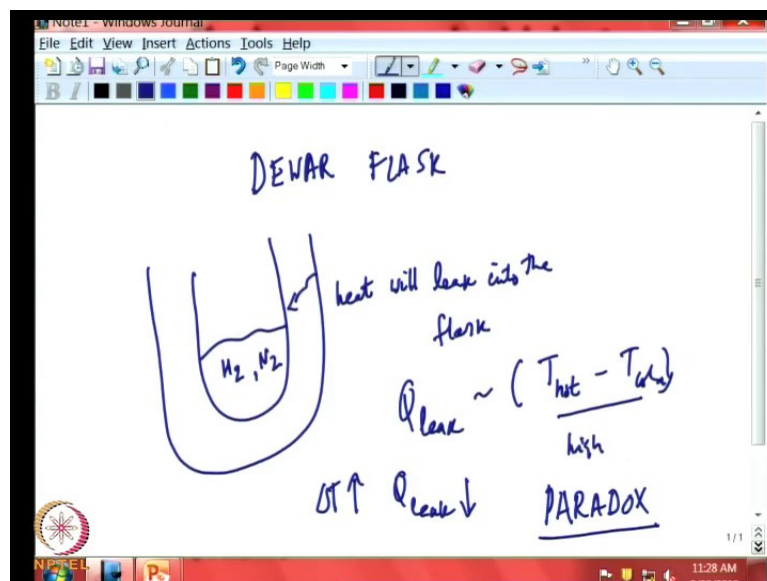


Radiation Heat Transfer
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Lecture - 10
Applications

In the last lecture, we looked at radiative transfer in an enclosure containing gray diffuse isotropic surfaces. We will now look at situations, where the assumption of a gray surface is not good. We saw earlier, when looking at radiative property of surfaces that there are many materials which cannot be assumed to be gray; that is their emissivity and absorptivity are functions of wavelength. Now, that immediately causes a problem. We will take a few examples today to illustrate how the spectral variation of properties of materials causes very unusual situations, which is not encountered when we deal with gray surfaces.

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The example we are going to take is a Dewar flask. All of us have seen it, in many situations, where people carry liquid nitrogen in these flasks to the laboratories. These are essentially containers, which contains liquid nitrogen or liquid oxygen or hydrogen, and they are like thermos flask, but of a larger size. They are used to let us say to store liquid hydrogen or liquid nitrogen. These are at very low temperatures, typically below 100 degrees Kelvin and so we will imagine that from the ambient, heat will leak into the flask.

The main purpose of this Dewar flask is to cut down the amount of heat leaking from the outside into the flask, because as the heat leaks in the liquid will start boiling and we will lose the liquid. Now normally, if we have looked at the heat transfer literature, generally it is believed that heat leakage in this case will be proportional to the difference between hot temperature and the cold temperature. This is the well known Newtonian cooling law, which says that the heat transfer from a hot surface to the cold surface is essentially proportional to the temperature difference.

If this is high, then we will expect the heat leakage to be high, and vice versa. This is true in most situations, as the temperature difference between the hot and the cold object increases, more heat is transferred. But today, we will see in the Dewar flask case, there is, a new situation, wherein the heat transfer is not proportional to the temperature difference. Not only that, that there are situations wherein, as the temperature difference increases, the heat loss actually decreases. So, as the temperature difference increases, the heat leakage decreases. This is completely counter-intuitive result; goes against all the previous understanding of the nature of heat transfer between two surfaces. This shows that, radiation heat transfer has some unusual features, which make it different from conduction and convection heat transfer.

In conduction and convection heat transfer, in most situations, we can be quite sure that the, if the temperature difference between the two objects increases, then the heat transfer between them by conduction or convection will be larger. To encounter situation here, radiative transfer can decrease, as the temperature difference increases, in what is called a paradox. Our aim will be to understand this paradox, in the context of what we have learnt so far about the way emissivity variation varies with wavelength and temperature, and how radiation is exchanged between two bodies.

(Refer Slide Time: 04:51)

Dewar flasks are used widely to store cryogenic liquids like Nitrogen, Hydrogen and Helium.

The boiling point of Hydrogen is 20 K when the pressure is 1 bar while the boiling point of Nitrogen at 1 bar is 76 K.

If the ambient temperature is 300 K, then the temperature difference between the outside and inside walls of the flask will be 280 K when liquid Hydrogen is stored in it but it will be 223 K when liquid Nitrogen is stored in the flask.



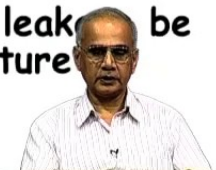

So, just to recap, Dewar flasks are used to store cryogenic fluids like nitrogen, hydrogen and helium. The boiling point of hydrogen is 20 degrees Kelvin, when the pressure is 1 bar, while that of nitrogen is around 76 Kelvin. If these liquids are carried in Dewar flask, and the ambient temperature, the surroundings is around 300 Kelvin, room temperature, then the difference between the ambient and the hydrogen contained in Dewar flask is 280 degree Kelvin, while that between the ambient and the flask containing liquid nitrogen is about 223. So, by normal wisdom, we would expect the heat transfer into the Dewar flask containing hydrogen to be larger than that containing liquid nitrogen, because the temperature difference is larger, but it was found by experiment that this is not so.

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Hence one should expect that the heat leakage into to Dewar flask with liquid Hydrogen should more than the in the flask with liquid Nitrogen.

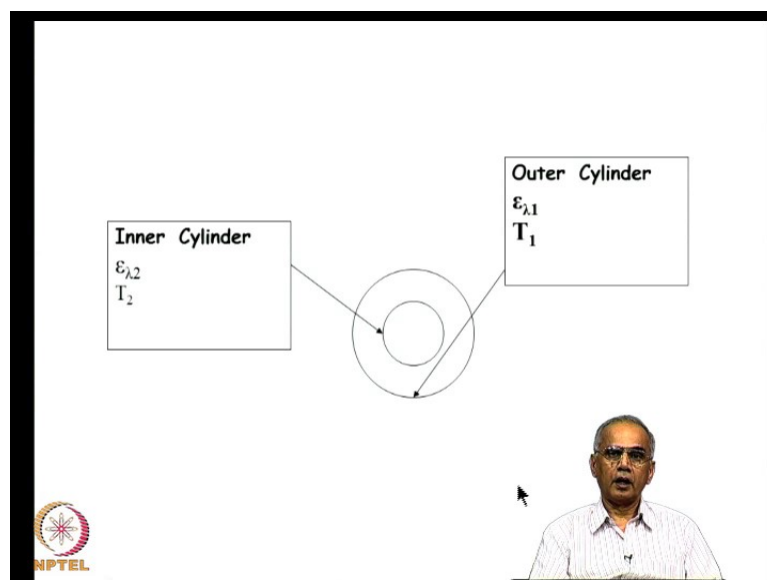
Laboratory observations show, however, that the heat leakage into the flask with liquid nitrogen in about 60% more than the heat leakage into the Dewar flask with liquid hydrogen in it .

Why should the heat leak be higher when the temperature difference is lower?



