Radiation Heat Transfer
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Lecture - 11
Non-gray enclosures

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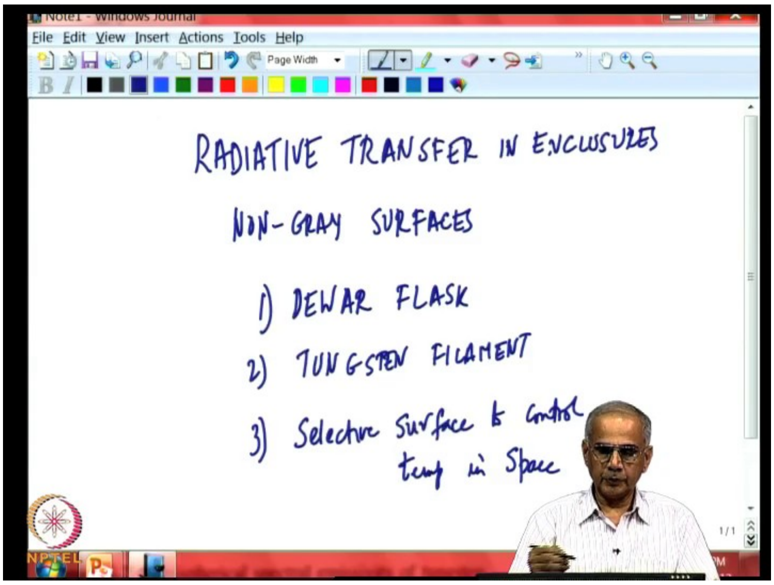

In the last lecture, we looked at radiative transfer in enclosure, and we looked a few examples of non-gray surfaces. We gave example of Dewar Flask, where a heat leakage into the flask was influenced by the spectral variation emissivity of the silver surface is on the on the inside the flask. The second example, we gave was related to the tungsten filament lab design, where also the design of the filament influenced by the fact that the emissivity tungsten was not really independent of wave length. Now these are very simple examples, we also looked at third example of selective surfaces to control temperature in the space. We saw how in space, we can maintain a surface at a very high temperature or very low temperature by controlling the spectral variation of absorptive surface.

All these are very simple examples involved only essentially one surface impeaching, so it is impeaching or two surfaces like Dewar Flask. Now let us go back and look at the Dewar flask example in terms of the spectral nature the radiation, and we draw simple analytical technology.

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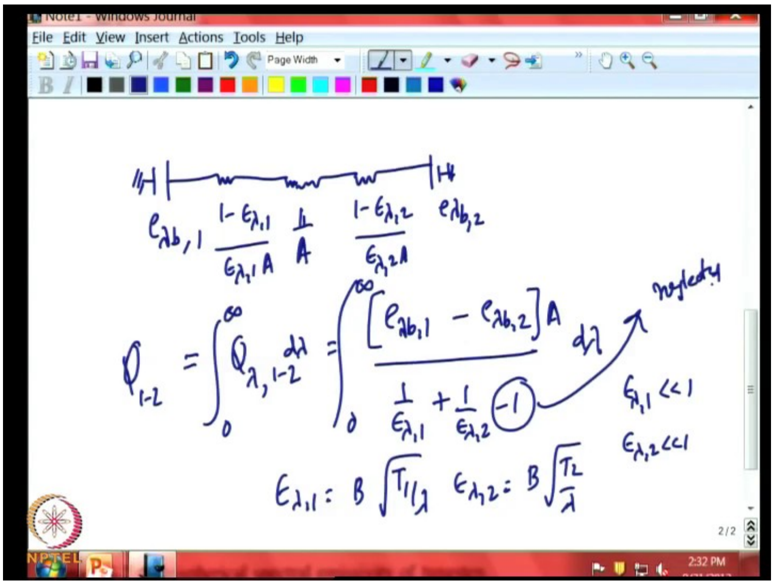

Now at a spectral level not at the total level, so the two surfaces, two parallel surfaces now, so f 1 to is 1 and let me guess. The first is 1 by epsilon lambda 1 epsilon lambda 1 A, then this is 1 over A, A is area of the parallel plate, and finally we have 1 minus epsilon lambda 2 by epsilon lambda 2 A, and finally we have the surface 2. This expression can be written as, heat transfer in the range lambda 2, lambda 2 as the lambda from 1 to 2 as e lambda b 1 minus e lambda b 2 divided by 1 by epsilon lambda 1 plus 1 by epsilon lambda 2 minus 1, of course into area of the plates.

This is for the interval lambda that if the total heat transfer over all wave length, then we are integrate this over d lambda. This is also has integrated and this integration can get quite complicated by the simple case, that we dealt in the last lecture. We assume that epsilon lambda 1 is B in to root T 1 by lambda and surface 2 are again the same material, but different are a temperature. Then we use in approximation that sends the emissivity of silver surfaces is very, very small their reflectivity is very large. This term and this term is very large we neglected this term.

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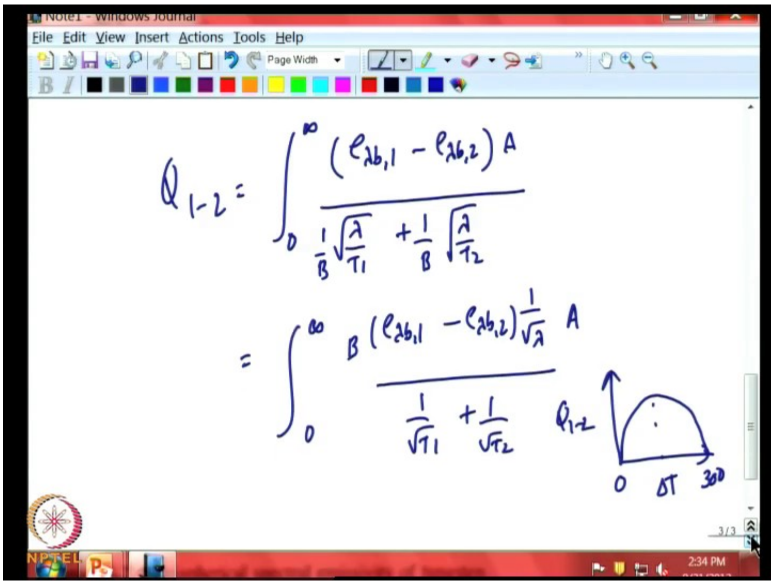

On neglecting this term be the expression we got between two parallel plates was, e lambda b 1 minus e lambda b 2 into A divided by 1 by epsilon lambda1 which goes as root lambda by T 1 1 over b plus 1 over b root lambda divided by T 2 and root lambda is common to them. This can be simplified to write as, 1 over root lambda goes above and below we have and we can write A above, then take also B above nothing but 1 over root T 1 plus 1 over root T 2.

