lambda, is even more rapid variation. Let me now plot for the typical situation alpha prime lambda is the only between 1 and 0 because of this definition here. There are many, many gases that we encounter. It will go almost transparent of m lambda 0 to 1 over a very short distance.

This is the complexity that we face that this quantity will fluctuate between 1 and 0 over a very short distance along the wavelength direction. So, most of the difficulty is that one has faced in solving problems, with respect to radiation heat transfer has been on account of the rapid variation of the directional spectral absorptivity, with wavelength. This is the problem, which people have been struggling to handle for the last 100 years. Today the main advantage we have is an access to high speed computer, which enables us to do large number of computations every second.

So, today the complexity introduced by this large variation of directional spectral absorptivity with wavelength is mostly handled by solving this problem on the computer. So, right now we want to look at simpler issues in front of us so that we understand the nature of this equation, that we have to solve and later we will talk about what are actual ways to tackle this problem.

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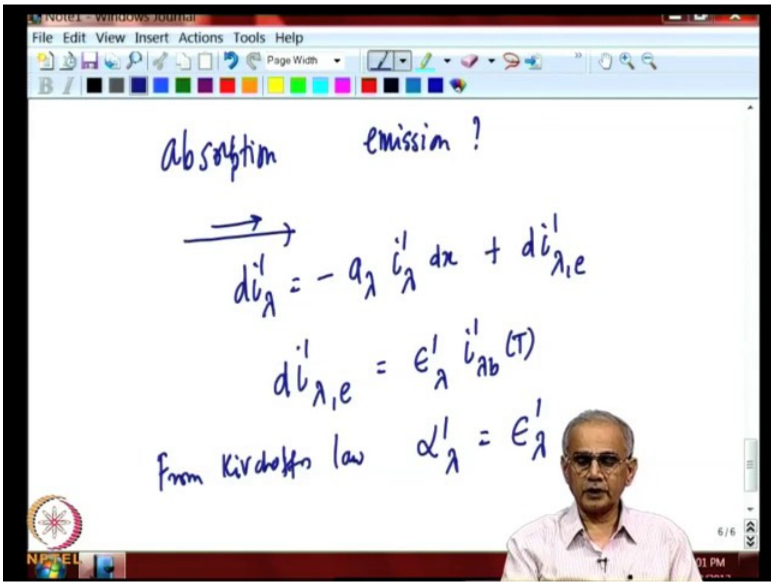

Now, in the discussion, so far we have only talked about absorption, in that radius scattering not absorbing. Note that a gas which absorbs will also emit. So, when you are going along a path, there is a decrease in the intensity on account of the absorption by the gas it is this quantity, but that must also be an increase in the number of photon, due to emission of gas along that direction.

The gas emits radiation along the direction in which you are interested and that depends upon the emissivity of the gas. So, from the basic definition of emissivity which we began in the first few lectures, we know it has to be this quantity notice the b here. The amount of emission in a given layer d x is the emissivity of that layer times the black body intensity at the temperature of the gas. This follows from the definition of emissivity. Now we involve Kirchoff's law and say from Kirchoff's law alpha prime lambda equals epsilon prime lambda which is always true, as long as you satisfy a condition called the local thermodynamic equilibrium. These are powerful result from Kirchoff's law.

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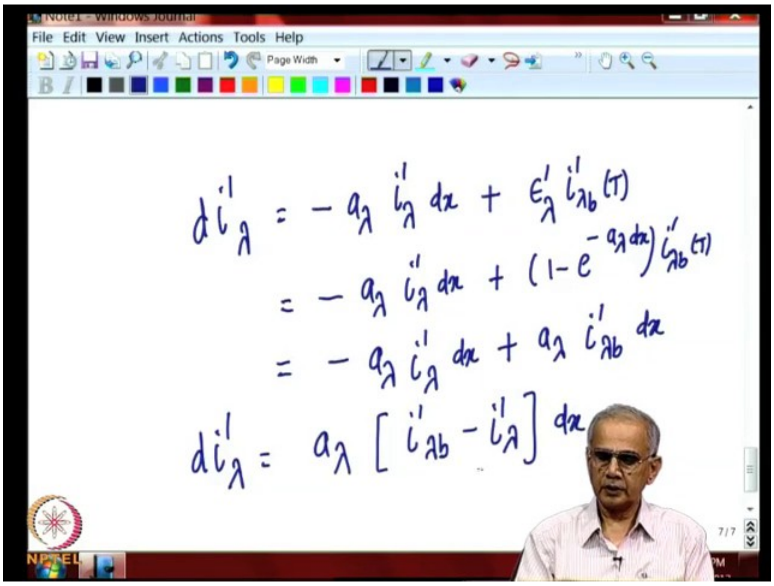

We substitute that there so we get the statement that change in intensity is equal to, a decrease on account of absorption and an increase on account of emission. We made this equal to absorptivity, so from the previous discussion this is 1 minus e to the minus a lambda d x the absorptivity equals emissivity. Yes, the d x will come later, so if we take the length d x to be small d x is quantity, we have chosen we can keep it as small as possible.