Laboratory observations show that, heat leakage into the flask with liquid nitrogen, which has a lower temperature difference, is about 60 percent more than the heat leakage into the Dewar flask which contains liquid hydrogen. The question is, why should heat leakage be higher, when the temperature difference is lower. This is the issue we want to address and understand, from radiative heat transfer.

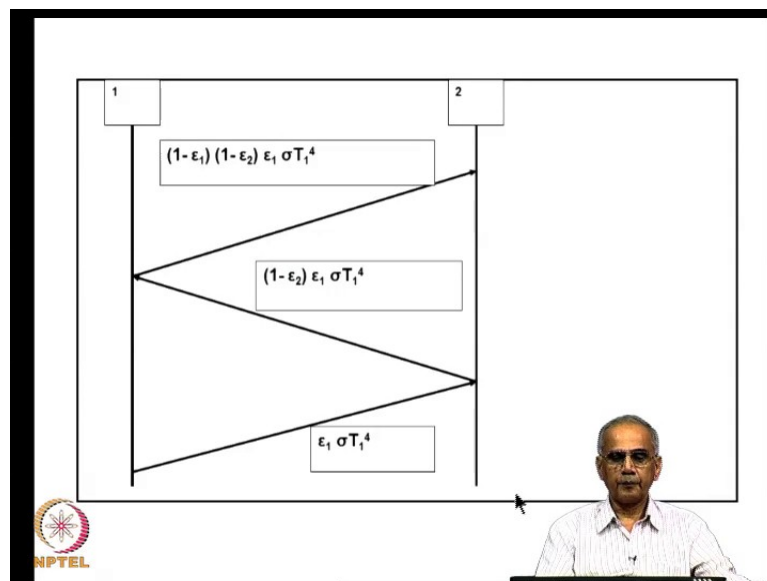
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To do that, we treat the Dewar flask as a simple, two parallel cylinders. The inner cylinder is at temperature T_2 , which is close to the boiling point of hydrogen or nitrogen and the

emissivity of the outside surface which is silvered is ϵ_1 . We are reminding ourselves that, the emissivity of the surface is not constant; it is not gray; it is a function of wavelength. Similarly, the inner surface of the outer cylinder which is also silvered is ϵ_2 and the outer temperature is T_1 . Our aim is to get an expression for the heat leakage from the outer to inner cylinder through radiation, when the emissivity of the two surfaces varies with wavelength. We have discussed this issue when discussing the radiative properties of surfaces and since we are talking about here, silvered surfaces, silver is a metal, we had discussed, quite in detail, how the emissivity of metals varies with temperature.



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The simplest way to understand the heat transfer between the two surfaces is to do a simple ray-tracing, the radiation emitted by surface 1 reaches surface 2; is partly absorbed; the remaining part is reflected and it is reflected back to surface 1 again. One can add up the total number of photons arriving from 1 to 2 and being absorbed at 2 as an infinite series; the direct contribution after 2 reflections, 4 reflections, 6 reflections so on and this standard ray-tracing technique.

(Refer Slide Time: 08:36)

Newton's cooling Law

$$Q_{\text{conv}} = h (T_1 - T_2)$$
$$Q_{\text{rad}} = \sigma (T_1 + T_2) (T_1^2 + T_2^2) (T_1 - T_2)$$
$$Q_{\text{rad}} = h_r (T_1 - T_2) \text{ where}$$
$$h_r = \sigma (T_1 + T_2) (T_1^2 + T_2^2)^{\uparrow}$$


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$$Q_{2a} = \epsilon_2 \epsilon_1 \sigma T_1^4 \{1 + (1 - \epsilon_1)(1 - \epsilon_2) + (1 - \epsilon_1)^2 (1 - \epsilon_2)^2 \dots\}$$
$$Q_{1a} = \epsilon_2 \epsilon_1 \sigma T_2^4 \{1 + (1 - \epsilon_1)(1 - \epsilon_2) + (1 - \epsilon_1)^2 (1 - \epsilon_2)^2 \dots\}$$
$$Q_{\text{net}} = Q_{2a} - Q_{1a}$$
$$= \epsilon_2 \epsilon_1 \{T_1^4 - T_2^4\} \{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2\}^{-1}$$


This is how it looks; the radiation absorbed by 2 from that emitted by 1, is an infinite series and finally, the net radiation exchange between two surfaces can be written as follows. These results are derived by ray-tracing. It could also be derived in, by using electrical analogy. This is a well-known result. This result is very simple and because in these results we have not allowed for the variation of emissivity with temperature and wavelength.

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Newton's cooling Law

$$Q_{\text{conv}} = h (T_1 - T_2)$$
$$Q_{\text{rad}} = \sigma (T_1 + T_2) (T_1^2 + T_2^2) (T_1 - T_2)$$



$Q_{\text{rad}} = h_r (T_1 - T_2)$ where

$$h_r = \sigma (T_1 + T_2) (T_1^2 + T_2^2)$$

The slide includes an NPTEL logo in the bottom left corner and a small video inset of a man speaking in the bottom right corner.

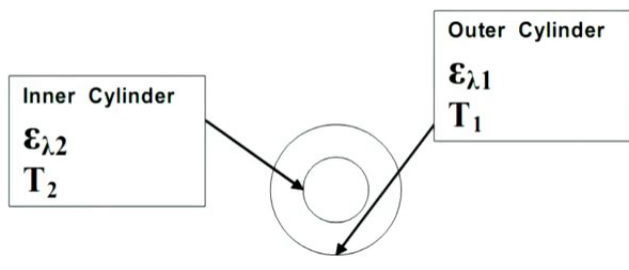


This result can actually be expressed in a simple way, in terms of Newton's cooling law. This standard law that comes from convection is, heat transfer coefficient times temperature difference; because it is radiation, if it is a, two black surfaces, it will be T_1 to the power of 4 minus T_2 to the power of 4; we can expand it and we can see that, these two terms do not vary that much when we vary T_1 , T_2 , because this is in degrees Kelvin, while the real difference comes in here. Many people write, Q radiation also as radiative heat transfer coefficient times the temperature difference. They make it look like convection. If this was true, then the heat transfer by radiation should be proportional to temperature difference. But this does not happen, in the case of Dewar flask. In a Dewar flask, it is found that, when the ΔT is higher, the Q radiation is lower.

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$$\begin{aligned} Q_{2a} &= \epsilon_2 \epsilon_1 \sigma T_1^4 \{1 + (1 - \epsilon_1)(1 - \epsilon_2) + (1 - \epsilon_1)^2 (1 - \epsilon_2)^2 \\ &\quad \dots\dots \} \\ Q_{1a} &= \epsilon_2 \epsilon_1 \sigma T_2^4 \{1 + (1 - \epsilon_1)(1 - \epsilon_2) + (1 - \epsilon_1)^2 (1 - \epsilon_2)^2 \\ &\quad \dots\dots \} \\ Q_{\text{net}} &= Q_{2a} - Q_{1a} \\ &= \epsilon_2 \epsilon_1 \{T_1^4 - T_2^4\} \{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2\}^{-1} \end{aligned}$$


The expression we just now derived, assuming gray diffuse isotropic surfaces is not quite accurate, and we need to look at an alternative derivation.