Now this can be integrated suitably and we also already discuss, the results obtain by this integration and we showed that, the heat transfer between two surfaces need not be proportional to temperature difference and can behave in unusual ways and in particular we showed that, as temperature different changes very low value to high value. In this case 0 to 300 actually the heat transfer rate, it should be 0 at 0 delta T went up and came down, reached its maximum at around 150 K. All this is occurred, because of the spectral dependence and temperature dependence of emissivity of the surfaces, but let me just take some more similar example there is little complicated.

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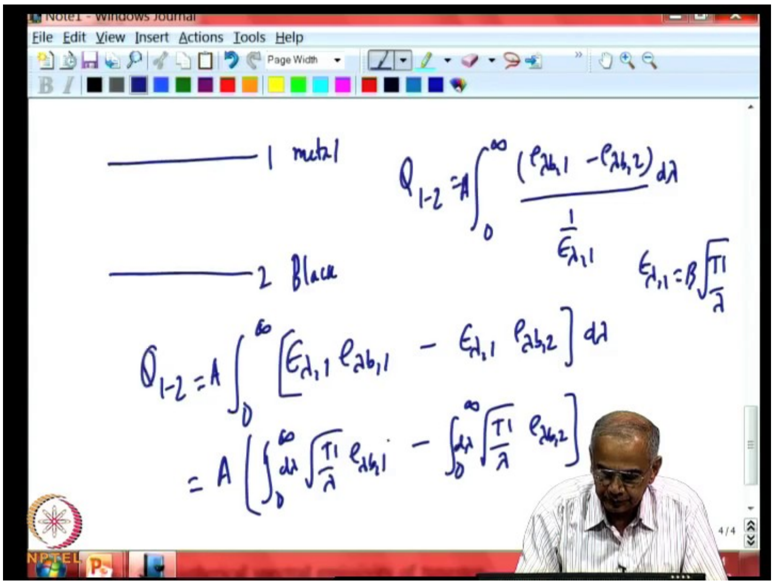

Suppose the surface in plate 1 and plate 2, let us say this was a metal and this was black. This is somewhat simple example than the previous one and what happens here is that, our equation simplifies to 1 epsilon lambda 2 1. So, that cancel out we have d lambda here. This can be written as, Q 1 2 as 0 to infinity epsilon lambda 1 e lambda b 1 minus epsilon lambda 1 e lambda b 2 into d lambda because, area is still there.

Now, this follows very close to what we are done earlier, when discussing the emissivity and absorptivity of metals. We can write e lambda is 1 as B times root T 1 by lambda. This equation here becomes 0 to infinity root T 1 by lambda e lambda b 1 minus 0 to infinity root T 1 by lambda e lambda b 2. This kind of analysis we are done before, we know that this is s a function primarily of lambda T when it divide by T to the power of 5 we saw that earlier. So, as before we will multiply up and down by T 1 and get this to be a function of T to the power of 5 and if we do all that, we get the result which should be quite familiar to us based on what we done earlier.

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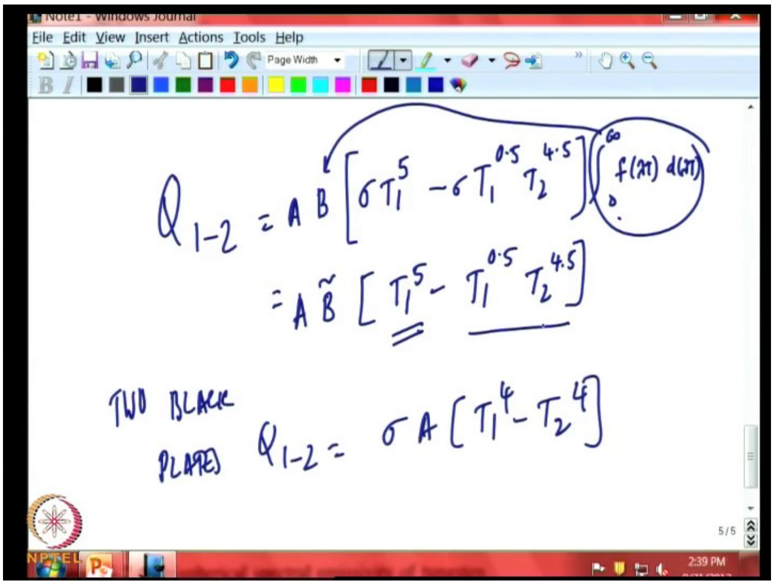

What will happen is we get B here and actually we get A B delta, actually to write down the whole thing B and we will get T 1 to the power of 5 sigma T 1 to the power of 5 minus T 1 to the power of 5 sigma here T to the power 4.5 and what we are inside in integral 0 to infinity, this integral which is function only of lambda T d lambda T. This is a number which can be evaluated very easily and we just combined with this we did earlier and we call it B delta along with sigma.

We have left finally, with T one to the power of 5 minus T 1 to the power of 0.5 into T 2 to the power of 4.5. This must be compared with this is metal and a black plate, if we have two black plates, this is nothing but sigma A T 1 to the power of 4 minus T 2 to the power of 4. So, that is the difference introduced by 1 metal being there in the in the top side. So, it increases the emissive power of the metal from 4 to 5, then on this side what is happening is, that the effect of changing T 1 is effecting the radiation absorbed from T 2 because, this can also be written; if we now temporarily remove this simple expression we have here.