As d x tends to 0, this will become 1 plus for a small a lambda d x, this is nothing but 1 plus a lambda d x 1 will cancel out. You have this equation. So finally, we have equation for change in intensity, due to both absorption and emission by the gases. This is a most important equation for non scattering medium. Non scattering gas, which say that the change in intensity along the path is equal to the absorption coefficient time the difference in intensity between the intensity of blackbody emission along the direction at the temperature of the gas minus the incoming intensity.

We notice the difference one of contains subscript b, which we can calculate from Planck's equation, the other one is whatever intensity is coming in to the gas. Let me visually portray that for you.

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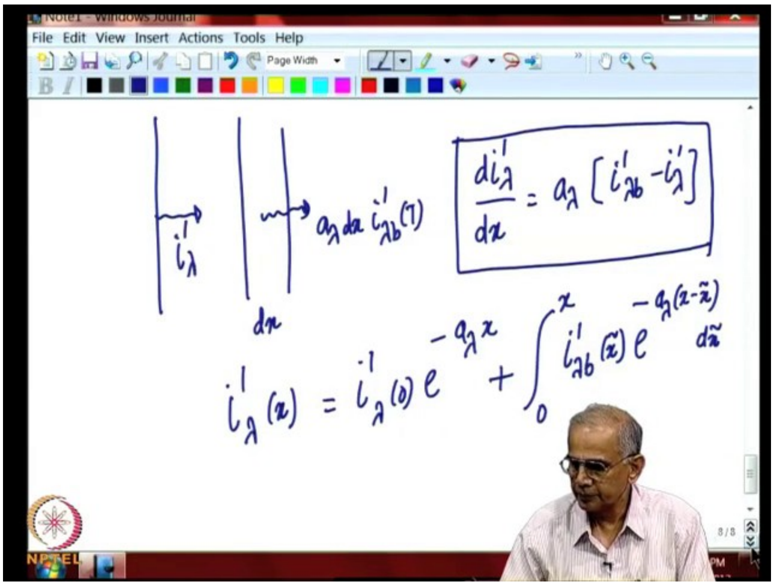

It is a surface from which this is coming and here is the layer of interest to you d x. This layer emits radiation and that radiation emission is a lambda d x i prime lambda b at the layer temperature while, this is nothing but i prime lambda from the surface. This intensity will change as it goes through the gas it will decrease because of absorption and increase because of emission. That is a main information that you get, so let me again write this as n o d e.

This o d is somewhat more complicated than the one we looked at earlier. We have now a non homogenous term. This is not a function of this but is an independent parameter, which can determine once the temperature of the gas is known. This kind of problem you must have solved in your first course in differential equation. We know that the answer will come out as i prime lambda x. It will be the first term which involves this which involves attenuation and the second term, which involves the non homogeneous term. This is a dummy variable, here with this solution to that equation. Now, this is a very important equation, which we will keep coming again and again. It is important that you fully understand, what this equation implies.

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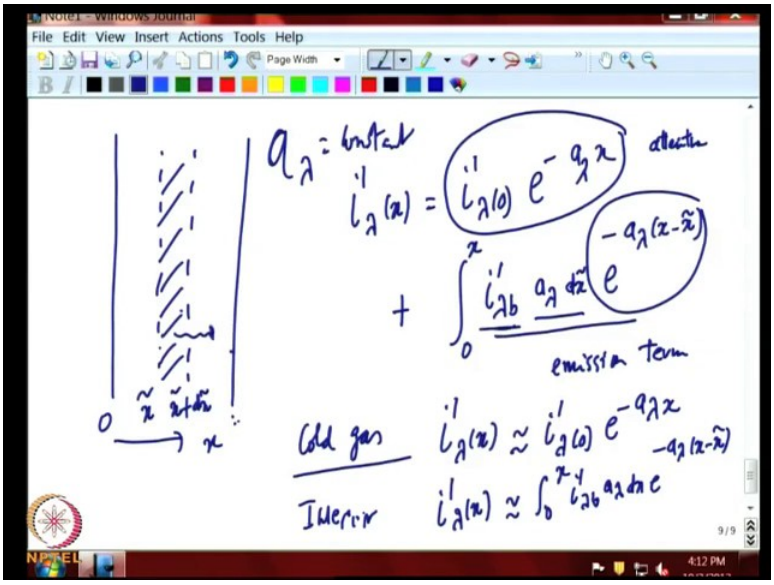