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$$\epsilon_{\lambda 1} = B(T_1/\lambda)^{0.5}$$


In the new derivation, now, we take into account the fact that, the spectral emissivity of metals, because the outer surface of the inner cylinder of the Dewar flask and inner surface of outer cylinder, are both silvered. Silver is a metal and for that, we can assume that emissivity is a constant times temperature of that surface divided by lambda to power of 0.5. This is the



expression which we had earlier looked at and showed how it affects both the total emissivity and total absorptivity of metals. Now, we are going to use this result.

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Spectral Radiation Heat Transfer

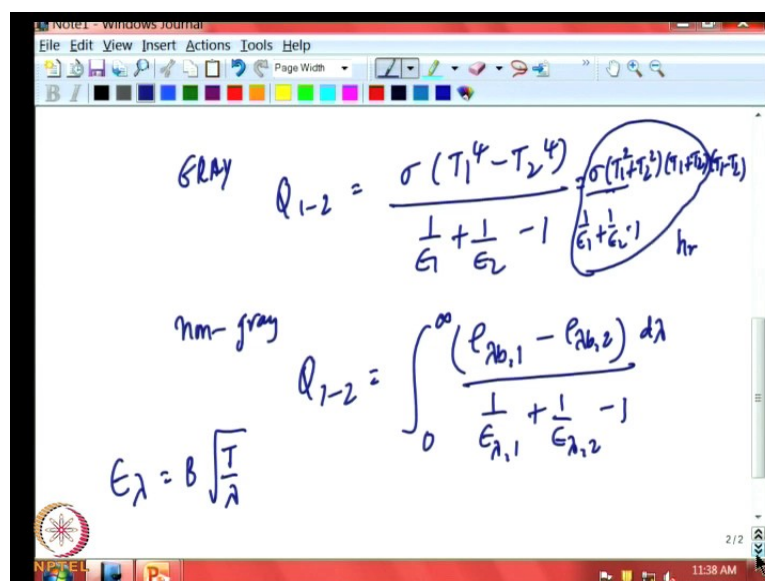
$$Q_\lambda = \epsilon_{\lambda 2} \epsilon_{\lambda 1} \{e_{\lambda 1} - e_{\lambda 2}\} \{\epsilon_{\lambda 1} + \epsilon_{\lambda 2} - \epsilon_{\lambda 1} \epsilon_{\lambda 2}\}^{-1}$$

$e_{\lambda 1}$ = Spectral Blackbody emissive power at T_1
 $e_{\lambda 2}$ = Spectral Blackbody emissive power at T_2
 $\epsilon_{\lambda 1}$ = Spectral Emissivity of Surface 1
 $\epsilon_{\lambda 2}$ = Spectral Emissivity of Surface 2

The expression for heat transfer in wavelength range λ to $\lambda + d\lambda$ will now become, as an extension of the previous derivation for gray surfaces, this is now a non-gray surface, can be written as $\epsilon_{\lambda 2} \epsilon_{\lambda 1} (e_{\lambda 1} - e_{\lambda 2})$ divided by this quantity. This result is very close to what we derived.

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GRAY $Q_{1-2} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \cdot \frac{\sigma (T_1^4 - T_2^4) (\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2)}{\epsilon_1 + \epsilon_2 - 1} \cdot hr$

non-gray $Q_{1-2} = \int_0^\infty \frac{(e_{\lambda b,1} - e_{\lambda b,2})}{\frac{1}{\epsilon_{\lambda,1}} + \frac{1}{\epsilon_{\lambda,2}} - 1} d\lambda$

$E_\lambda = b \sqrt{\frac{T}{\lambda}}$

If the surfaces are gray, then Q1-2 could be written like this. This is the expression we write, but if it is non-gray, then Q 1-2 becomes 0 to infinity, $e_{\lambda 1} - e_{\lambda 2}$ divided by $1 + \epsilon_{\lambda 1}$ plus $1 + \epsilon_{\lambda 2} - \epsilon_{\lambda 1} \epsilon_{\lambda 2}$. This expression will be same as the expression above, if these quantities are independent of wavelength and we can take it out; then we integrate this to get that result. Now, we cannot do that, because we just now saw that, ϵ_{λ} goes as B into \sqrt{T} by λ . ϵ_{λ} is a function of λ .



We have to do this integration very carefully to get the right result. This result at the top can be written as similar to what we have discussed for convection. If we treat this quantity as not a strong function of temperature, then this is what is called the radiative heat transfer coefficient. This kind of result is used frequently by many people and it is quite adequate in those situations where the surfaces are gray. But when we come to a non-gray surface, like a metal, this is not going to work and we saw that observation with Dewar flask shows clearly that, the heat transfer rate is not proportional to the temperature difference. This is the point we want to look at further.

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Spectral Radiation Heat Transfer

$$Q_{\lambda} = \epsilon_{\lambda 2} \epsilon_{\lambda 1} \{e_{\lambda 1} - e_{\lambda 2}\} \{\epsilon_{\lambda 1} + \epsilon_{\lambda 2} - \epsilon_{\lambda 1} \epsilon_{\lambda 2}\}^{-1}$$

$e_{\lambda 1}$ = Spectral Blackbody emissive power at T_1
 $e_{\lambda 2}$ = Spectral Blackbody emissive power at T_2
 $\epsilon_{\lambda 1}$ = Spectral Emissivity of Surface 1
 $\epsilon_{\lambda 2}$ = Spectral Emissivity of Surface 2





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$$Q_{\text{net}} = \int_0^{\infty} Q_{\text{net},\lambda} d\lambda$$
$$Q_{\text{net}} = \sigma B^* (T_1^5 T_2^{0.5} - T_2^5 T_1^{0.5}) (T_2^{0.5} + T_1^{0.5})^{-1}$$


$B^* = 4.26 \times 10^{-5} \text{ K}^{-1}$ for a silvered surface

For a flask containing hydrogen, $T_2 = 20 \text{ K}$, while for nitrogen it is 77 K . In both cases T_1 can be assumed to be 300 K (ambient temperature). Hence the ratio of radiation heat transfer to the flask containing liquid nitrogen to that containing liquid hydrogen can be written as (after assuming that $T_1 \gg T_2$)