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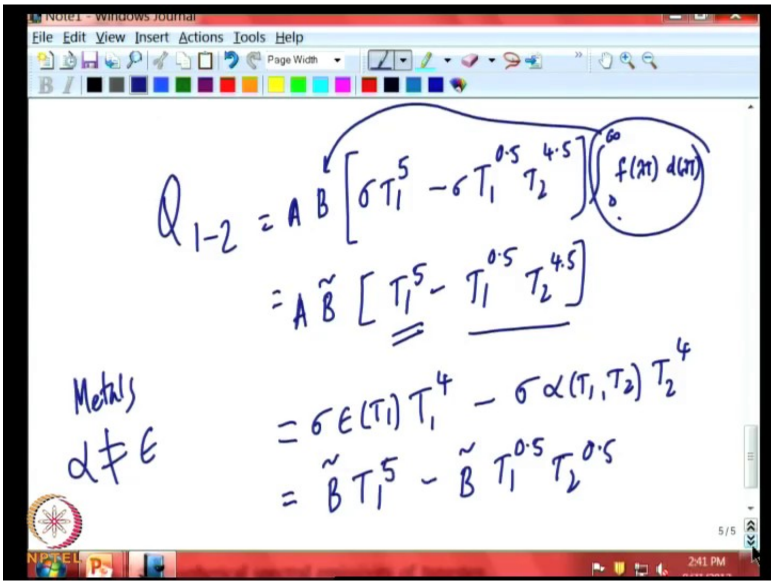

We now write this very simply as, epsilon function of T 1, T 1 to the power of 4 into sigma minus sigma alpha a function of T 1, T 2 into T 1, T 2 the power of 4 this how it should be because, in the case of metals we must recall that alpha is not equal to epsilon. That was the fundamental result, we are derived that metals being non-gray, we cannot assume that the total hemispherical absorptivity is equal to total hemispherical emissivity.

An alpha not equal to epsilon, then we have result which is of this kind, but we recall that of course, that this could be written and if sigma is absorbed in that definition as B delta into T 1 that becomes this and this is B delta into root T 1, T 2 and hence this becomes B delta T 1 to the power of 0.5 T 2 to the power of 0.5. So, that is how this emerges. This is also same really, but derived into two different ways and it clearly tells we that, the traditional assumption, that emissive power goes as fourth power of T is only true for black body; the bodies are not black and not grey. Then we can have emissive power, having powers larger than 4 or even less depending on how the emissivity and absorptive with that surface depends on the wavelength.

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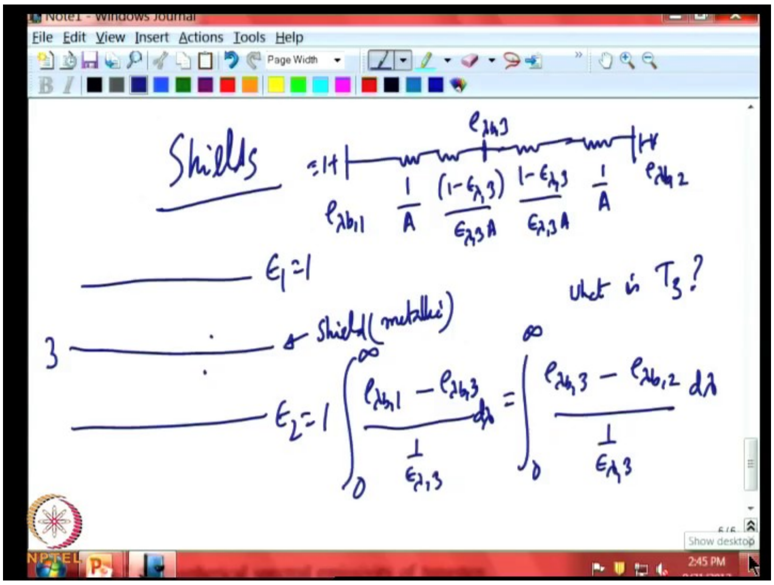

Let us take one more example to illustrate this and we will behave, let us look at shields. Suppose we have two black plates and we put a shield in between which is metallic. Now why we would put a metallic shield. Because, we know that metals are the property of having low emissivity They are good reflectors. They will reflect the radiation and reduce the hence reduce the emission. So, let us draw the electro technology here. We have e lambda b 1 here, there is no surface resistance, emissivity equals 1, straight away we go to the geometric resistance and then we come to the metal here, which will be 1 minus epsilon lambda 3 by epsilon lambda 3 A and then we have the other surface of metal both surface are have to be counted.

There is another surface of metal here, 1 minus lambda 3 by lambda 3 A and of course, finally, we have other surface also with no surfaces which is only geometry resistance only one over a finally, we come to epsilon lambda b 2. This is the electrical circuit and we can see that the simple way to understand this problem is that, epsilon lambda b 1 minus this point which is epsilon lambda b 3 point. This point the flow of current will depend on epsilon lambda 1 which is one. So, it will be divisible by 1 by epsilon lambda 3.

Similarly, on this side heat going from the metal to the other plate will only depend on emissivity of the metal because, the two plates are emissivity equal to 1. So, that is the interesting situation we are in, that the current flow from plate 1 to this is equal to that, but this we must do over the entire wavelength range not just think. So, now we have to ask, what is temperature of the shield T 3.

This surface 3 shield is, now the shield temperature is such that the heat flow from 1 to 3 has equal to heat flow from 3 to 1 because, in steady state the thin metal here cannot retain heat. So, whatever heat it gets from surface 1 to 3 has to go in to the other side. Now, if we recall the discussion we had about ten minutes ago, this expression now can be written.

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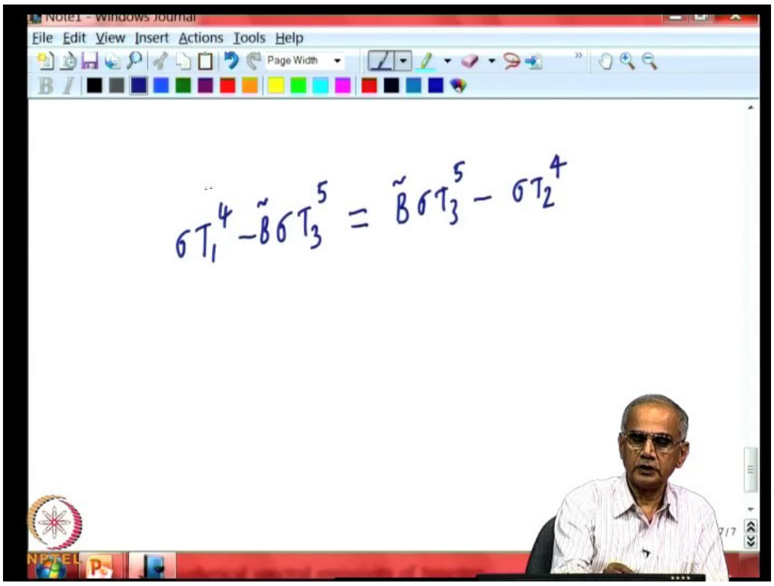

We will write the relation for emission from surface 1 is sigma T 1 to the power of 4 minus emitted by the plate and absorbed by the black plate, will be nothing but sigma T 3 to the power of 5 because, the emission from the metal is like; T to the power of 5 proportionality of course, there will be B tilda here which we have to remember and this has be equal to the energy that is emitted by 3 and that emission is absorbed by the plate 2 and the plate 2 emits back so much. The radiation that is emitted by 1, and it is absorbed by 3.