Let me then again draw a schematic to get you to, so this is the layer at a distance x and x plus d x. There is another layer also you have to worry about x delta and x delta plus dx delta. So, the radiation coming here is radiation coming from this surface e to the power of minus a lambda x. So radiation emitted from this surface is i prime lambda 0 reaches the plane x after attenuation to the gas plus emission by all these gas elements.

The gas elements will emit radiation in this direction reach here, so those gas elements will emit this much radiation and this layer is d x delta. We have to integrate for all gas elements from 0 to x and take care of their attenuation. This is the attenuation term and this is the emission term. The emission term is an integral. The attenuation term, is the decrease in intensity due to the absorption of the gas. The next term is emission by all gas elements between 0 and x, which reach x.

This is the blackbody emission, this is emissivity of that layer this layer and this is the attenuation which depends on the distance between this point and this point. We are able to express the intensity at location x in terms of what started from a surface 0 after attenuation through the gas and arrived at x. The next one is summation of all emissions from all layers between 0 and x, and their attenuation depending on the location of the gas layer with reference to this plane x.

This is a very important result, which we will see many times and one must clearly recognize the general formal equation, but there are a few approximation which will help us. Suppose, we had a cold gas, gas temperature is very, very low. Then this quantity will be very small emission by the gas, so we neglect the second term. Then we have a very simple approximation for attenuation where emission is small. This may be irrelevant in some situation where the gas temperature is very small compared to the temperature of the solid surface here.

We imagine there is a hot ball or furnace emitting into a cold gas in which case the emission term is small, only the absorption term plays the role. The other extreme is suppose, we have an interior of a very large medium extremely large medium, so that the surface 0 and surface l are very far away, then the intensity that you see will be dominated by the emission term. In the interior far away from the walls, you might be able to only represent the intensity by this kind provided, a lambda is large. Now we want to make this result little more general. This result was obtained assuming a lambda as constant independent of space.

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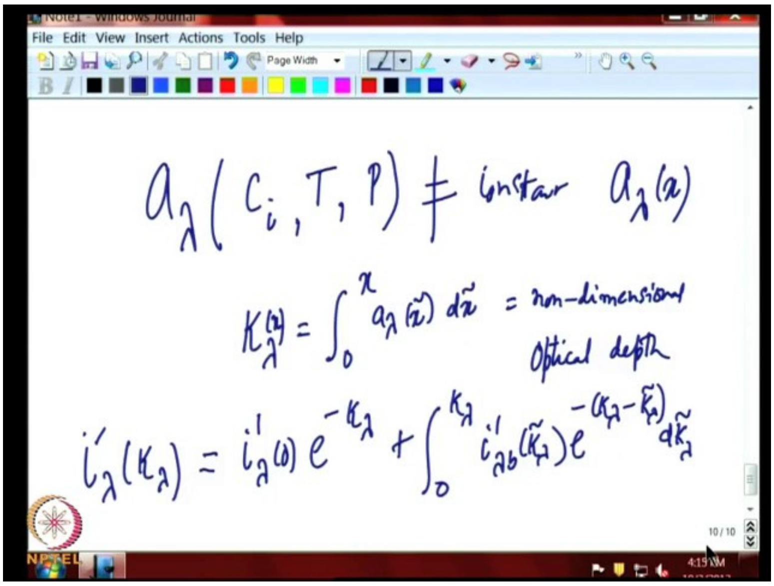

Now, it is not true. The absorption coefficient a lambda is the function of space. We can from basic physical background that this coefficient depends upon the concentration of the gas of the i species, depends on temperature depend on also total pressure. If along a certain path the temperature is varying pressure is varying the concentration of the gas are varying then this will not be a constant. So a lambda is a function of x and not a constant, we have to redo the calculation to allow for the fact that the absorbed coefficient varies along the path, in which your ray is travelling.