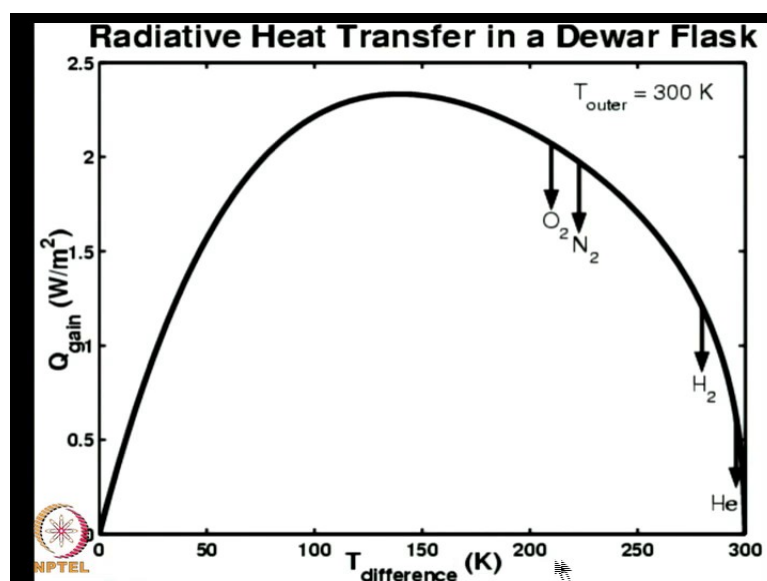
This expression which is written earlier. If we integrate that expression for the spectral flux, assuming the emissivity to be B times root T by λ , then we will get the following expression for the net radiation. Notice that, this is very different from the simple expression involving T to the power of 4 that we obtained for gray, diffuse, isotropic surfaces. This is much more complicated; involves T_1 to the power of 5, T_2 to the power of 0.5, T_2 to the power of 5 and so on, and we will not expect this expression to show a simple dependence of heat transfer rate to ΔT . Here we have a typical number for a silvered surface. We have to calculate this for case, when there is hydrogen; when T_2 is 20 degrees Kelvin; for nitrogen, it is 77; in both cases, T_1 will be taken as ambient temperature. And if we take this equation and remember the fact that, if T_1 is about 5 to 10 times larger than T_2 , so T_1 to the power of 5 will be much greater than T_2 to the power of 5; We can neglect these terms.

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$$Q_{\text{net,N}_2}/Q_{\text{net,H}_2} = (77)^{0.5}(20)^{-0.5}[(20)^{0.5} + (300)^{0.5}]$$
$$[(77)^{0.5} + (300)^{0.5}]^{-1} = 1.63.$$


We take the ratio of heat transfer to nitrogen to heat transfer to hydrogen, then we will get this kind of number, which will come out as around 1.63. This is very close to the observations which indicated that, heat leakage in nitrogen was about 60 percent more than that to hydrogen. This simple spectral, non-gray approximation has given a result very close to the observations. This clearly shows that, this is the approach we need to take, because if we had not taken this approach, but had assumed that, the emissivity is not a function of wavelength, as it is a gray surface, then we would not, we would get a number less than 1 here; because we then assume that, heat transfer is proportional to delta T.

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To make this point, we more dramatic, we take up the case of Dewar flask, where the outer cylinder has 300 degree K and I vary the temperature in the cylinder, so that delta T, when the inner cylinder is 0 degrees Kelvin, delta T is 300; when the inner cylinder is 300, delta T is 0. If the inner cylinder is 300, of course, there is no heat transfer, because the temperature of the outer and the inner cylinder are same. We start reducing into the inner cylinder, the heat transfer will go up and we see that, up to around 150 Kelvin, the heat transfer keeps going up. But then something unusual happens. As the temperature difference goes beyond 150, the heat transfer starts coming down. This is the portion which is of great interest to us. This is the portion where, if we store liquid oxygen, or liquid hydrogen, we will be in this part of the curve. In this part of the curve is where, as the delta T increases, the heat transfer goes down. This is the result which is counter-intuitive, which does not follow the traditional heat transfer texts, which say that, as we increase delta T between two surfaces, the heat transfer should go up; here, it is going down.

And, to the point that, if the inner surface was really at 0 K, then heat transfer is almost 0; although delta T is of the order of 300 K. This is a point which is really what the understanding that, when the temperature difference is small, then roughly, the heat gain is proportional to delta T; but when the temperature of the inner cylinder becomes very low, below 150 K, we start seeing another domain in which the heat transfer is going down, as delta T is going up.


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$$Q_{\text{net}} = \int_0^{\infty} Q_{\text{net},\lambda} d\lambda$$

$$Q_{\text{net}} = \sigma B^* (T_1^5 T_2^{0.5} - T_2^5 T_1^{0.5}) (T_2^{0.5} + T_1^{0.5})^{-1}$$

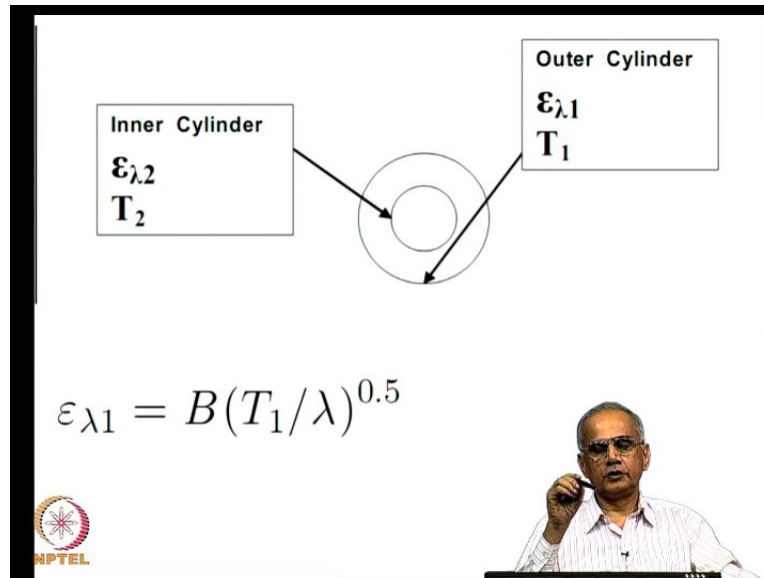
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This can only be explained by the fact, that radiation heat transfer and temperature in the case of radiative heat transfer, is quite complicated; especially, when the two surfaces are not gray, but metals.

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Because of the spectral variation of emissivity with temperature and wavelength, we can expect this unusual result. The simplest way to understand this, is the impact of emissivity on temperature. If the inner cylinder is truly at 0 Kelvin, according to this equation, emissivity is 0. Now, strictly speaking, we cannot use this equation when temperature approaches 0K, because the phenomena of radiation that occurs at such low temperatures, are different from what was assumed in obtaining this equation; but still, we can assume that, although this actual equation may not be valid, that as the temperature goes to 0, emissivity will indeed tend to 0.

This is one important feature and this is the primary reason why the heat transfer to hydrogen Dewar flask is lower than that of nitrogen. Because, hydrogen is at 20 K and at 20 K, the emissivity is about one fourth of the emissivity at, 77K for half of the emissivity at 77 K. What really happens is that. the Dewar flask containing hydrogen, being at a low temperature, absorbs about half the radiation compared to the Dewar flask containing liquid nitrogen, mainly because the emissivity decreases with temperature; that is the main, but if we really want to account for all other phenomena occurring there, including changes in the, how the emissivity, if we take all these into account, we get this 60 percent change; otherwise, we

should get around 2. Because of temperature change alone, the emissivity of the inner surface which is equal to absorptivity, assuming the surface is diffusive, isotropic, then the Dewar containing liquid hydrogen should absorb about half of the radiation emitted by the outer surface in comparison to liquid nitrogen, normally because of this phenomenon. But if we take into account both the temperature effect and the wavelength effect, then one can show that, the result will be somewhat more complicated than which we saw here.