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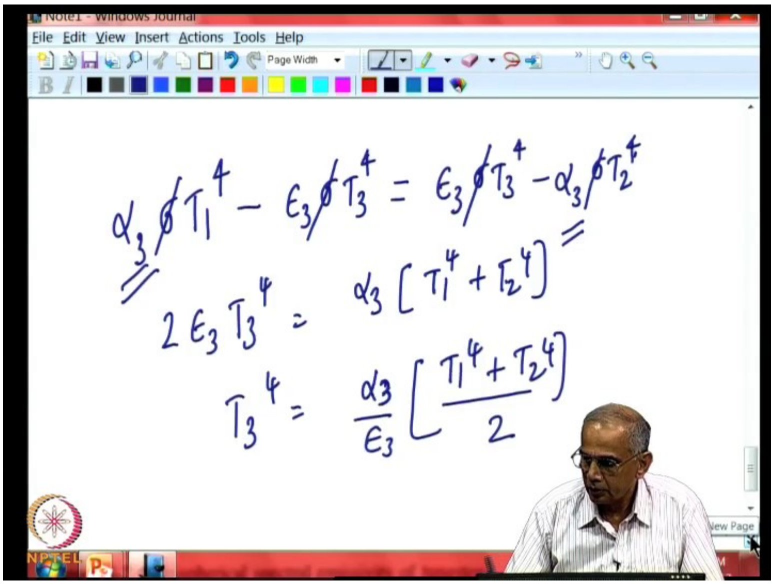

These are the radiation emitted by 1, what is absorbed with 3 is alpha 3 and the radiation emitted by 3 is epsilon 3 and all of here we absorbed by 1 because, 1 is a black body and here the radiation emitted by 3, all of it is absorbed by 1 and minus the radiation that is emitted by 2 and absorbed by 3. The balance is that radiation emitted by 1, say what we want to grow absorbed by 3 minus what is emitted back by 3, this flow has to be equal to radiation by 3 and what is coming back from 2 which is similar to 4 and absorbed by 3. If we finalize now what is T to the power of 4, notice that sigma will cancel out on both sides. So, T 3 to the power of 4 will come out as, here 2 epsilon 3 is there do not forget and we will have here alpha 3 into T 1 to the power of 4 plus T 2 the power of 4.

The final answer really is that, T 3 to the power of 4 is alpha 3 by epsilon 3 into T 1 to the power of 4 plus T 2 the power of 4 divided by 2. The metal shield between two black plates, will have temperature which depends on the temperature of the two plates, but depend upon the ratio of alpha 3 and epsilon 3.

Now do we know alpha 3, the answer is not quite as a metal fact, we have little bit of problem here because, this alpha 3 is not same as this alpha 3. There is an interesting issue must look at because, the radiation that is absorbed by 3 from 2 will be somewhat different from the radiation 3 absorbed by 3 from 1 because, 1 and 2 are not same temperature. We have to give a slightly different notification here; otherwise so we will go back and slightly modify this.

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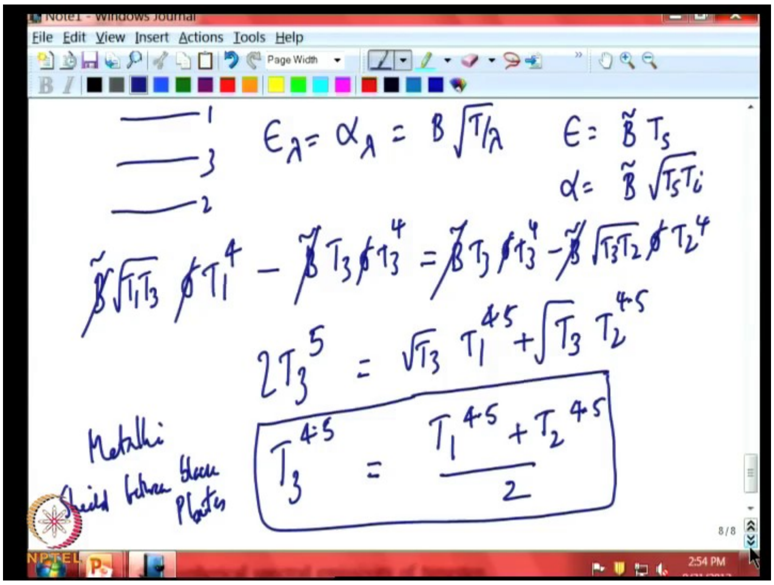

Let us now, recall that for metals emissivity equals absorptivity goes as B by root surface by lambda. Because of that we also derived this earlier, that alpha goes as a epsilon first lag. We know, this epsilon goes as a constant times T of the surface and alpha goes as the same constant times root of surface temperature and the incoming temperature of that radiation, assume that applies body radiation. Given this now we will write down, what is emitted by 1 as sigma T 1 to the power of 4 and already absorb it 2 will depend on temperature of 2 and temperature of 1.

This will be B delta root of T 1, T 2, T 3; T 3 is a temperature of the emitter 1,3 and 2. This 3 is the metal. So, what it absorbed depend on the temperature of the emitter and the absorb. Now minus what is emitted will only depend on B delta and T 3 sigma T 3 to the power of 4, this will be equal to on this side again what is emitted is only d delta emissivity times emission. What is absorbed here will depend on B delta and root T 3, T 2 because, these are the two temperatures involved here and what is emitted is T to the power of 4.