When that is happening, it is good to define another quantity called Kappa lambda, which is 0 to x a lambda the dummy variable. Now, this is a non dimensional quantity and is normally called the optical depth. It is a very convenient parameter or actually in this case variable whether it will be function of x note. We can redo your integration and easily derive that the intensity of the gas or certain location Kappa lambda is equal to i prime lambda 0 e to the power of minus Kappa lambda plus 0 to Kappa lambda i prime lambda b of Kappa lambda tilde at dummy variable e to the power of minus Kappa lambda minus Kappa lambda d Kappa lambda tilde.

This is the general equation when the absorption coefficient of the gas is a function of the path along which you are travelling. We define a quantity in terms of the non dimensional optical depth. We are implicitly accounted for the variation absorb coefficient with temperature pressure and concentration. This is the result, which is the basis of all our future discussions. The advantage of this generalized equation is that we can take care of variations of a lambda with path because it is built into the definition here, so this will be the basis for our calculation. Now, this equation gives you the directional spectral intensity in a given direction.

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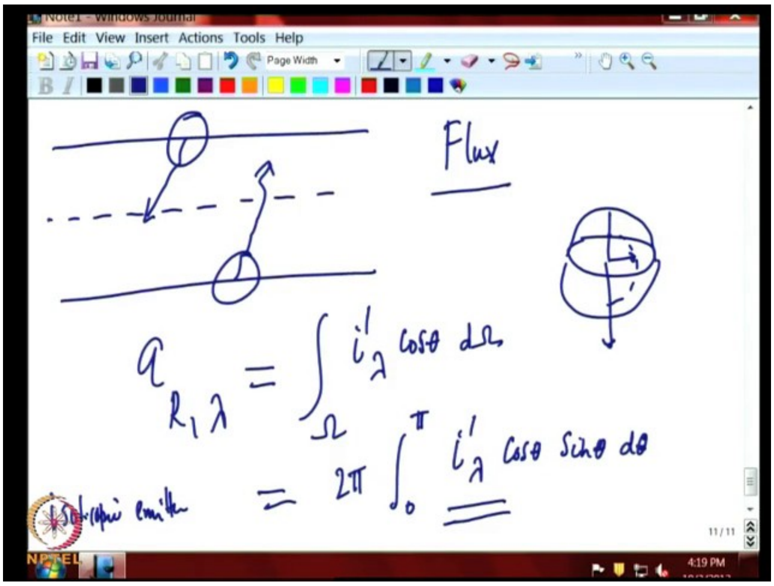

Just to give you example. Now, let us take two parallel planes. So this is a useful tool to understand in any given direction how intensity is changing. But finally, what we want is not i prime, but flux. The flux is obtained after averaging the intensity over all angles theta and phi. If we recall our definition of radiative flux it was nothing but integral omega of i prime lambda cos theta d omega. We look at the definition of i prime lambda and we will see that cos theta is built into the definition.

So, is the d omega, this follows from the definition of i prime lambda in terms of pure lambda. Now, this integration can be tedious, but if we assume gas is isotropic emitter, then integration is somewhat simplified in the direction phi everything is same. So, you integrate to get 2 pi, 0 to pi i prime lambda cos theta. Write this in terms of sine theta d theta. So integration of phi gives you 2 pi and then you are integrate over all angle 0 to pi. Let us visualize this for you, so your plane and you integrate over this and also this.

Integration is based on 0 to pi. So it covers in this plane parallel situation all intensity going upwards and downwards. So once we solved for the directional spectral intensity by solving these equations we then plug it into this expression to get the flux and finally, this is what matters q R lambda and its derivatives. Now, the question is how do you do the integration. Now, for convenience we will divide that again in two parts the upper and the lower and this becomes necessary, because if we look at a layer the center of the gas here the radiation going downwards is coming from the upper wall and radiation going upwards is coming from the lower wall.

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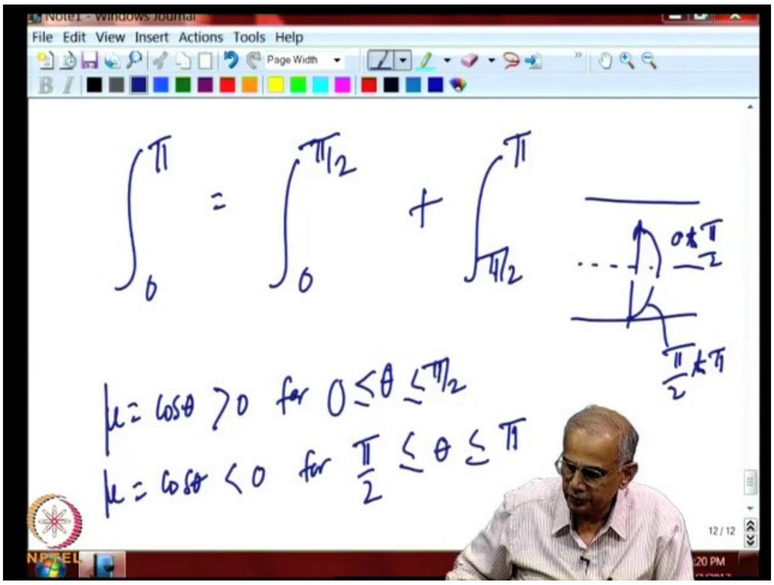