This is a very good example showing how radiation heat transfer behaves in ways, somewhat paradoxically, mainly because of the spectral variation of radiative properties, which can alter the general rule that heat transfer is proportional to temperature difference. We saw this interesting example that, as the temperature of the inner cylinder keeps falling, the heat transfer rate keeps falling, although the ΔT is going up. This is related to the fact that emissivity is a function both of temperature and wavelength, and that plays an important role. By looking at the non-gray nature of the radiative properties, we are able to explain this unusual feature of radiative heat transfer, which is quite from difference from what one encounters in either conduction or convective transfer.

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Temperature control in space through selective surfaces
A flat plate (insulated at the back) is subject to solar radiation normal to its surface
The plate is coated with a selective surface with different emissivities below and above the cut-off wavelength (λ_c)

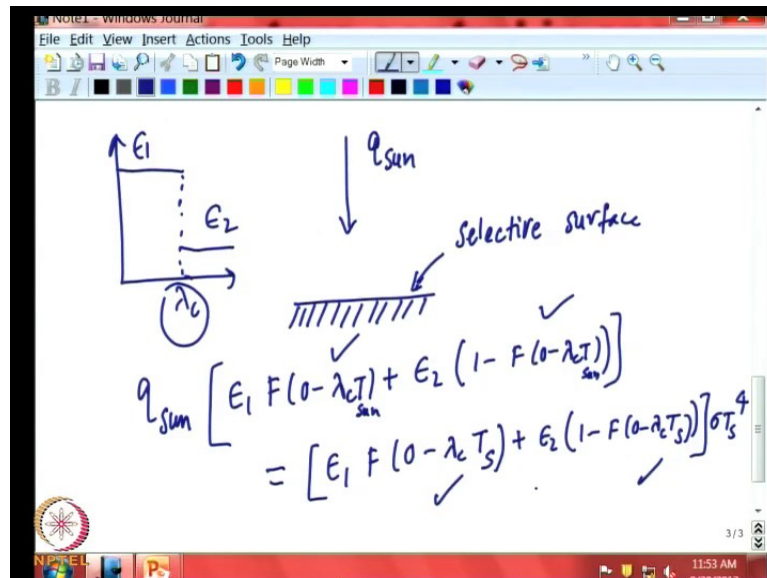
The slide features a graph with a vertical axis and a horizontal axis. The horizontal axis is labeled λ_c at a specific point. To the left of λ_c , the emissivity is constant at a higher value, labeled ϵ_1 . To the right of λ_c , the emissivity is constant at a lower value, labeled ϵ_2 . An inset image of a man in a white shirt is visible in the bottom right corner of the slide frame. The NPTEL logo is in the bottom left corner.

Now, we are going to take another example of the spectral variation of properties having influence on temperature, while looking at temperature control in space, through use of selective surfaces. We had talked about selective surfaces earlier, as surfaces whose radiative properties varies dramatically in two different wave length ranges. So, we have taken a very

simple example in which the emissivity of a surface is a large value; is of one value for a wavelength below a cutoff wavelength called λ_c and above that, it has a totally different value. There is one wavelength at which the property changes dramatically. This is seen quite often, when we coat a semi conductor on a metal, the semi conductors' properties will emerge in this region; while the properties of the underlying substrate metal will be appearing in this region. This unusual sharp change in the radiative property is related to the way in which radiation is absorbed in the semiconductor. Now, such surfaces are used quite frequently in satellite to control temperature of a surface. In a satellite, there may be a requirement to have the temperature of a surface to be very high, or very low; very high would be, in application where we are trying to generate power in a satellite using a high temperature source. In that case, we want to maintain the temperature of the surface very high; that means, we want to absorb as much of the solar radiation as possible in a surface and at the same time, reduce the emission of the surface, so that, the surface remains hot.

This is commonly also useful in solar collectors, where we want to get a very high temperature. We want your collector surface to absorb radiation in the solar region, which is between 0.4 micron and 4 micron; but at the same time, we do not want the surface to emit much radiation and most of the radiation emitted by the surface is usually in the range between a few microns to 100 micron. We choose a cutoff of your wavelength somewhere around 4 micron, so that, we maintain the emissivity of the surface low, in the region beyond a few micron; at the same time, in the region where solar radiation is incident, the low wavelength region, we try to main a high emissivity and high absorpion. This is done deliberately ; let us write down the energy balance for this case.

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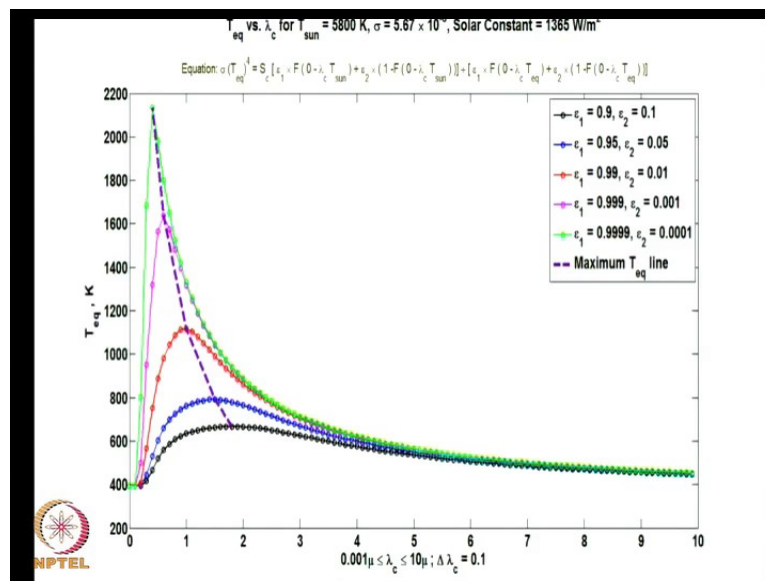
Let us write down the energy balance for this case. We will take a very simple case of a flat plate, insulated here and subjected to solar radiation. This is the selective surface we are talking about. The energy balance will tell us that, Q_{sun} and we already indicated that, our selective surface will be one in which, we will have one emissivity at one wavelength, and at some cutoff wavelength emissivity will go down to ϵ_2 . If that is a case, then the total energy absorbed will be the amount of radiation lying in this region, 0 to λ_c , plus the amount of radiation lying in the remaining region on the right.

This region has to be equal to the heat lost by the surface, which will be $\epsilon_1 F(0-\lambda_c T_{sun})$, this is T_{sun} here; this is 0 to $\lambda_c T$ of the surface; and ϵ_2 into $1 - F(0-\lambda_c T_{surface})$ into $\sigma T_{surface}^4$. This is the main balance. We have neglected here the cosmic background radiation coming on to the surface; we will include that subsequently; because that is very small; only two causes that we are getting is, photons coming from the sun are being absorbed by the selective surface and at the same time, the selective surface is emitting radiation, depending on this temperature T_s . Our aim is to find T_s as the function of this cutoff wavelength λ_c . We want to know, what is the best value of λ_c , which gives the highest temperature. This could be an application, where we are trying to generate power in a satellite, using a high temperature surface as a heat source. We want to know, what is the appropriate cutoff wavelength. This problem can be solved easily, but of course, since we have to calculate these quantities, we

have to integrate the function numerically; this, we have to obtain numerically and this equation is non-linear.

There is no simple, analytical solution available to you. This is solved numerically using standard software which calculates this integrals, and then assumes the value of, T s which will get this balance.