Once more sigma are cancelling out very nicely, B delta is also cancelling out is common to all of them. What we obtain as 2 T 3 to the power of 4 is equal to root T 3 T 1 to the power of 4.5 that, these 2 plus again do T 3 and T 2 the power 4.5 divided by 2. The n take root 3 out here so, absorbed by 2 is not necessary now. We take the 2 down and this T 3 this side. Finally, T 3 to the power of 5, this should be 5, because T 3 it is good T 3 to the power of 5, 4, 4 and 1 5. We take root 3 back here, now we will get T 3 to the power of 4.5 is equal to T 1 to the power 4.5 plus T 2 the power of 4.5 by 2.

This is the final result which is for a metallic shield between two parallel plates. The temperature of a metallic shield between black plates, then show that our understanding of the problem has been quite good, we will look at one more example which is the counter part of this.

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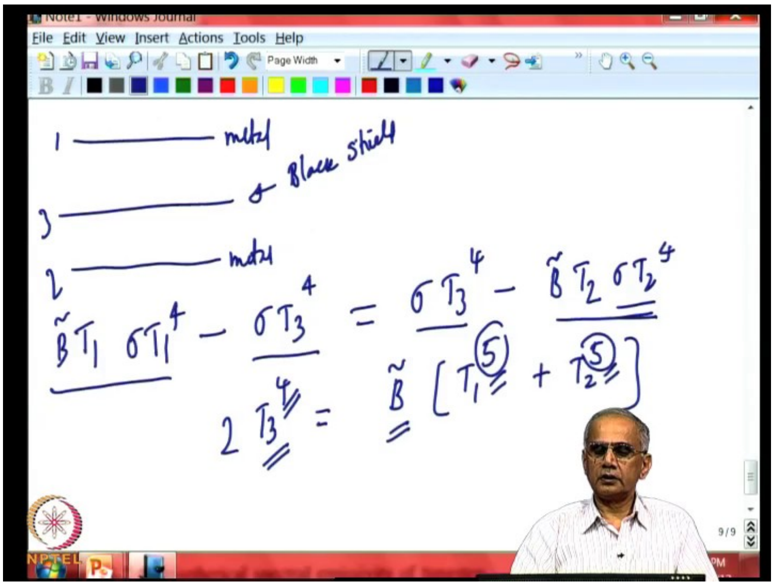

Now suppose, we have a metal, two metal plates and a black shield. See here two metal plates and a black shield, if we have got familiar with this approach We can write the instructions that radiation emitted by 1 and absorbed by the shield will be here T 1, T 3 minus what is emitted by shield 3 that is here. This is the radiation, that is emitted by 1 and absorbed by 3, 3 is the black shield. So, we have to correct this 3, the black shield. So, what is emitted by 1 is B delta T 1, that is the emissivity of metal times the black body emission minus what is emitted back by 3. So we are doing a heat balance of 3, this is what 3 will absorbed emitted.

Similarly, what is emitted by 3 is like this and what is coming back for the metal and absorbed black shield or we can have a metal is B tilda T 2 times sigma T 2 the power 4. So this is what the emitted black by the metal and the entire thing is absorbed by the black shield. So here we can see that, we have sigma cancelling out and we have T 3 to the power of 4 twice of that is equal to B tilda T 1 to the power of 4.5 plus T 2 the power of 4.5. We can see that in this case, the result is somewhat different from what we got for the a metal shield between two black plates and this is not surprising because, what we are getting here is the fact, that the black shield emits as T 3 the power of 4. Final result must depend on that while, I am sorry this is T 1 to the power 5 here I am sorry while the emission from the two metal plates top and bottom will be as T 1 to the power of 5, and the current flow will depend on both the value B tilda here and the way the metal emits, the two metal plates emit.

The very under of this problem is that, the metal plate emission will govern these value and the black plate emission is govern by T to the power of 4. This gives two example say one in which there were metal shield in between two black plates, the other whether the black shield between two metal plates and the results are somewhat different, indicating the important role played by the spectral variation emissivity and absorptivity of the material. Now it’s worth looking here and understanding the fact, that in this derivation the absorptivity of the metal plate now rolled because, we were doing heat balance on the black shield. The black shield observes all the emission from the metal, it is emits like a black body.

This difference is the current flow into the, into the black shield and when on this side, this what black shield emits and what the black shield absorbs is, all the radiation coming from the metal on the other side and This result is somewhat continuatively, but still have very useful result to have. Now so, far we have been dealing with cases where the surfaces are 1 or at the most 2, but we should also know, how to extend this derivation to situation, when the number of surfaces enclosure is more than 2.

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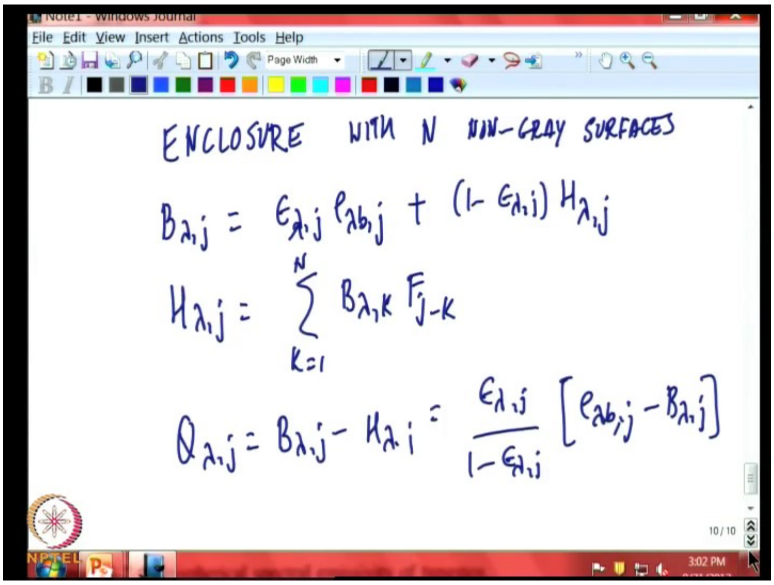

We want a general formulation which we must have and this general formulation of an enclosure with N non-grey surfaces. If we have enclosure with N non-gray surfaces then, for the jth surface the radiosity, will be defined as the emissivity of the jth surface times the black body emission of the jth surface plus what is reflected of the radiation irradiation on the jth surface and the radiation of the jth surface is nothing but, what is leaving the kth surface and after applying reciprocity we have F j k here and this is sigma k 1 to N. This is all similar to what we did for gray diffuse isotropic enclosure accept that now, we are treating everything at a specific wave length.