It makes sense to split this integration 0 to pi as 0 to pi by 2 plus pi by 2 to pi. The advantage is in the region 0 to pi by 2 if we define mu as cos theta this is greater than 0 for theta between 0 and pi by 2, where this quantity mu is negative for theta between pi by 2 to plus pi. These are the definition we are assuming that the layer we draw normal here and it is 0 to pi by 2. This is pi by 2 to pi. It will go through, all this somewhat tedious calculations.

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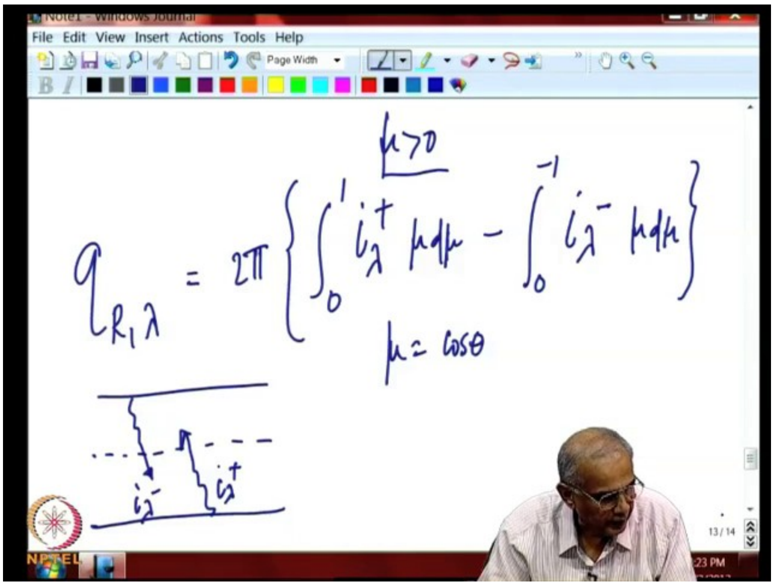

So our q R lambda becomes 2 pi into 0 to 1 i lambda plus upper going radiation mu d mu minus 0 to minus 1 i plus downward radiation mu d mu. We here are considering all rays moving upwards from the bottom plate to the top plate. The other case we are considering all rays moving downwards from the top plate to the bottom plate. So schematically the first direction, we draw this.

So, i lambda plus is intensity that is emerging from this plate through this gas a lambda i lambda minus is radiation coming through the top surface and top layer gas. This involves 0 to pi by 2, so where mu is cos theta. So, when theta has been 0 to pi by 2, this quantity is always positive and the second one involves the theta value between pi by 2 and pi because it will be negative. Now, we can write what is i lambda plus and i lambda minus. Let me first write what is i lambda plus.

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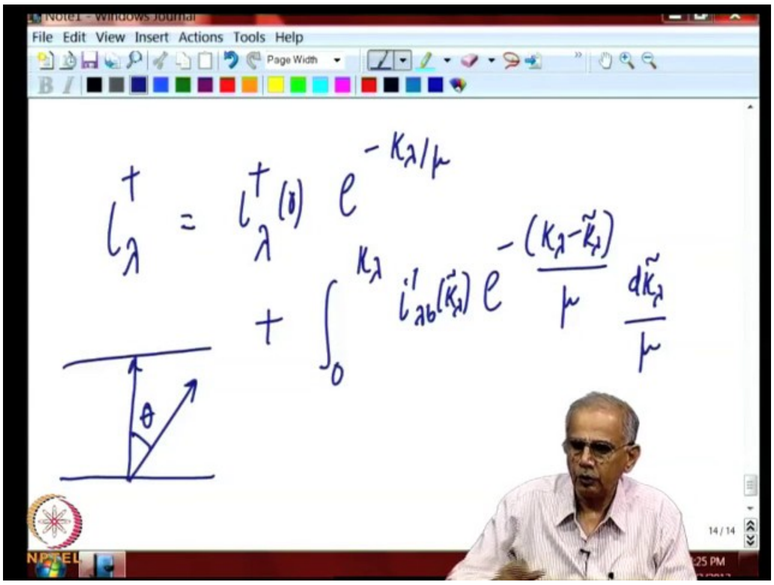

All intensity going upwards is equal to all intensity going upwards from the bottom surface and e to the power minus Kappa lambda by mu and the second term is very similar to what we have done earlier, only some notation has changed. We have explicitly accounted, we are measuring everything with respect to a vertical direction and this mu and this mu is coming in because of the difference between the direct radiation and the slanted radiation.