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Let us see what we get. This is the problem and here, at the top we see, this is the equation; this is called the equivalent temperature; this is surface temperature of the selective surface and what we have shown here is, how the equivalent temperature surface depends upon the cutoff wavelength, varied from 0.001 micron to 10 micron, steps of 0.1 micron, solved numerically, and we have taken various cases, one in which the emissivity is 0.1, 0.9 in the solar region and 0.1 in the infra red region; and I have gradually increased this value to, close to 1 and this one very close to 0; we realize, in this case, we cannot make it exactly 0, because if the emissivity of the surface is exactly 0, then it cannot have a steady state temperature, because then it cannot get rid of the heat, and so whatever heat is absorbed, cannot be lost; so temperature will go on increasing. We have to keep this non-zero; so we have kept a very low value. We can see, the lower the value, higher a temperature. In this example, we have gone from a value when emissivity is 0.1, a value close to 600 degrees Kelvin, and as we reduce the emissivity from 0.1 onwards to very low values, this temperature is going up, is increasing,

and has gone up to 2100 degrees Kelvin. And as we take emissivity lower and lower, it will keep going up.

We probably realized, the highest value it can go to is of course, the Sun's temperature, which is assumed to be 5800 Kelvin; we assume sun to be a black body at 5800 degrees Kelvin, and highest we can attain, if at all is that number; but we will not get anywhere near it; unless we make this extremely small. The reason why this temperature increases so rapidly, as we reduce the cutoff wavelength, is because of the fact that, the amount of radiation from the sun is around 1365 watts per meter square; this is quite large.

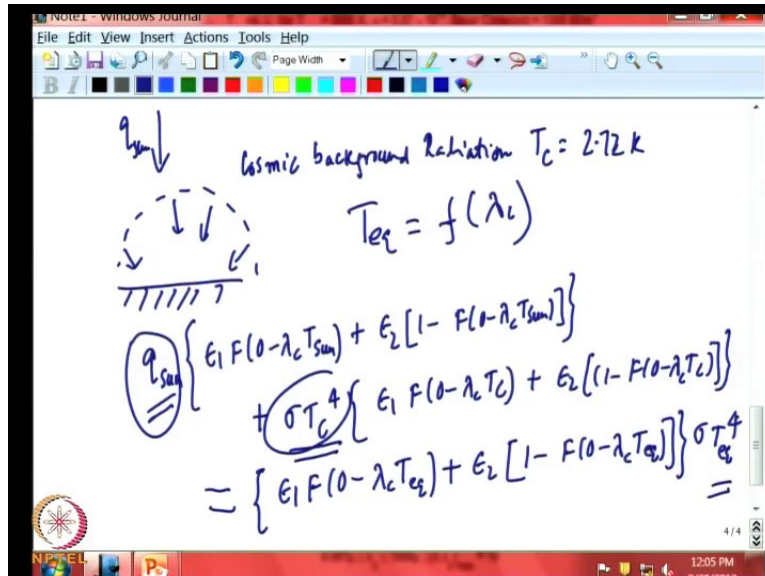
If we recall from the previous lectures, the sun's radiation lies between, mainly between 0.4 micron and 4 micron. If we keep the cutoff wavelength at 4 micron, we will see that, the temperature we get there is around 600K, whatever the values of these emissivity. What it means is that, at 4 micron, the high emissivity in this region, is leading to sufficiently high losses from the surface; the temperature cannot really go beyond 600. We need to push the cutoff wavelength to a lower, lower wavelength, of course, which ultimately will reduce the amount of solar radiation absorbed; that is why we got two lower value, like around less than a micron, then we can see temperature falls sharply, because we are not absorbing sufficient amount of solar radiation, in order to attain high temperature.

There is a trade-off between using a very high λ_c , which causes some of the emission from the surface to be in the region below the cutoff wavelength and lose heat; so to avoid that, we have to reduce the λ_c . But if we reduce λ_c to too low a value, then the sun's radiation plays a role. This line represents the optimal wavelength at which we are able to attain the highest temperature. This depends, of course, on the emissivity ϵ . It depends on this emissivity because this determines how much heat the surface is losing. If this can be cut very, very low, we can reach as high a temperature as we can; as said theoretically, the temperature we can reach is about 5800 degrees Kelvin, which is sun's temperature.

We cannot exceed sun's temperature, because if we did, we will violate the second law of Thermodynamics. Now, we might ask, why we did not go to lower temperature, and lower emissivity values. Actually, there is a problem with numerical convergence as we go to very low emissivity. We have to, we need more and more accurate numerical schemes to calculate this $F(0, \lambda, T)$ values. These are obtained by numerical integration. And those

integrations have to be pretty accurate to the sixth or seventh or eighth decimal place for we to get the right values. That is the main intention.

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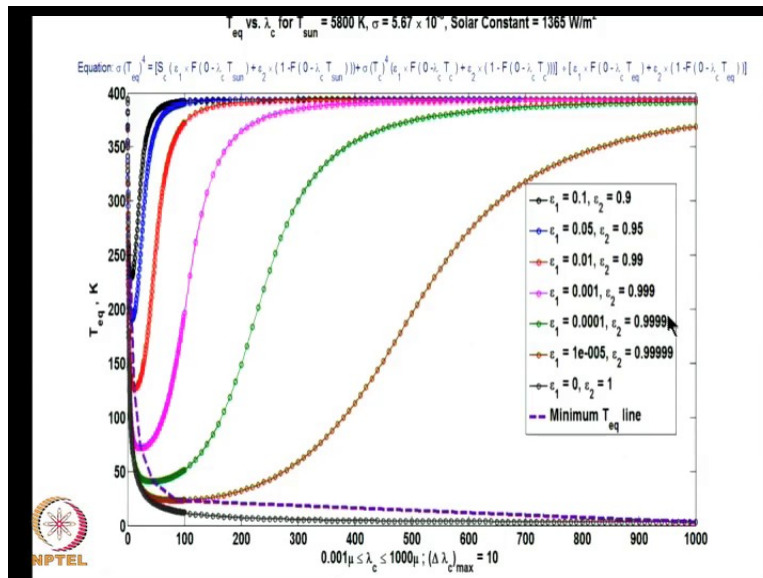
Now, let us look at the next example. Now here, let us rewrite this equation we wrote earlier. In this case, we are trying to keep the temperature cool. In this case we think of an application in space, where a certain surface has to be kept cool, although it is absorbing solar radiation. We want to make the problem a little more complicated. Here is the flat surface; it is subjected to sun's radiation; it is also subjected to radiation from all around. This is known as cosmic background radiation. This is from black body, at around 2.72 degrees Kelvin. This was discovered about fifty years ago. This is radiation coming from the, all around from the universe. It is very close to a black body at about 2.72. Normally, we will ignore this kind of temperatures; they are so low. But in this present example, where we are trying to keep the surface at low temperature, we have to account for this, for completeness sake. In this case, the radiation absorbed will be from the sun, which will be epsilon 1 times F (0 to lambda c T_{sun}) plus, epsilon 2(1- F (0 to lambda c T_{sun})), plus the second term here, is from the cosmic radiation, which will be sigma T_c to the power of 4; here, it will be epsilon 1 F 0 to lambda c T_{cosmic}, plus epsilon 2 into 1 minus F 0 to lambda c T_{cosmic}.