We can now derive what is Q lambda j which is B lambda j minus H lambda j and this is epsilon lambda j minus 1 minus epsilon lambda j into e lambda b j minus B lambda j. All this is very similar to the gray case accept that now we are treating results at every lambda.

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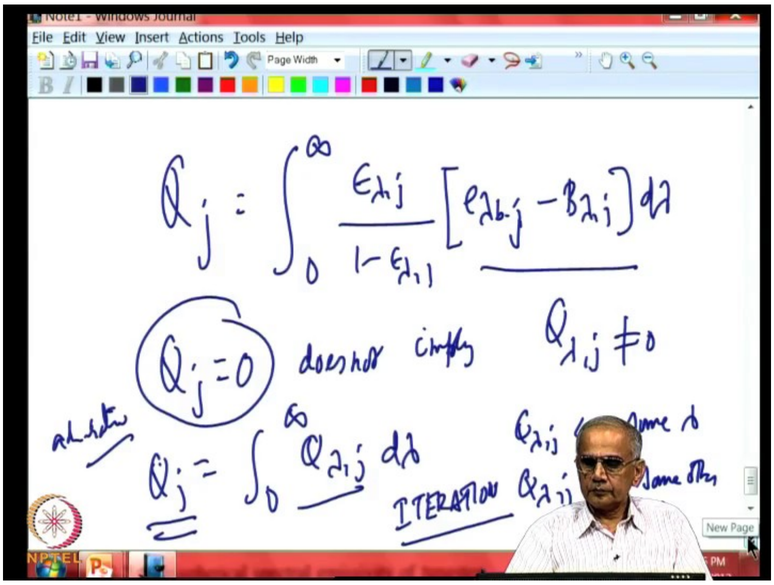

Now, what is Q j. Ultimately we are interested in Q j. The Q j is equal to 0 to infinity epsilon lambda j by 1 minus epsilon lambda j into e lambda b j minus B lambda j d lambda. So, although most of the results are very similar to the gray case, but we must remember, that Q j equal to 0 an adiabatic surface does not imply that Q lambda j is 0 because, Q j by definition is 0 to infinity Q lambda j d lambda, but this is 0 does not mean this is 0 because, what we are saying is that integral is 0, if Q lambda j is negative for some lambda and in some other range of wavelength it is posted. So, is possible for this integral to come to 0 because, Q lambda j is changing sign and this is quite common in many example like furnaces; that in certain wavelength the range the surface gains energy by radiation. In some other wavelength range it losses energy radiation. The integral of all wave length is 0. This shows what posses present challenge, in solving enclosure problem in non-grey surfaces, if one other surface in the enclosure is adiabatic.

Then, all we know that this integral is 0, that does not mean these two are equal. We cannot use this equation to infer anything about e lambda b j or b lambda j. All we can do this do an iteration. So, iteration techniques are the only technique available, we guess a temperature.

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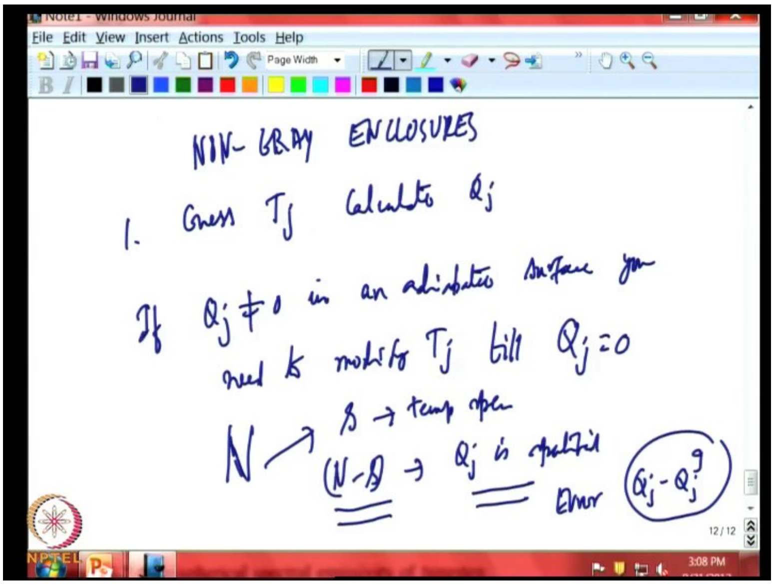

The non-gray enclosures the procedure is; first guess T j, calculate Q j then if Q j in a given surface is not 0, if Q j is not equal to 0 in an adiabatic surface, we need to modify T j. So, we go on modifying temperature until that surface which is adiabatic has Q j or to by till Q j is 0. This is a fairly tedious procedure and if there are N surfaces in a closure and in some other temperature is specified; let us say N surfaces temperature is specified and in N minus S surfaces Q j is specified.

Then we have to go on adjusting all this all this N minus 1 temperatures until Q j reach a specific value. So, we can imagine, that this procedure is not simple it involve a involves changing many temperatures in the iteration procedure, until Q j is either 0 or some other number which is provide in advance. This iteration may not convert easily, but there are powerful mathematical techniques available, to ensure that we are indeed converting to this specified value.

For example, we can give an error as value of Q j 1 minus Q j we have guessed and if the guess value is less than the value given, then we know that the temperature of the concerned surface is too low so, increase it. Then is possible that Q guess becomes larger than Q j, then we decrease it is like the root finding algorithm, that we all use for solving equations not in the equations. So, we have to ultimately make sure that the guest value of Q j and actual value of Q j specified converge rapidly, but today with availability of very fast computers.

It is possible to achieve that kind of convergence fairly rapidly because, it involves good irrational guess in all iteration techniques, we must remember that initially guess is very important. We cannot assume orbited temperatures and expect our technique of iteration to converts to the right answer. The rest be some experience and expertise in guessing, the temperatures of the surfaces with heat specified in a clever way. So, that we get the heat transfer value quite close to the number specified. So, it requires some expertise so, that is acquired quite easily after doing the iodine couple times.