All rays which are not travelling perpendicular to the 2 plates are going to travel longer distance and hence we divide by cos theta. That is the only thing we introduced here in our in our discussion, otherwise this is very similar to the earlier one. We plug everything in into these equations and calculate the upward and downward radiative flux.

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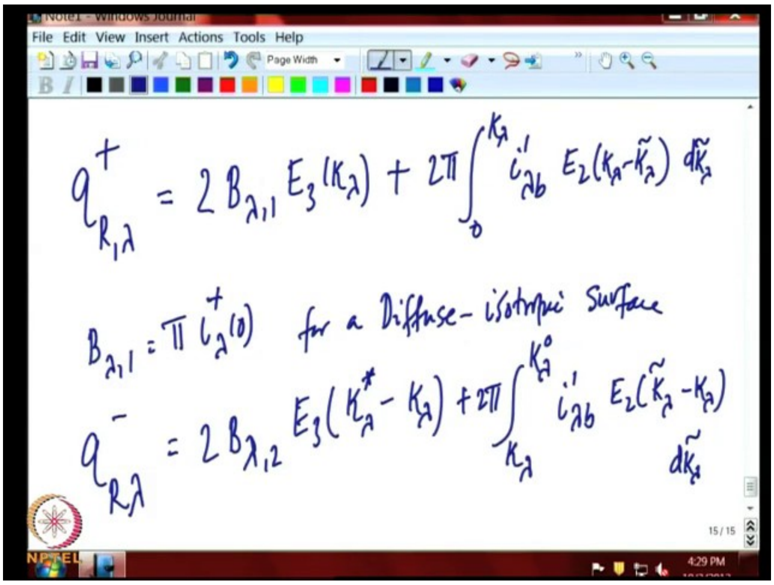

We do all this integrations, then we will get the upward radiative flux will be equal to we will write down the expression and explain what these terms are. This is an important result after integration over all angles. Notice that there are two terms and b lambda 1 by definition is pie times i plus lambda 0. This is the upward travelling intensity from the lower surface and which is assumed to be diffuse isotropic. So, pie times that is radiosity. That is radiation leaving this surface which is radiosity. The second term is the emission term which has not really changed that much.

That is q R lambda plus upward going ray. Then we come to downward coming radiation, which is coming from the top. This will have the radiation from the top surface times the distance which is the highest optical depth k lambda 0. The second equation right now here is, the radiation leaving surface 2 at the top going downwards after attenuation through all angles. The second term is gas radiation emitted downwards integrated over all angles taken into account the different attenuation of the ray depending on the angle, so angle plays a very important role here.

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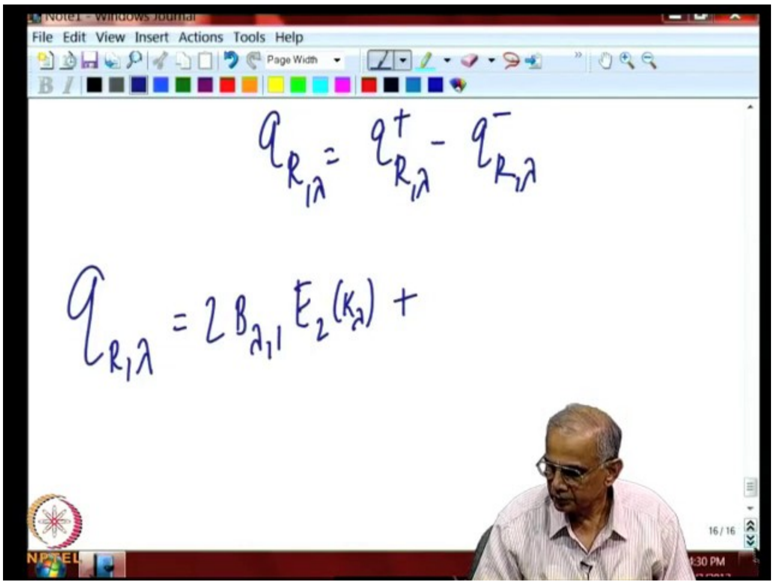

We have now list this net flux as up minus down. The final expression for radiative flux, which will be using quite frequently in the next few lectures, is written as follows. The net radiative flux d lambda and lambda plus B lambda as the radiosity of the top surface of invisible bottom surface e 2 of Kappa lambda. Let us define the functions e 3 and e 2. First that expression of the integral function has to be defined before we go further. We should define the e 2 emissivity; you can define e 2 over here.