These are the two terms of the incoming radiation absorbed by the surface. This will include emission. Emissivity of the surface is now is going to be epsilon 1 (F 0 to lambda c T equilibrium of the surface), plus epsilon 2 into 1 minus F (0 to lambda c T equilibrium), into

σT equilibrium to the power of 4. We want to find what is the equilibrium temperature of the so called radiative equilibrium temperature, that is the temperature attained by the surface, purely from radiative balances. This quantity as a function of the sun's radiation and the cosmic background radiation. Of course, we would expect that this will play a very important role; because the radiation emitted by the sun is 1365 watts per meter square. This will be very, very low, because at a temperature of around 2 degrees Kelvin, the radiation coming into the surface is very small. We still retain it, because in the limiting case, wherein we are able to prevent absorption of any solar radiation, this term will begin to play a role. So here, our aim is to find that λ_c which will give us the lowest temperature. So we want to find out the T equilibrium as a function of the cutoff wavelength. And in this case we are interested in that cutoff wavelength at which the temperature of the surface is the lowest; because in this application, we want to minimize the temperature of the surface; not maximize it, like we had last time.

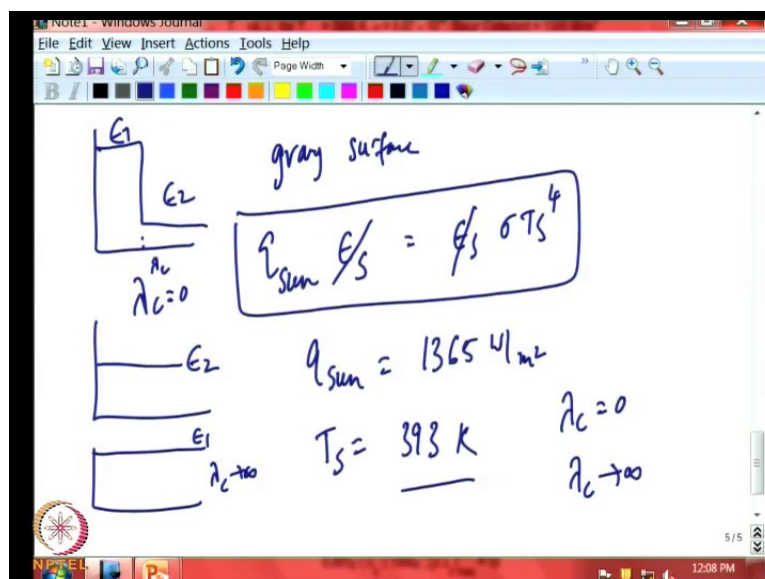
Again we have to solve this equation numerically, because all these factors F_0 , λ_c factors here, they are all obtained by accurate numerical integration. We know these two quantities. We know everything, except T equilibrium. This problem is solved iteratively; we assume some value of T equilibrium and see whether the left hand side, right hand side balance; if they do not, they are adjusted, until they come to balance. This is a process of iteration; has to be done carefully and accurately, because some of the numbers are quite small, but they are still very important. This is best done with commercial software available today in most computers and institutions. We use this to solve this problem iteratively. It is a good exercise in numerical analysis to solve these equations, because it is highly non-linear. It requires some kind of expertise as well as intelligence to ensure that your methodology converges quickly. We take it up and see, how good we are. Let me just show you the answers that we got. This is the variation of the equilibrium temperature with the cutoff wavelength. The cutoff wavelength is varied mostly from around 10, 20 microns to 1000 microns, because we want to reduce the absorption of solar radiation in order to keep the surface cool; so our cutoff wavelength has to be as high as is possible.

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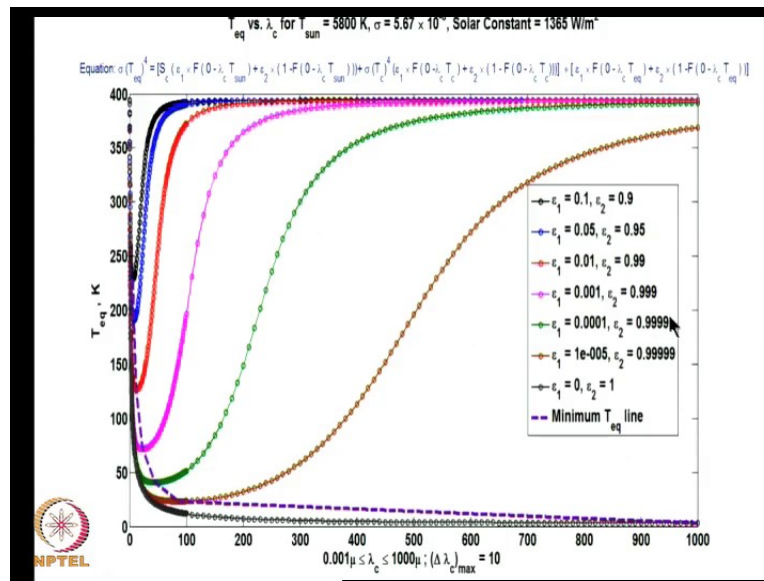
We can see, that as the cutoff wavelength is increased, from around 10 microns, all the way to a 1000 microns, the minimum temperature which is a the dot line here, goes on decreasing; and ultimately, will reach 2.72 degrees Kelvin, asymptotically. But before that, notice this interesting feature, that if your surface had, let us say the green one, emissivity only 0.0001 in the solar region, but a very high emissivity in the region beyond λ_c ; then, we see that, we get this kind of curve.

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That is, when λc is 0, or very large, in both cases, the temperature is close to 393 Kelvin. The 393 Kelvin comes in if the emissivity is, here, a gray surface, then your equation will be, Q_{sun} , $\epsilon_{\text{surface}}$ is equal to $\epsilon_{\text{surface}} \sigma T_s^4$. We can solve this equation very easily, and we will get that, for Q_{sun} of 1365 watts per meter square, we will get T_s is 393 degrees Kelvin. This is the solution when emissivity is not a function of wavelength. This happens for λc of 0, and λc tending to infinity. In both these cases, if we recall, λc tending to 0, then we will have only, it will be equal to ϵ_2 . This is a general function; so λc is equal to 0 here. On the other hand, if λc tends to infinity, we have only ϵ_1 . Both these are gray surfaces. For a gray surface, this cancels out, $\epsilon_{\text{surface}}$; We are left with a very simple balance from which we will get this number. Now, let us go back to the result we saw here.