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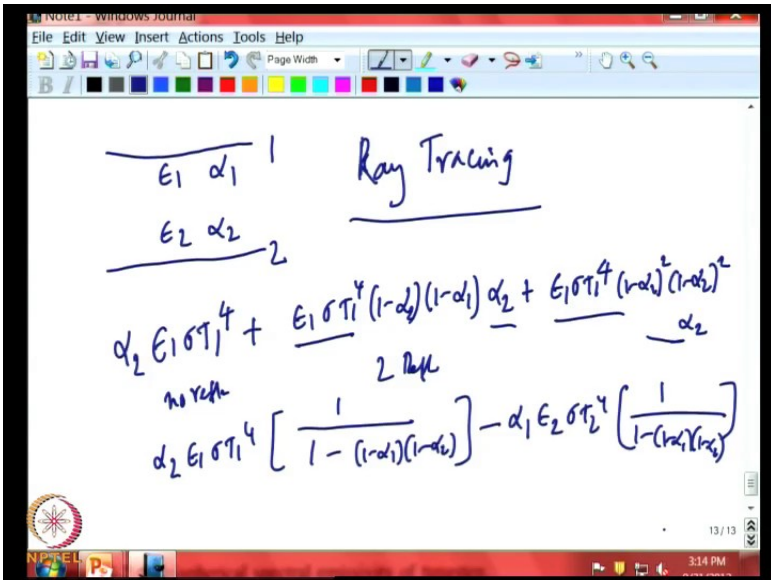

Now, to illustrate the importance of non-grey surfaces I like once more go ahead two infinite parallel plates which are not grey so, epsilon 1 and alpha 1, epsilon 2 and alpha 2. Now we can derive the heat transfer between these surfaces either by following this spectral model under of it or we can do it by ray tracing. Ray tracing is a powerful physically understandable concept, now ray tracing is getting very, very popular today because, of the fact that computers are quite often in terms the speed and so, is possible actually do radiative transformations just by following photons. Just tracing photons from the birth to the death; we just start to the photon emitted by one surface keep find the photon as it reflects and ultimately once its absorbed that life of the photon is over and we come the last place, where the photon was absorbed as having gain heat; then we look under a photon emitted by under surfaces. So, good geometric rate terracing software along with what it is called Monticello technique can solve any complex problem from nearly following a large number of photons; may be 10000, may be one other 1000 or a million photons and doing estuarial average. Now this technique does not give we some any physical incite, but it is a very useful technique to deal with from the complex geometry, from the complex spectral variation.

It is getting more popular, specially now there are standard Monticello packages available which will do the job of generating then the numbers to make a decision, whether certain photon is absorbed or not absorbed. So, it is good to understand a gain problem from the perspective of ray tracing so, that we will do with this example. Now no need that surface one is emitting the radiation This much radiation is emitted and its service to an there is a property that is absorbed so much, but if it is not absorbed this emission will be reflected by surface to once it reflected, we look at when we will come back again, we will come back with reflect surface 1; then it is absorbed the surface 2. This process can continue. This is no reflection, 2 reflections then we can add here 4 reflections which will be like this. This can expand. This all over we can see that ultimately we are left with the infinite series because, all are contain the common term and so, it will be 1 by 1 minus. So, that is radiation living 1 and it is absorbed by 2.

Similarly radiation living 2 absorbed 1 will be a similar expression like this. So, net heat transfer between the two surfaces, based on this result can be written in a very elegant and simple way.

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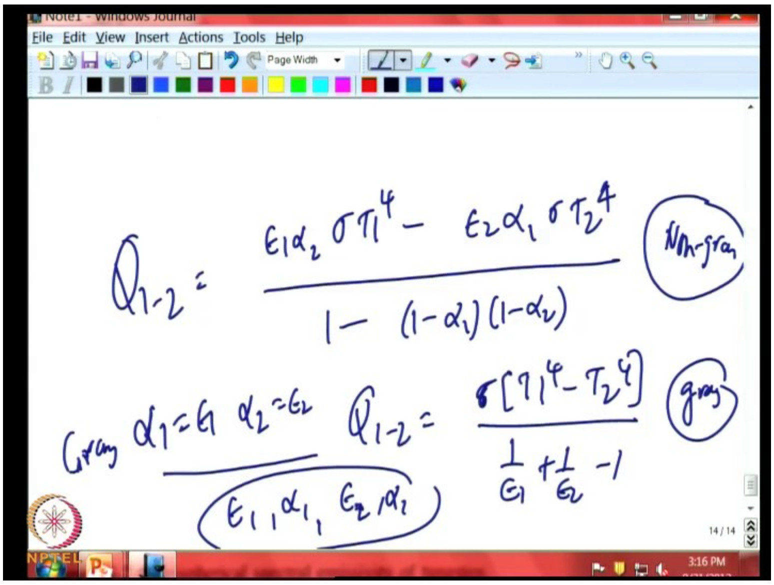

This general result for a non-gray surface, where the emissivity or absorptivity are not equal and clearly demonstrating the role played reflection in the denominator, emission observation with nominator is sport out very clearly. Suppose we have a gray surface then of course, alpha 1 equal to epsilon 1, alpha 2 is equal to epsilon 2. The epsilon 2 becomes common here, there one anywhere cancels out. Then we can easily show, that this is same as what we have derived for gray surfaces. So, what we have done really is the general relationship, when alpha 1 and epsilon 1 are not equal non-gray and ultimately looked at how this simplifies millimeter assumption.

The result is more generally the one above and if we can estimate from some properly data epsilon 1, alpha 1, epsilon 2, alpha 2 then of course, we have got a more accurate result then this which assumes gray surfaces. We can also derive the same result by so many other means, but each bond recognize that the non-gray situation occurs more commonly in the industry; there are very few truly gray surfaces, that we encounter in the real world and we have to deal with non-gray surface.