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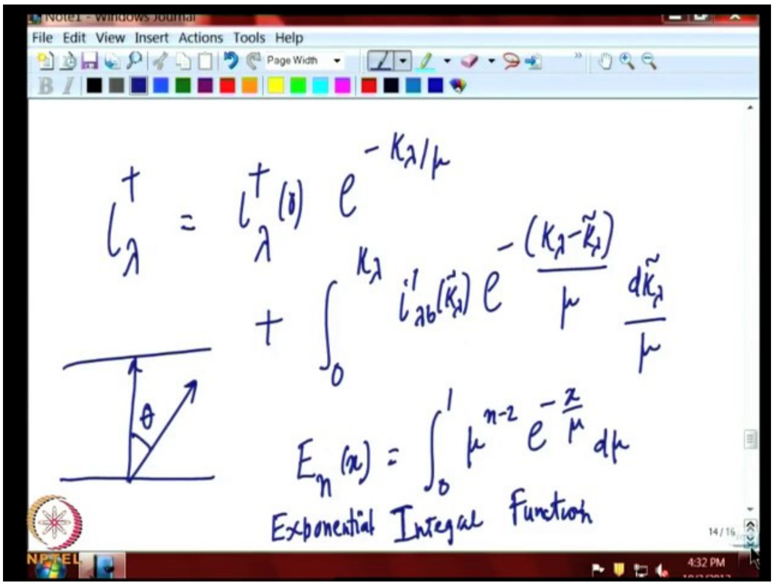

Now, e 2 had to be defined here, just we will define here, what is e 2 of x. It is nothing but 0 2 1 mu to the power of n minus 2 e to the power x by mu d mu. This is, e n f x. This is called the exponential integral function and plays a fundamental role in integrate transfer. It accounts for essentially angular integration. We are getting e 3 and e 2 both here.

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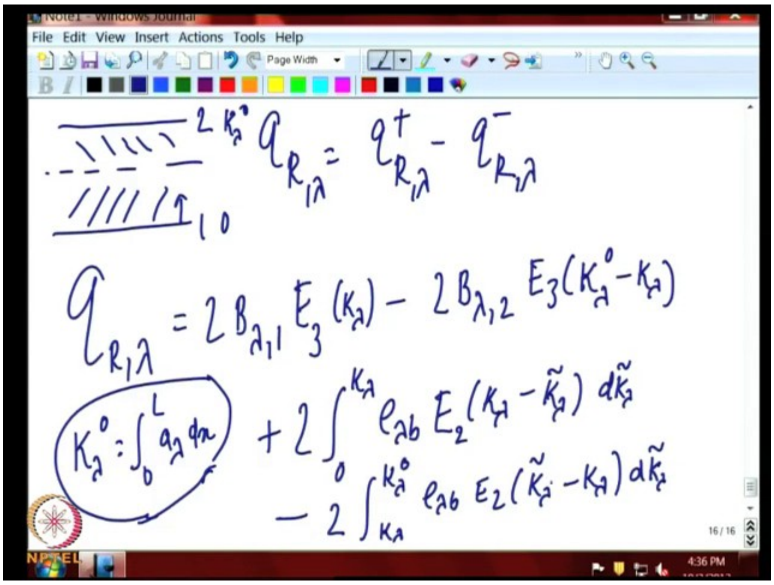

We will go to the next page, this has to be e 3 and then we will put a negative term here, which is a 2 b 2 lambda in the top surface. Let us put this as 1 and this is 2, so the radiation from the below goes up and radiation above comes down there is a negative flux and the distance travelled here is where k lambda 0 is equal to 0 to l a lambda d x the full optical depth of the medium. There are two more terms, one all gas radiation emitted from the bottom surface, another from the bottom layer. This is the radiation emitted by all gas elements lying in this region and upward region both are there. That will emit upward, that is positive.