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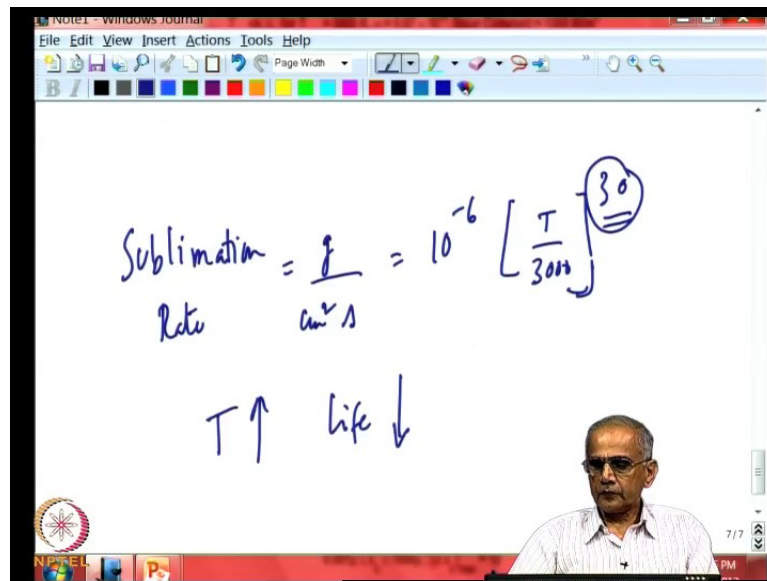
In both these limits, very low lambda c and high lambda c, the temperature approaches the 393 value. In between, it reaches a minimal. In this present case, for this particular case, for example, it reaches at about a 100 micron. At 100 micron, what we are able to do is, we are able to keep the amount of solar radiation that is absorbed by the surface to a minimum, at the same time, keeping the radiation emitted by the surface highest. Only when we do that, we get a minimum value, which around here, is 25 Kelvin. But if we really take this case, where emissivity in the region, one below lambda c is 0, and the emissivity at the higher region is 1, then the minimum temperature goes on decreasing and if we do your computation accurately, ultimately we will reach a temperature of 2.72 degrees Kelvin, which is the temperature of the cosmic background radiation. So, that is the result that we get in this case. These are very interesting problems, and the unusual behavior we see here, is primarily on account of the non-gray nature of the surface. Both in the case of Dewar flask as well as in this case of selective surfaces used in solar, in space applications, we see some unusual behavior, because of the fact that, we allow for the variation of spectral properties with wavelength. So, that brings in rather interesting issues into play. So, these two examples were brought in just to show that, there are situations, wherein the variation of emissivity and absorptivity with wavelength can get very paradoxical result, a result which are counter-intuitive, because of the strong variation of the properties with wavelength. These are two examples, which we wanted to take up.

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Tungsten filament Lamp
melting point 3655°K .
Sufficient radiates in the Visible
Emissive Power $\left(\frac{\text{W}}{\text{m}^2}\right) = 160.5 \left[\frac{T}{3000}\right]^4$
Meth. $e \sim T^5$ $e_b \sim T^4$

Now, before we move on to the detailed treatment of the, how to treat non-radiative, non-gray surfaces, we will give one more example of non-gray nature of surface. This is in the design of tungsten filament lamp. For more than 100 years, this was the traditional lamp used for lighting houses and industries. Of course, today, slowly this lamp is getting phased out, primarily because these lamps are not very efficient. Typical tungsten filament lamp converts around 5 to 8 percent of the electrical energy consumed into visible light. Tungsten filament lamp emits mostly in the infrared. Tungsten filament lamp is not a good way to get light energy; it is a very efficient way to get energy for heating; infrared radiation, for example, is used in industry for infrared heating of various materials. But it is not a very good source of visible light. And this is because tungsten is a material whose melting point is 3655 degrees Kelvin. When we use tungsten filament lamp, we have got to ensure, your temperature is sufficiently below this, so tungsten does not evaporate and disappear. The challenge in design of the tungsten filament lamp is to keep the temperature sufficiently low, so that, it does not lose material rapidly. But at the same time, the temperature has to be high enough, that is sufficient to emit sufficient radiation in the visible. This is the challenge.

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If we keep the temperature too low, then most of the radiation will be in the infrared and not enough in the visible. In order to get a lamp which gives sufficient visible light, we need a high temperature of the filament. But a too high temperature, will lead to destruction of the filament because of evaporation, or actually, strictly sublimation. This trade-off has to be there. The tungsten filament lamp is not a gray surface and that is illustrated, if we look at the emissive power of the tungsten filament lamp, measured in the laboratory, in Watts per centimeter square. Empirically, we see, it is around $160.5 T^{4.48}$. So, notice that, it is not to the power of 4, which is true for a gray body or a black body; it is somewhat, it is not like a true metal; a true metal, if it is a metal as in the manner in which we discussed in the properties of metals, then emissive power should go on as T^5 . Emissivity is proportional to temperature. But it is not following that; black body is to the power of 4. Tungsten material is somewhere between black and metal behavior, in between, 4.5.

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$$P = \frac{V^2}{R}$$

$$P_{\text{radiation}} = \epsilon_f \sigma T_f^4 \pi D L$$

$$\text{Resistivity (ohm-cm)} = 92 \left[\frac{T_f}{3000} \right]^{1.2}$$

Coiled coil
 TWO UNKNOWN L, D
 $D \sim 1 \text{ to } 10 \mu\text{m}$
 $L \sim \text{centimeters}$

But the real issue in the tungsten filament lamp, is the way in which it loses material by sublimation. The sublimation rate, that is conversion from solid to vapor, it has been measured in grams per centimeter square per second, and it is typically, 10 to the power minus 6 into T by 3000 to the power of 30 ; a huge power; that is, as the temperature increased, even 1 or 2 degrees, the sublimation rate goes up as T to the power of 30 . This is really what is going to control the design, because if the temperature goes up too rapidly in the design, then the sublimation rate goes up very rapidly, and hence, the life goes down. As the temperature of the filament increases, life will go down very rapidly. This is the challenge that is faced by the design of the tungsten filament lamp, is to maintain the lamp temperature sufficiently high to ensure that there is more radiation in the visible but at the same time, keep the life high. Essentially, the design involves realizing that the power consumed by the filament is equal to V squared by R ; and the power radiated is the power generated by electrical heating, is radiated is ϵ filament, σT filament to the power of 4 , into area of the filament which is $\pi D L$. These two have to be equal, at equilibrium, neglecting conduction and convection.

At the same time, we must remember that, the resistivity of the tungsten filament lamp in ohm – centimeters, is 92 ohm - centimeter at the standard value of 3000 degrees Kelvin, to the power of 1.2 . This also has to be accounted for. Once we know the temperature of the filament, we calculate the resistivity; from that, we calculate the resistance; then your power

generated; that has to be equal to the power radiated. So, there are two unknowns, here, in this problem. The two unknowns are the length of the filament and the diameter of the filament. We will find when we do the design, for a typical lamp, the diameter may be of the order of 1 to 10 micron; and the length will be of the order of centimeters.

In order to build this we do, what is known as coiled coil. We take a thin filament and then it is coiled many times to get the large length that is required within that small lamp. This is a coiled coil that we arrange inside the lamp; and we can see that, the tungsten filament lamp demands a very precise manufacturing technology, because the diameters of the filaments are very low. Any error in manufacture, will cause a short life of the lamp. Here is where, electric engineering and heat transfer engineering are interacting in the design of a filament lamp. The design is very sensitive to the details of the, emissivity of the filament and the resistivity of the filament. This illustrates how, many of the problems in real engineering are interface between many subjects. We have given a few examples, and we will continue in the next lecture with some more examples.