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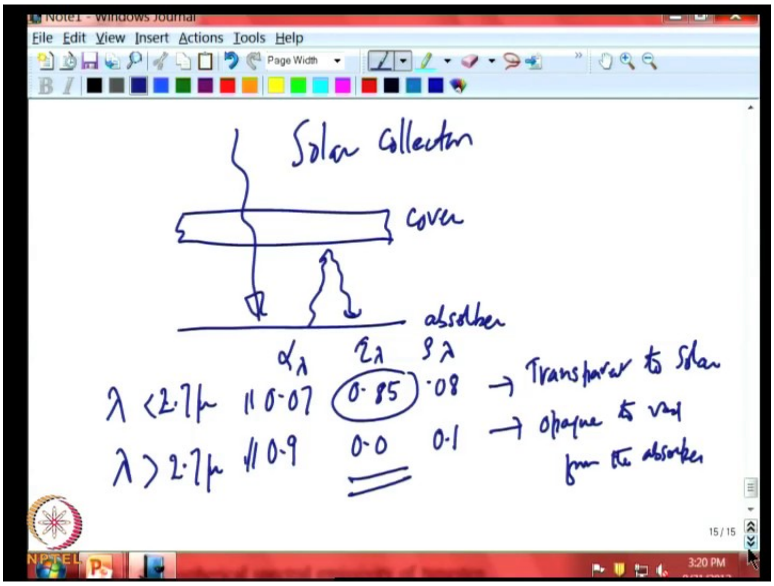

We will just illustrate example of a solar collector, all of us are familiar with solar collectors which are used for collecting solar radiation in solar water heaters. If we look at solar collector, most important common in solar collectors is, the cover which is usually glass, it can also be a plastic and we want to see the typical property of glass special property. If we are in a waiver length region bellow two point some micron and the absorptivity of glass is very small 0.07 transitivity, transfer both the radiation in this region below 2.75 and reflection which is bound to occur in the order of 0.8 some is equal to 1. This is the in the region where visible and infrared, but we go to further up in the infrared beyond 2.7 micron, then situation in changes total reflectivity increases 0.1 the absorptivity decreases to 0.9 and transitivity is 0.

This property of glass is what we tried explore in a solar collector. So, it is transparent to solar radiation, transfers almost 85 percent or can be even be more, while it is opaque to radiation from the absorber,. Absorptivity is typically a temperature of around 70, 80, 90 degree centigrade. At the temperature the absorber emission relation in the range from 4 micron to 100 micron in which range there is no transmission. So, interestingly the glass surface which is so, transparent to the visible, it totally opaque in the profile base like a black body in the fervent, but it is totally transparent in the solar region.

These are the kinds of unusual surfaces, which give us a special advantage for designing collectors, which will allow the radiation from sun to come through, but radiation from the plate is turned back. This is the advantage of non-gray surfaces which have very different properties between with some absorptivity 0.7 here two point some micron and its 0.9 above 2.7 and the transitivity is exactly opposite. This what is used in solar collectors to trap the solar radiation within the collector and hence increases temperature. So, same thing happens on earth.

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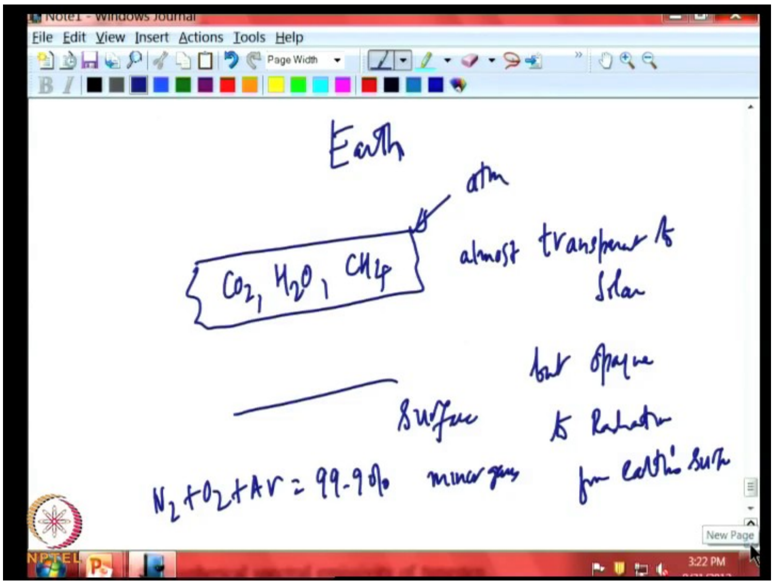

On earth the surface of the earth and we have an atmosphere containing gases like; carbon dioxide, water vapor, methane which are all almost transparent to solar radiation, but opaque to radiation from earth surface. The presence of this minor gas, there all minor as that earth contains mostly Nitrogen, Oxygen and Argon. These 3 gases makeup the most 99.9 percent of the Earth's surface, but they play no role in radiated transfer of either the sun's radiation or the earth's radiation. It is the minor gases which constitute a very small concentration, they are the ones enclosing the Earth's climate because of their ability to trap it.

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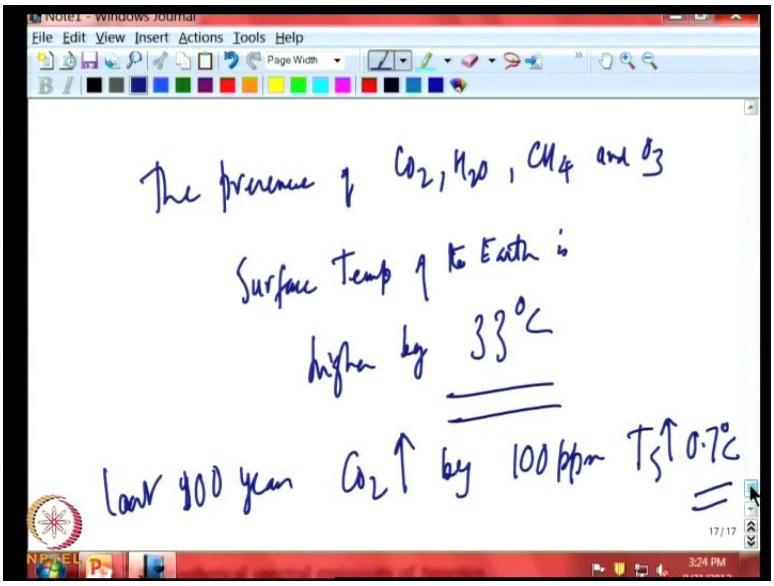

So, because of the presence of these gases like ozone. All these are gases which trap the Earth's radiation. The surface temperature Earth is higher by about 33 centigrade. This is a huge warming that occurred on Earth due to naturally occurring gases like carbon dioxide, water vapor, methane, but for the last 100 years we have increased C o 2 by around 100 p pm and this has increased the temperature by around 0.7 degree centigrade. This is because high amount of C o 2 traps more of the Earth's radiation and this leads to changes reaction there which water vapor also goes up, ice melts in the polar reason; all this combined together it increase the 0.7 and the prediction as that in the next 100 years the temperature make go up by another 4 to 5 degrees. This shows the power of the non-gray surface that exists in the Earth atmosphere, and so we will look at these issues later when we talk about gas radiation.