Then minus 2, this is integration from Kappa lambda to Kappa lambda 0 e lambda b e 2 of Kappa and tilde minus Kappa lambda d Kappa lambda tilde. This expression for radiative flux must be properly understood the first term is the emission from the photon, leaving the surface one and on attenuation reaching the point of interest to us. This attenuation term takes account angular distribution of photon or not all photons are coming vertically. Some of them are coming at an angle and you are integrating all angles while upward flux.

Then the downward flux is the radiation leaving the top surface and travelling that distance. The distance travelled now is counted because everything is counted this is x going upwards this is 0, this is Kappa lambda 0. So, when you come to this point your traveled distance Kappa lambda is assumed as Kappa lambda at certain radiation. That is the downward flux for the top surface. Then these are emission by the gas elements between 0 and Kappa lambda, which reach the surface, finally emission from all gas elements above this.

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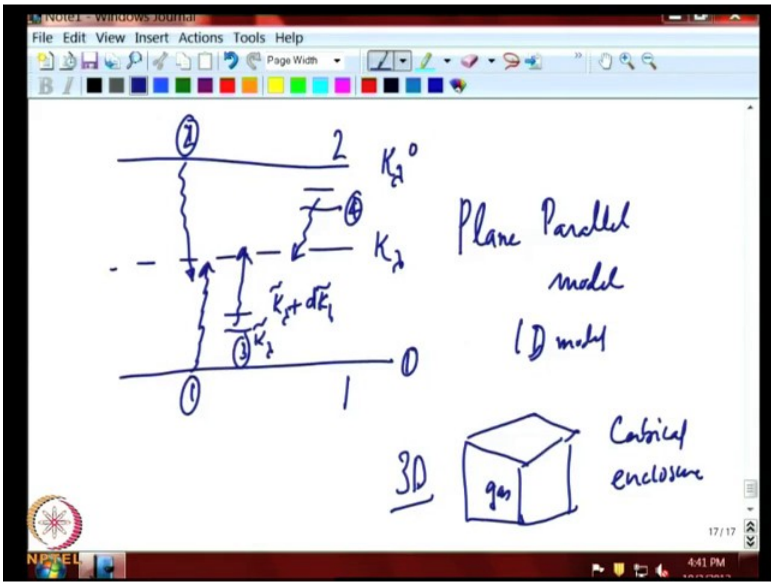

Let me show this schematically, so this is one term first term in the equation. Surface one, surface two, second term in the equation is radiation from this surface coming here. This is 2 and this is 1. This is now element two, this is element one, and the third element is emission from gas layers here, which comes here and fourth element is gas layer here, emitting radiation here.

One must fully understand the role played by these four streams, stream one is the radiation leaving surface 1 by either by emission of reflection and after attenuation, which is angle dependent integration reaches this layer, which is Kappa lambda. The second term is radiation leaving the top surface coming down to the level Kappa lambda. Hence, it travels the distance Kappa lambda 0 minus Kappa lambda to reach here.

The two of the terms are radiation emitted by all layers from Kappa lambda tilde to Kappa lambda tilde plus d Kappa lambda tilde, this layer and integrate all emissions from surface to Kappa lambda to get the third term. The fourth term is radiation emitted downwards by all gas layer, lying above Kappa lambda. They travel distance Kappa lambda or a Kappa lambda tilde, which is built into the exponential integral function. These four terms contribute to the flux in that flux at any layer is the addition of all these four terms and by our notation radiation moving downwards is positive radiation moving upwards is negative.

That is why terms two and three which are terms coming down and negative, while terms one and three which are terms going upwards is positive. We have at last got an equation and this geometry we used is called the plane parallel model or atmosphere. This is simplest formulation radiation problems it is essentially 1D model. We do not account for variations in y and in z. We only worry about variations in x, so this is an approximation which is alright as long as the distance between the plates is large compared to the length of the plates in which case we can neglect, what is happening in the ends.

We have simple plane parallel model, so this works quite well in many situations, but if we have a for example, a cubical enclosure with gas in it, then this is that about approximation this is a cubical enclosure with gas. Here the plane parallel model may not work well because the gas inside this cubical box is getting radiation from all the six surface of the cube. We have to go for the full 3D formulation for this case; that is fairly complicated and hence cannot be easily done anything in the class.

We will stick to simple one dimensional model of the plane parallel atmosphere to illustrate all the basic issues in during transfer. We presume that you know that if we had to deal with 3D model, it is similar to 2D model, except that there is more complexity and computation involved, which can be tackled in the real world, but we will be using only the 1D model for teaching purposes.

Thank you.